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A Model Describing the Neutrosophic Differential Equation and Its Application on Mine Safety

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Abstract. In the theory of uncertainty and approximation neutrosophy plays a significant role. Neutrosophy is tool emerged on standard or non-standard to measured the mathematical model of uncertainty, vagueness, ambiguity etc. In light of these major issues, the paper outlines of Neutrosophic Set, Single Valued Neutrosophic Set, Triangular Single Valued Neutrosophic Number and Trapezoidal Single Valued Neutrosophic Number. It also propose Neutrosophic Differential Equation and shown its solution in different conditions. Thereafter mining safety model via Single Valued neutrosophic number be epitomized. At last a mathematical experiment is done to exhibit its reality and use fullness of this Number.

Keywords: neutrosophic set(NS); single valued neutrosophic set(SVNS); triangular single valued neutrosophic number(TSVNNs); trapezoidal single valued neutrosophic number(TrSVNNs); neutrosophic differential equation(NDE); mining safety model

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1. Introduction

NS highlights the origin and nature of neutralise in different fields which is the generalization of classical set, fuzzy set(FS), intuitionistic fuzzy set(IFS) etc. Gradually varying value is used in FS theory rather than precise or sharp value. In 1965 [1], a famous paper was published by Prof. L.A. Zadeh as "Fuzzy sets" in "Information and Control" that provided some new mathematical tool which enable us to described and handle dubious or unclear notions. FS theory, only shows membership degree and do not provide any idea about non-membership degree. In reality, this linguistic statement don't fulfill the logical statement. When choosing the membership degree there may exist some types of doubtfulness or absence of information are present while defining the membership. Due to this doubtfulness, an idea of IFS as generalization of FS was introduced by Atanassov in 1983 [2]. IFS consider both membership and non-membership function. IFS only pick up incomplete information. In 2003 [3], A new concept, say, NS was innovated by Smarandache. It deals with the study of origin, nature and scope of neutralise, as well as their interaction with different idealism spectra. NS is the generalization of CS, FS, IFS and so on. A NS can be distinguish by a truth membership function 'μ_T', an indeterminacy membership function 'ν_I', and a falsity membership function 'σ_F'. In NS: μ_T, ν_I and σ_F are not dependent, which is useful in situations such as information fusion. In NS μ_T, ν_I, σ_F being the real standard or non standard subset of $^{-}]0, 1[^{+}$,moreover in SNVS, μ_T, ν_I, σ_F be the subset of [0,1]. From philosophical point of view, NS generalised the above mentioned sets but from scientific or engineering point of view, need to be defined. Else it's difficult to apply in many real application.

It is much noticed that when modeling some problems related to physical science and engineering, where the parameters are unknown but performed in an interval. Before, the application of interval arithmetic managed such circumstances, where mathematical calculation is done on intervals to get the estimate of target quantities in respective intervals. Fuzzy arithmetic is the generalization of the intervals arithmetic. As the principle definition of FS which approve gradation of membership for an element of the Universal set. So the situation of the modeling based on fuzzy arithmetic is awaited to publish more realistically. There are several types of fuzzy number are exist. These are applied in Decision-making problem and so on [4]. But it is not efficient for any application where the knowledge about membership degree is lacking. Latter generalization it to intuitionistic fuzzy number [5] were developed. In these paper we define several types of neutrosophic numbers and their cuts.

In the field of science & engineering, differentiation takes on an evidential role. Many problems stand up with uncertain or imprecise parameters. Due to this naiveness, we bear upon the differential equation with imprecise parameters. Fuzzy differential equation [6] has been proposed to model this uncertainty. However, it consider only membership value. Later, intuitionistic fuzzy differential [7] equation was founded with degree of membership and non-membership function. However, the term indeterminacy is absent in the above logic's. Hence, neutrosophic differential eqaution(NDE) [8–10] was developed to model indeterminacy. In this paper, a mining safety model describe [11], this model consist of three differential equations, those differential equations describe via Single Valued Neutrosophic Number(SVNNs). The solution of the equation is describe later.

In reality, the collected data, in many situations, it was observed that is insufficient and transmit some misinformation. As a result, the solution obtained from these data suffers with insufficiency and inconsistency. In these situations, the neutrosophic sets offer better result.

We have designed the paper in the following way: Section-2 gives some preliminaries concept and definition. Section-3 contains definition of NDE. Section-4 contains solution of NDE with numerical example. Section-5 contains Mining Safety model. Section-6 contains Mining safety model formulation. Section-7 described solution mode of the model. Section-8 contains numerical experiment and consequently, conclusions are discussed in Section-9. The references are shown in Section-10.

2. Preliminaries

2.1. *Definition of NS* [12]

Let $\mathfrak{U}$ be a Universal set. A NS $\tilde{\mathcal{A}}^{NS}$ of $\mathfrak{U}$ be defined by $\tilde{\mathcal{A}}^{NS} = \langle(\mathfrak{u}; \mu_T(\mathfrak{u}), \nu_I(\mathfrak{u}), \sigma_F(\mathfrak{u})) : \mathfrak{u} \in \mathfrak{U}\rangle$ where $\mu_T(\mathfrak{u}), \nu_I(\mathfrak{u}), \sigma_F(\mathfrak{u})$ be outlined as the truth membership, indeterminacy membership, falsity membership grade of $\mathfrak{u}$ in $\tilde{\mathcal{A}}^{NS}$ which are real standard or non-standard subsets of $^{-}]0, 1[^{+}$ & $\mu_T(\mathfrak{u}) + \nu_I(\mathfrak{u}) + \sigma_F(\mathfrak{u}) \leqslant 3^{+}$.

2.2. *Definition of SVNS* [12]

Let $\mathfrak{U}$ be a Universal set. A SVNS $\tilde{\mathcal{A}}^{Ne}$ of $\mathfrak{U}$ be defined by $\tilde{\mathcal{A}}^{Ne} = \langle(\mathfrak{u}; \mu_T(\mathfrak{u}), \nu_I(\mathfrak{u}), \sigma_F(\mathfrak{u})) : \mathfrak{u} \in \mathfrak{U}\rangle$ where $\mu_T(\mathfrak{u}), \nu_I(\mathfrak{u}), \sigma_F(\mathfrak{u})$ be outlined as the truth membership, indeterminacy membership, falsity membership grade of $\mathfrak{u}$ in $\tilde{\mathcal{A}}^{Ne}$ which are subset of $[0, 1]$ & $\mu_T(\mathfrak{u})+\nu_I(\mathfrak{u})+\sigma_F(\mathfrak{u}) \leqslant 3$.

2.3. *Definition of TSVNNs* [8]

A TSVNNs is denoted by $\tilde{\mathcal{A}}^{Ne} = \langle \mathfrak{a}'_1, \mathfrak{a}'_2, \mathfrak{a}'_3; w_\mu, w_\nu, w_\sigma\rangle$ whose truth, indeterminacy and falsity membership functions are defined by

$$\mu_T(\mathfrak{u}) = \begin{cases} (\frac{\mathfrak{u}-\mathfrak{a}'_1}{\mathfrak{a}'_2-\mathfrak{a}'_1})w_\mu & \text{when } \mathfrak{a}'_1 \leqslant \mathfrak{u} \leqslant \mathfrak{a}'_2 \\ w_\mu & \text{when } \mathfrak{u} = \mathfrak{a}'_2 \\ (\frac{\mathfrak{a}'_3-\mathfrak{u}}{\mathfrak{a}'_3-\mathfrak{a}'_2})w_\mu & \text{when } a'_2 \leqslant \mathfrak{u} \leqslant \mathfrak{a}'_3 \\ 0 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}'_1 \text{ or } \mathfrak{u} \geqslant \mathfrak{a}'_3 \end{cases}$$

$$\nu_I(\mathfrak{u}) = \begin{cases} \frac{(\mathfrak{a}'_2-\mathfrak{u})+(\mathfrak{u}-\mathfrak{a}'_1)w_\nu}{\mathfrak{a}'_2-\mathfrak{a}'_1} & \text{when } \mathfrak{a}'_1 \leqslant \mathfrak{u} \leqslant \mathfrak{a}'_2 \\ w_\nu & \text{when } \mathfrak{u} = \mathfrak{a}'_2 \\ \frac{(\mathfrak{u}-\mathfrak{a}'_2)+(\mathfrak{a}'_3-\mathfrak{u})w_\nu}{\mathfrak{a}'_3-\mathfrak{a}'_2} & \text{when } \mathfrak{a}'_2 \leqslant \mathfrak{u} \leqslant \mathfrak{a}'_3 \\ 1 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}'_1 \text{ or } \mathfrak{u} \geqslant \mathfrak{a}'_3 \end{cases}$$

$$\sigma_F(\mathfrak{u}) = \begin{cases} \frac{(\mathfrak{a}_2'-\mathfrak{u})+(\mathfrak{u}-\mathfrak{a}_1')w_\sigma}{\mathfrak{a}_2'-\mathfrak{a}_1'} & \text{when } \mathfrak{a}_1' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_2' \\ w_\sigma & \text{when } \mathfrak{u} = \mathfrak{a}_2' \\ \frac{(\mathfrak{u}-\mathfrak{a}_2')+(\mathfrak{a}_3'-\mathfrak{u})w_\sigma}{\mathfrak{a}_3'-\mathfrak{a}_2'} & \text{when } \mathfrak{a}_2' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_3' \\ 1 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}_1' \text{ or } \mathfrak{u} \geqslant \mathfrak{a}_3' \end{cases}$$

where $\mu_T(\mathfrak{u}) + \nu_I(\mathfrak{u}) + \sigma_F(\mathfrak{u}) \leqslant 3$ & $w_\mu \in (0,1]$, $w_\nu, w_\sigma \in [0,1)$.

2.4. ***Definition of TrSVNNs*** *[9]*

A TrSVNNs is denoted by $\tilde{\mathcal{A}}^{Ne} = \langle \mathfrak{a}_1', \mathfrak{a}_2', \mathfrak{a}_3', \mathfrak{a}_4'; w_\mu, w_\nu, w_\sigma \rangle$ whose truth, indeterminacy and falsity membership functions are defined by

$$\mu_T(\mathfrak{u}) = \begin{cases} (\frac{\mathfrak{u}-\mathfrak{a}_1'}{\mathfrak{a}_2'-\mathfrak{a}_1'})w_\mu & \text{when } \mathfrak{a}_1' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_2' \\ w_\mu & \text{when } \mathfrak{a}_2' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_3' \\ (\frac{\mathfrak{a}_4'-\mathfrak{u}}{\mathfrak{a}_4'-\mathfrak{a}_3'})w_\mu & \text{when } a_3' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_4' \\ 0 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}_1' \text{ or } \mathfrak{u} \geqslant \mathfrak{a}_4' \end{cases}$$

$$\nu_I(\mathfrak{u}) = \begin{cases} \frac{(\mathfrak{a}_2'-\mathfrak{u})+(\mathfrak{u}-\mathfrak{a}_1')w_\nu}{\mathfrak{a}_2'-\mathfrak{a}_1'} & \text{when } \mathfrak{a}_1' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_2' \\ w_\nu & \text{when } \mathfrak{a}_2' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_3' \\ \frac{(\mathfrak{u}-\mathfrak{a}_3')+(\mathfrak{a}_4'-\mathfrak{u})w_\nu}{\mathfrak{a}_4'-\mathfrak{a}_3'} & \text{when } \mathfrak{a}_3' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_4' \\ 1 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}_1' \text{ or } \mathfrak{u} \geqslant \mathfrak{a}_4' \end{cases}$$

$$\sigma_F(\mathfrak{u}) = \begin{cases} \frac{(\mathfrak{a}_2'-\mathfrak{u})+(\mathfrak{u}-\mathfrak{a}_1')w_\sigma}{\mathfrak{a}_2'-\mathfrak{a}_1'} & \text{when } \mathfrak{a}_1' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_2' \\ w_\sigma & \text{when } \mathfrak{a}_2' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_3' \\ \frac{(\mathfrak{u}-\mathfrak{a}_3')+(\mathfrak{a}_4'-\mathfrak{u})w_\sigma}{\mathfrak{a}_4'-\mathfrak{a}_3'} & \text{when } \mathfrak{a}_3' \leqslant \mathfrak{u} \leqslant \mathfrak{a}_4' \\ 1 & \text{when } \mathfrak{u} \leqslant \mathfrak{a}_1' \text{ or } \mathfrak{u} \geqslant \mathfrak{a}_4' \end{cases}$$

where $\mu_T(\mathfrak{u}) + \nu_I(\mathfrak{u}) + \sigma_F(\mathfrak{u}) \leqslant 3$ & $w_\mu \in (0,1]$, $w_\nu, w_\sigma \in [0,1)$.

2.5. ***Cut Set*** *[8]*

Let $\tilde{\mathcal{A}}^{Ne}$ be any SVNS, then (r,β,γ)-cut of SVNS is denoted by $\tilde{\mathcal{A}}^{Ne}(r,\beta,\gamma)$and it is defined by $\tilde{\mathcal{A}}^{Ne}(r,\beta,\gamma)$=$\langle \mathfrak{u} \in \mathfrak{U} : \mu_T(\mathfrak{u}) \geqslant r, \nu_I(\mathfrak{u}) \leqslant \beta, \sigma_F(\mathfrak{u}) \leqslant \gamma; 0 < r \leqslant 1, 0 \leqslant \beta < 1, 0 \leqslant \gamma < 1 \rangle$.

2.6. ***Operation Using SVNNs:*** *[13]*

Consider two TSVNNs, $\tilde{\mathcal{A}}^{Ne}$=$\langle \mathfrak{a}_1', \mathfrak{a}_2', \mathfrak{a}_3'; w_\mu, w_\nu, w_\sigma \rangle$; $\tilde{\mathcal{B}}^{Ne}$=$\langle \mathfrak{b}_1', \mathfrak{b}_2', \mathfrak{b}_3'; u_\mu, u_\nu, u_\sigma \rangle$, the following operation are:

• **Addition:**

$\tilde{\mathcal{A}}^{Ne}+\tilde{\mathcal{B}}^{Ne} = \langle[(\mathfrak{a}_1'+\mathfrak{b}_1', \mathfrak{a}_2'+\mathfrak{b}_2', \mathfrak{a}_3'+\mathfrak{b}_3'); w_\mu \wedge u_\mu, w_\nu \vee u_\nu, w_\sigma \vee u_\sigma]\rangle$

• **Substraction:**

$\tilde{\mathcal{A}}^{Ne}\text{-}\tilde{\mathcal{B}}^{Ne} = \langle[(\mathfrak{a}_1'-\mathfrak{b}_3', \mathfrak{a}_2'-\mathfrak{b}_2', \mathfrak{a}_3'-\mathfrak{b}_1'); w_\mu \wedge u_\mu, w_\nu \vee u_\nu, w_\sigma \vee u_\sigma]\rangle$

• **Multiplication:**

$\tilde{\mathcal{A}}^{Ne}.\tilde{\mathcal{B}}^{Ne} = \langle[(\mathfrak{a}_1'\mathfrak{b}_1', \mathfrak{a}_2'\mathfrak{b}_2', \mathfrak{a}_3'\mathfrak{b}_3'); w_\mu \wedge u_\mu, w_\nu \vee u_\nu, w_\sigma \vee u_\sigma]\rangle$

• **Division:**

$\dfrac{\tilde{\mathcal{A}}^{Ne}}{\tilde{\mathcal{B}}^{Ne}}=\langle[(\dfrac{\mathfrak{a}_1'}{\mathfrak{b}_3'}, \dfrac{\mathfrak{a}_2'}{\mathfrak{b}_2'}, \dfrac{\mathfrak{a}_3'}{\mathfrak{b}_1'}); w_\mu \wedge u_\mu, w_\nu \vee u_\nu, w_\sigma \vee u_\sigma]\rangle$

Where $\wedge = Min, \vee = Max$

3. **Definition of NDE:** [8]

Consider an Ordinary differential equation $\dfrac{d\mathbb{Y}}{d\mathfrak{t}} = \mathcal{K}\mathbb{Y}$, $\mathfrak{t} \in [0,\infty)$ with initial condition(IC) $\mathbb{Y}(\mathfrak{t}_0) = \mathbb{Y}_0$. The above ODE is called NDE if any one of the following three cases hold:

(i) $\tilde{\mathcal{K}}^{Ne}$ is SVNNs & $\mathbb{Y}_0$ is Crisp number.

(ii) $\mathcal{K}$ is Crisp number & $\tilde{\mathbb{Y}_0}^{Ne}$ is SVNNs.

(iii) Both $\tilde{\mathcal{K}}^{Ne}$ & $\tilde{\mathbb{Y}_0}^{Ne}$ are SVNNs.

Let the classical solution [14] be $\tilde{\mathbb{Y}}^{Ne}(\mathfrak{t})$ and its Cut be $\mathbb{Y}(\mathfrak{t},r,\beta,\gamma)$=$\langle[\mathbb{Y}_1(\mathfrak{t},r), \mathbb{Y}_2(\mathfrak{t},r)], [\mathbb{Y}_1'(\mathfrak{t},\beta), \mathbb{Y}_2'(\mathfrak{t},\beta)], [\mathbb{Y}_1''(\mathfrak{t},\gamma), \mathbb{Y}_2''(\mathfrak{t},\gamma)]\rangle$.

The solution is strong if

(i) $\dfrac{d\mathbb{Y}_1(\mathfrak{t},r)}{dr} > 0$, $\dfrac{d\mathbb{Y}_2(\mathfrak{t},r)}{dr} < 0 \;\forall\; r \in (0,1]$, $\mathbb{Y}_1(\mathfrak{t},1) \leqslant \mathbb{Y}_2(\mathfrak{t},1)$

(ii) $\dfrac{d\mathbb{Y}_1'(\mathfrak{t},\beta)}{d\beta} < 0$, $\dfrac{d\mathbb{Y}_2'(\mathfrak{t},\beta)}{d\beta} > 0 \;\forall\; \beta \in [0,1)$, $\mathbb{Y}_1'(\mathfrak{t},0) \leqslant \mathbb{Y}_2'(\mathfrak{t},0)$

(iii) $\dfrac{d\mathbb{Y}_1''(\mathfrak{t},\gamma)}{d\gamma} < 0$, $\dfrac{d\mathbb{Y}_2''(\mathfrak{t},\gamma)}{d\gamma} > 0 \;\forall\; \gamma \in [0,1)$, $\mathbb{Y}_1''(\mathfrak{t},0) \leqslant \mathbb{Y}_2''(\mathfrak{t},0)$

Otherwise the solution is weak solution.

4. **Solution of NDE**

(i) $\tilde{\mathcal{K}}^{Ne}$ is SVNNs & $\mathbb{Y}_0$ is Crisp number.

Case 1 When Sign of $\tilde{\mathcal{K}}^{Ne}$ is positive.

Therefore required solutions are

$\mathbb{Y}_1(\mathfrak{t},r) = \mathbb{Y}_0 e^{\mathbb{K}_1(r)(\mathfrak{t}-\mathfrak{t}_0)}$; $\mathbb{Y}_2(\mathfrak{t},r) = \mathbb{Y}_0 e^{\mathbb{K}_2(r)(\mathfrak{t}-\mathfrak{t}_0)}$

$\mathbb{Y}_1'(\mathfrak{t},\beta) = \mathbb{Y}_0 e^{\mathbb{K}_1'(\beta)(\mathfrak{t}-\mathfrak{t}_0)}$; $\mathbb{Y}_2'(\mathfrak{t},\beta) = \mathbb{Y}_0 e^{\mathbb{K}_2'(\beta)(\mathfrak{t}-\mathfrak{t}_0)}$

$\mathbb{Y}_1''(\mathfrak{t},\gamma) = \mathbb{Y}_0 e^{\mathbb{K}_1''(\gamma)(\mathfrak{t}-\mathfrak{t}_0)}$; $\mathbb{Y}_2''(\mathfrak{t},\gamma) = \mathbb{Y}_0 e^{\mathbb{K}_2''(\gamma)(\mathfrak{t}-\mathfrak{t}_0)}$

Case 2 When Sign of $\tilde{\mathcal{K}}^{Ne}$ is negative.

Therefore required solutions are

$$\mathbb{Y}_1(\mathfrak{t},r)=\frac{\mathbb{Y}_0}{2}[(1+\sqrt{\frac{\mathbb{K}_2(r)}{\mathbb{K}_1(r)}})e^{-\sqrt{\mathbb{K}_1(r)\mathbb{K}_2(r)}(\mathfrak{t}-\mathfrak{t}_\circ)}+(1-\sqrt{\frac{\mathbb{K}_2(r)}{\mathbb{K}_1(r)}})e^{\sqrt{\mathbb{K}_1(r)\mathbb{K}_2(r)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

$$\mathbb{Y}_2(\mathfrak{t},r)=\frac{\mathbb{Y}_0}{2}[(\sqrt{\frac{\mathbb{K}_1(r)}{\mathbb{K}_2(r)}}+1)e^{-\sqrt{\mathbb{K}_1(r)\mathbb{K}_2(r)}(\mathfrak{t}-\mathfrak{t}_\circ)}-(\sqrt{\frac{\mathbb{K}_1(r)}{\mathbb{K}_2(r)}}-1)e^{\sqrt{\mathbb{K}_1(r)\mathbb{K}_2(r)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

$$\mathbb{Y}_1'(\mathfrak{t},\beta)=\frac{\mathbb{Y}_0}{2}[(1+\sqrt{\frac{\mathbb{K}_2'(\beta)}{\mathbb{K}_1'(\beta)}})e^{-\sqrt{\mathbb{K}_1'(\beta)\mathbb{K}_2'(\beta)}(\mathfrak{t}-\mathfrak{t}_\circ)}+(1-\sqrt{\frac{\mathbb{K}_2'(\beta)}{\mathbb{K}_1'(\beta)}})e^{\sqrt{\mathbb{K}_1'(\beta)\mathbb{K}_2'(\beta)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

$$\mathbb{Y}_2'(\mathfrak{t},\beta)=\frac{\mathbb{Y}_0}{2}[(\sqrt{\frac{\mathbb{K}_1'(\beta)}{\mathbb{K}_2'(\beta)}}+1)e^{-\sqrt{\mathbb{K}_1'(\beta)\mathbb{K}_2'(\beta)}(\mathfrak{t}-\mathfrak{t}_\circ)}-(\sqrt{\frac{\mathbb{K}_1'(\beta)}{\mathbb{K}_2'(\beta)}}-1)e^{\sqrt{\mathbb{K}_1'(\beta)\mathbb{K}_2'(\beta)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

$$\mathbb{Y}_1''(\mathfrak{t},\gamma)=\frac{\mathbb{Y}_0}{2}[(1+\sqrt{\frac{\mathbb{K}_2''(\gamma)}{\mathbb{K}_1''(\gamma)}})e^{-\sqrt{\mathbb{K}_1''(\gamma)\mathbb{K}_2''(\gamma)}(\mathfrak{t}-\mathfrak{t}_\circ)}+(1-\sqrt{\frac{\mathbb{K}_2''(\gamma)}{\mathbb{K}_1''(\gamma)}})e^{\sqrt{\mathbb{K}_1''(\gamma)\mathbb{K}_2''(\gamma)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

$$\mathbb{Y}_2''(\mathfrak{t},\gamma)=\frac{\mathbb{Y}_0}{2}[(\sqrt{\frac{\mathbb{K}_1''(\gamma)}{\mathbb{K}_2''(\gamma)}}+1)e^{-\sqrt{\mathbb{K}_1''(\gamma)\mathbb{K}_2''(\gamma)}(\mathfrak{t}-\mathfrak{t}_\circ)}-(\sqrt{\frac{\mathbb{K}_1''(\gamma)}{\mathbb{K}_2''(\gamma)}}-1)e^{\sqrt{\mathbb{K}_1''(\gamma)\mathbb{K}_2''(\gamma)}(\mathfrak{t}-\mathfrak{t}_\circ)}]$$

Where $\langle[\mathbb{K}_1(r),\mathbb{K}_2(r)],[\mathbb{K}_1'(\beta),\mathbb{K}_2'(\beta)],[\mathbb{K}_1''(\gamma),\mathbb{K}_2''(\gamma)]\rangle$ is the cut set of $\tilde{\mathcal{K}}^{Ne}$. Solutions are strong or week if it satisfies the condition of NDE.

Similarly, we can get the solution of other two cases.

Numerical Example: Let us consider NDE $\frac{d\mathbb{Y}}{d\mathfrak{t}}=\mathcal{K}\mathbb{Y}$, with IC $\tilde{\mathbb{Y}}^{Ne}(0)=\langle 3,4,5;0.8,0.2,0.3\rangle$, $\mathcal{K}=\frac{1}{3}$.

Solution: Required (r,β,γ)-cut solution at $\mathfrak{t}=2$ we get $\mathbb{Y}_1(\mathfrak{t},r)=[3+1.25r]e^{\frac{2}{3}}$; $\mathbb{Y}_2(\mathfrak{t},r)=[5-1.25r]e^{\frac{2}{3}}$; $\mathbb{Y}_1'(\mathfrak{t},\beta)=[\frac{3.4-\beta}{0.8}]e^{\frac{2}{3}}$; $\mathbb{Y}_2'(\mathfrak{t},\beta)=[\frac{3+\beta}{0.8}]e^{\frac{2}{3}}$; $\mathbb{Y}_1''(\mathfrak{t},\gamma)=[\frac{3.1-\gamma}{0.7}]e^{\frac{2}{3}}$; $\mathbb{Y}_2''(\mathfrak{t},\gamma)=[\frac{2.5+\gamma}{0.7}]e^{\frac{2}{3}}$.

When we take $\mathfrak{t}=2$ and for different values of r,β,γ the solution is given in Table 1. The graphical interpretation of the table is also shown in the form of membership function in the Figure. 1.

5. **Mining Safety Model**

The miming industry has played an important role in development in the human civilization. Extraction of minerals from the underground system of work has involved a considerable amount of risks like roof fall over the workplace, inundation of the workplace due to the influx of water from the old working, explosion, influx of poisonous gases in the workplace, etc. Similarly, the opencast system of work has involved chances of runway of dumpers, sliding of benches in the workplace, striking by the fly rocks blasting, etc. These phenomenon's not only

r, β, γ	$\mathbb{Y}_1(\mathrm{t}, r)$	$\mathbb{Y}_2(\mathrm{t}, r)$	$\mathbb{Y}_1'(\mathrm{t}, \beta)$	$\mathbb{Y}_2'(\mathrm{t}, \beta)$	$\mathbb{Y}_1''(\mathrm{t}, \gamma)$	$\mathbb{Y}_2''(\mathrm{t}, \gamma)$
0	5.8432	9.7387	8.2778	7.3040	8.6257	6.9561
0.1	6.0866	9.4952	8.0344	7.5474	8.3474	7.2344
0.2	6.3301	9.2517	7.7909	7.7909	8.0692	7.5127
0.3	6.5736	9.0083	7.5475	8.0344	7.7909	7.7909
0.4	6.8171	8.7648	7.3040	8.2779	7.5127	8.0692
0.5	7.0605	8.5213	7.0605	8.5213	7.2344	8.3474
0.6	7.3040	8.2779	6.8171	8.7648	6.9562	8.6257
0.7	7.5475	8.0344	6.5736	9.0082	6.6779	8.9039
0.8	7.7909	7.7909	6.3301	9.2517	6.3997	9.1822
0.9	8.0344	7.5475	6.0867	9.4952	6.1214	9.4604
1.0	8.2779	7.3040	5.8432	9.7387	5.8432	9.7387

Table 1. Solution for $\mathrm{t} = 2$

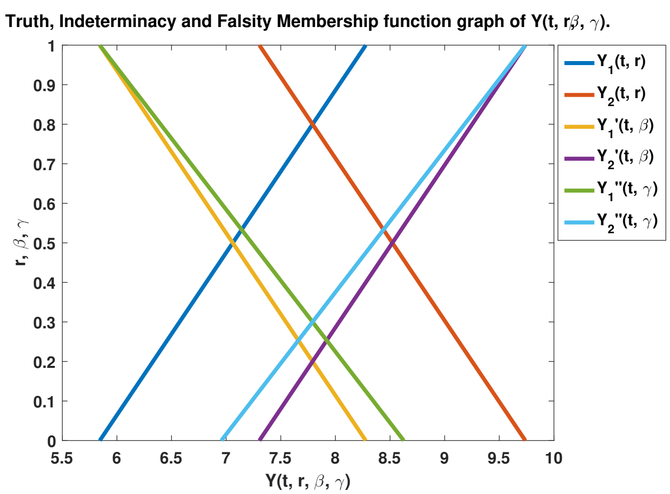

Figure 1. Membership Function Graph (at t=2).

causes injury to the workmen, sometimes lead to fatal. Improper used and malfunctioning mining equipment or system also results an accident.

The system fails safely is denoted by λ_1 and system fails unsafely is denoted by λ_2 for the mining safety model used here. Either λ_1 or λ_2 or both λ_1 and λ_2 are imprecise in nature. Our main interest in this paper are given below:

• Formulate Mining Safety model.

• Observe solution of the model in Crisp environment.

• Observe solution of the mining model in three ways:

(i) when λ_1 is SVNNs and λ_2 is crisp number.

(ii) when λ_1 is crisp number and λ_2 is SVNNs.

(iii) both λ_1 and λ_2 are SVNNs.

• Observe cut value in table form of the solution of the mining model in each of the cases mention above and show its graphical representation.

5.1. *Acceptation*

(I) All events are not dependent to one another.

(II) The probability of progression from one condition to another is $\Psi\delta\mathfrak{t}$; $\delta\mathfrak{t}$ indicates finite time interval, Ψ indicate the progression rate from one condition to another.

(III) $(\Psi\delta\mathfrak{t})(\Psi\delta\mathfrak{t}) \to 0$.

(IV) $\mathcal{P}\{\eta(\delta\mathfrak{t}) \geqslant 2\} = o(\delta\mathfrak{t})$, where $\eta(\delta\mathfrak{t})$ be the number of event that occur in $\delta\mathfrak{t}$.

(V) $\mathcal{P}\{\eta(\delta\mathfrak{t}) = 1\} = \Psi\delta\mathfrak{t} + o(\delta\mathfrak{t})$, where $\Psi > 0$.

(VI) $\lim_{\delta\mathfrak{t}\to 0} \frac{o(\delta\mathfrak{t})}{\delta\mathfrak{t}} = 0$.

5.2. *Input data*

$\mathfrak{t}$= time.

λ_1= mining system safe failure rate.

λ_2= mining system unsafe failure rate.

5.3. *Output data*

$\mathcal{P}_0(\mathfrak{t})$= Probability of Mining system operating normally.

$\mathcal{P}_1(\mathfrak{t})$= Probability of Mining system failed safely.

$\mathcal{P}_2(\mathfrak{t})$= Probability of Mining system failed unsafely.

5.4. *Modulator*

$\mathfrak{t}$	time.
$\delta\mathfrak{t}$	finite time intervall.
$\mathcal{P}_0(\mathfrak{t}+\delta\mathfrak{t})$	operating probability in state 0 at time $\mathfrak{t}+\delta\mathfrak{t}$.
$\mathcal{P}_1(\mathfrak{t}+\delta\mathfrak{t})$	safe fail probability in state 1 at time $\mathfrak{t}+\delta\mathfrak{t}$.
$\mathcal{P}_2(\mathfrak{t}+\delta\mathfrak{t})$	unsafe fail probability in state 2 at time $\mathfrak{t}+\delta\mathfrak{t}$.
j=0	state operating normal.
j=1	state fail safe.
j=2	state fail unsafe.
$\mathcal{P}_j(\mathfrak{t})$	probability in state j at time $\mathfrak{t}$.
$\lambda_1\delta\mathfrak{t}$	safe fail probability in finite time interval $\delta\mathfrak{t}$
$\lambda_2\delta\mathfrak{t}$	unsafe fail probability in $\delta\mathfrak{t}$
(1 - $\lambda_1\delta\mathfrak{t}$)	no safe fail probability in $\delta\mathfrak{t}$
(1 - $\lambda_1\delta\mathfrak{t}$)	no unsafe fail probability in $\delta\mathfrak{t}$

6. Model Formulation

Consider a mining system, the state space diagram is shown in Figure-2.

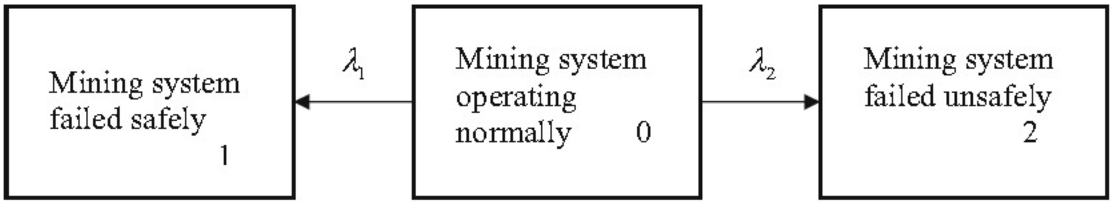

FIGURE 2. Mining system state space diagram

From Fig.2,we get following three equations

$$\mathcal{P}_0(\mathfrak{t}+\delta\mathfrak{t}) = \mathcal{P}_0(\mathfrak{t})(1-\lambda_1\delta\mathfrak{t})(1-\lambda_2\delta\mathfrak{t}) \tag{1}$$

$$\mathcal{P}_1(\mathfrak{t}+\delta\mathfrak{t}) = \mathcal{P}_1(\mathfrak{t})(1-o(\delta\mathfrak{t})) + \mathcal{P}_0(\mathfrak{t})\lambda_1\delta\mathfrak{t} \tag{2}$$

$$\mathcal{P}_2(\mathfrak{t}+\delta\mathfrak{t}) = \mathcal{P}_2(\mathfrak{t})(1-o(\delta\mathfrak{t})) + \mathcal{P}_0(\mathfrak{t})\lambda_2\delta\mathfrak{t} \tag{3}$$

From (1), (2), (3) we get

$$\therefore \frac{d\mathcal{P}_0(\mathfrak{t})}{d\mathfrak{t}} = -(\lambda_1+\lambda_2)\mathcal{P}_0(\mathfrak{t}) \tag{4}$$

$$\frac{d\mathcal{P}_1(\mathfrak{t})}{d\mathfrak{t}} = \lambda_1\mathcal{P}_0(\mathfrak{t}) \tag{5}$$

$$\frac{d\mathcal{P}_2(\mathfrak{t})}{d\mathfrak{t}} = \lambda_2\mathcal{P}_0(\mathfrak{t}) \tag{6}$$

with IC: $\mathcal{P}_j(0) = 1$ for j=0 & $\mathcal{P}_j(0) = 0$ for j=1,2.

7. Solution mode

7.1. ***Crisp Solution:***

Input data: Both λ_1 and λ_2 are Crisp number..

Output data: We get the values of $\mathcal{P}_0(\mathfrak{t})$, $\mathcal{P}_1(\mathfrak{t})$, $\mathcal{P}_2(\mathfrak{t})$.

7.2. ***Neutrosophic Solution:***

Input data: Three cases arise

Case-1: $\tilde{\lambda_1}^{Ne}$=$\langle \mathfrak{a}_1', \mathfrak{a}_2', \mathfrak{a}_3'; w_\mu, w_\nu, w_\sigma\rangle$ & λ_2 is Crisp number.

Case-2: λ_1 is Crisp number & $\tilde{\lambda_2}^{Ne}$=$\langle \mathfrak{b}_1', \mathfrak{b}_2', \mathfrak{b}_3'; u_\mu, u_\nu, u_\sigma\rangle$

Case-3: $\tilde{\lambda_1}^{Ne}$=$\langle \mathfrak{a}_1', \mathfrak{a}_2', \mathfrak{a}_3'; w_\mu, w_\nu, w_\sigma\rangle$ & $\tilde{\lambda_2}^{Ne}$=$\langle \mathfrak{b}_1', \mathfrak{b}_2', \mathfrak{b}_3'; u_\mu, u_\nu, u_\sigma\rangle$

Output data:

Let, $\tilde{\mathcal{P}_0}(\mathfrak{t})^{Ne}$,$\tilde{\mathcal{P}_1}(\mathfrak{t})^{Ne}$,$\tilde{\mathcal{P}_2}(\mathfrak{t})^{Ne}$ be the solution of the modified model with Cut
$\mathcal{P}_0(\mathfrak{t},r,\beta,\gamma)$=$\langle[\mathcal{P}_{01}(\mathfrak{t},r),\mathcal{P}_{02}(\mathfrak{t},r)],[\mathcal{P}'_{01}(\mathfrak{t},\beta),\mathcal{P}'_{02}(\mathfrak{t},\beta)],[\mathcal{P}''_{01}(\mathfrak{t},\gamma),\mathcal{P}''_{02}(\mathfrak{t},\gamma)]\rangle$

$\mathcal{P}_1(\mathfrak{t},r,\beta,\gamma)$=$\langle[\mathcal{P}_{11}(\mathfrak{t},r),\mathcal{P}_{12}(\mathfrak{t},r)],[\mathcal{P}'_{11}(\mathfrak{t},\beta),\mathcal{P}'_{12}(\mathfrak{t},\beta)],[\mathcal{P}''_{11}(\mathfrak{t},\gamma),\mathcal{P}''_{12}(\mathfrak{t},\gamma)]\rangle$

$\mathcal{P}_2(\mathfrak{t},r,\beta,\gamma)$=$\langle[\mathcal{P}_{21}(\mathfrak{t},r),\mathcal{P}_{22}(\mathfrak{t},r)],[\mathcal{P}'_{21}(\mathfrak{t},\beta),\mathcal{P}'_{22}(\mathfrak{t},\beta)],[\mathcal{P}''_{21}(\mathfrak{t},\gamma),\mathcal{P}''_{22}(\mathfrak{t},\gamma)]\rangle$

Solution is strong or weak if it satisfies the condition of NDE.

8. Numerical Experiment

8.1. *Crisp Solution*

Input data: $\lambda_1 = 0.009; \lambda_2 = 0.001$; $\mathfrak{t}$=20-h.

Output: $\mathcal{P}_2(20)$=0.018127

8.2. *NS Solution*

Case: 1
Input data: $\tilde{\lambda_1}^{Ne}$=$\langle 0.007, 0.009, 0.011; 0.5, 0.3, 0.2\rangle; \lambda_2 = 0.001$; $\mathfrak{t}$=20-h.

Output: When we take the value $\mathfrak{t}$=20-h the output of λ_1^{Ne} is TSVNNs & λ_2 is crisp number are shown in Table-2 and the corresponding membership function shown in Figure-3.

Case: 2
Input data: λ_1=0.009; $\tilde{\lambda_2}^{Ne} = \langle 0.0007, 0.001, 0.0013; 0.7, 0.5, 0.4\rangle$; $\mathfrak{t}$=20-h.

Output: When we take the value $\mathfrak{t}$=20-h the output of λ_1 is Crisp number and $\tilde{\lambda_2}^{Ne}$ is TSVNNs are shown in Table-3 and the corresponding membership function shown in Figure-4.

Case: 3

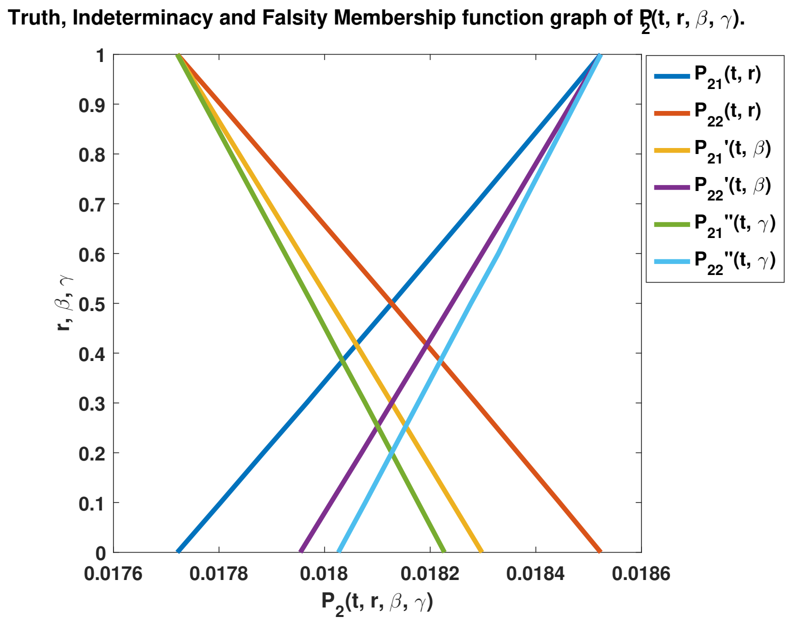

FIGURE 3. Membership Function Graph (at t=20).

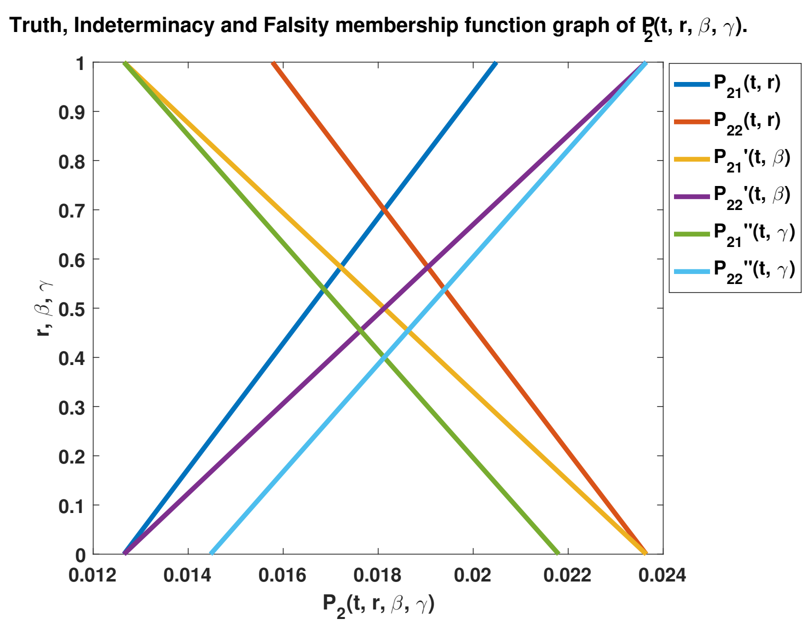

FIGURE 4. Membership Function Graph (at t=20).

r, β, γ	$\mathcal{P}_{21}(\mathfrak{t}, r)$	$\mathcal{P}_{22}(\mathfrak{t}, r)$	$\mathcal{P}'_{21}(\mathfrak{t}, \beta)$	$\mathcal{P}'_{22}(\mathfrak{t}, \beta)$	$\mathcal{P}''_{21}(\mathfrak{t}, \gamma)$	$\mathcal{P}''_{22}(\mathfrak{t}, \gamma)$
0	0.017721	0.018523	0.018298	0.017954	0.018227	0.018026
0.1	0.017803	0.018445	0.018241	0.018012	0.018177	0.018077
0.2	0.017884	0.018366	0.018184	0.018070	0.018127	0.018127
0.3	0.017966	0.018287	0.018127	0.018127	0.018077	0.018177
0.4	0.018046	0.018207	0.018070	0.018184	0.018026	0.018227
0.5	0.018127	0.018127	0.018012	0.018241	0.017976	0.018277
0.6	0.018207	0.018046	0.017954	0.018298	0.017925	0.018329
0.7	0.018287	0.017965	0.017896	0.018355	0.017874	0.018376
0.8	0.018366	0.017884	0.017838	0.018411	0.017823	0.018425
0.9	0.018445	0.017803	0.017780	0.018467	0.017772	0.018474
1.0	0.018523	0.017721	0.017721	0.018523	0.017721	0.018523

TABLE 2. $\tilde{\lambda_1}^{Ne}$ is TSVNNs & λ_2 is Crisp number.

r, β, γ	$\mathcal{P}_{21}(\mathfrak{t}, r)$	$\mathcal{P}_{22}(\mathfrak{t}, r)$	$\mathcal{P}'_{21}(\mathfrak{t}, \beta)$	$\mathcal{P}'_{22}(\mathfrak{t}, \beta)$	$\mathcal{P}''_{21}(\mathfrak{t}, \gamma)$	$\mathcal{P}''_{22}(\mathfrak{t}, \gamma)$
0	0.012647	0.023643	0.023643	0.012647	0.021800	0.014470
0.1	0.013427	0.022853	0.022537	0.013740	0.020881	0.015382
0.2	0.014209	0.022063	0.021432	0.014834	0.019962	0.016296
0.3	0.014991	0.021275	0.020329	0.015930	0.019044	0.017211
0.4	0.015774	0.020487	0.019227	0.017028	0.018127	0.018127
0.5	0.016557	0.019699	0.018127	0.018127	0.017211	0.019044
0.6	0.017342	0.018913	0.017028	0.019227	0.016296	0.019962
0.7	0.018127	0.018127	0.015930	0.020329	0.015382	0.020881
0.8	0.018913	0.017342	0.014834	0.021432	0.014470	0.021800
0.9	0.019699	0.016557	0.013740	0.022537	0.013558	0.022721
1.0	0.020487	0.015774	0.012647	0.023643	0.012647	0.023643

TABLE 3. λ_1 is Crisp number & $\tilde{\lambda_2}^{Ne}$ is TSVNNs

Input data:
$\tilde{\lambda_1}^{Ne}=\langle 0.007, 0.009, 0.011; 0.5, 0.3, 0.2\rangle; \tilde{\lambda_2}^{Ne} = \langle 0.0007, 0.001, 0.0013; 0.7, 0.5, 0.4\rangle$; $\mathfrak{t}$=20-h.
Output: When we take the value $\mathfrak{t}$=20-h the output of $\tilde{\lambda_1}^{Ne}$ & $\tilde{\lambda_2}^{Ne}$ are TSVNNs are shown in Table-4 and the corresponding membership function shown in Figure-5.

From the table values and graph, we see that
$\mathcal{P}_1(\mathfrak{t}, r)$ is increasing function and
$\mathcal{P}_2(\mathfrak{t}, r)$ is decreasing function, whereas

r,β,γ	$\mathcal{P}_{21}(\mathrm{t},r)$	$\mathcal{P}_{22}(\mathrm{t},r)$	$\mathcal{P}'_{21}(\mathrm{t},\beta)$	$\mathcal{P}'_{22}(\mathrm{t},\beta)$	$\mathcal{P}''_{21}(\mathrm{t},\gamma)$	$\mathcal{P}''_{22}(\mathrm{t},\gamma)$
0	0.012361	0.024156	0.024156	0.012361	0.022118	0.014253
0.1	0.013188	0.023247	0.022930	0.013493	0.021109	0.015210
0.2	0.014023	0.022345	0.021714	0.014635	0.020108	0.016175
0.3	0.014866	0.021449	0.020508	0.015788	0.019165	0.017147
0.4	0.015715	0.020561	0.019312	0.016952	0.018127	0.018127
0.5	0.016573	0.019681	0.018127	0.018127	0.017147	0.019114
0.6	0.017438	0.018807	0.016952	0.019312	0.016175	0.020108
0.7	0.018310	0.017941	0.015788	0.020508	0.015210	0.021109
0.8	0.019190	0.017083	0.014635	0.021714	0.014253	0.022118
0.9	0.020077	0.016232	0.013493	0.022930	0.013303	0.023134
1.0	0.020971	0.015388	0.012361	0.024156	0.012361	0.024156

TABLE 4. Both $\tilde{\lambda}_1^{Ne}$ & $\tilde{\lambda}_2^{Ne}$ are TSVNNs

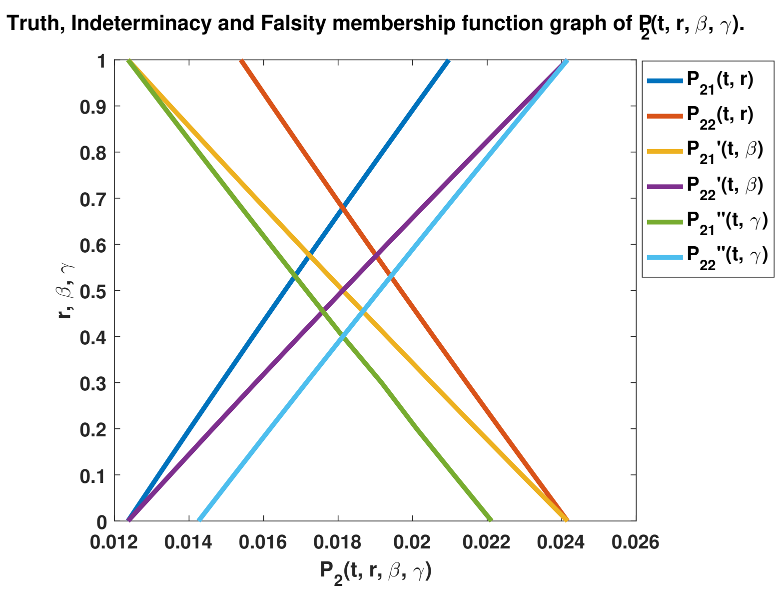

FIGURE 5. Membership Function Graph (at t=20).

$\mathcal{P}'_1(\mathrm{t},\beta)$, $\mathcal{P}''_1(\mathrm{t},\gamma)$ are decreasing functions and
$\mathcal{P}'_2(\mathrm{t},\beta)$, $\mathcal{P}''_2(\mathrm{t},\gamma)$ are increasing functions. Hence, the solution is strong solution.

9. Conclusion

- NS is a hot research topic and can be applied for solving the mathematical model of uncertainty, vagueness, ambiguity, etc.
- The mining safety model described in this paper with two parameters which satisfies the condition of NDE has got strong solutions.
- The solutions of the three differential equations of the mining safety model have been described via TSVNNs.
- The paper has also proposed numerical experiment and graphical representation of truth, indeterminacy and falsity membership function.

This will promote the future study of trapezoidal single valued neutrosophic numbers.

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