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# Introduction to NeutroNearrings

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**Abstract**. Algebraic concepts and structures are enriched with the special types of operations and axioms known as NeutroOperations and NeutroAxioms. Various types of NeutroAlgebras are studied using several such defined concepts. The objective of this paper is to introduce the concept of NeutroNearrings. Several interesting results and examples of NeutroNearrings, NeutroSubRings, NeutroQuotientNearrings and NeutroNearringHomomorphisms are presented.

**Keywords:** Nearring; NeutroRings; NeutroNearring; Neutrosophy.

## 1. Introduction

The NeutroDefined and AntiDefined Laws, as well as the NeutroAxioms and AntiAxioms was first time introduced in 2019 by Smarandache [3, 5]. This concept has given birth to new fields of research called NeutroStructures and AntiStructures. For basic and recent results on Neutrosophy, NeutroAlgebraic structures and AntiAlgebraic structures we refer [4–8].

In [2],Agboola formally presented the notion of NeutroGroups. In this he showed that in general, Lagrange's theorem and first isomorphism theorem of the classical groups do not hold in the NeutroGroups. Also in [1], Agboola studied NeutroRing, NeutroSubring, NeutroQuotientRings and he proved the 1st isomorphism theorem of the classical rings for this class of NeutroRing.

The present paper will be concerned with the introduction of NeutroNearrings.

## 2. Preliminaries

**Definition 2.1.** [7]

(i) A classical operation is an operation well defined for all the set's elements while a NeutroOperation is an operation partially well defined, partially indeterminate, and partially outer defined on the given set. An AntiOperation is an operation that is outer defined for all the set's elements.

(ii) A NeutroAlgebra is an algebra that has at least one NeutroOperation or one NeutroAxiom ( axiom that is true for some elements, indeterminate for other elements, and false for other elements), and no AntiOperation or AntiAxiom. An AntiAlgebra is an algebra endowed with at least one AntiOperation or at least one AntiAxiom.

**Definition 2.2.** A NeutroGroup is a nonempty set $G$ with binary operation $*$ satisfying following conditions:

(i) The $*$ is **NeutroAssociative** if there exists atleast one triplet $(a, b, c) \in G$ such that

$$a * (b * c) = (a * b) * c$$

and there exists atleast one triplet $(x, y, z) \in G$ such that

$$x * (y * z) \neq (x * y) * z$$

(ii) There exists a **NeutroNeutral element** in G if at least one of the below statements occurs:

- There exists at least one element $x$ that has no unit-element.
- There exists at least one element $b \in G$ that has at least two distinct unit-elements $e_1, e_2 \in G, e_1 \neq e_2$ such that:

$$b * e_1 = e_1 * b = b$$

$$b * e_2 = e_2 * b = b$$

- There exists at least two different elements $r, s \in G, r \neq s$, such that they have different unit elements $e_r, e_s \in G, e_r \neq e_s$, with $e_r * r = r * e_r = r$ and $e_s * s = s * e_s = s$

(iii) There exists a **NeutroInverse element** in $G$ if there is an element $a \in G$ that has an inverse $b \in G$ with respect to a unit element $e \in G$ that is

$$b * a = a * b = e$$

or there exists atleast one element $b \in G$ that has two or more inverses $c, d \in G$ with respect to some unit element $u \in G$ that is

$$b * c = c * b = u$$

$$b * d = d * b = u$$

In addition, if $*$ is **NeutroCommutative** that is there exists atleast a duplet $(a, b) \in G$ such that

$$a * b = b * a$$

and there exists atleast a duplet $(c, d) \in G$ such that

$$c * d \neq d * c$$

then $(G, *)$ is called a NeutroCommutative group or NeutroAbelian group.
If condition $(i)$ is satisfied, then $(G, *)$ is called a NeutroSemiGroup and if conditions $(i)$ and $(ii)$ are satisfied, then $(G, *)$ is called a NeutroMonoid [1].

**Definition 2.3.** Let $R$ be a nonempty set and let $+, \cdot : R \times R \to R$ be binary operations of ordinary addition and multiplication on $R$. Then $\cdot$ is both left and right NeutroDistributive over $+$ that is there exists atleast a triplet $(a, b, c) \in R$ and atleast a triplet $(d, e, f) \in R$ such that

$$a.(b + c) = a.b + a.c$$

$$d.(e + f) \neq d.e + d.f$$

then . is **left NeutroDistributive** over $+$ on a set$R$.
Suppose if there exists atleast a triplet $(p, q, r) \in R$ and atleast a triplet $(x, y, z) \in R$ such that

$$(p + q).r = p.r + q.r$$

$$(x + y).z \neq x.z + y.z$$

then the binary operation . is said to be **right NeutroDistributive** over $+$ on a set $R$.

A **right Nearring** is a set $N$ together with two binary operations $+$ and $\cdot$ such that:

(1) $(N, +)$ is a group (not necessarily abelian)
(2) $(N, \cdot)$ is a semigroup

(3) For all $n_1, n_2, n_3 \in N : (n_1 + n_2) \cdot n_3 = n_1 \cdot n_3 + n_2 \cdot n_3$ (right distribution law).

If $n_1 \cdot (n_2 + n_3) = n_1 \cdot n_2 + n_1 \cdot n_3$ instead of condition (3) then set $N$ is a **left Nearring**.

**NeutroNearring and their properties**

A NeutroNearring is a Nearring that has either a Neutro-operation or a Neutro-axiom. In this paper we define NeutroNearing as below.

**Definition 2.4.** Let N be a nonempty set and let $+, \cdot : N \times N \to N$ be binary operations of ordinary addition and multiplication on N. The triple $(N, +, \cdot)$ is called a **left NeutroNearring** if the following conditions are satisfied:

(i) $(N, +)$ is a NeutroGroup (not necessarily abelian)

(ii) $(N, \cdot)$ is a NeutroSemiGroup

(iii) the left NeutroDistributive law holds in $N$: that is there exists atleast one triplet $(a, b, c) \in N$ and atleast one triplet $(d, e, f) \in N$ such that:

- $a \cdot (b + c) = a \cdot b + a \cdot c$
- $d \cdot (e + f) \neq d \cdot e + d \cdot f$

**Remark 2.5.** If right NeutroDistributive law holds in $N$: that is there exists atleast one triplet $(p, q, r) \in N$ and atleast one triplet $(s, t, u) \in N$ such that:

- $(p + q) \cdot r = p \cdot r + q \cdot r$
- $(s + t) \cdot u \neq s \cdot u + t \cdot u$

then $N$ is called **right NeutroNearring**.

**Example 2.6.** Let $X = \mathbb{Z}_{12}$ and let $\oplus$ and $\odot$ be two binary operations on $X$ defined by $x \oplus y = x + 2y$ and $x \odot y = x + 4y$ for all $x, y \in X$ where "+" is addition modulo 12. Then $(X, \oplus, \odot)$ is a NeutroNearring.

**Example 2.7.** Let $X = \{a, b, c\}$ with "+" and "·" be binary operations defined on $X$ as shown in the Cayley tables below:

| + | a | b | c |
|---|---|---|---|
| a | c | c | b |
| b | c | b | c |
| c | c | c | b |

| · | a | b | c |
|---|---|---|---|
| a | b | a | a |
| b | a | c | a |
| c | a | a | b |

It is clear from the table that :

$$(a+b)+c = a+(b+c) = b,$$
$$(c+a)+b = c, \text{ but } c+(a+b) = b \neq c$$

This shows that $(X,+)$ is a NeutroSemiGroup.

Next, let $N_x$ and $I_x$ represent additive neutral and additive inverse element respectively with respect to any element $x \in X$. Then

$$N_b = b$$
$$I_b = b$$
$$N_a \text{ does not exist,}$$
$$I_b \text{ does not exist.}$$

Hence, $(X,+)$ is a NeutroGroup.

Next, consider

$$b(cb) = (bc)b$$
$$a(bc) = b \text{ but } (ab)c = a \neq b$$

This shows that $(X,\cdot)$ is NeutroAssociative.

Lastly, consider

$$b.(b+b) = b.b + b.b = c,$$
$$a.(b+c) = a, \text{but } a.b + a.c = c \neq a$$

This shows that "$\cdot$" is left distributive over "$+$". Hence, $(X,+,\cdot)$ is a left NeutroNearring.

**Note 2.8.** *Every NeutroRing is a NeutroNearring.*

Notation: Let N be a NeutroNearring $d \in N$ is called NeutroDistributive if there exist atleast two pairs $(n_1,n_2)$ *and* $(m_1,m_2) \in N$ such that $(n_1+n_2)d = n_1d + n_2d$ *and* $(m_1+m_2)d \neq m_1d + m_2d$. *Let* $N_d = \{d \in N|\text{d is NeutroDistributive }\}$.

**Remark 2.9.** Let $(N,+,\cdot)$ be left NeutroNearring

(i) If $(N,+)$ is NeutroAbelian, then $N$ is a NeutroAbelian NeutroNearring.
(ii) If $(N,.)$ is NeutroCommutative then $N$ is a NeutroCommutative NeutroNearring.
(iii) If $N = N_d$ then $N$ is said to be NeutroDistributive.
(iv) If $(N^*,\cdot)$ where $N^* = N \setminus \{0\}$ is a NeutroGroup then $N$ is called NeutroNearfield.

**Theorem 2.10.** *Let* $(N_i,+,),i=1,2,\ldots,n$ *be a family of NeutroNearrings. Then*

(1) $N = \cap_{i=1}^{n} N_i$ *is a NeutroNearring.*

(2) $N = \prod_{i=1}^{n} N_i$ *is a NeutroNearring.*

(1) *Proof.* Obvious □

(2) *Proof.* Proof is by induction on $n$.

For $n = 1$ result is trivial. Let $n = 2$. Consider $N = N_1 \times N_2$ then is closed with respect to coordinate wise addition and coordinate wise multiplication. Note that there exist $n_1 \in N_1$ such that $n_1 + e_1 = n_1$ and there exist $n_2 \in N_2$ such that $n_2 + e_2 = n_2$.

Also, there doesnot exist additive identity for $n_1^{'} \in N_1$ and $n_2^{'} \in N_2$.

But $(n_1, n_2) \in N$ such that $(n_1, n_2) + (e_1, e_2) = (n_1, n_2)$ and there doesnot exist additive identity for $(n_1^{'}, n_2^{'}) \in N$.

Similarly one can observe the existence of NeutroAdditive inverse in $N$.

Since $(N_1, +)$ and $(N_2, +)$ are NeutroAssociative, there exist $a_1, b_1, c_1, a_1^{'}, b_1^{'}, c_1^{'} \in N_1$ and $a_2, b_2, c_2, a_2^{'}, b_2^{'}, c_2^{'} \in N_2$ such that $a_1 + (b_1 + c_1) = (a_1 + b_1) + c_1$

$a_2 + (b_2 + c_2) = (a_2 + b_2) + c_2$

$a_1^{'} + (b_1^{'} + c_1^{'}) \neq (a_1^{'} + b_1^{'}) + c_1^{'}$

$a_2^{'} + (b_2^{'} + c_2^{'}) \neq (a_2^{'} + b_2^{'}) + c_2^{'}$

Now, $(a_1, a_2), (b_1, b_2), (c_1, c_2), (a_1^{'}, a_2^{'}), (b_1^{'}, b_2^{'}), (c_1^{'}, c_2^{'}) \in N$ such that

$(a_1, a_2) + [(b_1, b_2) + (c_1, c_2)] = (a_1, a_2) + [(b_1 + c_1, b_2 + c_2)] = (a_1 + (b_1 + c_1), a_2 + (b_2 + c_2))$

$= ((a_1 + b_1) + c_1, (a_2 + b_2) + c_2) = (a_1 + b_1, a_2 + b_2) + (c_1, c_2)$

$= [(a_1, a_2) + (b_1, b_2)] + (c_1, c_2)$

and $(a_1^{'}, a_2^{'}) + [(b_1{'}, b_2^{'}) + (c_1^{'}, c_2^{'})] \neq [(a_1^{'}, a_2^{'}) + (b_1{'}, b_2^{'})] + (c_1^{'}, c_2^{'})$

Similarly we prove $(N, .)$ is NeutroAssociative.

Further there exist $x_1, y_1, z_1, x_1^{'}, y_1^{'}, z_1^{'} \in N_1$ and $x_2, y_2, z_2, x_2^{'}, y_2^{'}, z_2^{'} \in N_2$ such that

$x_1.(y_1 + z_1) = x_1.y_1 + x_1.z_1$

$x_2.(y_2 + z_2) = x_2.y_2 + x_2.z_2$

$x_1^{'}.(y_1^{'} + z_1^{'}) \neq x_1^{'}.y_1^{'} + x_1^{'}.z_1^{'}$

$x_2^{'}.(y_2^{'} + z_2^{'}) \neq x_2^{'}.y_2^{'} + x_2^{'}.z_2^{'}$

But then, $(x_1, x_2), (y_1, y_2), (z_1, z_2), (x_1^{'}, x_2^{'}), (y_1^{'}, y_2^{'}), (z_1^{'}, z_2^{'}) \in N$ such that

$(x_1, x_2).[(y_1, y_2) + (z_1, z_2)] = (x_1, x_2).(y_1 + z_1, y_2 + z_2)$

$= (x_1(y_1 + z_1), x_2(y_2 + z_2)) = (x_1.y_1 + x_1.z_1, x_2.y_2 + x_2.z_2)$

$= (x_1.y_1, x_2.y_2) + (x_1.z_1, x_2.z_2) = (x_1, x_2).(y_1, y_2) + (x_1, x_2).(z_1, z_2)$

and $(x_1^{'}, x_2^{'}).[(y_1^{'}, y_2^{'}) + (z_1^{'}, z_2^{'})] \neq (x_1^{'}, x_2^{'}).(y_1^{'}, y_2^{'}) + (x_1^{'}, x_2^{'}).(z_1^{'}, z_2^{'})$

$\therefore$ N is a NeutroNearring for $n = 2$.

Let $n > 2$. Assume the result for $n - 1$. Note that $M = \prod_{i=1}^{n-1} N_i$ forms a NeutroNearring with respect to coordinate wise addition and coordinate wise multiplication.

But then, $N = M \times N_n$ forms a NeutroNearring with respect to coordinate wise addition and coordinate wise multiplication. □

**Definition 2.11.** Let $(N, +, .)$ be a NeutroNearring. A nonempty subset $S$ of $N$ is called a **NeutroNearSubring** of N if $(S, +, .)$ is also a NeutroNearring. The only trivial NeutroNear-Subring of N is N.

**Theorem 2.12.** *Let $(N, +, .)$ be a NeutroNearring and let $\{S_i\}, i = 1, 2, \ldots, n$ be a family of NeutroNearSubrings of N. Then*

(1) *$S = \cap_{i=1}^{n} S_i$ is a NeutroNearSubring of N.*
(2) *$S = \prod_{i=1}^{n} S_i$ is a NeutroNearSubring of N.*

*Proof.* Both result follows directly from Theorem 2.10. □

**Definition 2.13.** Let $(N, +, .)$ be a NeutroNearring. A nonempty subset I of R is called a **left NeutroNearIdeal** of $N$ if the following conditions hold:

(1) I is a NeutroNearSubring of N.
(2) There exist $x \in I$ such that $xr \in I, \forall r \in N$.

**Definition 2.14.** Let $(N, +, .)$ be a NeutroNearring. A nonempty subset I of N is called a **right NeutroNearIdeal** of N if the following conditions hold:

(1) I is a NeutroNearSubring of N.
(2) There exist $x \in I$ such that $rx \in I, \forall r \in N$

**Definition 2.15.** Let $(N, +, .)$ be a NeutroNearring. A nonempty subset I of N is called a **NeutroNearIdeal** of N if the following condition hold:

(1) I is a NeutroNearSubring of $N$.
(2) There exist $x \in I$ such that $xr, rx \in I, \forall r \in N$

**Theorem 2.16.** *Let $(N, +, .)$ be a NeutroNearring and let $\{I_i\}, i = 1, 2, \ldots, n$ be a family of NeutroNearIdeals of $N$. Then*

(1) *$I = \cap_{i=1}^{n} I_i$ is a NeutroNearIdeal of $N$.*
(2) *$I = \sum_{i=1}^{n} I_i$ is NeutroNearIdeal of $N$.*

(1) *Proof.* Since each $I_i, 1 \leq i \leq n$ is a NeutroNearSubring of $N$, it follows from Theorem 1.8 that $I = \cap_{i=1}^{n} I_i$ is a NeutroNearSubring of $N$.

Note that there exist $x_i \in I_i$ such that $x_i r, r x_i \in I_i, \ \forall r \in N$ and $\forall i, 1 \leq i \leq n$.
Let $y = x_1^2 x_2^2 \cdots x_n^2$. Then $y \in I_i, \ \forall i, 1 \leq i \leq n$.
For any $r \in N$ $ry, yr \in I_i, \ \forall i, 1 \leq i \leq n$
$\therefore y \in I$ with $ry, yr \in I$. □

(2) Obivous.

**Definition 2.17.** Let $(N, +, .)$ be a NeutroNearring and let I be a NeutroNearIdeal of N. The set $N/I$ is defined by

$$N/I = \{x + I : x \in N\}$$

.

for $x + I, y + I \in N/I$ with at least a pair $(x, y) \in N$, let $\oplus$ and $\odot$ be binary operations on $N/I$ defined as follows:

$$(x + I) \oplus (y + I) = (x + y) + I$$
$$(x + I) \odot (y + I) = xy + I$$

Then it can be shown that the tripple $(N/I, \oplus, \odot)$ is a NeutroNearring which we call NeutroQuotientNearring.

**Theorem 2.18.** *Let I be a NeutroNearIdeal of the NeutroNearring $(N, +, .)$. Suppose N is NeutroCommutative NeutroNearring with Neutro unity then so is $N/I$.*

*Proof.* There exist $a, b, c, d \in N$ such that $ab = ba$ and $cd \neq dc$.
But then $a + I, b + I, c + I, d + I \in N/I$ such that $(a + I)(b + I) = ab + I$
$= ba + I = (b + I)(a + I)$ and $(c + I)(d + I) = cd + I \neq dc + I = (d + I)(c + I)$
Let $e_y$ be a Neutro unity of N. Then there exist $y \in N$ such that $ye_y = e_y y = y$
But then $y + I \in N/I$ such that $(y + I)(e_y + I) = ye_y + I$
$= y + I = (e_y + I)(y + I)$
$\therefore N/I$ is NeutroCommutative NeutroNearring with Neutro unity $e_y + I$. □

**Definition 2.19.** Let $(N, +, .)$ and $(S, \oplus, \odot)$ be any two NeutroNearrings. The mapping $\phi : N \to S$ is called a NeutroNearringHomomorphism if $\phi$ preserves the binary operations of $N$ and $S$ that is if for at least a pair $(x, y) \in N$, we have:

$$\phi(x + y) = \phi(x) \oplus \phi(y)$$
$$\phi(x.y) = \phi(x) \odot \phi(y)$$

The kernel of $\phi$ denoted by ker$\phi$ is defined as $\ker\phi = \{x \in N : \phi(x) = e_N\}$
Where $e_N \in N$ is a neutral element for at least one $n \in N$. The image of $\phi$ denoted by $Im\phi$ is defined as

$$Im\phi = \{y \in S : y = \phi(x)\}$$

for at least one $x \in N$. If in addition $\phi$ is a NeutroBijection, then $\phi$ is called a NeutroNearringIsomorphism and we write $N \cong S$. NeutroNearringEpimorphism, NeutroNearringMonomorphism, NeutroNearringEndomorphism and NeutroNearringAutomorphism are defined similarly.

**Theorem 2.20.** *Let $R$ and $S$ be two NeutroNearrings. Let $N_x = e_R$ for at least one $x \in R$ and let $N_y = e_S$ for at least one $y \in S$. Suppose that $\phi : R \to S$ is a NeutroNearHomomorphism. Then:*

(1) *$\phi(e_R)$ is not necessarily equals $e_S$.*
(2) *Ker $\phi$ is a NeutroNearSubring of R.*
(3) *Im $\phi$ is not necessarily a Neutro Near Subring of S.*
(4) *$\phi$ is NeutroInjective if and only if Ker $\phi = \{e_R\}$ for at least one $e_R \in R$.*

**Theorem 2.21.** *Let $I$ be a NeutroNearIdeal of a NeutroNearring $(N, +, .)$. Then the mapping $\phi : N \to N/I$ defined by $\phi(x) = x + I$ is a NeutroNearringIsomorphism with $Ker\phi = I$*

*Proof.* For atleast one pair $x, y$ in $N$,
$\phi(x + y) = (x + y) + I = (x + I) + (y + I) = \phi(x) + \phi(y)$
and $\phi(xy) = xy + I = (x + I)(y + I) = \phi(x)\phi(y)$
$Ker\phi = \{x \in N | \phi(x) = e_{N/I}\}$, where $e_{N/I} \in N/I$ such that $N_r = e_{N/I}$ for atleast one $r \in N/I$
$. = \{x \in N | x + I = e_{N/I}\} = \{x \in N | x \in I\} = I$ □

**Theorem 2.22.** *Let $\phi : R \to S$ be a NeutroNearringHomomorphism and let $K = Ker\phi$. Then the mapping $\psi : R/K \to Im\phi$ defined by $\psi(x + K) = \phi(x)$ is a NeutroNearringIsomorphism.*

*Proof.* For atleast one pair $x, y \in R$
$\psi((x + K) + (y + K)) = \psi((x + y) + K) = \phi(x + y) = \phi(x) + \phi(y)$
$= \psi(x + K) + \psi(y + K)$ and $\psi((x + K)(y + K)) = \psi((xy) + K) = \phi(xy) = \phi(x)\phi(y)$
$= \psi(x + K)\psi(y + K)$
Also $Ker\psi = \{x + K \in R/K : \psi(x + K) = e_{Im\phi}\}$where $e_{Im\phi} \in Im\phi$ such that $N_r = e_{Im\phi}$ for atleast $r \in Im\phi$.
$= \{x + K \in R/K : \phi(x) = e_{Im\phi}\} = \{e_{R/K}\}$
Thus $\psi$ is a NeutroBijectiveNearringHomomorphism. □

**Note 2.23.** *The above map $\phi$ is an epimorphism. So, we can treat $\phi$ as NeutroNearringEpimorphism.*

**Theorem 2.24.** *NeutroNearringIsomorphism of NeutroNearrings is an equivalence relation.*

*Proof.* Define a relation $\sim$ as follows:
For any two NeutroNearrings $N$ and $N^{'}$, we say $N \sim N^{'}$ iff there exist a NeutroNearringIsomorphism between $N$ and $N^{'}$. Clearly $\sim$ is reflexive.
Suppose NeutroNearrings $N, N^{'}$ are such that $N \sim N^{'}$, let $f : N \to N^{'}$ be NeutroNearringIsomorphism.
Then there exist $x, y \in N$ such that $f(x+y) = f(x) + f(y)$ and $f(xy) = f(x)f(y)$.
Now, $f(x), f(y) \in N^{'}$ and
$f^{-1}(f(x)+f(y)) = f^{-1}(f(x+y)) = x+y = f^{-1}(f(x)) + f^{-1}(f(y))$
$f^{-1}(f(x)f(y)) = f^{-1}(f(xy)) = xy = f^{-1}(f(x))f^{-1}(f(y))$
$\therefore N^{'} \sim N$ and $f^{-1}$ is a NeutroNearringIsomorphism.
Let $f : N \to N^{'}$ and $g : N^{'} \to N^{''}$ be NeutroNearringIsomorphisms.
Then $g \circ f : N \to N^{''}$ is bijective and there exist $x^{'}, y^{'} \in N^{'}$ such that $g(x^{'}+y^{'}) = g(x^{'})+g(y^{'})$ and $g(x^{'}y^{'}) = g(x^{'})g(y^{'})$
Now, $x^{'} = f(x), y^{'} = f(y)$ for some $x, y \in N$ with $f(x+y) = f(x)+f(y)$ and $f(xy) = f(x)f(y)$
Consider,
$g \circ f(x+y) = g(f(x+y)) = g(f(x)+f(y)) = g(x^{'}+y^{'}) = g(x^{'}) + g(y^{'}) = g(f(x)) + g(f(y))$
$= g \circ f(x) + g \circ f(y)$
and $g \circ f(xy) = g(f(xy)) = g(f(x)f(y)) = g(x^{'}y^{'}) = g(x^{'})g(y^{'}) = g(f(x))g(f(y))$
$= g \circ f(x) g \circ f(y)$
$\therefore N \sim N^{''}$ □

## 3. **Conclusion**

We have introduced in this paper the concept of NeutroNearrings by considering three NeutroAxioms(NeutroGroup(additive)), NeutroSemigroup(multiplicative) and left and right NeutroDistributive laws(multiplication over addition). Several intresting results and examples on NeutroNearrings, NeutroSubrings, NeutroQuotientNearrings and NeutroNearringHomomorphisms are presented.

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