Neutrosophic Sets and Systems, Vol. 47, 2021
University of New Mexico

Statistical Development of the Neutrosophic Lognormal Model with Application to Environmental Data

Zahid Khan [1,*], Adnan Amin [2], Sajjad Ahmad Khan [2] and Muhammad Gulistan[1]

[1] Department of Mathematics and Statistics Hazara University Mansehra Pakistan, zahidkhan@hu.edu.pk, gulistanmath@hu.edu.pk
[2] Department of Statistics, Islamia College University Peshawar, Pakistan, sajjadkhan@icp.edu.pk, aminadnan@gmail.com

* Correspondence: zahidkhan@hu.edu.pk

Abstract: The identification of an appropriate probability model is always essential in environmental data analysis. This work presents the notion of the neutrosophic lognormal distribution (NLD) and its application to environmental data. The general structure for the density function of the NLD and its usefulness has been provided. Some more critical distributional properties such as moments, skewness and kurtosis coefficients have been derived. A methodology of estimating the distributional parameters under the neutrosophic environment is developed. The simulation study is conducted to validate the derived results for the proposed model. In the application part, a real dataset on Nitrogen oxides emissions has been analyzed using NLD to highlight the practical significance of the proposed model.

Keywords: Neutrosophic probability model; uncertainty; Lognormal model; Estimation; Simulation

1. Introduction

In pollution studies, observing environmental levels and quantifying the concentrations of different contaminants entering a particular environmental region are of considerable interest because of the potential for adverse side effects. The selection of appropriate statistical models is particularly vital in environmental studies [1]. None of the probability models containing the traditional lognormal has been superior to others [2]. These probability models could estimate the parameters needed to meet up changing information desires of an environmental quality organization [3]. Unfortunately, data on environmental pollution is often biased to the right, with a long slope towards high concentrations [4]. When using the normal distribution to certain types of data, the validity of the appropriateness may be called into question. The modelling of this type of distribution consists of transforming data values to bring changed values closer to the normal distribution [5]. In this context, logarithmic alteration is typically applied to pollution figures. Other distributions may be more appropriate for representing pollution concentration data, despite the lognormal distribution being the most commonly used [6]. Larsen [7] modified the lognormal distribution by including the third parameter, called an increment to deal with air quality data. In the Ghent region of Belgium, Berger et al. [8] analyzed fitness based on the extreme and moderate values in gamma distribution at the daily concentration of sulphur dioxide. Xiang et al. [9] also found that gamma models reflect acid-gas quantities in an industrial zone better than the lognormal model. Neither of the probability models, along with the traditional lognormal, has been preferable to others in a general logic in published literature [10].

Among these generic models, the NLD proposed in this work distribution offers a lot of promise for evaluating environmental contaminant data. Although the NLD is a relatively "unknown" distribution in environmental studies, as indicated above, its skewness and long-tail appear to create it suited for environmental pollutant data. Aside from that, the NLD group is extensive and incorporates many components that are very relatively frequent distribution when it comes to fitting pollutant concentration data. As a result, the NLD appears to be a potential candidate for environmental modeling. Smarandache's work on the idea of neutrosophy provides the inspiration for this generalization [11]. The analysis of false or true statements, but indeterminate, neutral, inconsistent, or something in between, is oriented by Neutrosophy logic [12-14]. In the actual world, there are numerous circumstances where the data that have any kind of indeterminacy [15]. The notion of neutrosophic statistics is used to cope with such data [16]. The term "neutosophic statistics" refers to the extension of conventional statistics [17]. As a result of its advantageous properties, the application of neutrosophic statistics has gained considerable attention in recent years, particularly because conventional statistics cannot be used when our data contains incomplete, vague, unclear or uncertain measurements [18-20]. The use of neutrosophic statistics in the applied research may be seen in literature such as [21–24]. The neutrosophic statistics has given rise to study areas dealing with indeterminacy effects in statistical process control [25-29].

The notion of NLD in the analysis of pollutant concentration data is proposed in the present paper when the moments, kurtosis, and skewness of other distributions do not correspond to log normality. The NLD model, which has a particular form, may offer an excellent alternative to the lognormal distribution and greater flexibility.

The rest of this work is arranged as follows: The neutrosophic extension of the lognormal distribution is established in Section 2. Section 3 explains the mathematical approach used to find unknown distributional parameters. A Monte Carlo simulation is conducted in Section 4 to validate the theoretical results of the neutrosophic model. Section 5 describes an application of the suggested model. Lastly, Section 6 outlines the main research findings.

2. Proposed Model

If $\widetilde{\mathrm{X}} = \ln \mathrm{Z}$ follows a neutrosophic normal distribution, a random variable $\mathrm{Z} > 0$ is said to follow the NLD with the density function:

$$\omega_n(z) = \frac{1}{z\sigma_n\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln z-\mu_n}{\sigma_n}\right)^2};\, z > 0,\ \mu_n,\ \sigma_n > 0 \qquad (1)$$

where $\mu_n = [\mu_l, \mu_u]$ is the neutrosophic location, $\sigma_n = [\sigma_l, \sigma_u]$ is neutrosophic shape parameters on the log scale and Z denotes neutrosophic random variable. For the selected values of $\mu_n = [0, 0.2]$ and $\sigma_n = [0.2, 0.8]$, the neutrosophic density $(\widetilde{\mathrm{PDF}})$ is graphically portrayed in Figure 1.

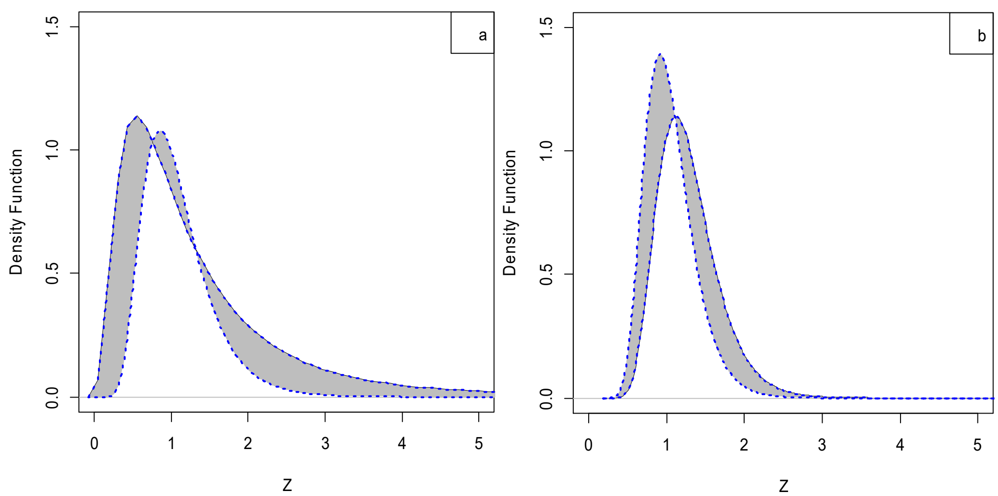

Figure 1. The density of NLD with neutrosophic parameters (a) $\sigma_n = [0.2, 0.8]$ and (b) $\mu_n = [0, 0.2]$

In Figure 1, the area under the curve indicates the interval in which the neutrosophic lognormal variable will fall. The entire area of the graph in this period equals the probability of Z occurring. The grey zone in Figure 1 represents the neutrosophic region due to uncertainties involved in distributional parameters.

Aside from specific pattern of the neutrosophic density, an analyst may be interested in seeking certain additional favourable distributional features of the NLD, which may be established in the form of some theorems below:

Theorem 1 Show that the mode of the NLD is $e^{\mu_n - \sigma_n}$

Proof: The mode of the NLD is the point at which function $\omega_n(z)$ reaches its highest or maximum value.

Therefore differentiating (1) with respect to z implies:

$$\frac{d}{dz}\,\omega_n(z) = \frac{d}{dz}\left[\frac{1}{z\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2}, \qquad \frac{1}{z\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2}\right]$$

$$= \left[\frac{d}{dz}\left\{\frac{1}{z\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2}\right\}, \qquad \frac{d}{dz}\left\{\frac{1}{z\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2}\right\}\right]$$

$$= \left[\left\{-\frac{1}{z^2\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2}\left(\left(\frac{\ln z-\mu_l}{\sigma_l}\right)+1\right)\right\}, \left\{-\frac{1}{z^2\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2}\left(\left(\frac{\ln z-\mu_u}{\sigma_u}\right)+1\right)\right\}\right] \quad (2)$$

To find maxima equating (2) to zero provides:

$$\left[\left\{-\frac{1}{z^2\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2}\left(\left(\frac{\ln z-{}_{l}}{\sigma_l}\right)+1\right)\right\},\ \left\{-\frac{1}{z^2\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-{}_{u}}{\sigma_u}\right)^2}\left(\left(\frac{\ln z-\mu_u}{\sigma_u}\right)+1\right)\right\}\right]=[0\ ,0] \quad (3)$$

Further simplification of (3) implies

$= [e^{\mu_l-\sigma_l},\ e^{\mu_u-\sigma_u}]$,

where $[e^{\mu_l-\sigma_l},\ e^{\mu_u-\sigma_u}] = e^{\mu_n-\sigma_n}$, hence Proved.

Theorem 2: Show that jth moment of the NLD is $e^{j\mu_n+j^2\frac{\sigma_n^2}{2}}$

Proof: By definition, the jth moment of the NLD is given by:

$$\mu'_{jn} = \int_0^\infty \frac{z^j}{z\sigma_n\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_n}{\sigma_n}\right)^2}dz$$

$$= \int_0^\infty z^j\left[\frac{1}{z\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-{}_{l}}{\sigma_l}\right)^2},\qquad \frac{1}{z\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2}\right]dz$$

$$= \left[\int_0^\infty \frac{z^j}{z\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2}dz,\ \ \int_0^\infty \frac{z^j}{z\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2}dz\right] \quad (4)$$

By substituting in (4)

$$y = \frac{\ln z - \mu_n}{\sigma_n}$$

This yielded:

$$\int_0^\infty \frac{z^j}{z\sigma_l\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_l}{\sigma_l}\right)^2} = e^{j\mu_l+j^2\frac{\sigma_l^2}{2}}$$

$$\int_0^\infty \frac{z^j}{z\sigma_u\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{\ln z-\mu_u}{\sigma_u}\right)^2} = e^{j\mu_u+j^2\frac{\sigma_u^2}{2}}$$

Thus (4) becomes

$$\mu_{jn} = \left[e^{j\mu_l+j^2\frac{\sigma_l^2}{2}},\qquad e^{j\mu_u+j^2\frac{\sigma_u^2}{2}}\right]$$

Hence,

$\mu_{jn} = e^{j\mu_n + j^2\frac{\sigma_n^2}{2}}$ where $j = 1,2,3, \ldots$ is a general expression for the jth moment about the origin of the NLD.

By using the following relations, moments about mean for NLD can be derived as:

$$\mu_{1n} = \mu_{1n}^{'} = e^{\mu_n} e^{\frac{\sigma_n^2}{2}}$$

$$\mu_{2n} = \mu_{2n}^{'} - \left(\mu_{1n}^{'}\right)^2 = e^{2\mu_n} e^{\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)$$

$$\mu_{3n} = \mu_{3n}^{'} - 3\mu_{2n}^{'}\mu_{1n}^{'} + 2\left(\mu_{1n}^{'}\right)^3 = e^{3\mu_n} e^{3\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)^2\left(e^{\sigma_n^2} - 2\right)$$

$$\mu_{4n} = \mu_{4n}^{'} - 4\mu_{3n}^{'}\mu_{1n}^{'} + 6\mu_{2n}^{'}\left(\mu_{1n}^{'}\right)^2 - 3\left(\mu_{1n}^{'}\right)^4 = e^{4\mu_n} e^{4\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)^2\left(e^{4\sigma_n^2} + 2e^{3\sigma_n^2} + 3e^{2\sigma_n^2} - 3\right)$$

Theorem 3 Show that the coefficient of skewness for the NLD is $\left(e^{\sigma_n^2} + 2\right)\left(\sqrt{e^{\sigma_n^2} - 1}\right)$

Proof: The coefficient of skewness for NLD is given by:

$$\hat{\gamma}_{1n} = \frac{\mu_{3n}}{(\mu_{2n})^{3/2}}, \quad (5)$$

where $\mu_{3n} = e^{3\mu_n} e^{3\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)^2\left(e^{\sigma_n^2} - 2\right)$ and $\mu_{2n} = e^{2\mu_n} e^{\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)$

Substituting in (5) provides:

$$\hat{\gamma}_{1n} = (e^{\sigma_n^2} + 2)\left(\sqrt{e^{\sigma_n^2} - 1}\right),$$

where $\hat{\gamma}_{1n} \in [\gamma_{1l}, \gamma_{1u}]$.

Theorem 4 Show that the coefficient of kurtosis for NLD is $e^{4\sigma_n^2} + 2e^{3\sigma_n^2} + 3e^{2\sigma_n^2} - 3$

Proof: By definition, the coefficient of kurtosis is NLD given by:

$$\hat{\gamma}_{2n} = \frac{\mu_{4n}}{\mu_{2n}^2} \quad (6)$$

where $\mu_{4n} = e^{4\mu_n} e^{4\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)^2\left(e^{4\sigma_n^2} + 2e^{3\sigma_n^2} + 3e^{2\sigma_n^2} - 3\right)$ and $\mu_{2n} = e^{2\mu_n} e^{\sigma_n^2}\left(e^{\sigma_n^2} - 1\right)$

Substituting in (6) and further simplification implies:

$$\hat{\gamma}_{2n} = e^{4\sigma_n^2} + 2e^{3\sigma_n^2} + 3e^{2\sigma_n^2} - 3,$$

where $\hat{\gamma}_{2n} = [\gamma_{2l}, \ \gamma_{2u}]$.

Similarly other characteristics of the defined model may be established in a neutrosophic environment.

3. Estimation Procedure

In this part, a method for estimating the NLD parameters, known as neutrosophic maximum likelihood estimation, is devised. Let us take n samples of $Z_j, j = 1,2,..n$ values from the NLD. The question is, for an observed sample, which values of the neutrosophic parameters should be used? The likelihood function of the neutrosophic model may be used to calculate these values. Because neutrosophy is included in the distributional parameters, joint function of the NLD is given by:

$$\tau_n(\mu_n, \sigma^2{}_n|z) = \prod_{j=1}^{n} \omega_n(z_j) \quad (7)$$

The likelihood function of (7) can be written as:

$$\tau_n(\mu_n, \sigma^2{}_n|z) = \sum_{j=1}^{n} \ln(z_j) - \frac{n\ln(2\pi\sigma^2{}_n)}{2} - \frac{n\mu_n{}^2}{2\sigma^2{}_n} + \frac{\sum_{j=1}^{n} \ln(z_j)}{\sigma^2{}_n} - \frac{\sum_{j=1}^{n} \ln(z_j{}^2)}{2\sigma^2{}_n} \quad (8)$$

The gradient involving unknown values μ_n and $\sigma^2{}_n$, in order to maximize τ_n is given by:

$$\frac{\partial\tau_n(\mu_n,\sigma^2{}_n|z)}{\partial\mu_n} = \frac{\sum_{j=1}^{n} \ln(z_j)}{\sigma^2{}_n} - \frac{2n\mu_n}{2\sigma^2{}_n} \quad (9)$$

$$\frac{\partial\tau_n(\mu_n,\sigma^2{}_n|z)}{\partial\sigma^2{}_n} = -\frac{n}{2\sigma^2{}_n} + \frac{\sum_{j=1}^{n}(\ln(z_j)-\mu_n)^2}{2(\sigma^2{}_n)^2} \quad (10)$$

The simultaneous solution for unknown is obtained by setting the gradians (9) and (10) to zero as:

$$\hat{\mu}_n = \frac{\sum_{j=1}^{n} \ln(z_j)}{n}$$

$$\hat{\sigma}^2{}_n = \frac{\sum_{j=1}^{n}\left(\ln(z_j) - \frac{\sum_{j=1}^{n} \ln(z_j)}{n}\right)^2}{n}$$

where $\hat{\mu}_n = [\hat{\mu}_l, \hat{\mu}_u]$ and $\hat{\sigma}^2{}_n = [\hat{\sigma}^2{}_l, \hat{\sigma}^2{}_u]$ are the required neutrosophic estimators of the parameters μ_n and $\sigma^2{}_n$ respectively.

4. Simulation Study

In this section, analytical results of the NLD for moments, skewness, and kurtosis have been validated using the Monte Carlo simulation. The NLD can be readily simulated in R software to assess the validity of theory-based results. For this, let us set the neutrosophic parameters $\mu_n = [0.5, 1.5]$ and $\sigma_n = [0.25, 0.5]$ in the NLD and 10^5 samples are randomly generated from $U[0, 1]$. Then according to (11), 10^5 pseudo neutrosophic random samples are generated from the NLD.

$$\frac{\ln(Z_i)-\mu_n}{\sigma_n} = F^{-1}(u_i) \qquad (11)$$

where $u_i \sim U[0,1]$ for $i = 1,2, ...$

As mentioned in Section 2, these simulated data are used to validate the analytical characteristics. Table 1 shows the exact findings for the mean, variance, mode, skewness, and kurtosis coefficients of the NLD beside the simulated values.

Table 1. Comparisons of the simulated findings with the NLD analytical results

Characteristics	Expected Result	Simulated Result
Mean	[1.700, 5.075]	[1.700, 5.072]
Standard deviation	[0.432, 2.707]	[0.431, 2.709]
Mode	[1.548 , 3.490]	[1.546, 3.488]
Skewness Coefficient	[0.778, 1.750]	[0.771, 1.749]
Kurtosis Coefficient	[4.095, 8.898]	[4.090, 8.891]

Results in Table 1 indicate that the simulated findings match quite well with those obtained from the analytical properties of the NLD.

5. Real Application

To demonstrate the computational method of the proposed NLD model, an actual dataset on yearly Nitrogen oxides emissions for Denmark is provided. The United Nations Statistics Divisions (UNSD) have calculated Nitrogen oxides emissions per capita for the period 1990 to 2018, and the data is accessible on the site [30]. Nitrogen oxides are general names for the two most important air pollutants, nitric oxide and nitrogen dioxide. Smog and acid rain are caused by these substances, which also have an impact on tropospheric ozone. As a result of their presence in the Earth's atmosphere, they constitute one of the most significant pollutants. Naturally, nitric oxide is created during thunderstorms, but it may also be produced during agricultural fertilization. Nitrogen oxides emissions are often calculated using an international methodology based on country information on industrial, energy, waste management and agricultural production. The probability plot and essential CDF plot of the original data are depicted in Figure 2 and Figure 3, respectively.

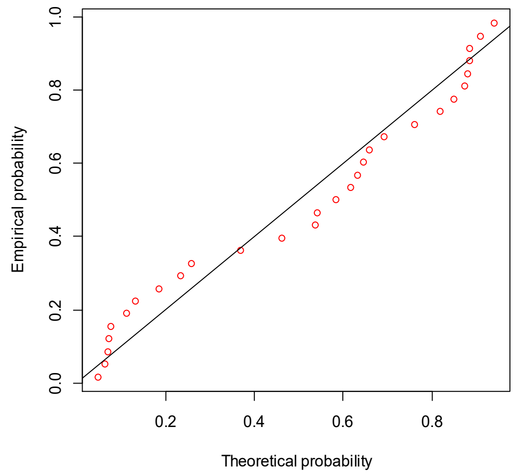

Figure 2. Probability plot for Nitrogen oxides emissions

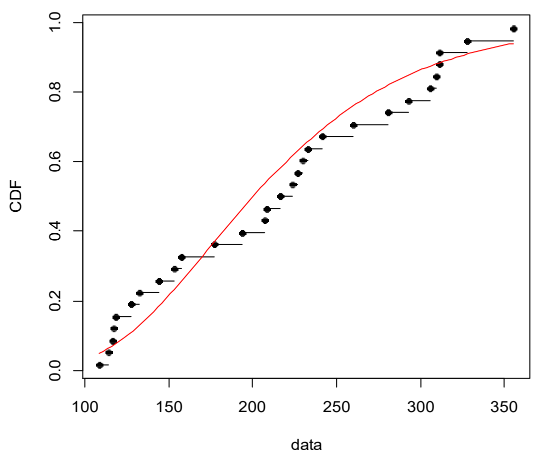

Figure 3. Empirical and theoretical CDF-plots for Nitrogen oxides emissions

Clearly, Figure 3 and Figure 4 indicate that data are skewed to the right and how well the lognormal model fitted the emission measurements. Initially, data are the crisp values however, for the sake of demonstration, we regard data as uncertain sample values for some emission values, as shown in Table 2.

Table 2. Nitrogen oxides emissions for Denmark for the period 1990-2018

Nitrogen oxides emissions
[304.12, 307.82], 355.34, 310.93, 309.47, [**309.12, 312.10**], 292.80, 327.49, 280.33, 259.99, **[238.19, 242.45]**, 229.98, 226.77, 223.57, 233.12, 216.37, 208.16, **[206.30, 209.14]**, 193.44, 177.31, 157.88, 153.18, 143.93, 132.78, 127.87, 118.28, 116.75, 116.96, 114.21, **[106.86, 110.62]**

Table 2 indicates that Nitrogen oxides emissions such as [304.12, 307.82], [309.12, 312.10], [238.19, 242.45], [206.30, 209.14] and [106.86, 110.62] are not accurately recorded to precise values but are given in intervals. Indeed, the existing lognormal model is ineffective due to ambiguity or uncertainties in the sample. On the other hand, the proposed model can easily be employed to analyze neutrosophic set of measurements. The descriptive measures using the proposed NLD are given in Table 3.

Table 3. Numerical characteristics of the for Nitrogen oxides emissions

Descriptive Measures	
Mean $_1$)	[214.106, 214.671]
Mode (m)	[174.617, 175.282]
Skewness ($_{1n}$)	[1.196, 1.200]
Kurtosis ($_{2n}$)	[5.647, 5.666]

Results in Table 3 show that the essential descriptive statistics of the Nitrogen oxides emissions are in ranges because of vagueness in the observed sample. Thus the proposed model can be applied to analyze the uncertainties involving data, which follows the NLD.

6. Conclusions

In this paper, a new generalization of the lognormal model under the neutrosophic environment, so-called the neutrosophic lognormal distribution has been proposed. This generalization is rooted in the methodology of neutrosophic algebra. The statistical characteristics of the new suggested distribution, such as moments, mode, skewness, and kurtosis, have been studied in detail. A strategy for estimating the neutrosophic distributional parameters has been developed. To investigate the validity of the analytical results produced for the suggested model, a simulation analysis has been performed. Simulated findings matched quite well with those obtained from the analytical properties of the NLD. Due to the variety of statistical properties proposed under the neutrosophic calculus, NLD can effectively be employed in analyzing real-dataset involving uncertainties as described in the application section.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Basu, S., Lokesh, K. S. Application of multiple linear regression and manova to evaluate health Impacts due to changing river water quality. Appl. Math., 2014, vol.05, no. 05, pp. 799–807.
2. Gajbhiye, S., Sharma, S. K., Awasthi, M. K. Application of principal components analysis for interpretation and grouping of water quality parameters. Int. J. Hybrid Inf, 2015, vol. 8, no. 4, pp. 89–96.
3. Zhong, M., Zhang, H., Sun, X., Wang, Z., Tian, W., Huang, H. Analyzing the significant environmental factors on the spatial and temporal distribution of water quality utilizing multivariate statistical techniques: a case study in the Balihe Lake, China. Environ. Sci. Pollut., 2018, vol. 25, no. 29, pp. 29418–29432.
4. Oguntunde P. E., Odetunmibi O. A., Adejumo A. O. A study of probability models in monitoring environmental pollution in Nigeria. J. Probab. Stat., 2014, Article ID 86496, doi:10.1155/2014/864965.
5. Rygg, B. Heavy-metal pollution and log-normal distribution of individuals among species in benthic communities. Mar. Pollut. Bull., 1986, vol. 17, no. 1, pp. 31–36.
6. Blackwood, L.G. The lognormal distribution, environmental data, and radiological monitoring. Environ. Monit. Assess, 1992, vol. 21, no. 3, pp. 193–210.
7. Larsen, R. I. and Heck, W. W. An air quality data analysis system for interrelating effects, standards, and needed source reductions: part 3. Vegetation Injury. J Air Pollut Contr Assoc., 1976, vol. 26, no. 4, pp. 325–333.
8. Berger, A., Melice, J. L. and Demuth, C. L. Statistical distributions of daily and high atmospheric SO2-concentrations. Atmos. Environ., 1982, vol. 16, no. 12, pp. 2863–2877.
9. Li, X., Peng, L., Yao, X., Cui, S., Hu, Y., You, C., Chi, T. Long short-term memory neural network for air pollutant concentration predictions: Method development and evaluation. Environ. Pollut., 2017, vol. 231, pp. 997–1004.
10. Langat, P. K., Kumar, L. and Koech, R. Identification of the most suitable probability distribution models for maximum, minimum, and mean stream flow. Water, 2019, vol. 11, no. 4, p. 734.
11. Smarandache, F: Neutrosophic logic: generalization of the Intuitionistic fuzzy logic. Extr. Metal. Nickel Cobalt Platin., Group Metals, 2016, vol.269, no. 50, pp. 49–53
12. Smarandache, F. A unifying field in logics: neutrosophic logic. neutrosophy neutrosophic set neutrosophic probability., Rehoboth, NM, USA:American Research Press, 1999.
13. Smarandache, F. and Khalid, H. E. Neutrosophic precalculus and neutrosophic calculus, Infinite Study, 2018.
14. Smarandache. F. Neutrosophy, Neutrosophic Probability, Set, and Logic. Rehoboth: Amer. Res. Press, 1998.
15. F. Smarandache, Introduction to Neutrosophic Statistics. Craiova: Sitech & Education Publishing, 2014.
16. Duan, W. Q., Khan, Z., Gulistan, M. and Khurshid, A. Neutrosophic Exponential Distribution: Modeling and Applications for Complex Data Analysis. Complexity, 2021, vol. 2021 doi.org/10.1155/2021/5970613
17. Patro. S. K and Smarandache.F. The neutrosophic statistical distribution: more problems, more solutions, Neutrosophic Sets and Systems, 2016, vol. 12, pp. 73-79.
18. Aslam. M. A variable acceptance sampling plan under neutrosophic statistical interval method, Symmetry, 2019, vol. 11, p. 114.
19. Schweizer, P. Uncertainty: two probabilities for the three states of neutrosophy. Int. J. Neutrosophic Sci., 2020, vol. 2, no.1, pp.18-26.
20. Arif, O. H. and Aslam, M. A new sudden death chart for the Weibull distribution under complexity. Compl. Int. Sys., 2021, pp.1-9.
21. Aslam, M. Analyzing wind power data using analysis of means under neutrosophic statistics. Soft Computing, 2021, vol. 25, no.10, pp.7087-7093.
22. Aslam, M., Arif, O.H. and Sherwani, R.A.K. New diagnosis test under the neutrosophic statistics: an application to diabetic patients. BioMed Research International, 2020, vol. 2020. doi: org/10.1155/2020/2086185

23. Chen, J., Ye, J., Du, S. and Yong, R.. Expressions of rock joint roughness coefficient using neutrosophic interval statistical numbers. Symmetry, 2017, vol.9, no.7, 9(7), p.123.
24. Smarandache, F. and Savoiu, G. Neutrosophic index numbers: neutrosophic logic applied in the statistical indicators theory. A Publication of Society for Mathematics of Uncertainty, 2015, vol. 10, p.67.
25. Aslam, M., Bantan, R .A. and Khan, N. Design of a new attribute control chart under neutrosophic statistics. Int. J. Fuzzy Syst., 2019, vol. 21 no.2, pp.433-440.
26. Khan, Z., Gulistan, M., Kadry, S., Chu, Y., Lane-Krebs. K. On scale parameter monitoring of the Rayleigh distributed data using a new design. IEEE Access, 2020, vol. 8, pp. 188390–188400.
27. Khan, Z., Gulistan, M., Chammam, W., Kadry, S., Nam, Y. A new dispersion control chart for handling the neutrosophic data. IEEE Access, 2020, vol. 8, pp. 96006–96015.
28. Khan, Z., Gulistan, M., Hashim, R., Yaqoob, N. and Chammam, W. Design of S-control chart for neutrosophic data: An application to manufacturing industry, J. Intell. Fuzzy Syst., 2020, vol. 38, no. 4, pp. 4743–4751.
29. Khan, Z., Gulistan, M., Kausar, N. and Park, C. Neutrosophic Rayleigh model with some basic characteristics and engineering applications. IEEE Access, 2021,vol. 9, pp. 71277–71283.
30. UN Enviromental Statistics https://di.unfccc.int/time_series

Received: Aug 1, 2021, Accepted: Dec 1, 2021