# Neutrosophic Triplet Partial Bipolar Metric Spaces

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**Abstract:** In this article, neutrosophic triplet partial bipolar metric spaces are obtained. Then some definitions and examples are given for neutrosophic triplet partial bipolar metric space. Based on these definitions, new theorems are given and proved. In addition, neutrosophic triplet partial bipolar metric spaces have been shown to be different from classical partial metric space, neutrosophic triplet partial metric space and neutrosophic triplet metric space. Thus, we add a new structure in neutrosophic triplet theory.

**Keywords:** triplet set, neutrosophic triplet metric space, bipolar metric space, neutrosophic triplet bipolar metric space, neutrosophic triplet partial metric space, neutrosophic triplet partial bipolar metric

## 1 Introduction

Smarandache obtained neutrosophic logic and set [1]. In neutrosophic theory, there is a degree of membership (t), there is a degree of indeterminacy (ı) and there is a degree of non-membership (f). These degrees are defined independently of each other. Therefore, neutrosophic logic and neutrosophic set help us to explain many uncertainties in our lives. In addition, many researchers have made studies on this theory [2-27]. Recently, some researchers studied neutrosophic theory [50 - 53]. Also, Tey et al. studied novel neutrosophic data analytic hierarchy process for multi-criteria decision making method [54], Son et al. obtained on the stabilizability for a class of linear time-invariant systems under uncertainty [55], Tanuwijaya et al. introduced novel single valued neutrosophic hesitant fuzzy time series model [56].

In fact, neutrosophic set is a generalized state of fuzzy [28] and intuitionistic fuzzy set [29].

Also, Smarandache and Ali obtained neutrosophic triplet set (NTS) and neutrosophic triplet groups (NTG) [30]. For every element "x" in NTS A, there exist a neutral of "x" and an opposite of "x". Also, neutral of "x" must be different from the classical neutral element. Therefore, the NTS is different from the classical set. Furthermore, a neutrosophic triplet (NT) "x" is showed by <x, neut(x), anti(x)>. Also, many researchers have obtained NT structures [31-44]. Recently, Şahin, Kargın Uz and Kılıç have discussed neutrosophic triplet bipolar metric space [45].

Mutlu and Gürdal introduced bipolar metric space [46] in 2016. Bipolar metric space is a generalization of metric space. Also, bipolar metric spaces have an important role in fixed point theory. Recently, Mutlu, Özkan and Gürdal studied fixed point theorems on bipolar metric spaces [47]; Kishore, Agarwal, Rao, and Rao introduced contraction and fixed point theorems in bipolar metric spaces with applications [48]; Rao, Kishore and Kumar obtained Geraghty type contraction and common coupled fixed point theorems in bipolar metric spaces with applications to homotopy [49].

In this section, neutrosophic triplet partial bipolar metric space is introduced. Chapter 2 provides definitions and properties for bipolar metric space [46], neutrophic triplet sets [30], neutrophic triplet metric spaces [32], neutrosophic triplet partial metric space [36] and neutrosophic triplet b - metric space [45]. In chapter 3, neutrosophic triplet partial bipolar metric space is described and some properties are given for neutrosophic triplet partial bipolar metric space. In addition, neutrosophic triplet partial bipolar metric spaces are shown to be different from classical partial metric space, neutrosophic triplet partial metric space and neutrosophic triplet metric space. We give conclusions in Chapter 4.

## 2 Preliminaries

**Definition 2.1: [30]** Let # be a binary operation. An NTS (X, #) is a set such that for x ∈ X,

i) There exists neutral of "x" such that x#neut(x) = neut(x)#x = x,

ii) There exists anti of "x" such that x#anti(x) = anti(x)#x = neut(x).

Also, a neutrosophic triplet "x" is denoted by (x, neut(x), anti(x)).

**Definition 2.2: [32]** Let (N,*) be an NTS and $d_N$:NxN→ $\mathbb{R}^+ \cup \{0\}$ be a function. If $d_N$:NxN→ $\mathbb{R}^+ \cup \{0\}$ and (N, *) satisfies the following conditions, then $d_N$ is called NTM.

a) x*y ∈ N ,

b) $d_N$(x, y) ≥ 0 ,

c) If x = y, then $d_N$(x, y) = 0,

d) $d_N$(x, y) = $d_N$(y, x) ,

e) If there exits at least a y ∈ N for each x, z ∈N such that $d_N$(x, z) ≤ $d_N$(x, z*neut(y)), then

$d_N$(x, z*neut(y)) ≤ $d_N$(x, y) + $d_N$(y, z).

In this case, ((N,*), $d_N$) is called an NTMS.

**Definition 2.3: [36]** Let (N,*) be a NTS. If $d_p$:NxN→ $\mathbb{R}^+ \cup \{0\}$ satisfies the following conditions, then $d_p$ is a NTpM. For all x, y, z ∈ N,

a) x*y ∈ N,

b) $d_p$ (x, y) ≥ $d_p$ (x, x) ≥ 0,

c) If $d_p$ (x, y) = $d_p$ (x, x) = $d_p$ (y, y) = 0, then there exists at least one pair of elements x, y ∈N such that x = y.

d) $d_p$ (x, y) = $d_p$ (y, x),

e) If for each pair of x, z ∈N, there exists at least one y ∈N such that $d_p$ (x, z) ≤ $d_p$ (x, z*neut(y)), then $d_p$ (x, z*neut(y)) ≤ $d_p$ (x, y) + $d_p$ (y, z) - $d_p$ (y, y).

In this case, ((N,*), $d_p$) is called a NTpMS.

**Definition 2.4: [46]** Let $X$ and $Y$ be nonempty sets and $d$:NxN→ $\mathbb{R}^+ \cup \{0\}$ be a function. If d satisfies the following conditions, then d is called a bipolar metric (bM).

i) For $\forall (x, y) \in X \times Y$, if *d(x, y)* = 0, then $x = y$,

ii) For $\forall u \in X \cap Y$, *d(u, u)* = 0,

iii) For $\forall u \in X \cap Y$, $d(u, v) = d(v, u)$,

iv) For $(x, y), (x', y') \in X \times Y$, $d(x, y) \leq d(x, y') + d(x', y') + d(x', y)$.

In this case, ($X$, $Y$, $d$) is called a bipolar metric space (bMS).

**Definition 2.5: [45]** Let $(X,*)$ and $(Y,*)$ be two NTSs and let $d: X \times Y \to \mathbb{R}^+ \cup \{0\}$ be a function. If d, $(X, *)$ and $(Y,*)$ satisfy the following conditions, then $d$ is called a neutrosophic triplet bipolar metric (NTbM).

i) For $\forall a, b \in X$, $a * b \in X$,

for $\forall c, d \in Y$, $c * d \in Y$,

ii) For $\forall a \in X$ and $\forall b \in Y$, if $d(a,b) = 0$, then $a = b$,

iii) For $\forall u \in X \cap Y$, d(u,u)=0,

iv) For $\forall u, v \in X \cap Y$, $d(u,v) = d(v,u)$.

v) Let $(x,y), (x',y') \in X \times Y$. For each $(x,y)$, if there exists at least one $(x',y')$ such that

$d(x,y) \le d(x, y * neut(y')) \le d(x * neut(x'), y * neut(y'))$ and

$d(x,y) \le d(x * neut(x'), y) \le d(x * neut(x'), y * neut(y'))$,

then

$d(x * neut(x'), y * neut(y')) \le d(x,y') + d(x',y') + d(x',y)$.

In this case, $\left(((X,Y),*), d\right)$ is called a neutrosophic triplet bipolar metric space (NTbMS).

**Definition 2.6: [45]** Let $(((X, Y), *), d)$ be a NTbMS. A left sequence $(x_n)$ converges to a right point y (symbolically $(x_n) \rightharpoondown y$ or $\lim_{n\to\infty}(x_n) = y$ ) if and only if for every ε > 0 there exists an $n_0 \in \mathbb{N}$ , such that $d(x_n, y) < \varepsilon$ for all $n \ge n_0$. Similarly, a right sequence $(y_n)$ converges to a left point x (denoted as $y_n \rightharpoonup x$ or $\lim_{n\to\infty}(y_n) = x$) if and only if, for every ε > 0 there exists an $n_0 \in \mathbb{N}$ such that, whenever $n \ge n_0$, $d(x, y_n) < \varepsilon$. Also, if $(u_n) \rightharpoondown u$ and $(u_n) \rightharpoonup u$, then $(u_n)$ converges to point u ($(u_n)$ is a central sequence).

**Definition 2.7: [45]** Let $(((X, Y), *), d)$ be an NTbMS, $(x_n)$ be a left sequence and $(y_n)$ be a right sequence in this space. $(x_n, y_n)$ is called an NT bisequence. Furthermore, if $(x_n)$ and $(y_n)$ are convergent, then $(x_n, y_n)$ is called an NT convergent bisequence. Also, if $(x_n)$ and $(y_n)$ converge to the same point, then $(x_n, y_n)$ is called an NT biconvergent bisequence.

**Definition 2.8: [45]** Let $(((X, Y), *), d)$ be an NTbMS and $(x_n, y_n)$ be an NT bisequence. $(x_n, y_n)$ is called an NT Cauchy bisequence if and only if for every ε > 0 there exists an $n_0 \in \mathbb{N}$ , such that $d(x_n, y_n) < \varepsilon$ for all $n \ge n_0$.

## 3 Neutrosophic Triplet Partial Bipolar Metric Space

**Definition 3.1:** Let $(X, *)$ and $(Y,*)$ be two NTSs and let $d_{pb}: X \times Y \to \mathbb{R}^+ \cup \{0\}$ be a function. If $d_{pb}$, (X, $*$) and (Y, $*$) satisfy the following conditions, then $d_{pb}$ is called a NT partial bipolar metric (NTpbM).

i-) For all $a, b \in X$ , $a * b \in X$,

for all $c, d \in Y$ , $c * d \in Y$,

ii-) For all $x \in X$ and $y \in Y$,

$d_{pb}(x,y) \ge d_{pb}(x,x) \ge 0$ and $d_{pb}(x,y) \ge d_{pb}(y,y) \ge 0$,

iii-) If $d_{pb}(x,y) = d_{pb}(x,x) = d_{pb}(y,y) = 0$, there exists at least one pair of elements $x, y \in X \cap Y$ such that $d_{pb}(x,y) = 0$, iv-) For all $x, y \in X \cap Y$, $d_{pb}(x, y) = d_{pb}(y, x)$,

v-) Let $(x, y), (x', y') \in X \times Y$. For each $(x, y)$, if there exists at least one $(x', y')$ such that

$d_{pb}(x, y) \leq d_{pb}\big(x, y * neut(y')\big) \leq d_{pb}(x * neut(x'), y * neut(y'))$ and

$d_{pb}(x, y) \leq d_{pb}(x * neut(x'), y) \leq d_{pb}(x * neut(x'), y * neut(y'))$,

then

$d_{pb}(x * neut(x'), y * neut(y')) \leq d_{pb}(x, y') + d_{pb}(x', y') + d_{pb}(x', y) - \min\{d_{pb}(x', x'), d_{pb}(y', y')\}$.

In this case, $\big(((X, Y), *), d_{pb}\big)$ is called a NTpbM space (NTpbMS).

**Example 3.2:** Let $X = \{0, 3, 6, 9, 10, 12\}$ and $Y = \{0, 5, 6, 10\}$. We show that $(X, .)$ and $(Y, .)$ are NTSs in $(\mathbb{Z}_{15}, .)$.

For $(X, .)$, NTs are (0, 0, 0), (3, 6, 12), (6, 6, 6), (9, 6, 9), (10,10,10), (12, 6, 3).

Also, for $(Y, .)$, NTs are (0, 0, 0), (5, 10, 5), (6, 6, 6), (10, 10, 10).

Thus, $(X, .)$ and $(Y, .)$ are NTSs.

Furthermore, we define the $d_{pb}: X \times Y \to \mathbb{R}^{+} \cup \{0\}$ function such that $d_{pb}(s, r) = max\{|3^s - 1|, |3^r - 1|\}$. We show that d is a NTpbM.

i-) $0.0 = 0 \in$ , $0.3 = 0 \in X$ , $0.6 = 0 \in X$ , $0.9 = 0 \in X$ , $0.10 = 0 \in X$ , $0.12 = 0 \in X$ , $3.3 = 9 \in X$ , $3.6 = 3 \in X$, $3.9 = 12 \in X$, $3.10 = 0 \in X$, $3.12 = 6 \in X$, $6.6 = 6 \in X$, $6.9 = 9 \in X$, $6.10 = 0 \in X$, $6.12 = 12 \in X$, $9.9 = 6 \in X$, $9.10 = 0 \in X$, $9.12 = 3 \in X$, $10.10 = 10 \in X$,$12.10 = 0 \in X$, $12.12 = 9 \in X$.

Thus, for all $a, b \in X$, $a.b \in X$.

Also, $0.0 = 0 \in Y$ , $0.5 = 0 \in Y$ , $0.6 = 0 \in Y$ , $0.10 = 0 \in Y$ , $5.5 = 10 \in Y$ , $5.10 = 5 \in Y$ , $5.6 = 0 \in Y$ , $10.10 = 10 \in Y$, $10.6 = 0 \in Y$, $6.6 = 6 \in Y$

Thus, for all $c, d \in Y$, $c.d \in Y$.

ii-)For all $x \in X$, $y \in Y$, if

$d_{pb}(x, y) = \max\{|3^x - 1|, |3^y - 1|\}$,

$d_{pb}(x, x) = \max\{|3^x - 1|, |3^x - 1|\}$,

$d_{pb}(y, y) = \max\{|3^y - 1|, |3^y - 1|\}$,

then it is clear that

$d_{pb}(x, y) \geq d_{pb}(x, x) \geq 0$ and

$d_{pb}(x, y) \geq d_{pb}(y, y) \geq 0$.

iii-)For $d_{pb}(x, y) = d_{pb}(x, x) = d_{pb}(y, y) = 0$ , if $d_{pb}(x, y) = 0$ , then there exists at least one pair $x, y \in X \cap Y$.

If $d_{pb}(x, y) = \max\{|3^x - 1|, |3^y - 1|\} = 0$, then $3^x - 1 = 0$ and $3^y - 1 = 0$.

If $3^x = 1$ and $3^y = 1$ , $x, y \in X \cap Y$ are pairs of elements, since $x = 0 \in X$ and $y = 0 \in Y$.

iv) For all $, y \in X \cap Y$ , $d_{pb}(x, y) = d_{pb}(y, x)$ .

$d_{pb}(x, y) = \max\{|3^x - 1|, |3^y - 1|\} = \max\{|3^y - 1|, |3^x - 1|\} = d_{pb}(y, x)$ .

v) It is clear that

$$d_{pb}(0,0) = 0 \le d_{pb}\big(0\,,0.neut(6)\big) = 0 \le d_{pb}\big(0.neut(3), 0.neut(6)\big) = d_{pb}(0,0) = 0,$$
$$d_{pb}(0,0) = 0 \le d_{pb}(0.neut(3), 0) = 0 \le d_{pb}\big(0.neut(3), 0.neut(6)\big) = d_{pb}(0,0) = 0.$$

Also,

$d_{pb}\big(0.neut(3), 0.neut(6)\big) = d_{pb}(0,0) = 0 \le\ d_{pb}(0,6) + d_{pb}(3,6) + d_{pb}(3,0) -$
$\min\big\{d_{pb}(3,3), d_{pb}(6,6)\big\}$.

It is clear that

$$d_{pb}(0,5) = 0 \le d_{pb}\big(0,5.neut(10)\big) = d_{pb}(0,5) = max\{|3^0 - 1|, |3^5 - 1|\}$$
$$\le d_{pb}\big(0.neut(6), 5.neut(10)\big) = d_{pb}(0,5),$$
$$d_{pb}(0,5) = 0 \le d_{pb}(0.neut(6), 5) = d_{pb}(0,5) = max\{|3^0 - 1|, |3^5 - 1|\}$$
$$\le d_{pb}\big(0.neut(6), 5.neut(10)\big) = d_{pb}(0,5).$$

Also,

$d_{pb}\big(0.neut(6), 5.neut(10)\big) = d_{pb}(0,5) \le$
$d_{pb}\ (0,10) + d_{pb}(6,10) + d_{pb}(6,5) -\ min\big\{d_{pb}(6,6), d_{pb}(10,10)\big\}$.

It is clear that

$$d_{pb}(0,10) \le d_{pb}\big(0,10.neut(5)\big) = d_{pb}(0,10) = max\{|3^0 - 1|, |3^{10} - 1|\}$$
$$\le d_{pb}\big(0.neut(3), 10.neut(5)\big) = d_p(0,10),$$
$$d_{pb}(0,10) = |3^{10} - 1| \le d_{pb}(0.neut(3), 10) = d_{pb}(0,10) = max\{|3^0 - 1|, |3^{10} - 1|\}$$
$$\le d_{pb}\big(0.neut(3), 10.neut(5)\big) = d_{pb}(0,10)$$

Also,

$d_{pb}\big(0.neut(3), 10.neut(5)\big) = d_{pb}(0,10) \le\ d_{pb}(0,5) + d_{pb}(3,5) + d_p(3,10) -\ min\big\{d_{pb}(3,3), d_{pb}(5,5)\big\}$

It is clear that

$$d_{pb}(0,6) = |3^6 - 1| \le d_{pb}\big(0,6.neut(6)\big) = d_{pb}(0,6) = max\{|3^0 - 1|, |3^6 - 1|\}$$
$$\le d_{pb}\big(0.neut(3), 6.neut(6)\big) = d_{pb}(0,6),$$
$$d_{pb}(0,6) = |3^6 - 1| \le d_{pb}(0.neut(3), 6) = d_{pb}(0,6) = max\{|3^0 - 1|, |3^6 - 1|\}$$
$$\le d_{pb}\big(0.neut(3), 6.neut(6)\big) = d_{pb}(0,6).$$

Also,

$$d_{pb}\big(0.neut(3), 6.neut(6)\big) = d_{pb}(0,6) = 728$$
$$\le\ d_{pb}(0,6) + d_{pb}(3,6) + d_{pb}(3,6) -\ min\big\{d_{pb}(3,3),\ d_{pb}(6,6)\big\}$$

It is clear that

$$d_{pb}(3,0) = |3^3 - 1| \le d_{pb}\big(3, 0.neut(5)\big) = d_{pb}(3,0) = max\{|3^3 - 1|, |3^0 - 1|\}$$
$$\le d_{pb}\big(3.neut(6), 0.neut(5)\big) = d_{pb}(3,0),$$
$$d_{pb}(3,0) = |3^3 - 1| = 26 \le d_{pb}(3.neut(6), 0) = d_{pb}(3,0) = max\{|3^3 - 1|, |3^0 - 1|\} = 26$$
$$\le d_{pb}\big(3.neut(6), 0.neut(5)\big) = d_{pb}(3,0).$$

Also,

$$d_{pb}(3.neut(6), 0.neut(5)) = d_{pb}(3,0) = 26$$
$$\leq d_{pb}(3,5) + d_{pb}(6,5) + d_{pb}(6,0) - min\{d_{pb}(6,6), d_{pb}(5,5)\}.$$

It is clear that

$$d_{pb}(3,5) = |3^5 - 1| \leq d_{pb}(3, 5.neut(10)) = d_{pb}(3,5) = max\{|3^3 - 1|, |3^5 - 1|\}$$
$$\leq d_{pb}(3.neut(6), 5.neut(10)) = d_{pb}(3,5),$$
$$d_{pb}(3,5) = |3^5 - 1| \leq d_{pb}(3.neut(6), 5) = d_{pb}(3,5) = max\{|3^3 - 1|, |3^5 - 1|\}$$
$$\leq d_{pb}(3.neut(6), 5.neut(10)) = d_{pb}(3,5).$$

Also,

$$d_{pb}(3.neut(6), 5.neut(10)) \leq d_{pb}(3,10) + d_{pb}(6,10) + d_{pb}(6,5) - min\{d_{pb}(6,6), d_{pb}(10,10)\}.$$

It is clear that

$$d_{pb}(3,10) = |3^{10} - 1| \leq d_{pb}(3, 10.neut(5)) = d_{pb}(3,10) = max\{|3^3 - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}(3.neut(9), 10.neut(5)) = d_{pb}(3,10),$$
$$d_{pb}(3,10) = |3^{10} - 1| \leq d_{pb}(3.neut(9), 10) = d_{pb}(3,10) = max\{|3^3 - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}(3.neut(9), 10.neut(5)) = d_{pb}(3,10).$$

Also, $d_{pb}(3.neut(9), 10.neut(5)) \leq d_{pb}(3,5) + d_{pb}(9,5) + d_{pb}(9,10) - min\{d_{pb}(9,9), d_{pb}(5,5)\}$

It is clear that

$$d_{pb}(3,6) = |3^6 - 1| \leq d_{pb}(3, 6.neut(6)) = d_{pb}(3,6) = max\{|3^3 - 1|, |3^6 - 1|\}$$
$$\leq d_{pb}(3.neut(9), 6.neut(6)) = d_{pb}(3,6),$$
$$d_{pb}(3,6) = |3^6 - 1| \leq d_{pb}(3.neut(9), 6) = d_{pb}(3,6) = max\{|3^3 - 1|, |3^6 - 1|\}$$
$$\leq d_{pb}(3.neut(9), 6.neut(6)) = d_{pb}(3,6).$$

Also,

$$d_{pb}(3.neut(9), 6.neut(6)) = d_{pb}(3,6) \leq d_{pb}(3,6) + d_{pb}(9,6) + d_{pb}(9,6) - min\{d_{pb}(9,9), d_{pb}(6,6)\}$$

It is clear that

$$d_{pb}(6,0) = |3^6 - 1| \leq d_{pb}(6, 0.neut(5)) = d_{pb}(6,0) = max\{|3^6 - 1|, |3^0 - 1|\}$$
$$\leq d_{pb}(6.neut(9), 0.neut(5)) = d_{pb}(6,0),$$
$$d_{pb}(6,0) = |3^6 - 1| \leq d_{pb}(6.neut(9), 0) = d_{pb}(6,0) = max\{|3^6 - 1|, |3^0 - 1|\}$$
$$\leq d_{pb}(6.neut(9), 0.neut(5)) = d_{pb}(6,0).$$

Also,

$$d_{pb}(6.neut(9), 0.neut(5)) = d_{pb}(6,0) \leq d_{pb}(6,5) + d_{pb}(9,5) + d_{pb}(9,0) - min\{d_{pb}(9,9), d_{pb}(5,5)\}.$$

It is clear that

$$d_{pb}(6,5) = |3^6 - 1| \leq d_{pb}(6, 5.neut(0)) = d_{pb}(6,0) = max\{|3^6 - 1|, |3^0 - 1|\}$$
$$\leq d_{pb}(6.neut(12), 5.neut(0)) = d_{pb}(6,0),$$

$$d_{pb}(6,5) = |3^6 - 1| \le d_{pb}(6.neut(12),5) = d_{pb}(6,5) = max\{|3^6 - 1|, |3^5 - 1|\}$$
$$\le d_{pb}\big(6.neut(12), 5.neut(0)\big) = d_{pb}(6,0).$$

Also,

$$d_{pb}\big(6.neut(12), 5.neut(0)\big)$$
$$\le d_{pb}(6,0)\ d_{pb}(6,0) + d_{pb}(12,0) + d_{pb}(12,5) - \ min\big\{d_{pb}(12,12), d_{pb}(0,0)\big\}.$$

It is clear that

$$d_{pb}(6,10) = |3^{10} - 1| \le d_{pb}\big(6, 10.neut(5)\big) = d_{pb}(6,10) = max\{|3^6 - 1|, |3^{10} - 1|\}$$
$$\le d_{pb}\big(6.neut(0), 10.neut(5)\big) = d_{pb}(0,10),$$
$$d_{pb}(6,10) = |3^{10} - 1| \le d_{pb}(6.neut(0),10) = d_{pb}(0,10) = max\{|3^0 - 1|, |3^{10} - 1|\}$$
$$\le d_{pb}\big(6.neut(0), 10.neut(5)\big) = d_{pb}(0,10).$$

Also,

$$d_{pb}\big(6.neut(0), 10.neut(5)\big) = d_{pb}(0,10) \le\ d_{pb}(6,5) + d_p(0,5) + d_{pb}(0,10) - \ min\big\{d_{pb}(0,0), d_{pb}(5,5)\big\}$$

It is clear that

$$d_{pb}(6,6) = |3^6 - 1| \le d_{pb}\big(6, 6.neut(0)\big) = d_{pb}(6,0) = max\{|3^6 - 1|, |3^0 - 1|\}$$
$$\le d_{pb}\big(6.neut(3), 6.neut(0)\big) = d_{pb}(6,0),$$
$$d_{pb}(6,6) = |3^6 - 1| \le d_{pb}(6.neut(3),6) = d_{pb}(6,6) = max\{|3^6 - 1|, |3^6 - 1|\}$$
$$\le d_{pb}\big(6.neut(3), 6.neut(0)\big) = d_{pb}(6,0).$$

Also,

$$d_{pb}\big(6.neut(3), 6.neut(0)\big) = d_{pb}(6,0) \le\ d_{pb}(6,0) + d_{pb}(3,0) + d_{pb}(3,6) - \ min\big\{d_{pb}(3,3), d_{pb}(0,0)\big\}$$

It is clear that

$$d_{pb}(9,0) = |3^9 - 1| \le d_{pb}\big(9, 0.neut(5)\big) = d_{pb}(9,0) = max\{|3^9 - 1|, |3^0 - 1|\}$$
$$\le d_{pb}\big(9.neut(12), 0.neut(5)\big) = d_p(9,0),$$
$$d_{pb}(9,0) = |3^9 - 1| \le d_{pb}(9.neut(12),0) = d_{pb}(9,0) = max\{|3^9 - 1|, |3^0 - 1|\}$$
$$\le d_{pb}\big(9.neut(12), 0.neut(5)\big) = d_{pb}(9,0).$$

Also,

$$d_{pb}\big(9.neut(12), 0.neut(5)\big) = d_{pb}(9,0) \le$$
$$d_{pb}(9,5) + d_{pb}(12,5) + d_{pb}(12,0) - \ min\big\{d_{pb}(12,12), d_{pb}(5,5)\big\}.$$

It is clear that

$$d_{pb}(9,5) = |3^9 - 1| \le d_{pb}\big(9, 5.neut(0)\big) = d_{pb}(9,0) = max\{|3^9 - 1|, |3^0 - 1|\}$$
$$\le d_{pb}\big(9.neut(12), 5.neut(0)\big) = d_{pb}(9,0),$$
$$d_{pb}(9,5) = |3^9 - 1| \le d_{pb}(9.neut(12),5) = d_{pb}(9,5) = max\{|3^9 - 1|, |3^5 - 1|\}$$
$$\le d_{pb}\big(9.neut(12), 5.neut(0)\big) = d_{pb}(9,0).$$

Also,

$$d_{pb}\big(9.neut(12), 5.neut(0)\big) = d_{pb}(9, 0)$$
$$\leq\ d_{pb}(9, 0) + d_{pb}(12, 0) + d_{pb}(12, 5) -\ min\big\{d_{pb}(12, 12), d_{pb}(0, 0)\big\}.$$

It is clear that

$$d_{pb}(9, 10) = |3^{10} - 1| \leq d_{pb}\big(9, 10.neut(5)\big) = d_{pb}(9, 10) = max\{|3^{9} - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}\big(9.neut(0), 10.neut(5)\big) = d_{pb}(0, 10),$$
$$d_{pb}(9, 10) = |3^{10} - 1| \leq d_{pb}(9.neut(0), 10) = d_{pb}(0, 10) = max\{|3^{0} - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}\big(9.neut(0), 10.neut(5)\big) = d_{pb}(0, 10).$$

Also,

$$d_{pb}\big(9.neut(0), 10.neut(5)\big) = d_{pb}(0, 10) \leq$$
$$d_{pb}(9, 5) + d_{pb}(0, 5) + d_{pb}(0, 10) -\ min\big\{d_{pb}(0, 0), d_{pb}(5, 5)\big\}.$$

It is clear that

$$d_{pb}(9, 6) = |3^{9} - 1| \leq d_{pb}\big(9, 6.neut(5)\big) = d_{pb}(9, 0) = max\{|3^{9} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(9.neut(3), 6.neut(5)\big) = d_{pb}(9, 0),$$
$$d_{pb}(9, 6) = |3^{9} - 1| \leq d_{pb}(9.neut(3), 6) = d_{pb}(9, 6) = max\{|3^{9} - 1|, |3^{6} - 1|\}$$
$$\leq d_{pb}\big(9.neut(3), 6.neut(5)\big) = d_{pb}(9, 0).$$

Also,

$$d_{pb}\big(9.neut(3), 6.neut(5)\big) = d_{pb}(9, 0) \leq\ d_{pb}(9, 5) + d_{pb}(3, 5) + d_{pb}(3, 6) -\ min\big\{d_{pb}(3, 3), d_{pb}(5, 5)\big\}$$

It is clear that

$$d_{pb}(10, 0) = |3^{10} - 1| \leq d_{pb}\big(10, 0.neut(6)\big) = d_{pb}(10, 0) = max\{|3^{10} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(10.neut(10), 0.neut(6)\big) = d_{pb}(10, 0),$$
$$d_{pb}(10, 0) = |3^{10} - 1| \leq d_{pb}(10.neut(10), 0) = d_{pb}(10, 0) = max\{|3^{10} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(10.neut(10), 0.neut(6)\big) = d_{pb}(10, 0).$$

Also,

$$d_{pb}\big(10.neut(10), 0.neut(6)\big) = d_{pb}(10, 0) \leq$$
$$d_{pb}(10, 6) + d_{pb}(10, 6) + d_{pb}(10, 0) -\ min\big\{d_{pb}(10, 10), d_{pb}(6, 6)\big\}.$$

It is clear that

$$d_{pb}(10, 5) = |3^{10} - 1| \leq d_{pb}\big(10, 5.neut(6)\big) = d_{pb}(10, 0) = max\{|3^{10} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(10.neut(10), 5.neut(6)\big) = d_{pb}(10, 0),$$
$$d_{pb}(10, 5) = |3^{10} - 1| \leq d_{pb}(10.neut(10), 5) = d_{pb}(10, 5) = max\{|3^{10} - 1|, |3^{5} - 1|\}$$
$$\leq d_{pb}\big(10.neut(10), 5.neut(6)\big) = d_{pb}(10, 0).$$

Also,

$$d_{pb}\big(10.neut(10), 5.neut(6)\big) = d_{pb}(10, 0) \leq$$
$$d_{pb}(10, 6) + d_{pb}(10, 6) + d_{pb}(10, 5) -\ min\big\{d_{pb}(10, 10), d_{pb}(6, 6)\big\}.$$

It is clear that

$$d_{pb}(10,10) = |3^{10} - 1| \leq d_{pb}\big(10, 10.neut(5)\big) = d_{pb}(10,10) = max\{|3^{10} - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}\big(10.neut(3), 10.neut(5)\big) = d_{pb}(0,10),$$
$$d_{pb}(10,10) = |3^{10} - 1| \leq d_{pb}(10.neut(3), 10) = d_{pb}(0,10) = max\{|3^{0} - 1|, |3^{10} - 1|\}$$
$$\leq d_{pb}\big(10.neut(3), 10.neut(5)\big) = d_{pb}(0,10).$$

Also,

$d_{pb}\big(10.neut(3), 10.neut(5)\big) = d_{pb}(0,10) \leq$

$d_{pb}(10,5) + d_{pb}(3,5) + d_{pb}(3,10) - \ min\{d_{pb}(3,3), d_{pb}(5,5)\}$.

It is clear that

$$d_{pb}(10,6) = |3^{10} - 1| \leq d_{pb}\big(10, 6.neut(0)\big) = d_{pb}(10,0) = max\{|3^{10} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(10.neut(5), 6.neut(0)\big) = d_{pb}(10,0),$$
$$d_{pb}(10,6) = |3^{10} - 1| \leq d_{pb}(10.neut(5), 6) = d_{pb}(10,6) = max\{|3^{10} - 1|, |3^{6} - 1|\}$$
$$\leq d_{pb}\big(10.neut(5), 6.neut(0)\big) = d_{pb}(10,0).$$

Also,

$d_{pb}\big(10.neut(5), 6.neut(0)\big) = d_{pb}(10,0) \leq$

$d_{pb}(10,0) + d_{pb}(5,0) + d_{pb}(5,6) - \ min\{d_{pb}(5,5), d_{pb}(0,0)\}$.

It is clear that

$$d_{pb}(12,0) = |3^{12} - 1| \leq d_{pb}\big(12, 0.neut(5)\big) = d_{pb}(12,0) = max\{|3^{12} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(12.neut(6), 0.neut(5)\big) = d_{pb}(12,0),$$
$$d_{pb}(12,0) = |3^{12} - 1| \leq d_{pb}(12.neut(6), 0) = d_{pb}(12,0) = max\{|3^{12} - 1|, |3^{0} - 1|\}$$
$$\leq d_{pb}\big(12.neut(6), 0.neut(5)\big) = d_{pb}(12,0).$$

Also,

$d_{pb}\big(12.neut(6), 0.neut(5)\big) = d_{pb}(12,0) \leq$

$d_{pb}(12,5) + d_{pb}(6,5) + d_{pb}(6,0) - \ min\{d_{pb}(6,6), d_{pb}(5,5)\}$.

It is clear that

$$d_{pb}(12,5) = |3^{12} - 1| \leq d_{pb}\big(12, 5.neut(10)\big) = d_{pb}(12,5) = max\{|3^{12} - 1|, |3^{5} - 1|\}$$
$$\leq d_{pb}\big(12.neut(3), 5.neut(10)\big) = d_{pb}(12,5),$$
$$d_{pb}(12,5) = |3^{12} - 1| \leq d_{pb}(12.neut(3), 5) = d_{pb}(12,5) = max\{|3^{12} - 1|, |3^{5} - 1|\}$$
$$\leq d_{pb}\big(12.neut(3), 5.neut(10)\big) = d_{pb}(12,5).$$

Also,

$d_{pb}\big(12.neut(3), 5.neut(10)\big) = d_{pb}(12{,}5) \leq$

$d_{d_{pb}}(12{,}10) + d_{pb}(3{,}10) + d_{pb}(3{,}5) - \ min\{d_{pb}(3{,}3), d_{pb}(10{,}10)\}$.

It is clear that

$$d_{pb}(12,10) = |3^{12}-1| \le d_{pb}\big(12, 10.neut(0)\big) = d_{pb}(12,0) = max\{|3^{12}-1|, |3^{0}-1|\}$$
$$\le d_{pb}\big(12.neut(6), 10.neut(0)\big) = d_{pb}(12,0),$$
$$d_{pb}(12,10) = |3^{12}-1| \le d_{pb}(12.neut(6), 10) = d_{pb}(12,10) = max\{|3^{12}-1|, |3^{10}-1|\}$$
$$\le d_{pb}\big(12.neut(6), 10.neut(0)\big) = d_{pb}(12,0).$$

Also,

$d_{pb}\big(12.neut(6), 10.neut(0)\big) = d_{pb}(12,0) \le$

$d_{pb}(12,0) + d_{pb}(6,0) + d_{pb}(6,10) - \ min\{d_{pb}(6,6), d_{pb}(10,10)\}$.

It is clear that

$$d_{pb}(12,6) = |3^{12}-1| \le d_{pb}\big(12, 6.neut(10)\big) = d_{pb}(12,0) = max\{|3^{12}-1|, |3^{0}-1|\}$$
$$\le d_{pb}\big(12.neut(9), 6.neut(10)\big) = d_{pb}(12,0),$$
$$d_{pb}(12,6) = |3^{12}-1| \le d_{pb}(12.neut(9), 6) = d_{pb}(12,6) = max\{|3^{12}-1|, |3^{6}-1|\}$$
$$\le d_{pb}\big(12.neut(9), 6.neut(10)\big) = d_{pb}(12,0).$$

Also,

$d_{pb}\big(12.neut(9), 6.neut(10)\big) = d_{pb}(12,0) \le$

$d_{pb}(12,10) + d_{pb}(9,10) + d_{pb}(9,6) - \ min\{d_{pb}(9,9), d_{pb}(10,10)\}$.

Thus, for each $(x, y)$, if there exists at least a $(x', y')$ such that

$d_{pb}(x,y) \le d_{pb}\big(x, y * neut(y')\big) \ \le d_{pb}(x * neut(x'), y * neut(y'))$

$d_{pb}(x,y) \le d_{pb}(x * neut(x'), y) \le d_{pb}(x * neut(x'), y * neut(y'))$,

then

$d_{pb}(x * neut(x'), y * neut(y')) \le \ d_{pb}(x, y') + d_{pb}(x', y') + d_{pb}(x', y) - min\{d_{pb}(x', x'), d_{pb}(y', y')\}$.

Therefore, $d_{pb}$ is an NTpbM and $\big(((X,Y),*), d_{pb}\big)$ is an NTpbMS.

**Corollary 3.3:**

**1)** The NTpbMS differs from the NTPMS due to the i -), ii-) and v-) conditions in the NTpbMS.

**2)** The NTpbMS differs from the NTMS. Because the triangle inequality in the NTMS differs from the triangle inequality in the NTpbMS.

**3)** The NTpbMS differs from the NTbMS. Because the triangle inequality in the NTbMS differs from the triangle inequality in the NTpbMS. Also, in a NTpbMS, it can be that $d_{pb}$(x, x) ≠ 0.

**Theorem 3.4:** Let $(((X,Y),*),\ d_{pb})$ be a NTpbMS. If the following conditions are satisfied, then $((X,*),\ d_{pb})$ is a NTpMS.

a) $Y = X$.

b) $y' = x'$, by the triangle equality in Definition 3.1.

**Proof:**

i) $(((X,Y),*), d_{pb})$ is a NTpbMS implies that for all $a, b \in X$, $a * b \in X$ and for all $c, d \in Y$, $c * d \in Y$. Also, from condition a) it is clear that for all $a, c \in X$ = Y, $a * c \in X$ = Y.

ii) Since $(((X,Y),\ *),d_{pb})$ is a NTpbMS, for all $a \in X$ and for all $b \in Y$,

$d_{pb}(a,b) \geq d_{pb}(a,a) \geq 0$ and

$d_{pb}(a,b) \geq d_{pb}(b,b) \geq 0$ and is obvious by condition (a)

For all , $b \in X$ , $d_{pb}(a,b) \geq d_{pb}(a,a) \geq 0$.

iii) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, for $d_{pb}(a,b) = d_{pb}(a,a) = d_{pb}(b,b) = 0$, if $d_{pb}(a,b) = 0$; then there exists at least one pair of $a,b \in X \cap Y$, and also from condition a), if $y = x$, then $d_{pb}(a,b) = d_{pb}(a,a) = d_{pb}(b,b) = 0$ there exists at least one pair of $a,b \in X \cap X = X$ such that $d_{pb}(a,b) = 0$.

iv) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, we have for all $a,b \in X \cap Y$, $d_{pb}(a,b) = d_{pb}(b,a)$. Also, from condition a), we can write $X \cap Y$ = X. Thus, for all $x,y \in X$, $d_{pb}(a,b) = d_{pb}(b,a)$.

v) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, for each $(a,b)$, if there exists at least a $(a',b')$ such that

$d_{pb}(a,b) \leq d_{pb}\big(a,b * neut(b')\big) \leq d_{pb}(a * neut(a'), b * neut(b'))$ and

$d_{pb}(a,b) \leq d_{pb}(a * neut(a'), b) \leq d_{pb}(a * neut(a'), b * neut(b'))$,

then

$d_{pb}(a * neut(a'), b * neut(b')) \leq d_{pb}(a,b') + d_{pb}(a',b') + d_{pb}(a',b) - \min\{d_{pb}(a',a'),\ d_{pb}(b',b')\}$.

From condition b), we can write that

$$d_{pb}(a,b) \leq d_{pb}\big(a, b * neut(b')\big) \leq$$

$d_{pb}\big(a * neut(a'), b * neut(b')\big) \leq d_{pb}(a,b') + d_p(a',a') + d_{pb}(a',b) - \min\{d_{pb}(a',a'), d_{pb}(b',b')\} = d_{pb}(a,b') + d_{pb}(a',b) - \min\{d_{pb}(a',a'), d_{pb}(b',b')\}$ .

Also, from condition a), if there exists at least a $b' \in$ Y = X for each $a$, b ∈ Y = X such that

$d_{pb}(a$, b) ≤ $d_{pb}(a$, b*neut($b'$)), then

$d_{pb}(a$, b*neut($b'$)) ≤ $d_{pb}(a, a')$ + $d_{pb}(a'$, b)- min $\{d_{pb}(a',a'),\ d_{pb}(a',a')\}$ = $d_{pb}(a, a')$ + $d_{pb}(a'$, b) .

Thus, $((X,*),d_{pb})$ is a NTpMS.

**Theorem 3.5:** Let $(((X,\ Y),\ *),\ d_{pb})$ be a NTpbMS. If (X∩Y, ∗) is a NTS, then $(((X \cap Y,\ X \cap Y),\ *),d_{pb})$ is a NTpbMS.

**Proof**: We suppose that $(X \cap Y, *)$ is a NTS.

i) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, for all $a,b \in X$, $a * b \in X$ and for all $c,d \in Y$, $c * d \in Y$. Thus, it is clear that $\forall a,c \in X \cap Y,\ a * c \in X \cap Y$.

ii) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, for all $a \in X$ and for all $c \in Y$, if $d_{pb}(a,c) \geq d_{pb}(a,a) \geq 0$, then $d_{pb}(a,c) \geq d_{pb}(c,c) \geq 0$.

Thus, it is clear that for all $a \in X \cap Y,\ c \in X \cap Y$;

$d_{pb}(a,c) \geq d_{pb}(a,a) \geq 0$ and $d_{pb}(a,c) \geq d_{pb}(c,c) \geq 0$.

iii) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, if $d_{pb}(a,b) = d_{pb}(a,a) = d_{pb}(b,b) = 0$, then there exists at least one pair of elements $a,b \in X \cap Y$ such that $d_{pb}(a,b) = 0$. Thus, it is clear that for $d_{pb}(a,b) = d_{pb}(a,a) = d_{pb}(b,b) = 0$, there exists at least one pair of $a,b \in (X \cap Y) \times (X \cap Y)$.

iv) Since $(((X,Y),*),d_{pb})$ is a NTpbMS, we have for all $a,b \in X \cap Y$, $d_p b(a,b) = d_{pb}(b,a)$. Thus, it is clear that for all $a,b \in (X \cap Y) \cap (X \cap Y) = X \cap Y$, $d_{pb}(a,b) = d_{pb}(b,a)$.

v) Now, let $(a,b),(a',b') \in X \times Y$. For each $(a,b)$, if there exists at least one $(a',b')$ such that

$d_{pb}(a,b) \le d_{pb}(a, b * neut(b')) \le d_{pb}(a * neut(a'), b * neut(b'))$ and

$d_{pb}(a,b) \le d_{pb}(a * neut(a'), b) \le d_{pb}(a * neut(a'), b * neut(b'))$,

then

$d_{pb}(a * neut(a'), b * neut(b')) \le d_{pb}(a,b') + d_{pb}(a',b') + d_{pb}(a',b) - \min\{d_{pb}(a',a'), d_{pb}(b',b')\}$.

Thus, it is clear that for each $(a,b) \in (X \cap Y) \times (X \cap Y)$, if there exists at least one $(a',b') \in (X \cap Y) \times (X \cap Y)$ such that

$d_{pb}(a,b) \le d_{pb}(a, b * neut(b'))$,

$d_{pb}(a,b) \le d_{pb}(a * neut(a'), b)$ and

$d_{pb}(a,b) \le d_{pb}(a * neut(a'), b * neut(b'))$, then

$d_{pb}(a * neut(a'), b * neut(b')) \le d_{pb}(a,b') + d_{pb}(a',b') + d_{pb}(a',b) - \min\{d_{pb}(a',a'), d_{pb}(b',b')\}$.

Thus, $(((X \cap Y, X \cap Y), *), d_{pb})$ is an NTpbMS**.**

**Theorem 3.6:** Let $(((X,Y),*),d)$ a NTbMS. Then, for $k \in \mathbb{R}^+$, $d_{kp}(x,y) = d(x,y) + k$ is a NTpbM.

**Proof:**

i) For all $,b \in X$ , $a * b \in X$ and for all $c,d \in Y$ ,$c * d \in Y$ since d is a neutrosophic triplet bipolar metric.

ii) For all $x \in X$ and for all $y \in Y$,

$d_{kp}(x,y) \ge d(x,x) \ge 0$ and

$d_{kp}(x,y) \ge d(y,y) \ge 0$ .

Because;

$d_{kp}(x,y) = d(x,y) + k$,

$d_{kp}(x,x) = d(x,x) + k$,

$d_{kp}(y,y) \ge d(y,y) + k$,

$d(x,x) = 0$ and $d(y,y) = 0$ for all $x,y \in X \cap Y$.

iii) If $d_{kp}(x,y) = d_{kp}(x,x) = d_{kp}(y,y) \ne 0$ the proof is straightforward, since

$d_{kp}(x,x) = d(x,x) + k > 0$ $(d(x,x) = 0)$

$d_{kp}(y,y) = d(y,y) + k > 0$ $(d(y,y) = 0)$ .

iv) For all $x,y \in X \cap Y$ , $d_{kp}(x,y) = d_{kp}(y,x)$ .

This is because of the fact that $d_{kp}(x,y) = d(x,y) + k$ and $d(x,y) = d(y,x)$ for $\forall x,y \in X \cap Y$ .

v) Let $\forall (x,y),(x',y') \in X \times Y$ . For each $(x,y) \in X \times Y$, If there exists at least one $(x',y') \in X \times Y$ such that

$d_{kp}(x,y) \le d_{kp}(x, y * neut(y'))$,

$d_{kp}(x,y) \le d_{kp}(x * neut(x'), y)$,

$d_{kp}(x,y) \le d_{kp}(x * neut(x'), y * neut(y'))$,

then

$d_{kp}(x,y) \leq d_{kp}\big(x * neut(x'), y * neut(y')\big) \leq$

$d_{kp}(x,y') + d_{kp}(x',y') + d_{kp}(x',y) - \min\{d_{kp}(x',x'), d_{kp}(y',y')\}$ .

As $d_{kp}(x,y) = d(x,y) + k$ and $d(x * neut(x'), y * neut(y')) \leq d(x,y') + d(x',y') + d(x',y)$, we obtain that

$d_{kp}\big(x * neut(x'), y * neut(y')\big) \leq$

$d(x,y') + k + d(x',y') + k + d(x',y) + k - \min\{d_{kp}(x',x'), d_{kp}(y',y')\}$ =

$d(x,y') + d(x',y') + d(x',y) + 3k - \min\{d_{kp}(x',x'), d_{kp}(y',y')\}$ =

$d(x,y') + d(x',y') + d(x',y) + 3k - \min\{d(x',x') + k, d(y',y') + k\}$ =

$d(x,y') + d(x',y') + d(x',y) + 3k - \min\{k,k\}$ =

$d(x,y') + d(x',y') + d(x',y) + 3k - k$ =

$d(x,y') + d(x',y') + d(x',y) + 2k$ .

In this case,

$d_{kp}\big(x * neut(x'), y * neut(y')\big) \leq d_{kp}(x,y') + d_{kp}(x',y') + d_{kp}(x',y) - \min\{d_{kp}(x',x'), d_{kp}(y',y')\}$ .

**Corollary 3.7:** A NTpbMS can be obtained from a NTbMS.

**Definition 3.8:** Let $\big(((X,Y),*), d_{pb}\big)$ be a NTpbMS. A left sequence $(x_n)$ converges to a right point y (symbolically $(x_n) \rightharpoonup y$ or $\lim_{n\to\infty}(x_n) = y$) if and only if for every ε > 0 there exists an $n_0 \in \mathbb{N}$ , such that $d_{pb}(x_n, y) < \varepsilon$ -min $\{d_{pb}(x,x), d_{pb}(y,y)\}$for all $n \geq n_0$. Similarly, a right sequence $(y_n)$ converges to a left point x (denoted as $y_n \rightharpoonup x$ or $\lim_{n\to\infty}(y_n) = x$) if and only if, for every ε > 0 there exists an $n_0 \in \mathbb{N}$ such that, whenever $n \geq n_0$ , $d_{pb}(x, y_n) < \varepsilon$ - min $\{d_{pb}(x,x), d_{pb}(y,y)\}$. Also, if $(u_n) \rightharpoonup u$ and $(u_n) \rightharpoonup u$, then $(u_n)$ converges to point u ($(u_n)$ is a central sequence).

**Definition 3.9:** Let $(((X, Y), *), d_{pb})$ be a NTpbMS, $(x_n)$ be a left sequence and $(y_n)$ be a right sequence in this space. $(x_n, y_n)$ is called a NT partial bisequence. Furthermore, if $(x_n)$ and $(y_n)$ are convergent, then $(x_n, y_n)$ is called a NT partial convergent bisequence. Also, if $(x_n)$ and $(y_n)$ converge to same point, then $(x_n, y_n)$ is called a NT partial biconvergent bisequence.

**Definition 3.10:** Let $(((X, Y), *), d_{pb})$ be a NTpbMS and $(x_n, y_n)$ be a NT partial bisequence. $(x_n, y_n)$ is called an NT partial Cauchy bisequence if and only if for every ε > 0, there exists an $n_0 \in \mathbb{N}$ , such that $d_{pb}(x_n, y_m) < \varepsilon$ -min $\{d_{pb}(x,x), d_{pb}(y,y)\}$ for all $n, m \geq n_0$.

**Definition 3.11:** Let $\big(((X,Y),*), d_{pb}\big)$ be a NTpbMS. In this space, if each $(x_n, y_n)$ NT partial Cauchy bisequence is a NT partial convergent Cauchy bisequence, then $\big(((X,Y),*), d_{pb}\big)$ is called complete NT partial bipolar metric space.

## Conclusion

In this study we first obtained NTpbMS. We show that NTpbMS is different from NTpMS and NTMS. Also, we show that a NTpbMS will provide the properties of a NTbMS under which conditions are met. Thus, we added a new structure to neutrosophic triple structures. Also, thanks to this study, researchers can obtain new fixed point theories, neutrosophic triplet partial bipolar normed space, neutrosophic triplet partial bipolar inner product space.

**Abbreviations**

bM: bipolar metric

bMS: bipolar metric space

pMS: partial metric space

NT: Neutrosophic triplet

NTS: Neutrosophic triplet set

NTM: Neutrosophic triplet metric

NTMS: Neutrosophic triplet metric space

NTpM: Neutrosophic triplet partial metric

NTpMS: Neutrosophic triplet partial metric space

NTbM: Neutrosophic triplet bipolar metric

NTbMS: Neutrosophic triplet bipolar metric space

NTpbMS: Neutrosophic triplet partial bipolar metric space

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