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Pythagorean Neutrosophic Dombi Fuzzy Graphs with an Application to MCDM

P. Chellamani [1] and D. Ajay [2,*]

[1]Department of Mathematics, Sacred Heart College, Tamil Nadu, India; joshmani238@gmail.com

[2]Department of Mathematics, Sacred Heart College, Tamil Nadu, India; dajaypravin@gmail.com

*Correspondence: dajaypravin@gmail.com

Abstract. Pythagorean Neutrosophic fuzzy graph, an extension of Pythagorean and Neutrosophic graph, is more efficient in representing relationship between various objects where the relation between the objects is uncertain, while the Dombi operators with adaptable operational parameter is very useful by taking distinct values. The Pythagorean Neutrosophic Dombi fuzzy graphs (PNDFG) is a novel concept proposed in this research paper by integrating the concepts Pythagorean Neutrosophic fuzzy graph and Dombi operator. Various basic graphical ideas using Dombi operator have been introduced for Pythagorean Neutrosophic fuzzy graphs. The main important part is the MCDM model which is proposed for the developed PNDFG and demonstrated with an illustrative example for choosing the best alternative.

Keywords: Pythagorean Neutrosophic sets; Pythagorean Neutrosophic Dombi fuzzy graphs; Dombi; Pythagorean fuzzy graphs.

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1. **Introduction**

In real-life situations, fuzzy set theory [1] plays an important role in resolving incomplete and ambiguous information. A fuzzy set is a variant of a regular set in which elements have a membership degree between 0 and 1. Fuzzy set and its conceptual development have wide-ranging applications in fields like engineering, computer science, mathematics, artificial intelligence,decision making and image analysis. The passage presented below gives some of the recent advancements in fuzzy set theory.

Atanassov [2] extended the fuzzy set to intuitionistic fuzzy set, which gives each element a membership and non-membership degree. An intuitionistic fuzzy set is one that meets the requirement that the sum of both membership and non-membership values is between 0 and 1. Smarandache's Neutrosophic set [3] is a generalization of the theory of fuzzy and intuitionistic fuzzy sets [1, 4] which deals with imprecise information. Elements with truth, indeterminacy, and false membership degrees that lie within the interval [0,1] characterize the single valued neutrosophic set which was introduced by Wang et al [5].

Yager [6-8] introduced Pythagorean fuzzy sets as an extension of intuitionistic fuzzy sets to deal with complex imprecision and uncertainty when the sum of squares of membership and non-membership degrees is between 0 and 1. Hence, Pythagorean fuzzy set accounts for larger amount of uncertainty than intuitionistic fuzzy set. The degree of dependence among components of fuzzy sets and neutrosophic sets was introduced by Smarandache and was developed further. Out of three membership functions of neutrosophic sets, one special case with independent indeterminacy and dependent truth and falsity are chosen with the constraint that the total of squares of membership, indeterminacy and non-membership lies between 0 and 2 and it is termed as Pythagorean Neutrosophic set [23].

Graphs are pictorial representations of objects and their relationships. More uncertainties occur in relations among objects which results in the need for framing fuzzy graph model rather than ordinary graph, which has the same structure. Using Zadeh's fuzzy relation, Kaufmann [9] introduced the concept of fuzzy graphs. Various fundamental and theoretical ideas like bridges, cycles and connectedness were defined and developed by Rosenfeld [10]. Karunambigai and Parvathi [11] instituted the intuitionistic fuzzy graphs which was further extended to intuitionistic fuzzy hypergraph and its applications have been explored [12]. Broumi et al. presented single-valued neutrosophic graphs [13] with examples and properties, and the properties of degree and regular single valued neutrosophic graphs were also examined [14]. The concept of fuzzy graph was advanced to pythagorean fuzzy graphs in [15]. The new emerging concept of Pythagorean neutrosophic fuzzy graph [16] were advanced by blending the concept of Pythagorean Neutrosophic sets and fuzzy graphs. Ashraf et al. [17] proposed the idea of Dombi fuzzy graphs. Subsequently, many researchers worked on this Dombi fuzzy graphs and made advancements like interval valued Dombi fuzzy neutrosophic graph [18], decision making using Dombi fuzzy graphs [19], Pythagorean Dombi fuzzy graphs [20], picture Dombi fuzzy graph [21] and Dombi bipolar fuzzy graph [22]. Application of the fuzzy theory in decision making is an effective way for solving real-life problems, and it is advanced using all the new concept developments most recently in [24-30]. This paper presents Pythagorean neutrosophic Dombi graphs as a generalization of Pythagorean and Neutrosophic Dombi fuzzy graphs (PNDFG).

The following is the layout of the research paper: In section 2, the paper's basic terminologies are explained. In section 3, we define the complement, homomorphism, isomorphism, strength, and completeness of PNDFG. Section 4 proposes an algorithm for Multi-criteria decision making based on the PNDFG, which is demonstrated with an example and concluded in section 5.

2. Preliminaries

Definition 2.1.[1] On a universe $\mathfrak{U}$, $\mathfrak{A} = \{\langle \mathfrak{s}, \mu_{\mathfrak{A}}(\mathfrak{s})\rangle \,|\mathfrak{s} \in \mathfrak{U}\}$ is a fuzzy set (FS) where $\mu_{\mathfrak{A}} : \mathfrak{U} \to [0,1]$ symbolizes the membership grade of $\mathfrak{A}$.

Definition 2.2.[1] A fuzzy relation on a fuzzy set $\mathfrak{X}$ is $\mathfrak{X} \times \mathfrak{X}$, represented by $\mathfrak{B} = \{\langle \mathfrak{st}, \mu_{\mathfrak{B}}(\mathfrak{st})\rangle \,|\mathfrak{st} \in \mathfrak{X} \times \mathfrak{X}\}$, where $\mu_{\mathfrak{B}} : \mathfrak{X} \times \mathfrak{X} \to [0,1]$ is the membership grades of $\mathfrak{A}$.

Definition 2.3.[9] A fuzzy graph is a duo $G = (\mathfrak{A}, \mathfrak{B})$ on $\mathfrak{X}$ with A a FS on $\mathfrak{X}$ and $\mathfrak{B}$ a FR on $\mathfrak{X}$ such that $\mu_{\mathfrak{B}}(\mathfrak{st}) \leq \mu_{\mathfrak{A}}(\mathfrak{s}) \wedge \mu_{\mathfrak{A}}(\mathfrak{t})\ \forall\ s,t \in \mathfrak{X}$, where $\mathfrak{A} : \mathfrak{X} \to [0,1]$ and $\mathfrak{B}$ from $\mathfrak{X} \times \mathfrak{X}$ to $[0,1]$.

Definition 2.4.[6] A Pythagorean fuzzy set (PFS) on a universe $\mathfrak{X}$ is $\mathfrak{A} = \{\langle s, \mu_{\mathfrak{A}}(\mathfrak{s}), \vartheta_{\mathfrak{A}}(\mathfrak{s})\rangle \,|\mathfrak{s} \in \mathfrak{X}\}$, where $\mu_{\mathfrak{A}} : \mathfrak{X} \to [0,1]$ and $\vartheta_{\mathfrak{A}} : \mathfrak{X} \to [0,1]$ signify the membership and non-membership grades of $\mathfrak{A}$, and $\mu_{\mathfrak{A}}, \vartheta_{\mathfrak{A}}$ satisfying $0 \leq \mu^2_{\mathfrak{A}}(s) + \vartheta^2_{\mathfrak{A}}(\mathfrak{s}) \leq 1\ \forall\ \mathfrak{s} \in \mathfrak{X}$.

Definition 2.5.[6] A Pythagorean fuzzy set on $\mathfrak{X} \times \mathfrak{X}$ is called a Pythagorean fuzzy relation (PFR) on $\mathfrak{X}$, represented by $\mathfrak{B} = \{\langle \mathfrak{st}, \mu_{\mathfrak{B}}(\mathfrak{st}), \vartheta_{\mathfrak{B}}(\mathfrak{st})\rangle \,|\mathfrak{s} \in \mathfrak{X}\}$, where $\mu_{\mathfrak{B}} : \mathfrak{X} \times \mathfrak{X} \to [0,1]$ and $\vartheta_{\mathfrak{A}} : \mathfrak{X} \times \mathfrak{X} \to [0,1]$ signify the membership and non-membership grades of $\mathfrak{B}$, correspondingly, such that $0 \leq \mu^2_{\mathfrak{B}}(\mathfrak{st}) + \vartheta^2_{\mathfrak{B}}(\mathfrak{st}) \leq 1\ \forall\ \mathfrak{s}, \mathfrak{t} \in \mathfrak{X}$.

Definition 2.6.[15] A Pythagorean fuzzy graph (PFG) on a non-empty set $\mathfrak{X}$ is a pair $G = (\mathfrak{A}, \mathfrak{B})$ with $\mathfrak{A}$ a PFS on $\mathfrak{X}$ and $\mathfrak{B}$ a PFR on $\mathfrak{X}$ such that $\mu_{\mathfrak{B}}(\mathfrak{st}) \leq \mu_{\mathfrak{A}}(\mathfrak{s}) \wedge \mu_{\mathfrak{A}}(\mathfrak{t})$, $\vartheta_{\mathfrak{B}}(\mathfrak{st}) \geq \vartheta_{\mathfrak{A}}(\mathfrak{s}) \vee \vartheta_{\mathfrak{A}}(\mathfrak{t})$ and $0 \leq \mu^2_{\mathfrak{B}}(\mathfrak{st}) + \vartheta^2_{\mathfrak{B}}(\mathfrak{st}) \leq 1\ \forall\ \mathfrak{s}, \mathfrak{t} \in \mathfrak{X}$. where $\mu_{\mathfrak{B}} : \mathfrak{X} \times \mathfrak{X} \to [0,1]$ and $\vartheta_{\mathfrak{B}} : \mathfrak{X} \times \mathfrak{X} \to [0,1]$ symbolize the membership and non-membership grades of $\mathfrak{B}$, correspondingly.

Definition 2.7.[19] A binary function $\mathfrak{T} : [0,1] \times [0,1] \to [0,1]$ is named as t-norm if $\forall$ $\mathfrak{a}, \mathfrak{b}, \mathfrak{u} \in [0,1]$, it fulfills the following criteria:

1. $\mathfrak{T}(\mathfrak{a}, 1) = \mathfrak{a}$,
2. $\mathfrak{T}(\mathfrak{a}, \mathfrak{b}) = \mathfrak{T}(\mathfrak{b}, \mathfrak{a})$,

3. $\mathfrak{T}(\mathfrak{a}, \mathfrak{T}(\mathfrak{b}, u)) = \mathfrak{T}(\mathfrak{T}(\mathfrak{a}, \mathfrak{b}), \mathfrak{u})$,
4. $\mathfrak{T}(\mathfrak{a}, \mathfrak{b}) \leq \mathfrak{T}(\mathfrak{u}, \mathfrak{v})$ if $\mathfrak{a} \leq \mathfrak{u}$ and $\mathfrak{b} \leq \mathfrak{v}$.

The Dombi's t-norm and t-conorm are given by $\frac{1}{1+[(\frac{1-\mathfrak{a}}{\mathfrak{a}})^{\gamma}+(\frac{1-\mathfrak{b}}{\mathfrak{b}})^{\gamma}]^{\frac{1}{\gamma}}}, \gamma > 0.$ and $\frac{1}{1+[(\frac{1-\mathfrak{a}}{\mathfrak{a}})^{-\gamma}+(\frac{1-\mathfrak{b}}{\mathfrak{b}})^{-\gamma}]^{\frac{1}{-\gamma}}}, \gamma > 0$ respectively.
By putting $\gamma = 1$ in Dombi's t-norm and t-conorm one gets the other set of T-operators $\mathfrak{T}(\mathfrak{a}, \mathfrak{b}) = \frac{\mathfrak{ab}}{\mathfrak{a}+\mathfrak{b}-\mathfrak{ab}}$ and $P(\mathfrak{a}, t) = \frac{\mathfrak{a}+\mathfrak{b}-2\mathfrak{ab}}{1-\mathfrak{ab}}$,.

Definition 2.8.[16] Pythagorean Neutrosophic Fuzzy Graph (PNFG) is $G = (V, E)$, where $V = \{v_1, v_2, ..., v_n\}$ such that μ_1, β_1, σ_1 are from V to $[0, 1]$ with $0 \leq \mu_1(v_i)^2+\beta_1(v_i)^2+\sigma_1(v_i)^2 \leq 2 \ \forall v_i \in V$ indicates the membership, indeterminacy and non-membership functions and μ_2, β_2, σ_2 are from $V \times V$ to $[0, 1]$ such that $\mu_2(v_iv_j) \leq \mu_1(v_i) \wedge \mu_1(v_j)$,$\beta_2(v_iv_j) \leq \beta_1(v_i) \wedge \beta_1(v_j)$ and $\sigma_2(v_iv_j) \leq \sigma_1(v_i) \vee \sigma_1(v_j)$ with $0 \leq \mu_2(v_iv_j)^2 + \beta_2(v_iv_j)^2 + \sigma_2(v_iv_j)^2 \leq 2 \ \forall v_iv_j \in V \times V$.

Definition 2.9.[17] A Dombi fuzzy graph on V is a pair that has been ordered as $G = (\mathfrak{A}, \mathfrak{B})$, where $\mathfrak{A} : V \to [0, 1]$ is contained in V and $\mathfrak{B} : V \times V \to [0, 1]$ is a symmetric fuzzy relation on $\mathfrak{A}$ such that $\mu_{\mathfrak{B}}(st) \leq \frac{\mu_{\mathfrak{A}}(s)\mu_{\mathfrak{A}}(t)}{\mu_{\mathfrak{A}}(s)+\mu_{\mathfrak{A}}(t)-\mu_{\mathfrak{A}}(s)\mu_A(t)} \ \forall \ s, t \in V$, where $\mu_{\mathfrak{A}}$ and $\mu_{\mathfrak{B}}$ symbolize the membership grades of $\mathfrak{A}$ and $\mathfrak{B}$, correspondingly.

3. Pythagorean Neutrosophic Dombi Fuzzy Graphs

Definition 3.1. A Pythagorean Neutrosophic Dombi Fuzzy Graph (PNDFG) with finite underlying set $\mathfrak{V}$ is a pair $G = (\eta, \zeta)$, where $\eta = (\mu_1, \beta_1, \sigma_1)$ from $\mathfrak{V}$ to $[0, 1]$ is a Pythagorean neutrosophic subset in $\mathfrak{V}$ and $\zeta = (\mu_2, \beta_2, \sigma_2)$ from $\mathfrak{V} \times \mathfrak{V}$ to $[0, 1]$ is a symmetric Pythagorean Neutrosophic fuzzy relation on η such that

$$\mu_2(\mathfrak{gh}) \leq \frac{\mu_1(\mathfrak{g})\mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g}) \cdot \mu_1(\mathfrak{gh})}$$

$$\beta_2(\mathfrak{gh}) \leq \frac{\beta_1(\mathfrak{g})\beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g}) \cdot \beta_1(\mathfrak{gh})}$$

$$\sigma_2(\mathfrak{gh}) \leq \frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}$$

and $0 \leq \mu_2^2(\mathfrak{gh}) + \beta_2^2(\mathfrak{gh}) + \sigma_2^2(\mathfrak{gh}) \leq 2$ for all $\mathfrak{g}, \mathfrak{h} \in \mathfrak{V}$.
η and ζ denote the Pythagorean Neutrosophic Dombi fuzzy vertex and edge sets of G.

Example 1. Consider a PNDFG over $\mathfrak{V} = \{a, b, c, d, e, f\}$ defined by
$\langle(a, .5, .6, .5), (b, .8, .4, .2), (c, .4, .6, .2), (d, .6, .5, .7), (e, .7, .4, .3), (f, .3, .6, .7)\rangle$
$\langle(ab, .44, .316, .556), (af, .231, .429, .769), (bc, .432, .316, .33), (cd, .316, .376, .721), (de, .477, .286, .734), (ef, .266, .316, .734), (ad, .375, .375, .769), (be, .596, .25, .404), (cf, .207, .429, .721)\rangle$

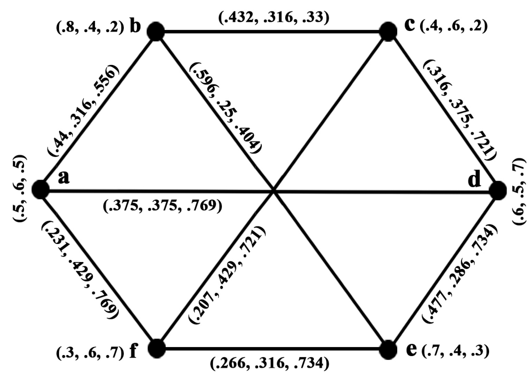

FIGURE 1. Pythagorean Neutrosophic Dombi Fuzzy graph

Definition 3.2. Let $\zeta = \{(\mathfrak{gh}, \mu_2(\mathfrak{gh}), \beta_2(\mathfrak{gh}), \sigma_2(\mathfrak{gh})) / \mathfrak{gh} \in \mathfrak{E}\}$ be a PN Dombi fuzzy edge set in PNDFG G; then

1. The order of G is represented by

$$O(G) = \left(\sum_{\mathfrak{g}\in\mathfrak{V}} \mu_1(\mathfrak{g}), \sum_{\mathfrak{g}\in\mathfrak{V}} \beta_1(\mathfrak{g}), \sum_{\mathfrak{g}\in\mathfrak{V}} \sigma_1(\mathfrak{g}) \right)$$

2. The size of G is symbolized as $S(G)$ and is defined by

$$S(G) = \left(\sum_{\mathfrak{gh}\in\mathfrak{E}} \mu_2(\mathfrak{gh}), \sum_{\mathfrak{gh}\in\mathfrak{E}} \beta_2(\mathfrak{gh}), \sum_{\mathfrak{gh}\in\mathfrak{E}} \sigma_2(\mathfrak{gh}) \right)$$

Example 2. For the PNDFG in Example 1, the order of G as (3.3, 3.1, 2.6) and size of the PNDFG as (3.34, 3.092, 5.738).

Definition 3.3. Let $\zeta = \{(\mathfrak{gh}, \mu_2(\mathfrak{gh}), \beta_2(\mathfrak{gh}), \sigma_2(\mathfrak{gh})) / \mathfrak{gh} \in \mathfrak{E}\}$ be a PN Dombi fuzzy edge set in PNDFG G; then the degree of vertex $\mathfrak{g} \in \mathfrak{V}$ is symbolized by $(D)_G(\mathfrak{g})$ and defined as $(D)_G(\mathfrak{g}) = ((D)_\mu(\mathfrak{g}), (D)_\beta(\mathfrak{g}), (D)_\sigma(\mathfrak{g}))$, where

$$(D)_\mu(\mathfrak{g}) = \sum_{\mathfrak{g},h\neq\mathfrak{g}\in\mathfrak{V}} \mu_2(gh) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}$$
$$(D)_\beta(\mathfrak{g}) = \sum_{\mathfrak{g},h\neq\mathfrak{g}\in\mathfrak{V}} \beta_2(\mathfrak{gh}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}$$

$$(D)_{\sigma}(\mathfrak{g}) = \sum_{\mathfrak{g},h\neq\mathfrak{g}\in\mathfrak{V}} \sigma_2(\mathfrak{g}\mathfrak{h}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}$$

The total degree of vertex $\mathfrak{g} \in \mathfrak{V}$ is symbolized by $(TD)_G(\mathfrak{g})$ and defined as $(TD)_G(\mathfrak{g}) = ((TD)_{\mu}(\mathfrak{g}), (TD)_{\beta}(\mathfrak{g}), (TD)_{\sigma}(\mathfrak{g}))$, where

$$(TD)_{\mu}(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \mu_2(\mathfrak{g}\mathfrak{h}) + \mu_1(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})} + \mu_1(\mathfrak{g}),$$
$$(TD)_{\beta}(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \beta_2(\mathfrak{g}\mathfrak{h}) + \beta_1(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})} + \beta_1(\mathfrak{g}),$$
$$(TD)_{\sigma}(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \sigma_2(\mathfrak{g}\mathfrak{h}) + \sigma_1(\mathfrak{g}) = \sum_{\mathfrak{g},\mathfrak{h}\neq\mathfrak{g}\in\mathfrak{V}} \frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})} + \sigma_1(\mathfrak{g}).$$

Example 3. For the PNDFG in Figure 1, the degree and the total degree of the vertices are
$D_G(a) = (1.046, 1.12, 2.094)$, $TD_G(a) = (1.546, 1.72, 2.594)$
$D_G(b) = (1.468, 0.882, 1.29)$, $TD_G(b) = (2.268, 1.282, 1.49)$
$D_G(c) = (0.995, 1.12, 1.772)$, $TD_G(c) = (1.355, 1.72, 1.972)$
$D_G(d) = (1.168, 1.036, 2.224)$, $TD_G(d) = (1.768, 1.536, 2.924)$
$D_G(e) = (1.339, 0.852, 1.872)$, $TD_G(e) = (2.039, 1.252, 2.172)$
$D_G(f) = (0.704, 1.174, 2.224)$, $TD_G(f) = (1.004, 1.774, 2.924)$

Definition 3.4. The complement of a PNDFG $G = (\eta, \zeta)$ is a PNDFG $\bar{G} = (\bar{\eta}, \bar{\zeta})$ which is defined by

1. $\overline{\mu_1(\mathfrak{g})} = \mu_1(\mathfrak{g})$, $\overline{\beta_1(\mathfrak{g})} = \beta_1(\mathfrak{g})$ and $\overline{\sigma_1(\mathfrak{g})} = \sigma_1(\mathfrak{g})$.
2. $\overline{\mu_2(\mathfrak{g}\mathfrak{h})} = \begin{cases} \frac{\mu_1(\mathfrak{g})\mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g})+\mu_1(\mathfrak{h})-\mu_1(\mathfrak{g})\mu_1(\mathfrak{h})} & if\ \mu_2(\mathfrak{g}\mathfrak{h}) = 0, \\ \frac{\mu_1(\mathfrak{g})\mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g})+\mu_1(\mathfrak{h})-\mu_1(\mathfrak{g})\mu_1(\mathfrak{h})} - \mu_2(\mathfrak{g}\mathfrak{h}) & if\ 0 < \mu_2(\mathfrak{g}\mathfrak{h}) \leq 1 \end{cases}$
3. $\overline{\beta_2(\mathfrak{g}\mathfrak{h})} = \begin{cases} \frac{\beta_1(\mathfrak{g})\beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g})+\beta_1(\mathfrak{h})-\beta_1(\mathfrak{g})\beta_1(\mathfrak{h})} & if\ \beta_2(\mathfrak{g}\mathfrak{h}) = 0, \\ \frac{\beta_1(\mathfrak{g})\beta_1(h)}{\beta_1(\mathfrak{g})+\beta_1(\mathfrak{h})-\beta_1(\mathfrak{g})\beta_1(\mathfrak{h})} - \beta_2(\mathfrak{g}\mathfrak{h}) & if\ 0 < \beta_2(\mathfrak{g}\mathfrak{h}) \leq 1 \end{cases}$
4. $\overline{\sigma_2(\mathfrak{g}\mathfrak{h})} = \begin{cases} \frac{\sigma_1(\mathfrak{g})+\sigma_1(\mathfrak{h})-2\sigma_1(\mathfrak{g})\sigma_1(\mathfrak{h})}{1-\sigma_1(\mathfrak{g})\sigma_1(\mathfrak{h})} & if\ \sigma_2(\mathfrak{g}\mathfrak{h}) = 0, \\ \frac{\sigma_1(\mathfrak{g})+\sigma_1(\mathfrak{h})-2\sigma_1(\mathfrak{g})\sigma_1(\mathfrak{h})}{1-\sigma_1(\mathfrak{g})\sigma_1(\mathfrak{h})} - \sigma_2(\mathfrak{g}\mathfrak{h}) & if\ 0 < \sigma_2(\mathfrak{g}\mathfrak{h}) \leq 1 \end{cases}$

Theorem 1. If $G = (\eta, \zeta)$ is a PNDFG, then $\overline{\overline{G}} = G$.

Proof: Consider G as a PNDFG. By definition of complement of PNDFG, we have
$\overline{\overline{\mu_1(\mathfrak{g})}} = \overline{\mu_1(\mathfrak{g})} = \mu_1(\mathfrak{g})$, $\overline{\overline{\beta_1(\mathfrak{g})}} = \overline{\beta_1(\mathfrak{g})} = \beta_1(\mathfrak{g})$, $\overline{\overline{\sigma_1(\mathfrak{g})}} = \overline{\sigma_1(\mathfrak{g})} = \sigma_1(\mathfrak{g})$, for all $\mathfrak{g} \in \mathfrak{V}$.
If $\mu_2(\mathfrak{g}\mathfrak{h}) = 0$, $\beta_2(\mathfrak{g}\mathfrak{h}) = 0$, $\sigma_2(\mathfrak{g}\mathfrak{h}) = 0$, then

$$\overline{\overline{\mu_2(\mathfrak{g}\mathfrak{h})}} = \frac{\overline{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}}{\overline{\mu_1(\mathfrak{g})} + \overline{\mu_1(\mathfrak{h})} - \overline{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}} = \frac{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})} = \mu_2(\mathfrak{g}\mathfrak{h}),$$

$$\overline{\overline{\beta_2(\mathfrak{gh})}} = \frac{\overline{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}}{\overline{\beta_1(\mathfrak{g})} + \overline{\beta_1(\mathfrak{h})} - \overline{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}} = \frac{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})} = \beta_2(\mathfrak{gh}),$$

$$\overline{\overline{\sigma_2(\mathfrak{gh})}} = \frac{\overline{\sigma_1(\mathfrak{g})} + \overline{\sigma_1(\mathfrak{h})} - 2\ \overline{\sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}}{1 - \overline{\sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}} = \frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})} = \sigma_2(\mathfrak{gh}).$$

If $0 < \mu_2(\mathfrak{gh}), \beta_2(\mathfrak{gh}), \sigma_2(\mathfrak{gh}) \leq 1$, then

$$\overline{\overline{\mu_2(\mathfrak{gh})}} = \frac{\overline{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}}{\overline{\mu_1(\mathfrak{g})} + \overline{\mu_1(\mathfrak{h})} - \overline{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}} - \overline{\mu_2(\mathfrak{gh})}$$

$$= \frac{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})} - [\frac{\mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})}{\mu_1(\mathfrak{g}) + \mu_1(\mathfrak{h}) - \mu_1(\mathfrak{g})\ \mu_1(\mathfrak{h})} - \mu_2(\mathfrak{gh})] = \mu_2(\mathfrak{gh}),$$

$$\overline{\overline{\beta_2(\mathfrak{gh})}} = \frac{\overline{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}}{\overline{\beta_1(\mathfrak{g})} + \overline{\beta_1(\mathfrak{h})} - \overline{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}} - \overline{\beta_2(\mathfrak{gh})}$$

$$= \frac{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})} - [\frac{\beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})}{\beta_1(\mathfrak{g}) + \beta_1(\mathfrak{h}) - \beta_1(\mathfrak{g})\ \beta_1(\mathfrak{h})} - \beta_2(\mathfrak{gh})] = \beta_2(\mathfrak{gh}),$$

$$\overline{\overline{\sigma_2(\mathfrak{gh})}} = \frac{\overline{\sigma_1(\mathfrak{g})} + \overline{\sigma_1(\mathfrak{h})} - 2\ \overline{\sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}}{1 - \overline{\sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}} - \sigma_2(\mathfrak{gh})$$

$$= \frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})} - [\frac{\sigma_1(\mathfrak{g}) + \sigma_1(\mathfrak{h}) - 2\ \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})}{1 - \sigma_1(\mathfrak{g})\ \sigma_1(\mathfrak{h})} - \sigma_2(\mathfrak{gh})] = \sigma_2(\mathfrak{gh})\ \forall \mathfrak{g}, \mathfrak{h} \in \mathfrak{V}.$$

Hence, the complement of a complement PNDFG is a PNDFG itself.

Definition 3.5. A homomorphism $H : G_1 \to G_2$ of two PNDFGs $G_1 = (\eta_1, \zeta_1)$ and $G_2 = (\eta_2, \zeta_2)$ is a mapping $H : \mathfrak{V}_1 \to \mathfrak{V}_2$ satisfying

(1) $\mu_{\eta_1}(\mathfrak{g}) \leq \mu_{\eta_2}(H(\mathfrak{g}))$,

$\beta_{\eta_1}(\mathfrak{g}) \leq \beta_{\eta_2}(H(\mathfrak{g}))$,

$\sigma_{\eta_1}(\mathfrak{g}) \leq \sigma_{\eta_2}(H(\mathfrak{g}))$.

(2) $\mu_{\zeta_1}(\mathfrak{g}\mathfrak{t}) \leq \mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$,

$\beta_{\zeta_1}(\mathfrak{g}\mathfrak{t}) \leq \beta_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$,

$\sigma_{\zeta_1}(\mathfrak{g}\mathfrak{t}) \leq \sigma_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t})) \ \forall \ \mathfrak{g} \in \mathfrak{V}_1, \mathfrak{g}\mathfrak{t} \in \mathfrak{E}_1$.

Definition 3.6. An isomorphism $H : G_1 \to G_2$ of two PNDFGs $G_1 = (\eta_1, \zeta_1)$ and $G_2 = (\eta_2, \zeta_2)$ is a bijective mapping $H : \mathfrak{V}_1 \to \mathfrak{V}_2$ satisfying

(1) $\mu_{\eta_1}(\mathfrak{g}) = \mu_{\eta_2}(H(\mathfrak{g}))$,

$\beta_{\eta_1}(\mathfrak{g}) = \beta_{\eta_2}(H(\mathfrak{g}))$,

$\sigma_{\eta_1}(\mathfrak{g}) = \sigma_{\eta_2}(H(\mathfrak{g}))$.

(2) $\mu_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$,

$\beta_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \beta_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$,

$\sigma_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \sigma_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t})) \ \forall \ \mathfrak{g} \in \mathfrak{V}_1, \mathfrak{g}\mathfrak{t} \in \mathfrak{E}_1$.

Definition 3.7. A weak isomorphism $H : G_1 \to G_2$ of two PNDFGs $G_1 = (\eta_1, \zeta_1)$ and $G_2 = (\eta_2, \zeta_2)$ is a bijective mapping $H : \mathfrak{V}_1 \to \mathfrak{V}_2$ satisfying

(1) H is a homomorphism.

(2) $\mu_{\eta_1}(\mathfrak{g}) = \mu_{\eta_2}(H(\mathfrak{g}))$,
$\beta_{\eta_1}(\mathfrak{g}) = \beta_{\eta_2}(H(\mathfrak{g}))$,
$\sigma_{\eta_1}(\mathfrak{g}) = \sigma_{\eta_2}(H(\mathfrak{g})) \ \mathfrak{g} \in \mathfrak{V}_1$.

Definition 3.8. A co-weak isomorphism $H : G_1 \to G_2$ of two PNDFGs $G_1 = (\eta_1, \zeta_1)$ and $G_2 = (\eta_2, \zeta_2)$ is a bijective mapping $H : \mathfrak{V}_1 \to \mathfrak{V}_2$ satisfying

(1) H is a homomorphism.

(2) $\mu_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t})),$

$\beta_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \beta_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t})),$

$\sigma_{\zeta_1}(\mathfrak{g}\mathfrak{t}) = \sigma_{\zeta_2}(H(\mathfrak{g})H(t)) \ \forall \ \mathfrak{g} \in \mathfrak{V}_1, \mathfrak{g}\mathfrak{t} \in \mathfrak{E}_1.$

Definition 3.9. A PNDFG $G = (\eta, \zeta)$ is called self-complement if $\overline{G} \cong G$.

Proposition 1. If $G = (\eta, \zeta)$ is a self-complementary PNDFG, then

$$\sum_{\mathfrak{g}\neq t} \mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2} \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})},$$

$$\sum_{\mathfrak{g}\neq \mathfrak{t}} \beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2} \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\beta_\eta(\mathfrak{g}) \ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g}) \ \beta_\eta(\mathfrak{t})},$$

$$\sum_{\mathfrak{g}\neq \mathfrak{t}} \sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2} \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(\mathfrak{t}) - 2 \ \sigma_\eta(\mathfrak{g}) \ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g}) \ \sigma_\eta(\mathfrak{t})}.$$

Proof: Assume that G is a self-complementary PNDFG; then there exists an isomorphism $H : \mathfrak{V} \to \mathfrak{V}$ such that

$\overline{\mu_\eta(H(\mathfrak{g}))} = \mu_\eta(\mathfrak{g})$, $\overline{\beta_\eta(H(\mathfrak{g}))} = \beta_\eta(\mathfrak{g})$, $\overline{\sigma_\eta(H(\mathfrak{g}))} = \sigma_\eta(\mathfrak{g})$. $\forall \ \mathfrak{g} \in \mathfrak{V}$

$\overline{\mu_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \mu_\zeta(\mathfrak{g}\mathfrak{t})$, $\overline{\beta_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \beta_\zeta(\mathfrak{g}\mathfrak{t})$, $\overline{\sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \sigma_\zeta(\mathfrak{g}\mathfrak{t})$. $\forall \ \mathfrak{g}\mathfrak{t} \in \mathfrak{E}$

By definition of complement of G, we have

$$\overline{\mu_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \frac{\overline{\mu_\eta(H(\mathfrak{g})) \ \mu_\eta(H(\mathfrak{t}))}}{\overline{\mu_\eta(H(\mathfrak{g}))} + \overline{\mu_\eta(H(\mathfrak{t}))} - \overline{\mu_\eta(H(\mathfrak{g})) \ \mu_\eta(H(\mathfrak{t}))}} - \mu_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})} - \mu_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sum_{\mathfrak{g}\neq \mathfrak{t}} \mu_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})} - \sum_{\mathfrak{g}\neq \mathfrak{t}} \mu_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sum_{\mathfrak{g}\neq \mathfrak{t}} \mu_\zeta(\mathfrak{g}\mathfrak{t}) + \sum_{\mathfrak{g}\neq \mathfrak{t}} \mu_\zeta(H(\mathfrak{g})H(\mathfrak{t})) = \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}$$

$$2 \sum_{\mathfrak{g}\neq \mathfrak{t}} \mu_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq \mathfrak{t}} \frac{\mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g}) \ \mu_\eta(\mathfrak{t})}$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g}) + \mu_\eta(\mathfrak{t}) - \mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}$$

Similarly for indeterminacy membership grade, we have

$$\overline{\beta_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \frac{\overline{\beta_\eta(H(\mathfrak{g}))\ \beta_\eta(H(t))}}{\overline{\beta_\eta(H(\mathfrak{g}))} + \overline{\beta_\eta(H(\mathfrak{t}))} - \overline{\beta_\eta(H(\mathfrak{g}))\ \beta_\eta(H(\mathfrak{t}))}} - \beta_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g})\ \beta_\eta(t)} - \beta_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})} - \sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(H(\mathfrak{g})H(t))$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(\mathfrak{g}\mathfrak{t}) + \sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(H(\mathfrak{g})H(\mathfrak{t})) = \sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}$$

$$2\sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g}) + \beta_\eta(\mathfrak{t}) - \beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}$$

Likewise, for non-membership, we have

$$\overline{\sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t}))} = \frac{\overline{\sigma_\eta(H(\mathfrak{g}))} + \overline{\sigma_\eta(H(\mathfrak{t}))} - \overline{2\sigma_\eta(H(\mathfrak{g}))\ \sigma_\eta(H(\mathfrak{t}))}}{\overline{1 - \sigma_\eta(H(\mathfrak{g}))\ \sigma_\eta(H(\mathfrak{t}))}} - \sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(\mathfrak{t}) - 2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})} - \sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq t} \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(\mathfrak{t}) - 2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})} - \sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t}))$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(\mathfrak{g}\mathfrak{t}) + \sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(H(\mathfrak{g})H(\mathfrak{t})) = \sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(\mathfrak{t}) - 2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}$$

$$2\sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(\mathfrak{t}) - 2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g})\ \sigma_\eta(t)}$$

$$\sum_{\mathfrak{g}\neq\mathfrak{t}} \sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\sum_{\mathfrak{g}\neq\mathfrak{t}} \frac{\sigma_\eta(\mathfrak{g}) + \sigma_\eta(t) - 2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1 - \sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}$$

This completes the proof.

Proposition 2. Let $G = (\eta, \zeta)$ be a PNDFG. If

$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}\right)$,

$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}\right)$,

$\sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\sigma_\eta(\mathfrak{g})+\sigma_\eta(\mathfrak{t})-2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1-\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}\right)$ $\forall\ \mathfrak{g},\mathfrak{t} \in \mathfrak{V}$ then G is self-complementary.

Proof: Assume that G is a PNDFG that satisfies

$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}\right)$,

$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}\right)$,

$\sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\sigma_\eta(\mathfrak{g})+\sigma_\eta(\mathfrak{t})-2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1-\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}\right)$ $\forall\ \mathfrak{g}, t \in \mathfrak{V}$.

Then the identify mapping $I : \mathfrak{V} \to \mathfrak{V}$ is an isomorphism from G to $\overline{G}$ that satisfies the following condition:

$\mu_\eta(\mathfrak{g}) = \overline{\mu_\eta(I(\mathfrak{g}))}$, $\beta_\eta(\mathfrak{g}) = \overline{\beta_\eta(I(\mathfrak{g}))}$, and $\sigma_\eta(\mathfrak{g}) = \overline{\sigma_\eta(I(\mathfrak{g}))}$ $\forall\ \mathfrak{g} \in \mathfrak{V}$.

The membership grade of an edge $\mathfrak{g}\mathfrak{t}$ is given by

$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}\right)$ $\forall\ \mathfrak{g},\mathfrak{t} \in \mathfrak{V}$.

we have $\overline{\mu_\zeta(I(\mathfrak{g})I(\mathfrak{t}))} = \overline{\mu_\zeta(\mathfrak{g}\mathfrak{t})}$

$= \frac{\overline{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}}{\overline{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}} - \mu_\zeta(\mathfrak{g}\mathfrak{t})$

$= \frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})} - \frac{1}{2}\left(\frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}\right)$

$= \frac{1}{2}\left(\frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}\right) = \mu_\zeta(\mathfrak{g}\mathfrak{t})$

Likewise, the indeterminacy grade of an edge $\mathfrak{g}\mathfrak{t}$ is

$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{1}{2}\left(\frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}\right)$ $\forall\ \mathfrak{g},\mathfrak{t} \in \mathfrak{V}$.

we have $\overline{\beta_\zeta(I(\mathfrak{g})I(\mathfrak{t}))} = \overline{\beta_\zeta(\mathfrak{g}\mathfrak{t})}$

$= \frac{\overline{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}}{\overline{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}} - \beta_\zeta(\mathfrak{g}\mathfrak{t})$

$= \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})} - \frac{1}{2}\left(\frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}\right)$

$= \frac{1}{2}\left(\frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}\right) = \beta_\zeta(\mathfrak{g}\mathfrak{t})$

Similarly for the non-membership grade of an edge $\mathfrak{g}\mathfrak{t}$ is,

$\sigma_{\zeta}(\mathfrak{gt}) = \frac{1}{2}\left(\frac{\sigma_{\eta}(\mathfrak{g})+\sigma_{\eta}(\mathfrak{t})-2\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}{1-\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}\right)\ \forall\ \mathfrak{g},\mathfrak{t} \in \mathfrak{V}.$

So, we have

$\overline{\sigma_{\zeta}(I(\mathfrak{g})I(\mathfrak{t}))} = \overline{\sigma_{\zeta}(\mathfrak{gt})} = \frac{\overline{\sigma_{\eta}(\mathfrak{g})}+\overline{\sigma_{\eta}(\mathfrak{t})}-2\ \overline{\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}}{1-\overline{\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}} - \sigma_{\zeta}(\mathfrak{gt})$

$= \frac{\sigma_{\eta}(\mathfrak{g})+\sigma_{\eta}(\mathfrak{t})-2\ \sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}{1-\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})} - \frac{1}{2}\left(\frac{\sigma_{\eta}(\mathfrak{g})+\sigma_{\eta}(\mathfrak{t})-2\ \sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(t)}{1-\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}\right)$

$= \frac{1}{2}\left(\frac{\sigma_{\eta}(\mathfrak{g})+\sigma_{\eta}(\mathfrak{t})-2\ \sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}{1-\sigma_{\eta}(\mathfrak{g})\ \sigma_{\eta}(\mathfrak{t})}\right) = \sigma_{\zeta}(\mathfrak{gt})$

Since the conditions of isomorphism $\overline{\mu_{\zeta}(I(\mathfrak{g})I(\mathfrak{t}))} = \mu_{\zeta}(\mathfrak{gt})$, $\overline{\beta_{\zeta}(I(\mathfrak{g})I(\mathfrak{t}))} = \beta_{\zeta}(\mathfrak{gt})$ and $\overline{\sigma_{\zeta}(I(\mathfrak{g})I(\mathfrak{t}))} = \sigma_{\zeta}(\mathfrak{gt})$ are satisfied by I, $G = (\eta, \zeta)$ is self-complementary.

Proposition 3. If $G_1 = (\eta_1, \zeta_1)$ and $G_2 = (\eta_2, \zeta_2)$ are two isomorphic PNDFGs, then the complement of G_1 and G_2 are also isomorphic to each other and the converse also holds.

Proof: Assume that G_1 and G_2 are two isomorphic PNDFGs. Then by definition of isomorphism, there exists a bijective mapping $H : \mathfrak{V}_1 \to \mathfrak{V}_2$ that satisfies

$\mu_{\eta_1}(\mathfrak{g}) = \mu_{\eta_2}(H(\mathfrak{g}))$, $\beta_{\eta_1}(\mathfrak{g}) = \beta_{\eta_2}(H(\mathfrak{g}))$ and $\sigma_{\eta_1}(\mathfrak{g}) = \sigma_{\eta_2}(H(\mathfrak{g}))$. $\forall\ \mathfrak{g} \in \mathfrak{V}_1$,

$\mu_{\zeta_1}(\mathfrak{gt}) = \mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$, $\beta_{\zeta_1}(\mathfrak{gt}) = \beta_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$ and $\sigma_{\zeta_1}(\mathfrak{gt}) = \sigma_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$. $\forall\ \mathfrak{gt} \in \mathfrak{E}_1$.

By using the definition of complement of PNDFG, the membership grade of an edge $\mathfrak{gt}$ is

$\overline{\mu_{\zeta_1}(\mathfrak{gt})} = \frac{\mu_{\eta_1}(\mathfrak{g})\ \mu_{\eta_1}(\mathfrak{t})}{\mu_{\eta_1}(\mathfrak{g})+\mu_{\eta_1}(t)-\mu_{\eta_1}(\mathfrak{g})\ \mu_{\eta_1}(\mathfrak{t})} - \mu_{\zeta_1}(\mathfrak{gt})$

$= \frac{\mu_{\eta_2}(H(\mathfrak{g}))\ \mu_{\eta_2}(H(\mathfrak{t}))}{\mu_{\eta_2}(H(\mathfrak{g}))+\mu_{\eta_2}(H(\mathfrak{t}))-\mu_{\eta_2}(H(\mathfrak{g}))\ \mu_{\eta_2}(H(t))} - \mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$

$= \overline{\mu_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))}.$

Similarly,

$\overline{\beta_{\zeta_1}(\mathfrak{gt})} = \frac{\beta_{\eta_1}(\mathfrak{g})\ \beta_{\eta_1}(\mathfrak{t})}{\beta_{\eta_1}(\mathfrak{g})+\beta_{\eta_1}(\mathfrak{t})-\beta_{\eta_1}(\mathfrak{g})\ \beta_{\eta_1}(\mathfrak{t})} - \beta_{\zeta_1}(\mathfrak{gt})$

$= \frac{\beta_{\eta_2}(H(\mathfrak{g}))\ \beta_{\eta_2}(H(t))}{\beta_{\eta_2}(H(\mathfrak{g}))+\beta_{\eta_2}(H(\mathfrak{t}))-\beta_{\eta_2}(H(\mathfrak{g}))\ \beta_{\eta_2}(H(\mathfrak{t}))} - \beta_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$

$= \overline{\beta_{\zeta_2}(H(\mathfrak{g})H(t))}.$

Also, the non-membership grade of an edge $\mathfrak{gt}$ is,

$\overline{\sigma_{\zeta_1}(\mathfrak{gt})} = \frac{\sigma_{\eta_1}(\mathfrak{g})+\sigma_{\eta_1}(\mathfrak{t})-2\sigma_{\eta_1}(\mathfrak{g})\ \sigma_{\eta_1}(\mathfrak{t})}{1-\sigma_{\eta_1}(\mathfrak{g})\ \sigma_{\eta_1}(\mathfrak{t})} - \sigma_{\zeta_1}(\mathfrak{gt})$

$= \frac{\sigma_{\eta_2}(H(\mathfrak{g}))+\sigma_{\eta_2}(H(\mathfrak{t}))-2\ \sigma_{\eta_2}(H(\mathfrak{g}))\ \sigma_{\eta_2}(H(\mathfrak{t}))}{1-\sigma_{\eta_2}(H(\mathfrak{g}))\ \sigma_{\eta_2}(H(\mathfrak{t}))} - \sigma_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))$

$= \overline{\sigma_{\zeta_2}(H(\mathfrak{g})H(\mathfrak{t}))}$.

Hence, the complement of G_1 is isomorphic to the complement of G_2. Similarly, the converse can be proved.

Definition 3.10. A PNDFG is said to be complete if

$$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})},$$

$$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})},$$

$$\sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\sigma_\eta(\mathfrak{g})+\sigma_\eta(\mathfrak{t})-2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1-\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}\ \forall\ \mathfrak{g},\mathfrak{t}\in\mathfrak{V}.$$

The above mentioned properties are satisfied for the PNDFG in Example 1, thus the PNDFG is a complete PNDFG.

Definition 3.11. A PNDFG is said to be strong if

$$\mu_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})}{\mu_\eta(\mathfrak{g})+\mu_\eta(\mathfrak{t})-\mu_\eta(\mathfrak{g})\ \mu_\eta(\mathfrak{t})},$$

$$\beta_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})}{\beta_\eta(\mathfrak{g})+\beta_\eta(\mathfrak{t})-\beta_\eta(\mathfrak{g})\ \beta_\eta(\mathfrak{t})},$$

$$\sigma_\zeta(\mathfrak{g}\mathfrak{t}) = \frac{\sigma_\eta(\mathfrak{g})+\sigma_\eta(\mathfrak{t})-2\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}{1-\sigma_\eta(\mathfrak{g})\ \sigma_\eta(\mathfrak{t})}\ \forall\ \mathfrak{g}\mathfrak{t}\in\mathfrak{E}.$$

Example 4. The PNDFG over $\mathfrak{V} = \{m_1, m_2, m_3, m_4, m_5, m_6\}$
$\langle(m_1, .5, .6, .5), (m_2, .8, .4, .2), (m_3, .4, .6, .2), (m_4, .6, .5, .7), (m_5, .7, .4, .3), (m_6, .3, .6, .7)\rangle$
and the edge set
$\langle(m_1m_2, .44, .316, .556), (m_1m_6, .231, .429, .769), (m_2m_3, .432, .316, .33), (m_2m_5, .596, .25, .404),$
$(m_3m_4, .316, .375, .721), (m_4m_5, .477, .286, .734), (m_5m_6, .266, .316, .734)\rangle$ is strong.

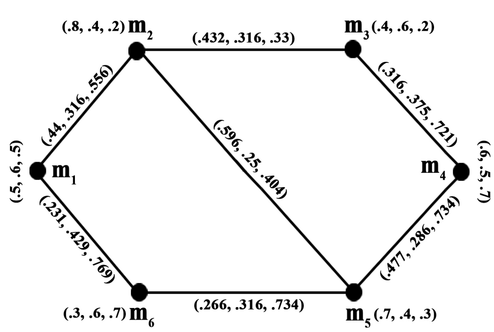

FIGURE 2. Strong PNDFG

4. Numerical Approach

We solve a decision-making problem involving the selection of the best money-transfer applications in this section to demonstrate the suitability of the proposed Pythagorean Neutrosophic Dombi fuzzy graphs concept in a real-world scenario.

4.1 Algorithm:

The following algorithm depicts our proposed technique for multi-criteria decision making. The algorithm for the selection of the best money transferring application.

Step 1: Input the attributes $M = \{m_1, m_2, ...m_k\}$ and set of criteria $C = \{c_1, c_2, ...c_n\}$ with weight vector $W = \{w_1, w_2, ...w_n\}$ and construct Pythagorean fuzzy relation $Q^{(g)} = (q_{lp}^{(g)})_{k\times k}$ corresponding to each criterion.

Step 2: Aggregate all $q_{lp}^{(g)} = (\mu_{lp}^{(g)}, \beta_{lp}^{(g)}, \sigma_{lp}^{(g)})$ $(l, p = 1, 2, .., k)$ regarding criteria $c_l(l = 1, 2, 3, 4, 5)$ and get $Q = (q_{lp})_{k\times k}$, where $q_{lp} = (\mu_{lp}, \beta_{lp}, \sigma_{lp})$ is the value assigned for the alternative m_l over m_p with respect to all the considered criteria C_l by using Pythagorean Neutrosophic Dombi fuzzy weighted arithmetic averaging (PNDFWAA) operator given by

$$q_{lp} = PNDFWAA(q_{lp}^{(1)}, q_{lp}^{(2)}, ..., q_{lp}^{(n)},) =$$

$$\left(\sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\beta_{lp}^{j})^2}{1-(\beta_{lp}^{g})^2}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\mu_{lp}^{j})^2}{1-(\beta_{lp}^{g})^2}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{1-(\sigma_{lp}^{j})}{(\sigma_{lp}^{g})}\right)^{\rho}\right]^{\frac{1}{\rho}}}\right)$$

Step 3: According to Q, draw the Pythagorean Neutrosophic fuzzy directed graph.

Step 4: By considering the condition $\mu_{lp} \geq 0.5$ $(l, p = 1, 2, ..., k)$, draw the Pythagorean Neutrosophic fuzzy partial directed graph.

Step 5: Calculate the out degrees $out - d(M_i)$ $(i = 1, 2, ..k)$ of all the alternatives M_i in the Pythagorean Neutrosophic fuzzy partial directed graph.

Step 6: Arrange the alternatives according to the diminishing value of the membership degrees of $out - d(M_i)$.

Step 7: The optimal alternative is the alternative with the maximum membership degree of $out - d(M_i)$.

4.2 Selection of the best money transferring application:

In this modern technology-filled world, everything is turning to online and digital cards. There is no need of searching for money exchanges as money can be easily transferred in online mode or through online transferring. According to existing trends, the fastest and easiest way of transferring is pivotal. Let us consider the following case. A person who has created a new

bank account wants to sync with this existing trends and techniques and want to have a money transferring app. Let us consider five money transferring applications $M_i(i = 1, 2, 3, 4, 5)$ that are doing really well on the market. The decision-making expert makes a comparison between five money transferring apps with respect to five criteria $C_l(l = 1, 2, 3, 4, 5)$ which are given as
C_1 = Safety & security
C_2 = Fast transfer without delay
C_3 = Money remittance
C_4 = User friendly
C_5 = Offers & gifts
with the respective weight $W = (0.3, 0.2, 0.3, 0.1, 0.1)$ and presents preferable information
$Q^{(g)} = (q_{lp}^{(g)})_{5\times5}$ $(g = 1, 2, 3, 4, 5)$, where
$q_{lp}^{(g)} = (\mu_{lp}^{(g)}, \beta_{lp}^{(g)}, \sigma_{lp}^{(g)})$ is the Pythagorean Neutrosophic number assigned by decision-making expert $\mu_{lp}^{(g)}, \beta_{lp}^{(g)}$ and $\sigma_{lp}^{(g)}$ are the degree to which money transferring application M_l is preferred and not preferred over the application M_p regarding the given criteria, respectively. The relations $Q^{(g)} = (q_{lp}^{(g)})_{5\times5}$ are given in the following tables (1-5).

Table 1. Comparision for Criteria 1

$Q^{(1)}$	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.8,.4,.2)	(.7,.3,.1)	(.6,.2,.4)	(.4,.4,.6)
M_2	(.2,.4,.8)	(.5,.5,.5)	(.6,.5,.4)	(.8,.2,.6)	(.6,.4,.7)
M_3	(.1,.3,.7)	(.4,.5,.6)	(.5,.5,.5)	(.8,.4,.3)	(.7,.5,.2)
M_4	(.4,.2,.6)	(.6,.2,.8)	(.3,.4,.8)	(.5,.5,.5)	(.8,.4,.7)
M_5	(.6,.4,.4)	(.7,.4,.6)	(.2,.5,.7)	(.7,.4,.8)	(.5,.5,.5)

Table 2. Comparision for Criteria 2

$Q^{(2)}$	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.9,.8,.5)	(.7,.4,.4)	(.9,.4,.7)	(.8,.6,.6)
M_2	(.5,.8,.9)	(.5,.5,.5)	(.8,.4,.5)	(.5,.4,.6)	(.7,.6,.8)
M_3	(.4,.4,.7)	(.5,.4,.8)	(.5,.5,.5)	(.7,.8,.6)	(.7,.7,.8)
M_4	(.7,.4,.9)	(.6,.4,.5)	(.6,.8,.7)	(.5,.5,.5)	(.7,.8,.6)
M_5	(.6,.6,.8)	(.8,.6,.7)	(.8,.7,.7)	(.6,.8,.7)	(.5,.5,.5)

Table 3. Comparision for Criteria 3

$Q^{(3)}$	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.9,.6,.5)	(.8,.4,.6)	(.9,.8,.4)	(.7,.6,.5)
M_2	(.5,.6,.9)	(.5,.5,.5)	(.8,.7,.6)	(.9,.8,.7)	(.6,.6,.5)
M_3	(.6,.4,.8)	(.6,.7,.8)	(.5,.5,.5)	(.8,.2,.7)	(.7,.8,.7)
M_4	(.4,.8,.9)	(.7,.8,.9)	(.7,.2,.8)	(.5,.5,.5)	(.9,.6,.7)
M_5	(.5,.6,.7)	(.5,.6,.6)	(.7,.8,.7)	(.7,.6,.9)	(.5,.5,.5)

Table 4. Comparision for Criteria 4

$Q^{(4)}$	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.9,.7,.5)	(.6,.3,.4)	(.7,.5,.6)	(.8,.3,.6)
M_2	(.5,.7,.9)	(.5,.5,.5)	(.9,.6,.5)	(.6,.4,.7)	(.7,.6,.6)
M_3	(.4,.3,.6)	(.5,.6,.9)	(.5,.5,.5)	(.9,.6,.4)	(.6,.7,.8)
M_4	(.6,.5,.7)	(.7,.4,.6)	(.4,.6,.9)	(.5,.5,.5)	(.9,.7,.6)
M_5	(.6,.3,.8)	(.6,.6,.7)	(.8,.7,.6)	(.6,.7,.9)	(.5,.5,.5)

Table 5. Comparision for Criteria 5

$Q^{(5)}$	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.8,.6,.4)	(.7,.6,.8)	(.6,.6,.8)	(.9,.3,.6)
M_2	(.4,.6,.8)	(.5,.5,.5)	(.8,.7,.6)	(.7,.5,.4)	(.8,.6,.5)
M_3	(.8,.6,.7)	(.6,.7,.8)	(.5,.5,.5)	(.9,.5,.4)	(.8,.5,.7)
M_4	(.8,.6,.6)	(.4,.5,.7)	(.4,.5,.9)	(.5,.5,.5)	(.9,.8,.7)
M_5	(.6,.3,.9)	(.5,.6,.8)	(.7,.5,.8)	(.7,.8,.9)	(.5,.5,.5)

The Pythagorean Neutrosophic directed graphs for $Q^{(g)}(g = 1, 2, 3, 4, 5)$ in Tables 1-5 are displayed in Figure 3.

With the purpose to complete the grouped $q_{lp} = (\mu_{lp}, \beta_{lp}, \sigma_{lp})$ $(e, p = 1, 2, 3, 4, 5)$ of the money transferring application M_l over M_p regarding all considered criteria $e^{(g)}(g = 1, 2, 3, 4, 5)$, the PNDFWAA operator is defined as

$$q_{lp} = PNDFWAA(q_{lp}^{(1)}, q_{lp}^{(2)}, ..., q_{lp}^{(n)},) =$$
$$\left(\sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\beta_{lp}^{j})^2}{1-(\beta_{lp}^{g})^2}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \sqrt{1 - \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{(\mu_{lp}^{j})^2}{1-(\beta_{lp}^{g})^2}\right)^{\rho}\right]^{\frac{1}{\rho}}}}, \frac{1}{1+\left[\sum_{j=1}^{n} w_j \left(\frac{1-(\sigma_{lp}^{j})}{(\sigma_{lp}^{g})}\right)^{\rho}\right]^{\frac{1}{\rho}}} \right)$$

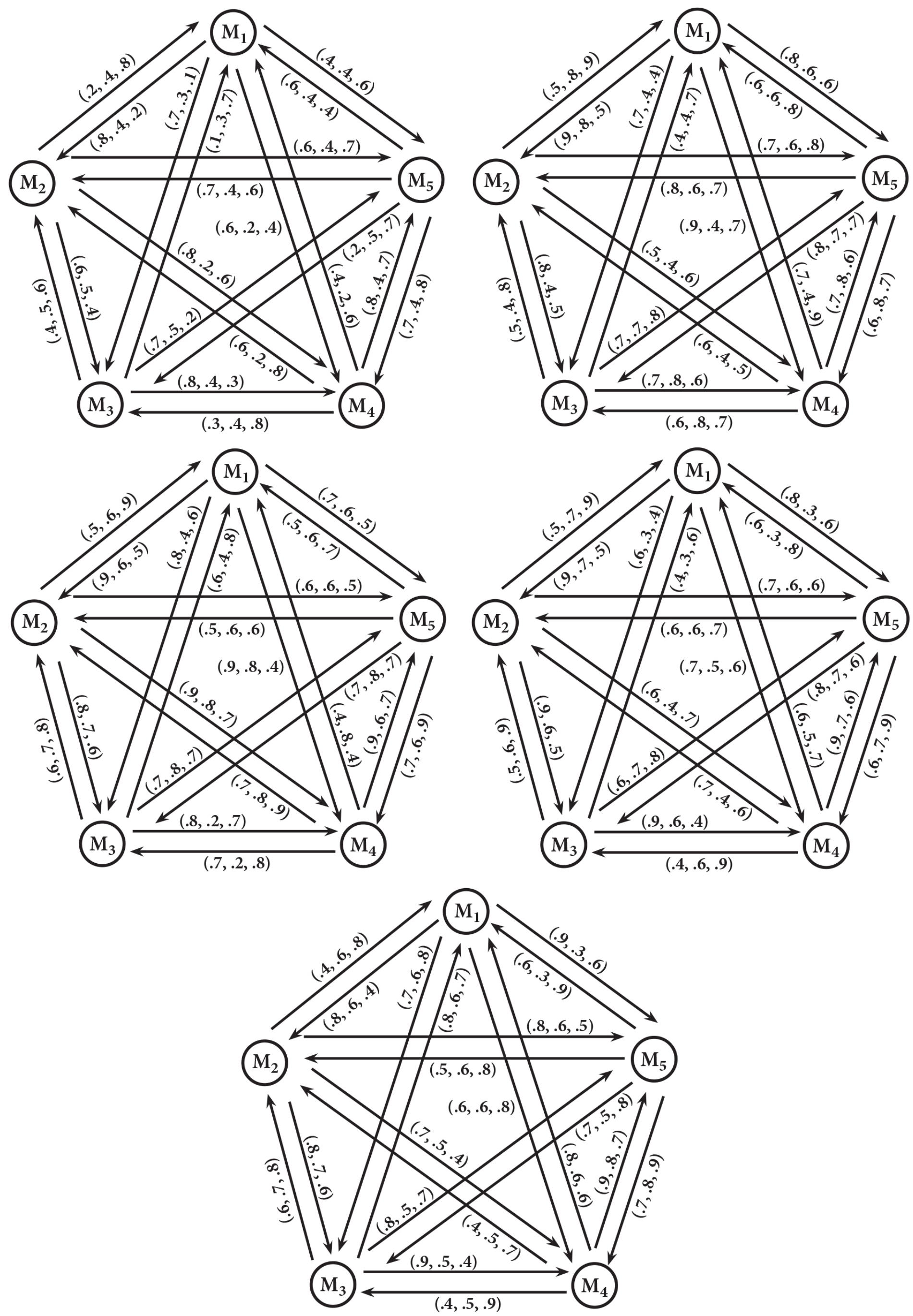

FIGURE 3. Pythagorean Neutrosophic directed graph for $Q^{(g)}(g = 1, 2, 3, 4, 5)$

In the above equation, we consider $\rho = 1$ as in Dombi's t-norm and t-conorm to obtain the corresponding $Q^g = (q_{lp}^{(g)})_{5\times 5}$, which is shown in Table 6.

Table 6. Combined Pythagorean Neutrosophic fuzzy relation

Q	M_1	M_2	M_3	M_4	M_5
M_1	(.5,.5,.5)	(.875,.651,.357)	(.734,.606,.239)	(.843,.635,.481)	(.752,.514,.565)
M_2	(.432,.651,.858)	(.5,.5,.5)	(.79,.605,.496)	(.819,.624,.602)	(.671,.557,.545)
M_3	(.538,.401,.715)	(.525,.605,.734)	(.5,.5,.5)	(.824,.583,.444)	(.708,.644,.409)
M_4	(.593,.635,.732)	(.637,.624,.705)	(.565,.583,.795)	(.5,.5,.5)	(.861,.679,.666)
M_5	(.575,.514,.601)	(.674,.558,.64)	(.694,.699,.697)	(.676,.679,.822)	(.5,.5,.5)

The Pythagorean Neutrosophic directed graphs according to Q, is in Figure 4.

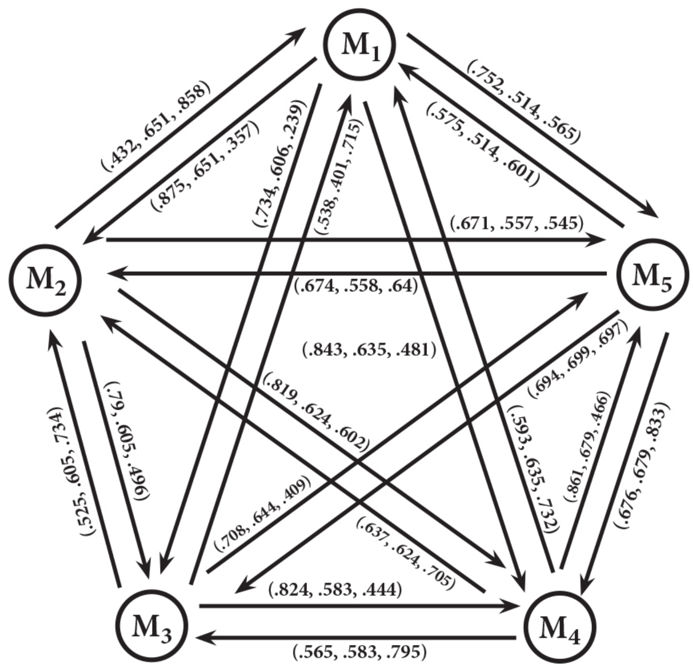

FIGURE 4. Pythagorean Neutrosophic directed graph according to Q

We consider the condition of $\mu_{lp} \geq 0.5$ $(l, p = 1, 2, 3, 4, 5)$ a partial directed graph is drawn in Figure 5.

The out-degrees $out - d(M_l)$ $(l = 1, 2, 3, 4, 5)$ of the money transferring app in the partial graph are calculated as

$out - d(M_1) = (3.204, 2.4061, 1.642)$

$out - d(M_2) = (2.28, 1.786, 1.643)$

$out - d(M_3) = (1.532, 1.227, 0.853)$

$out - d(M_4) = (0.861, 0.679, 0.666)$

$out - d(M_5) = (0, 0, 0)$

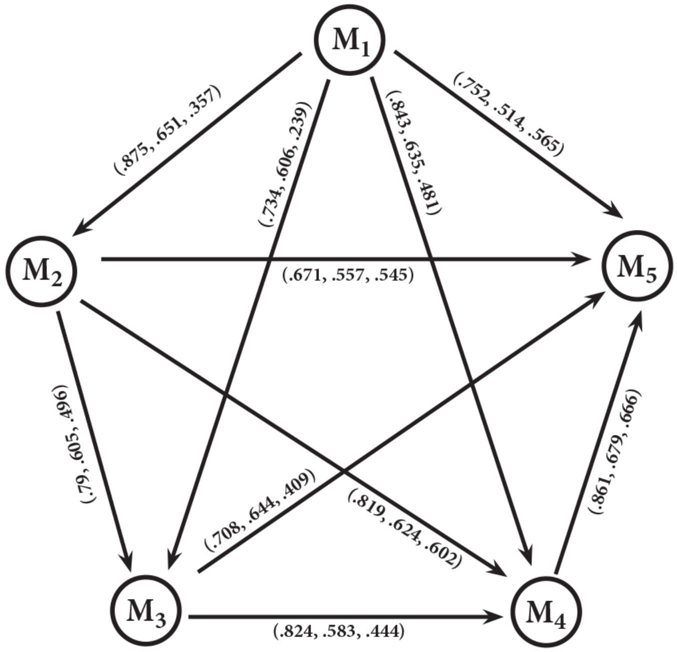

FIGURE 5. Partial directed Pythagorean Neutrosophic directed graph

According to the membership value of out-degrees of $M_l(l = 1, 2, 3, 4, 5)$ we get the optimal ranking order as:

$$M_1 \succ M_2 \succ M_3 \succ M_4 \succ M_5$$

On the basis of ranking, we conclude that M_1 is the best money transferring application.

5. **Conclusion**

In this paper, the concept of Pythagorean Neutrosophic Dombi Fuzzy graph has been introduced. Along with the introduction of this new model of Pythagorean neutrosophic Dombi fuzzy graph some of its definitions and few properties have been discussed. An application in decision making has been done by using the graph model of Pythagorean Neutrosophic Dombi Fuzzy graphs using the newly defined PNDFWAA operator. In the proposed MCDM, the limitation of μ_{lp} is restrained to be greater than 0.5, because the value less than 0.5 will have very low chances to be in the obtained alternative and it is defined to be more precise for making decision. This work on the concept of Pythgorean Neutrosophic Dombi Fuzzy graph can be extended further to investigate the operations of PNDFG and bipolar PNDFG along with some real life applications.

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