Single Valued Neutrosophic Hypersoft Expert Set with Application in Decision Making

Muhammad Ihsan[1,*], Atiqe Ur Rahman[1], Muhammad Saeed[2]

[1,1,*] University of Management and Technology, Lahore, Pakistan. 1;mihkhb@gmail.com, aurkhb@gmail.com

[2] University of Management and Technology, Lahore, Pakistan. 2; muhammad.saeed@umt.edu.pk

*Correspondence: mihkhb@gmail.com

Abstract. Soft set deals with single set of attributes whereas its generalization i.e. hypersoft set deals with multiple disjoint attribute-valued sets corresponding to distinct attributes. In this paper, we first introduced the concept of single valued neutrosophic hypersoft expert sets (SVNHESs) which combines single valued neutrosophic sets and hypersoft expert sets. Some fundamental properties (i.e. subset, not set and equal set), results (i.e. commutative, associative, distributive and D' Morgan Laws) and set-theoretic operations (i.e. complement, union intersection AND, and OR) are discussed. An algorithm is proposed to solve decision-making problems and applied to select the best product.

Keywords: Soft Set; Soft Expert Set; Neutrosophic set; Single Valued Neutrosophic set; Hypersoft Set; Single Valued Neutrosophic Hypersoft Expert Set.

---

1. Introduction

Neutrosophy has been introduced by Smarandache [1–3] as a new branch of philosophy and generalization of fuzzy logic, intuitionistic fuzzy logic, para-consistent logic. Fuzzy sets [4] and intuitionistic fuzzy sets [5] are defined by membership functions while intuitionistic fuzzy sets are characterized by membership and nonmembership functions, respectively. In some real life problems for proper description of an object in uncertain and ambiguous environment, we need to handle the indeterminate and incomplete information. But fuzzy sets and intuitionistic fuzzy sets do not handle the indeterminate and inconsistent information. Thus neutrosophic set (NS) is defined by Smarandache, as a new mathematical tool for dealing with problems involving incomplete, indeterminacy, inconsistent knowledge. In NS, the indeterminacy is quantified explicitly and truth-membership, indeterminacy membership, and false-membership are completely independent. From scientific or engineering point of view,

Muhammad Ihsan, Atiqe Ur Rahman, Muhammad Saeed, Single Valued Neutrosophic Hypersoft Expert Set with Application in Decision Making

1

the neutrosophic set and set- theoretic view, operators need to be specified. Otherwise, it will be difficult to apply in the real applications. Therefore, Wang et al [6] defined a single valued neutrosophic set (SVNS) and then provided the set theoretic operations and various properties of single valued neutrosophic sets. Broumi et al. [7] defined single valued neutrosophic soft expert sets and applied it in decision making.

Molodtsov [8] conceptualized soft set theory as a new parameterized family of subsets of the universe of discourse. Maji et al. [9] developed fuzzy soft set as a parametrization tool to deal with uncertainty. The fundamentals of soft set like subset, union, intersection, relations, functions etc., have been investigated by researchers [10–15]. Alkhazaleh et al. [16,17] conceptualized soft expert set and fuzzy soft expert set. They discussed their applications in decision making. Broumi et al. [18] conceptualized intuitionistic fuzzy soft expert sets and presented its application in decision making.

In 2018, Smarandache [19] generalized soft set to hypersoft set by replacing single attribute-valued function to multi-attribute valued function. Saeed et al. [20] and Mujahid et al. [21] discussed the rudiments of hypersoft sets along with illustrative examples. Rahman et al. [22–30] discussed the notions of complex set, convex set, parameterization, bijection, neutrosophic graph and rough set under hypersoft set environment. Saeed et al. [31–36] explored the concepts of complex multi-fuzzy set, mappings and neutrosophic graph with hypersoft settings. They discussed application of these models in decision-making problems. Ihsan et al. [37,38] introduced the expert system with multi-decisive opinions embedded with hypersoft set scenario. Some decision-making techniques i.e. TOPSIS etc. have been discussed for hypersoft set and its hybrids by researchers [39–43].

Having motivation from above literature, new notions of single valued neutrosophic hypersoft expert set are developed and an application is discussed in decision making through a proposed method. The pattern of rest of the paper is: section 2 reviews the notions of soft sets, fuzzy soft set, fuzzy soft expert set, hypersoft set and relevant definitions used in the proposed work. Section 3, presents notions of single valued neutrosophic hypersoft expert set with properties. Section 4, demonstrates an application of this concept in a decision-making problem. Section 5, concludes the paper.

2. **Preliminaries**

In this section, some basic definitions and terms regarding the main study, are presented from the literature.

Definition 2.1. [8]
Let $P(\Omega)$ denote power set of Ω(universe of discourse) and F be a collection of parameters

defining Ω. A *soft set* Ψ_M is defined by mapping

$$\Psi_M : F \rightarrow P(\Omega).$$

Definition 2.2. [9] Suppose Ω be a set of universe, while F is a set of parameters. Here I^Ω represents the power set of all fuzzy subsets of Ω. Let $C \subseteq F$. A pair (R, C) is called a fuzzy soft set with R is a mapping given by

$$R : C \rightarrow I^\Omega.$$

Definition 2.3. [16]
Assume that Y be a set of specialists (operators) and $\ddot{O}$ be a set of conclusions, $T = F \times Y \times \ddot{O}$ with $S \subseteq T$ where Ω denotes the universe , F a set of parameters.
A pair(Φ, S) is known as a *soft expert set* over Ω, where H is a mapping given by

$$\Phi : S \rightarrow P(\Omega).$$

Definition 2.4. [17] A pair (H, C) is called a fuzzy soft expert set over Ω where F is a mapping given by

$$H : C \rightarrow I^\Omega$$

where I^Ω the set of all fuzzy subsets of Ω.

Definition 2.5. [2] Suppose Ω denotes the universe of discourse then the neutrosophic set N is an object with the form
N $= \{< \beta : \mu_N(\beta), \nu_N(\beta), \omega_N(\beta) >, \beta \in \Omega\}$
While the functions $\mu_N(\beta), \nu_N(\beta), \omega_N(\beta) : \Omega \rightarrow]^-0, 1^+[$ denote the degree of membership, indeterminacy and non membership respectively for all $\beta \in \Omega$ with the condition
$^-0 \leq \mu_N(\beta) + \nu_N(\beta) + \omega_N(\beta) \leq 3^+$.

Definition 2.6. [6] Let Ω be a set of points (objects), with a generic element in Ω denoted by β. A single valued neutrosophic set (SVNS) N in Ω is defined by truth-membership function T_N, indeterminacy-membership function I_N and falsity-membership function F_N.
T_N, I_N, $F_N \in [0, 1]$ for all β in Ω with the condition
$0 \leq T_N(\beta) + I_N(\beta) + F_N(\beta) \leq 3$.

Definition 2.7. [19]
Let $h_1, h_2, h_3, ..., h_m,$ for $m \geq 1$, be m distinct attributes, whose corresponding attribute values are respectively the sets $H_1, H_2, H_3, ..., H_m$, with $H_i \cap H_j = \emptyset$, for $i \neq j$, and $i, j \in \{1, 2, 3, ..., m\}$. Then the pair (Ψ, G), where $G = H_1 \times H_2 \times H_3 \times ..... \times H_m$ and $\Psi : G \rightarrow P(\Omega)$ is called a *hypersoft Set* over Ω.

3. **Single Valued Neutrosophic Hypersoft Expert set (SVNHSE-Set)**

In this section, a new structure of single valued neutrosophic hypersoft expert set is developed and some properties are discussed.

Definition 3.1. Fuzzy Hypersoft Expert set (FHSE-Set)

A pair$(\xi, \mathbb{S})$ is known as a *fuzzy hypersoft expert set* over $\coprod$, where

$$\xi : \mathbb{S} \to I^{\coprod}$$

where

- $I^{\coprod}$ is collection of all fuzzy subsets of $\coprod$
- $\mathbb{S} \subseteq \mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$

- $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3 \times .... \times \mathcal{G}_p$ where $\mathcal{G}_i$ are disjoint attributive-valued sets corresponding to distinct attributes $g_i, i = 1, 2, 3, ..., p$

- $\mathcal{D}$ be a set of specialists (operators)

- $\mathbb{C}$ be a set of conclusions

For simplicity, $\mathbb{C} = \{0 = disagree, 1 = agree\}$.

Definition 3.2. Single Valued Neutrosophic Hypersoft Expert set (SVNHSE-Set)

A pair$(\xi, \mathbb{S})$ in definition 3.2, is known as a *single valued neutrosophic hypersoft expert set* over $\coprod$ if

$$\xi : \mathbb{S} \to SVNF^{\coprod}$$

with $SVNF^{\coprod}$ is collection of all single valued neutrosophic subsets of $\coprod$

Example 3.3. Suppose that a multi-national company aims to proceed the evaluation of certain specialists about its certain products. Let $\coprod = \{m_1, m_2, m_3, m_4\}$ be a set of products and

$\mathcal{G}_1 = \{q_{11}, q_{12}\}$
$\mathcal{G}_2 = \{q_{21}, q_{22}\}$
$\mathcal{G}_3 = \{q_{31}, q_{32}\}$

be disjoint attributive sets for distinct attributes $q_1 =$ simple to utilize, $q_2 =$ nature, $q_3 =$ modest.
Now $\mathcal{G} = \mathcal{G}_1 \times \mathcal{G}_2 \times \mathcal{G}_3$

$$\mathcal{G} = \left\{ \begin{array}{l} \mu_1 = (q_{11}, q_{21}, q_{31}), \mu_2 = (q_{11}, q_{21}, q_{32}), \mu_3 = (q_{11}, q_{22}, q_{31}), \mu_4 = (q_{11}, q_{22}, q_{32}), \\ \mu_5 = (q_{12}, q_{21}, q_{31}), \mu_6 = (q_{12}, q_{21}, q_{32}), \mu_7 = (q_{12}, q_{22}, q_{31}), \mu_8 = (q_{12}, q_{22}, q_{32}) \end{array} \right\}$$

Now $\mathcal{H} = \mathcal{G} \times \mathcal{D} \times \mathbb{C}$

$$\mathcal{H} = \left\{ \begin{array}{l} (\mu_1, s, 0), (\mu_1, s, 1), (\mu_1, t, 0), (\mu_1, t, 1), (\mu_1, u, 0), (\mu_1, u, 1), \\ (\mu_2, s, 0), (\mu_2, s, 1), (\mu_2, t, 0), (\mu_2, t, 1), (\mu_2, u, 0), (\mu_2, u, 1), \\ (\mu_3, s, 0), (\mu_3, s, 1), (\mu_3, t, 0), (\mu_3, t, 1), (\mu_3, u, 0), (\mu_3, u, 1), \\ (\mu_4, s, 0), (\mu_4, s, 1), (\mu_4, t, 0), (\mu_4, t, 1), (\mu_4, u, 0), (\mu_4, u, 1), \\ (\mu_5, s, 0), (\mu_5, s, 1), (\mu_5, t, 0), (\mu_5, t, 1), (\mu_5, u, 0), (\mu_5, u, 1), \\ (\mu_6, s, 0), (\mu_6, s, 1), (\mu_6, t, 0), (\mu_6, t, 1), (\mu_6, u, 0), (\mu_6, u, 1), \\ (\mu_7, s, 0), (\mu_7, s, 1), (\mu_7, t, 0), (\mu_7, t, 1), (\mu_7, u, 0), (\mu_7, u, 1), \\ (\mu_8, s, 0), (\mu_8, s, 1), (\mu_8, t, 0), (\mu_8, t, 1), (\mu_8, u, 0), (\mu_8, u, 1) \end{array} \right\}$$

let

$$\mathbb{S} = \left\{ \begin{array}{l} (\mu_1, s, 0), (\mu_1, s, 1), (\mu_1, t, 0), (\mu_1, t, 1), (\mu_1, u, 0), (\mu_1, u, 1), \\ (\mu_2, s, 0), (\mu_2, s, 1), (\mu_2, t, 0), (\mu_2, t, 1), (\mu_2, u, 0), (\mu_2, u, 1) \\ (\mu_3, s, 0), (\mu_3, s, 1), (\mu_3, t, 0), (\mu_3, t, 1), (\mu_3, u, 0), (\mu_3, u, 1), \end{array} \right\}$$

be a subset of $\mathcal{H}$ and $\mathcal{D} = \{s, t, u, \}$ be a set of specialists.

Following survey depicts choices of three specialists:

$$\xi_1 = \xi(\mu_1, s, 1) = \left\{ \frac{m_1}{< 0.2, 0.5, 0.4 >}, \frac{m_2}{< 0.7, 0.2, 0.5 >}, \frac{m_3}{< 0.5, 0.4, 0.6 >}, \frac{m_4}{< 0.1, 0.3, 0.6 >} \right\},$$

$$\xi_2 = \xi(\mu_1, t, 1) = \left\{ \frac{m_1}{< 0.4, 0.2, 0.3 >}, \frac{m_2}{< 0.8, 0.1, 0.5 >}, \frac{m_3}{< 0.4, 0.5, 0.6 >}, \frac{m_4}{< 0.2, 0.5, 0.3 >} \right\},$$

$$\xi_3 = \xi(\mu_1, u, 1) = \left\{ \frac{m_1}{< 0.7, 0.2, 0.3 >}, \frac{m_2}{< 0.5, 0.3, 0.6 >}, \frac{m_3}{< 0.6, 0.3, 0.7 >}, \frac{m_4}{< 0.3, 0.5, 0.6 >} \right\},$$

$$\xi_4 = \xi(\mu_2, s, 1) = \left\{ \frac{m_1}{< 0.9, 0.1, 0.3 >}, \frac{m_2}{< 0.4, 0.5, 0.4 >}, \frac{m_3}{< 0.7, 0.2, 0.6 >}, \frac{m_4}{< 0.3, 0.4, 0.8 >} \right\},$$

$$\xi_5 = \xi(\mu_2, t, 1) = \left\{ \frac{m_1}{< 0.4, 0.5, 0.6 >}, \frac{m_2}{< 0.8, 0.1, 0.7 >}, \frac{m_3}{< 0.3, 0.6, 0.5 >}, \frac{m_4}{< 0.2, 0.6, 0.7 >} \right\},$$

$$\xi_6 = \xi(\mu_2, u, 1) = \left\{ \frac{m_1}{< 0.5, 0.4, 0.7 >}, \frac{m_2}{< 0.3, 0.6, 0.4 >}, \frac{m_3}{< 0.6, 0.2, 0.5 >}, \frac{m_4}{< 0.8, 0.1, 0.6 >} \right\},$$

$$\xi_7 = \xi(\mu_3, s, 1) = \left\{ \frac{m_1}{< 0.2, 0.7, 0.5 >}, \frac{m_2}{< 0.9, 0.1, 0.4 >}, \frac{m_3}{< 0.4, 0.5, 0.7 >}, \frac{m_4}{< 0.5, 0.4, 0.8 >} \right\},$$

$$\xi_8 = \xi(\mu_3, t, 1) = \left\{ \frac{m_1}{< 0.4, 0.3, 0.2 >}, \frac{m_2}{< 0.6, 0.3, 0.1 >}, \frac{m_3}{< 0.7, 0.2, 0.3 >}, \frac{m_4}{< 0.9, 0.1, 0.4 >} \right\},$$

$$\xi_9 = \xi(\mu_3, u, 1) = \left\{ \frac{m_1}{< 0.7, 0.2, 0.6 >}, \frac{m_2}{< 0.3, 0.5, 0.7 >}, \frac{m_3}{< 0.5, 0.4, 0.5 >}, \frac{m_4}{< 0.2, 0.7, 0.8 >} \right\},$$

$$\xi_{10} = \xi(\mu_1, s, 0) = \left\{ \frac{m_1}{< 0.3, 0.2, 0.1 >}, \frac{m_2}{< 0.2, 0.4, 0.5 >}, \frac{m_3}{< 0.4, 0.5, 0.8 >}, \frac{m_4}{< 0.1, 0.8, 0.3 >} \right\},$$

$$\xi_{11} = \xi(\mu_1, t, 0) = \left\{ \frac{m_1}{< 0.1, 0.8, 0.4 >}, \frac{m_2}{< 0.9, 0.1, 0.2 >}, \frac{m_3}{< 0.6, 0.3, 0.4 >}, \frac{m_4}{< 0.2, 0.7, 0.5 >} \right\},$$

Muhammad Ihsan, Atiqe Ur Rahman, Muhammad Saeed, Single Valued Neutrosophic Hypersoft Expert Set with Application in Decision Making

$$\xi_{12} = \xi(\mu_1, u, 0) = \left\{ \frac{m_1}{<0.2, 0.7, 0.5>}, \frac{m_2}{<0.1, 0.8, 0.6>}, \frac{m_3}{<0.3, 0.5, 0.7>}, \frac{m_4}{<0.5, 0.4, 0.6>} \right\},$$

$$\xi_{13} = \xi(\mu_2, s, 0) = \left\{ \frac{m_1}{<0.8, 0.1, 0.6>}, \frac{m_2}{<0.3, 0.6, 0.7>}, \frac{m_3}{<0.5, 0.4, 0.8>}, \frac{m_4}{<0.7, 0.2, 0.9>} \right\},$$

$$\xi_{14} = \xi(\mu_2, t, 0) = \left\{ \frac{m_1}{<0.7, 0.2, 0.5>}, \frac{m_2}{<0.2, 0.6, 0.4>}, \frac{m_3}{<0.9, 0.1, 0.6>}, \frac{m_4}{<0.4, 0.5, 0.7>} \right\},$$

$$\xi_{15} = \xi(\mu_2, u, 0) = \left\{ \frac{m_1}{<0.6, 0.2, 0.5>}, \frac{m_2}{<0.7, 0.2, 0.4>}, \frac{m_3}{<0.3, 0.5, 0.4>}, \frac{m_4}{<0.2, 0.7, 0.6>} \right\},$$

$$\xi_{16} = \xi(\mu_3, s, 0) = \left\{ \frac{m_1}{<0.1, 0.7, 0.5>}, \frac{m_2}{<0.4, 0.5, 0.7>}, \frac{m_3}{<0.7, 0.2, 0.9>}, \frac{m_4}{<0.8, 0.2, 0.4>} \right\},$$

$$\xi_{17} = \xi(\mu_3, t, 0) = \left\{ \frac{m_1}{<0.2, 0.7, 0.4>}, \frac{m_2}{<0.9, 0.1, 0.6>}, \frac{m_3}{<0.8, 0.2, 0.4>}, \frac{m_4}{<0.3, 0.5, 0.7>} \right\},$$

$$\xi_{18} = \xi(\mu_3, u, 0) = \left\{ \frac{m_1}{<0.5, 0.4, 0.2>}, \frac{m_2}{<0.3, 0.6, 0.1>}, \frac{m_3}{<0.6, 0.3, 0.2>}, \frac{m_4}{<0.1, 0.8, 0.3>} \right\}$$

The single valued neutrosophic hypersoft expert set can be described as

$$(\xi, \mathbb{S}) = \left\{ \begin{array}{l}
\left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.7,0.2,0.5>}, \frac{m_3}{<0.5,0.4,0.6>}, \frac{m_4}{<0.1,0.3,0.6>} \right\} \right), \\
\left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.8,0.1,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.2,0.5,0.3>} \right\} \right), \\
\left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.3>}, \frac{m_2}{<0.5,0.3,0.6>}, \frac{m_3}{<0.6,0.3,0.7>}, \frac{m_4}{<0.3,0.5,0.6>} \right\} \right), \\
\left((\mu_2, s, 1), \left\{ \frac{m_1}{<0.9,0.1,0.3>}, \frac{m_2}{<0.4,0.5,0.4>}, \frac{m_3}{<0.7,0.2,0.6>}, \frac{m_4}{<0.3,0.4,0.8>} \right\} \right), \\
\left((\mu_2, t, 1), \left\{ \frac{m_1}{<0.4,0.5,0.6>}, \frac{m_2}{<0.8,0.1,0.7>}, \frac{m_3}{<0.3,0.6,0.5>}, \frac{m_4}{<0.2,0.6,0.7>} \right\} \right), \\
\left((\mu_2, u, 1), \left\{ \frac{m_1}{<0.5,0.4,0.7>}, \frac{m_2}{<0.3,0.6,0.4>}, \frac{m_3}{<0.6,0.2,0.5>}, \frac{m_4}{<0.8,0.1,0.6>} \right\} \right), \\
\left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.9,0.1,0.4>}, \frac{m_3}{<0.4,0.5,0.7>}, \frac{m_4}{<0.5,0.4,0.8>} \right\} \right), \\
\left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.2>}, \frac{m_2}{<0.6,0.3,0.1>}, \frac{m_3}{<0.7,0.2,0.3>}, \frac{m_4}{<0.9,0.1,0.4>} \right\} \right), \\
\left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.6>}, \frac{m_2}{<0.3,0.5,0.7>}, \frac{m_3}{<0.5,0.4,0.5>}, \frac{m_4}{<0.2,0.7,0.8>} \right\} \right), \\
\left((\mu_1, s, 0), \left\{ \frac{m_1}{<0.3,0.2,0.1>}, \frac{m_2}{<0.2,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.8>}, \frac{m_4}{<0.1,0.8,0.3>} \right\} \right), \\
\left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.8,0.4>}, \frac{m_2}{<0.9,0.1,0.2>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.2,0.7,0.5>} \right\} \right), \\
\left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.1,0.8,0.6>}, \frac{m_3}{<0.3,0.5,0.7>}, \frac{m_4}{<0.5,0.4,0.6>} \right\} \right), \\
\left((\mu_2, s, 0), \left\{ \frac{m_1}{<0.8,0.1,0.6>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.5,0.4,0.8>}, \frac{m_4}{<0.7,0.2,0.9>} \right\} \right), \\
\left((\mu_2, t, 0), \left\{ \frac{m_1}{<0.7,0.2,0.5>}, \frac{m_2}{<0.2,0.6,0.4>}, \frac{m_3}{<0.9,0.1,0.6>}, \frac{m_4}{<0.4,0.5,0.7>} \right\} \right), \\
\left((\mu_2, u, 0), \left\{ \frac{m_1}{<0.6,0.2,0.5>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.3,0.5,0.4>}, \frac{m_4}{<0.2,0.7,0.6>} \right\} \right), \\
\left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.7,0.5>}, \frac{m_2}{<0.4,0.5,0.7>}, \frac{m_3}{<0.7,0.2,0.9>}, \frac{m_4}{<0.8,0.2,0.4>} \right\} \right), \\
\left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.7,0.4>}, \frac{m_2}{<0.9,0.1,0.6>}, \frac{m_3}{<0.8,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.7>} \right\} \right), \\
\left((\mu_3, u, 0), \left\{ \frac{m_1}{<0.5,0.4,0.2>}, \frac{m_2}{<0.3,0.6,0.1>}, \frac{m_3}{<0.6,0.3,0.2>}, \frac{m_4}{<0.1,0.8,0.3>} \right\} \right)
\end{array} \right\}$$

Definition 3.4. Single Valued Neutrosophic Hypersoft Expert subset

A single valued neutrosophic hypersoft expert set $(\xi_1, \mathbb{S})$ is said to be single valued neutrosophic hypersoft expert subset of (ξ_2, R) over $\coprod$, if

(i) $\mathbb{S} \subseteq R$,

(ii) $\forall\ \alpha \in \mathbb{S}, \xi_1(\alpha) \subseteq \xi_2(\alpha).$

and denoted by $(\xi_1, \mathbb{S}) \subseteq (\xi_2, R).$ Similarly (ξ_2, R) is said to be *single valued neutrosophic hypersoft expert superset* of $(\xi_1, \mathbb{S}).$

Example 3.5. Considering Example 3.3, Suppose

$$A_1 = \Big\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1) \Big\}$$

$$A_2 = \Big\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_3, s, 1), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_1, t, 0), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1), (\mu_1, u, 1) \Big\}$$

It is clear that $A_1 \subset A_2.$ Suppose (ξ_1, A_1) and (ξ_2, A_2) be defined as following

$$(\xi_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.7>}, \frac{m_2}{<0.6,0.5,0.8>}, \frac{m_3}{<0.4,0.6,0.9>}, \frac{m_4}{<0.1,0.8,0.6>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.5>}, \frac{m_2}{<0.6,0.4,0.6>}, \frac{m_3}{<0.2,0.5,0.7>}, \frac{m_4}{<0.1,0.5,0.6>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.2,0.6,0.4>}, \frac{m_2}{<0.5,0.4,0.7>}, \frac{m_3}{<0.6,0.5,0.8>}, \frac{m_4}{<0.8,0.6,0.4>} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.6,0.4,0.3>}, \frac{m_2}{<0.2,0.7,0.6>}, \frac{m_3}{<0.4,0.5,0.3>}, \frac{m_4}{<0.1,0.7,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.1,0.6,0.3>}, \frac{m_2}{<0.1,0.7,0.4>}, \frac{m_3}{<0.2,0.7,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.8,0.6>}, \frac{m_2}{<0.3,0.6,0.5>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.7,0.2,0.6>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.1,0.7,0.4>}, \frac{m_2}{<0.6,0.3,0.6>}, \frac{m_3}{<0.7,0.2,0.5>}, \frac{m_4}{<0.2,0.7,0.4>} \right\} \right) \end{array} \right\}$$

$$(\xi_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.3,0.6>}, \frac{m_2}{<0.7,0.4,0.7>}, \frac{m_3}{<0.5,0.4,0.8>}, \frac{m_4}{<0.2,0.4,0.5>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.4>}, \frac{m_2}{<0.8,0.3,0.5>}, \frac{m_3}{<0.4,0.3,0.6>}, \frac{m_4}{<0.2,0.6,0.5>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.3,0.4>}, \frac{m_2}{<0.9,0.1,0.3>}, \frac{m_3}{<0.4,0.5,0.4>}, \frac{m_4}{<0.5,0.3,0.4>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.6,0.3,0.6>}, \frac{m_3}{<0.7,0.4,0.5>}, \frac{m_4}{<0.9,0.5,0.2} \right\} \right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.4>}, \frac{m_2}{<0.5,0.2,0.6>}, \frac{m_3}{<0.6,0.2,0.7>}, \frac{m_4}{<0.3,0.5,0.8} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.3,0.1>}, \frac{m_2}{<0.3,0.5,0.4>}, \frac{m_3}{<0.5,0.4,0.2>}, \frac{m_4}{<0.2,0.6,0.3>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.5,0.1>}, \frac{m_2}{<0.2,0.6,0.3>}, \frac{m_3}{<0.3,0.5,0.4>}, \frac{m_4}{<0.5,0.3,0.5>} \right\} \right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.9,0.1,0.6>}, \frac{m_3}{<0.6,0.3,0.8>}, \frac{m_4}{<0.2,0.6,0.8>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.2,0.7,0.4>}, \frac{m_2}{<0.4,0.5,0.3>}, \frac{m_3}{<0.7,0.2,0.1>}, \frac{m_4}{<0.8,0.1,0.5>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.5,0.1>}, \frac{m_2}{<0.7,0.2,0.3>}, \frac{m_3}{<0.8,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.2>} \right\} \right) \end{array} \right\}$$

which implies that $(\xi_1, A_1) \subseteq (\xi_2, A_2).$

Definition 3.6. Two single valued neutrosophic hypersoft expert sets (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are said to be equal if (ξ_1, A_1) is a single valued neutrosophic hypersoft expert subset of (ξ_2, A_2) and (ξ_2, A_2) is a single valued neutrosophic hypersoft expert subset of $(\xi_1, A_1).$

Definition 3.7. The complement of a single valued neutrosophic hypersoft expert set $(\xi, \mathbb{S}),$ denoted by $(\xi, \mathbb{S})^c,$ is defined by

$(\xi, \mathbb{S})^c = \tilde{c}(\xi(\beta)) \ \forall \ \beta \in \coprod$ while $\tilde{c}$ is a NF complement.

Example 3.8. Taking complement of single valued neutrosophic hypersoft expert set determined in 3.3, we have

$$
(\xi, \mathbb{S})^c = \left\{
\begin{aligned}
&\left((\mu_1, s, 1), \left\{\frac{m_1}{<0.4,0.5,0.2>}, \frac{m_2}{<0.5,0.8,0.7>}, \frac{m_3}{<0.6,0.6,0.5>}, \frac{m_4}{<0.6,0.7,0.1>}\right\}\right), \\
&\left((\mu_1, t, 1), \left\{\frac{m_1}{<0.3,0.8,0.6>}, \frac{m_2}{<0.5,0.9,0.8>}, \frac{m_3}{<0.6,0.5,0.4>}, \frac{m_4}{<0.3,0.5,0.2>}\right\}\right), \\
&\left((\mu_1, u, 1), \left\{\frac{m_1}{<0.3,0.8,0.7>}, \frac{m_2}{<0.6,0.7,0.5>}, \frac{m_3}{<0.7,0.7,0.6>}, \frac{m_4}{<0.6,0.5,0.3>}\right\}\right), \\
&\left((\mu_2, s, 1), \left\{\frac{m_1}{<0.3,0.9,0.9>}, \frac{m_2}{<0.4,0.5,0.4>}, \frac{m_3}{<0.6,0.8,0.7>}, \frac{m_4}{<0.8,0.6,0.3>}\right\}\right), \\
&\left((\mu_2, t, 1), \left\{\frac{m_1}{<0.6,0.5,0.4>}, \frac{m_2}{<0.7,0.9,0.8>}, \frac{m_3}{<0.5,0.4,0.3>}, \frac{m_4}{<0.7,0.4,0.2>}\right\}\right), \\
&\left((\mu_2, u, 1), \left\{\frac{m_1}{<0.7,0.6,0.5>}, \frac{m_2}{<0.4,0.4,0.3>}, \frac{m_3}{<0.5,0.8,0.6>}, \frac{m_4}{<0.6,0.9,0.8>}\right\}\right), \\
&\left((\mu_3, s, 1), \left\{\frac{m_1}{<0.5,0.3,0.2>}, \frac{m_2}{<0.4,0.9,0.9>}, \frac{m_3}{<0.7,0.5,0.4>}, \frac{m_4}{<0.8,0.6,0.5>}\right\}\right), \\
&\left((\mu_3, t, 1), \left\{\frac{m_1}{<0.2,0.7,0.4>}, \frac{m_2}{<0.1,0.7,0.6>}, \frac{m_3}{<0.3,0.8,0.7>}, \frac{m_4}{<0.4,0.9,0.9>}\right\}\right), \\
&\left((\mu_3, u, 1), \left\{\frac{m_1}{<0.6,0.8,0.7>}, \frac{m_2}{<0.7,0.5,0.3>}, \frac{m_3}{<0.5,0.6,0.5>}, \frac{m_4}{<0.8,0.3,0.2>}\right\}\right), \\
&\left((\mu_1, s, 0), \left\{\frac{m_1}{<0.1,0.8,0.3>}, \frac{m_2}{<0.5,0.6,0.2>}, \frac{m_3}{<0.8,0.5,0.4>}, \frac{m_4}{<0.3,0.2,0.1>}\right\}\right), \\
&\left((\mu_1, t, 0), \left\{\frac{m_1}{<0.4,0.2,0.1>}, \frac{m_2}{<0.2,0.9,0.9>}, \frac{m_3}{<0.4,0.7,0.6>}, \frac{m_4}{<0.5,0.3,0.2>}\right\}\right), \\
&\left((\mu_1, u, 0), \left\{\frac{m_1}{<0.5,0.3,0.2>}, \frac{m_2}{<0.6,0.2,0.1>}, \frac{m_3}{<0.7,0.5,0.3>}, \frac{m_4}{<0.6,0.6,0.5>}\right\}\right), \\
&\left((\mu_2, s, 0), \left\{\frac{m_1}{<0.6,0.9,0.8>}, \frac{m_2}{<0.7,0.4,0.3>}, \frac{m_3}{<0.8,0.6,0.5>}, \frac{m_4}{<0.9,0.8,0.7>}\right\}\right), \\
&\left((\mu_2, t, 0), \left\{\frac{m_1}{<0.5,0.8,0.7>}, \frac{m_2}{<0.4,0.4,0.2>}, \frac{m_3}{<0.6,0.9,0.9>}, \frac{m_4}{<0.7,0.5,0.4>}\right\}\right), \\
&\left((\mu_2, u, 0), \left\{\frac{m_1}{<0.5,0.8,0.6>}, \frac{m_2}{<0.4,0.8,0.7>}, \frac{m_3}{<0.4,0.5,0.3>}, \frac{m_4}{<0.6,0.3,0.2>}\right\}\right), \\
&\left((\mu_3, s, 0), \left\{\frac{m_1}{<0.5,0.3,0.1>}, \frac{m_2}{<0.7,0.5,0.3>}, \frac{m_3}{<0.9,0.8,0.8>}, \frac{m_4}{<0.4,0.8,0.8>}\right\}\right), \\
&\left((\mu_3, t, 0), \left\{\frac{m_1}{<0.4,0.3,0.2>}, \frac{m_2}{<0.6,0.9,0.9>}, \frac{m_3}{<0.4,0.8,0.8>}, \frac{m_4}{<0.7,0.5,0.3>}\right\}\right), \\
&\left((\mu_3, u, 0), \left\{\frac{m_1}{<0.2,0.6,0.5>}, \frac{m_2}{<0.1,0.4,0.3>}, \frac{m_3}{<0.2,0.7,0.6>}, \frac{m_4}{<0.3,0.2,0.1>}\right\}\right)
\end{aligned}
\right\}
$$

Definition 3.9. An agree-single valued neutrosophic hypersoft expert set $(\xi, \mathbb{S})_{ag}$ over $\coprod$, is a single valued neutrosophic hypersoft expert subset of $(\xi, \mathbb{S})$ and is characterized as $(\xi, \mathbb{S})_{ag} = \{\xi_{ag}(\beta) : \beta \in \mathcal{G} \times \mathcal{D} \times \{1\}\}$.

Example 3.10. Finding agree-single valued neutrosophic hypersoft expert set determined in 3.3, we get

$$(\xi, \mathbb{S}) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.7,0.2,0.5>}, \frac{m_3}{<0.5,0.4,0.6>}, \frac{m_4}{<0.1,0.3,0.6>} \right\}\right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.8,0.1,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.2,0.5,0.3>} \right\}\right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.3>}, \frac{m_2}{<0.5,0.3,0.6>}, \frac{m_3}{<0.6,0.3,0.7>}, \frac{m_4}{<0.3,0.5,0.6>} \right\}\right), \\ \left((\mu_2, s, 1), \left\{ \frac{m_1}{<0.9,0.1,0.3>}, \frac{m_2}{<0.4,0.5,0.4>}, \frac{m_3}{<0.7,0.2,0.6>}, \frac{m_4}{<0.3,0.4,0.8>} \right\}\right), \\ \left((\mu_2, t, 1), \left\{ \frac{m_1}{<0.4,0.5,0.6>}, \frac{m_2}{<0.8,0.1,0.7>}, \frac{m_3}{<0.3,0.6,0.5>}, \frac{m_4}{<0.2,0.6,0.7>} \right\}\right), \\ \left((\mu_2, u, 1), \left\{ \frac{m_1}{<0.5,0.4,0.7>}, \frac{m_2}{<0.3,0.6,0.4>}, \frac{m_3}{<0.6,0.2,0.5>}, \frac{m_4}{<0.8,0.1,0.6>} \right\}\right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.9,0.1,0.4>}, \frac{m_3}{<0.4,0.5,0.7>}, \frac{m_4}{<0.5,0.4,0.8>} \right\}\right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.2>}, \frac{m_2}{<0.6,0.3,0.1>}, \frac{m_3}{<0.7,0.2,0.3>}, \frac{m_4}{<0.9,0.1,0.4>} \right\}\right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.6>}, \frac{m_2}{<0.3,0.5,0.7>}, \frac{m_3}{<0.5,0.4,0.5>}, \frac{m_4}{<0.2,0.7,0.8>} \right\}\right), \end{array} \right\}$$

Definition 3.11. A disagree-single valued neutrosophic hypersoft expert set $(\xi, \mathbb{S})_{dag}$ over $\coprod$, is a single valued neutrosophic hypersoft expert subset of $(\xi, \mathbb{S})$ and is characterized as $(\xi, \mathbb{S})_{dag} = \{\xi_{dag}(\beta) : \beta \in \mathbb{G} \times \mathcal{D} \times \{0\}\}$.

Example 3.12. Getting disagree-single valued neutrosophic hypersoft expert set determined in 3.3,

$$(\xi, \mathbb{S}) = \left\{ \begin{array}{l} \left((\mu_1, s, 0), \left\{ \frac{m_1}{<0.3,0.2,0.1>}, \frac{m_2}{<0.2,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.8>}, \frac{m_4}{<0.1,0.8,0.3>} \right\}\right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.8,0.4>}, \frac{m_2}{<0.9,0.1,0.2>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.2,0.7,0.5>} \right\}\right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.1,0.8,0.6>}, \frac{m_3}{<0.3,0.5,0.7>}, \frac{m_4}{<0.5,0.4,0.6>} \right\}\right), \\ \left((\mu_2, s, 0), \left\{ \frac{m_1}{<0.8,0.1,0.6>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.5,0.4,0.8>}, \frac{m_4}{<0.7,0.2,0.9>} \right\}\right), \\ \left((\mu_2, t, 0), \left\{ \frac{m_1}{<0.7,0.2,0.5>}, \frac{m_2}{<0.2,0.6,0.4>}, \frac{m_3}{<0.9,0.1,0.6>}, \frac{m_4}{<0.4,0.5,0.7>} \right\}\right), \\ \left((\mu_2, u, 0), \left\{ \frac{m_1}{<0.6,0.2,0.5>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.3,0.5,0.4>}, \frac{m_4}{<0.2,0.7,0.6>} \right\}\right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.7,0.5>}, \frac{m_2}{<0.4,0.5,0.7>}, \frac{m_3}{<0.7,0.2,0.9>}, \frac{m_4}{<0.8,0.2,0.4>} \right\}\right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.7,0.4>}, \frac{m_2}{<0.9,0.1,0.6>}, \frac{m_3}{<0.8,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.7>} \right\}\right), \\ \left((\mu_3, u, 0), \left\{ \frac{m_1}{<0.5,0.4,0.2>}, \frac{m_2}{<0.3,0.6,0.1>}, \frac{m_3}{<0.6,0.3,0.2>}, \frac{m_4}{<0.1,0.8,0.3>} \right\}\right), \end{array} \right\}$$

Proposition 3.13. *If $(\xi, \mathbb{S})$ is a single valued neutrosophic hypersoft expert set over $\coprod$, then*

(1). $((\xi, \mathbb{S})^c)^c = (\xi, \mathbb{S})$

(2). $(\xi, \mathbb{S})_{ag}^c = (\xi, \mathbb{S})_{dag}$

(3). $(\xi, \mathbb{S})_{dag}^c = (\xi, \mathbb{S})_{ag}$

Definition 3.14. The union of $(\xi_1, \mathbb{S})$ and $(\xi_2, \mathbb{R})$ over $\coprod$ is (ξ_3, L) with $L = \mathbb{S} \cup \mathbb{R}$, defined as

$$\xi_3(\beta) = \left\{ \begin{array}{ll} \xi_1(\beta) & ; \beta \in \mathbb{S} - \mathbb{R} \\ \xi_2(\beta) & ; \beta \in \mathbb{R} - \mathbb{S} \\ \cup(\xi_1(\beta), \xi_2(\beta)) & ; \beta \in \mathbb{S} \cap \mathbb{R} \end{array} \right.$$

where $\cup(\xi_1(\beta), \xi_2(\beta)) = \{< u, \max\{\mu_1(\beta), \mu_2(\beta)\}, \min\{\nu_1(\beta), \nu_2(\beta)\}, \min\{\omega_1(\beta), \omega_2(\beta)\} >:$
$u \in U\}$.

Example 3.15. Taking into consideration the concept of example 3.3, consider the following two sets

$$A_1 = \left\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1) \right\}$$

$$A_2 = \left\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_3, s, 1), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_1, u, 1), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1), (\mu_1, t, 0) \right\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two single valued neutrosophic hypersoft expert sets such that

$$(\xi_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.3,0.2>}, \frac{m_3}{<0.4,0.5,0.1>}, \frac{m_4}{<0.1,0.8,0.5>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.5>}, \frac{m_2}{<0.6,0.2,0.3>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.5,0.3>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.2,0.6,0.7>}, \frac{m_2}{<0.5,0.2,0.3>}, \frac{m_3}{<0.6,0.3,0.5>}, \frac{m_4}{<0.8,0.1,0.9>} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.6,0.2,0.4>}, \frac{m_2}{<0.2,0.7,0.6>}, \frac{m_3}{<0.4,0.3,0.5>}, \frac{m_4}{<0.1,0.5,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.1,0.3,0.5>}, \frac{m_2}{<0.1,0.7,0.6>}, \frac{m_3}{<0.2,0.7,0.4>}, \frac{m_4}{<0.4,0.6,0.8>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.3>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.1,0.7,0.3>}, \frac{m_2}{<0.8,0.1,0.2>}, \frac{m_3}{<0.7,0.2,0.4>}, \frac{m_4}{<0.2,0.7,0.6>} \right\} \right) \end{array} \right\}$$

$$(\xi_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.3,0.4>}, \frac{m_2}{<0.7,0.4,0.5>}, \frac{m_3}{<0.5,0.4,0.6>}, \frac{m_4}{<0.2,0.4,0.7>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.8>}, \frac{m_2}{<0.8,0.3,0.5>}, \frac{m_3}{<0.4,0.3,0.5>}, \frac{m_4}{<0.2,0.6,0.7>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.3,0.6>}, \frac{m_2}{<0.9,0.1,0.7>}, \frac{m_3}{<0.4,0.5,0.8>}, \frac{m_4}{<0.5,0.3,0.5>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.7,0.4,0.5>}, \frac{m_4}{<0.9,0.5,0.7} \right\} \right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.3>}, \frac{m_2}{<0.5,0.2,0.4>}, \frac{m_3}{<0.6,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.6} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.3,0.7>}, \frac{m_2}{<0.3,0.5,0.6>}, \frac{m_3}{<0.5,0.4,0.3>}, \frac{m_4}{<0.2,0.6,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.2,0.6,0.3>}, \frac{m_3}{<0.3,0.5,0.6>}, \frac{m_4}{<0.5,0.3,0.7>} \right\} \right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.6,0.3>}, \frac{m_2}{<0.9,0.1,0.2>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.2,0.6,0.3>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.4,0.5,0.6>}, \frac{m_3}{<0.7,0.2,0.3>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.7,0.2,0.3>}, \frac{m_3}{<0.8,0.2,0.6>}, \frac{m_4}{<0.3,0.5,0.7>} \right\} \right) \end{array} \right\}$$

Then $(\xi_1, A_1) \cup (\xi_2, A_2) = (\xi_3, A_3)$

$$(\xi_3, A_3) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.3,0.4>}, \frac{m_2}{<0.7,0.3,0.2>}, \frac{m_3}{<0.5,0.4,0.1>}, \frac{m_4}{<0.2,0.4,0.5>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.5>}, \frac{m_2}{<0.8,0.2,0.3>}, \frac{m_3}{<0.4,0.3,0.5>}, \frac{m_4}{<0.2,0.5,0.3>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.3,0.6>}, \frac{m_2}{<0.9,0.1,0.7>}, \frac{m_3}{<0.4,0.5,0.8>}, \frac{m_4}{<0.5,0.3,0.5>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.6,0.2,0.3>}, \frac{m_3}{<0.7,0.3,0.5>}, \frac{m_4}{<0.9,0.1,0.7} \right\} \right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.3>}, \frac{m_2}{<0.5,0.2,0.4>}, \frac{m_3}{<0.6,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.6} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.4>}, \frac{m_2}{<0.3,0.5,0.6>}, \frac{m_3}{<0.5,0.3,0.3>}, \frac{m_4}{<0.2,0.5,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.3,0.4>}, \frac{m_2}{<0.2,0.6,0.3>}, \frac{m_3}{<0.3,0.5,0.4>}, \frac{m_4}{<0.5,0.3,0.7>} \right\} \right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.6,0.3>}, \frac{m_2}{<0.9,0.1,0.2>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.2,0.6,0.3>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.2,0.6,0.5>}, \frac{m_2}{<0.4,0.5,0.6>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.3>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.5,0.3>}, \frac{m_2}{<0.8,0.1,0.2>}, \frac{m_3}{<0.8,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.6>} \right\} \right) \end{array} \right\}$$

Proposition 3.16. *If $(\xi_1, A_1), (\xi_2, A_2)$ and (ξ_3, A_3) are three single valued neutrosophic hypersoft expert sets over $\coprod$, then*

(1). $(\xi_1, A_1) \cup (\xi_2, A_2) = (\xi_2, A_2) \cup (\xi_1, A_1)$

(2). $((\xi_1, A_1) \cup (\xi_2, A_2)) \cup (\xi_3, A_3) = (\xi_1, A_1) \cup ((\xi_2, A_2) \cup (\xi_3, A_3))$

Definition 3.17. The intersection of $(\xi_1, \mathbb{S})$ and $(\xi_2, \mathbb{R})$ over $\coprod$ is (ξ_3, L) with $L = \mathbb{S} \cap \mathbb{R}$, defined as

$$\xi_3(\beta) = \begin{cases} \xi_1(\beta) & ; \beta \in \mathbb{S} - \mathbb{R} \\ \xi_2(\beta) & ; \beta \in \mathbb{R} - \mathbb{S} \\ \cap(\xi_1(\beta), \xi_2(\beta)) & ; \beta \in \mathbb{S} \cap \mathbb{R} \end{cases}$$

where $\cap(\xi_1(\beta), \xi_2(\beta)) = \{< u, \min\{\mu_1(\beta), \mu_2(\beta)\}, \max\{\nu_1(\beta), \nu_2(\beta)\}, \max\{\omega_1(\beta), \omega_2(\beta)\} >: u \in U\}$.

Example 3.18. Taking into consideration the concept of example 3.3, consider the following two sets

$$A_1 = \left\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1) \right\}$$

$$A_2 = \left\{ (\mu_1, s, 1), (\mu_3, s, 0), (\mu_3, s, 1), (\mu_1, t, 1), (\mu_3, t, 1), (\mu_1, t, 0), (\mu_3, t, 0), (\mu_1, u, 0), (\mu_3, u, 1), , (\mu_1, u, 1) \right\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two single valued neutrosophic hypersoft expert sets such that

$$(\xi_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.3,0.2>}, \frac{m_3}{<0.4,0.5,0.1>}, \frac{m_4}{<0.1,0.8,0.5>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.5>}, \frac{m_2}{<0.6,0.2,0.3>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.5,0.3>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.2,0.6,0.7>}, \frac{m_2}{<0.5,0.2,0.3>}, \frac{m_3}{<0.6,0.3,0.5>}, \frac{m_4}{<0.8,0.1,0.9>} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.6,0.2,0.4>}, \frac{m_2}{<0.2,0.7,0.6>}, \frac{m_3}{<0.4,0.3,0.5>}, \frac{m_4}{<0.1,0.5,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.1,0.3,0.5>}, \frac{m_2}{<0.1,0.7,0.6>}, \frac{m_3}{<0.2,0.7,0.4>}, \frac{m_4}{<0.4,0.6,0.8>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.3>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.1,0.7,0.3>}, \frac{m_2}{<0.8,0.1,0.2>}, \frac{m_3}{<0.7,0.2,0.4>}, \frac{m_4}{<0.2,0.7,0.6>} \right\} \right) \end{array} \right\}$$

$$(\xi_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.2,0.3,0.4>}, \frac{m_2}{<0.7,0.4,0.5>}, \frac{m_3}{<0.5,0.4,0.6>}, \frac{m_4}{<0.2,0.4,0.7>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.4,0.3,0.8>}, \frac{m_2}{<0.8,0.3,0.5>}, \frac{m_3}{<0.4,0.3,0.5>}, \frac{m_4}{<0.2,0.6,0.7>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.3,0.6>}, \frac{m_2}{<0.9,0.1,0.7>}, \frac{m_3}{<0.4,0.5,0.8>}, \frac{m_4}{<0.5,0.3,0.5>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.4,0.2,0.3>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.7,0.4,0.5>}, \frac{m_4}{<0.9,0.5,0.7>} \right\} \right), \\ \left((\mu_1, u, 1), \left\{ \frac{m_1}{<0.7,0.2,0.3>}, \frac{m_2}{<0.5,0.2,0.4>}, \frac{m_3}{<0.6,0.2,0.4>}, \frac{m_4}{<0.3,0.5,0.6>} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.7,0.3,0.7>}, \frac{m_2}{<0.3,0.5,0.6>}, \frac{m_3}{<0.5,0.4,0.3>}, \frac{m_4}{<0.2,0.6,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.2,0.6,0.3>}, \frac{m_3}{<0.3,0.5,0.6>}, \frac{m_4}{<0.5,0.3,0.7>} \right\} \right), \\ \left((\mu_1, t, 0), \left\{ \frac{m_1}{<0.1,0.6,0.3>}, \frac{m_2}{<0.9,0.1,0.2>}, \frac{m_3}{<0.6,0.3,0.4>}, \frac{m_4}{<0.2,0.6,0.3>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.2,0.7,0.5>}, \frac{m_2}{<0.4,0.5,0.6>}, \frac{m_3}{<0.7,0.2,0.3>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.2,0.5,0.4>}, \frac{m_2}{<0.7,0.2,0.3>}, \frac{m_3}{<0.8,0.2,0.6>}, \frac{m_4}{<0.3,0.5,0.7>} \right\} \right) \end{array} \right\}$$

Then $(\xi_1, A_1) \cap (\xi_2, A_2) = (\xi_3, A_3)$

$$(\xi_3, A_3) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.1,0.8,0.7>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.8>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left((\mu_3, t, 1), \left\{ \frac{m_1}{<0.2,0.6,0.7>}, \frac{m_2}{<0.5,0.4,0.5>}, \frac{m_3}{<0.6,0.4,0.5>}, \frac{m_4}{<0.8,0.1,0.7>} \right\} \right), \\ \left((\mu_3, u, 1), \left\{ \frac{m_1}{<0.6,0.3,0.7>}, \frac{m_2}{<0.2,0.7,0.6>}, \frac{m_3}{<0.4,0.4,0.5>}, \frac{m_4}{<0.1,0.6,0.4>} \right\} \right), \\ \left((\mu_1, u, 0), \left\{ \frac{m_1}{<0.1,0.5,0.5>}, \frac{m_2}{<0.1,0.6,0.6>}, \frac{m_3}{<0.2,0.7,0.6>}, \frac{m_4}{<0.4,0.6,0.8>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.7,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.2,0.3>}, \frac{m_4}{<0.7,0.2,0.4>} \right\} \right), \\ \left((\mu_3, t, 0), \left\{ \frac{m_1}{<0.1,0.7,0.4>}, \frac{m_2}{<0.8,0.2,0.3>}, \frac{m_3}{<0.7,0.2,0.6>}, \frac{m_4}{<0.2,0.7,0.7>} \right\} \right) \end{array} \right\}$$

Proposition 3.19. If $(\xi_1, A_1), (\xi_2, A_2)$ and (ξ_3, A_3) are three single valued neutrosophic hypersoft expert sets over $\coprod$, then

(1). $(\xi_1, A_1) \cap (\xi_2, A_2) = (\xi_2, A_2) \cap (\xi_1, A_1)$

(2). $((\xi_1, A_1) \cap (\xi_2, A_2)) \cap (\xi_3, A_3) = (\xi_1, A_1) \cap ((\xi_2, A_2) \cap (\xi_3, A_3))$

Proposition 3.20. *If* $(\xi_1, A_1),(\xi_2, A_2)$ *and* (ξ_3, A_3) *are three single valued neutrosophic hypersoft expert sets over* $\coprod$*, then*

(1). $(\xi_1, A_1) \cup ((\xi_2, A_2) \cap (\xi_3, A_3)) = ((\xi_1, A_1) \cup ((\xi_2, A_2)) \cap ((\xi_1, A_1) \cup (\xi_3, A_3))$

(2). $(\xi_1, A_1) \cap ((\xi_2, A_2) \cup (\xi_3, A_3)) = ((\xi_1, A_1) \cap ((\xi_2, A_2)) \cup ((\xi_1, A_1) \cap (\xi_3, A_3))$

Definition 3.21. If (ξ_1, A_1) and (ξ_2, A_2) are two single valued neutrosophic hypersoft expert sets over $\coprod$ then (ξ_1, A_1) AND (ξ_2, A_2) denoted by $(\xi_1, A_1) \wedge (\xi_2, A_2)$ is defined by
$(\xi_1, A_1) \wedge (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,
while $\xi_3(\beta, \gamma) = \xi_1(\beta) \cap \xi_2(\gamma), \forall (\beta, \gamma) \in A_1 \times A_2$.

Example 3.22. Taking into consideration the concept of example 3.3, let two sets

$$A_1 = \left\{ (\mu_1, s, 1), (\mu_1, t, 1), (\mu_3, s, 0) \right\}$$

$$A_2 = \left\{ (\mu_1, s, 0), (\mu_3, s, 1) \right\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two single valued neutrosophic hypersoft expert sets such that

$$(\xi_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.1,0.8,0.7>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.8>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.3>} \right\} \right) \end{array} \right\}$$

$$(\xi_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 0), \left\{ \frac{m_1}{<0.2,0.1,0.3>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.5,0.2,0.5>}, \frac{m_4}{<0.2,0.3,0.6>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.5,0.6>}, \frac{m_2}{<0.4,0.2,0.5>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \end{array} \right\}$$

Then $(\xi_3, A_3) \wedge (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,

$$(\xi_3, A_1 \times A_2) = \left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.1,0.8,0.7>} \right\} \right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.2,0.4,0.8>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.1,0.5,0.8>}, \frac{m_2}{<0.4,0.3,0.5>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.1,0.6,0.6>}, \frac{m_2}{<0.4,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.1,0.8,0.7>} \right\} \right), \\ \left(((\mu_3, s, 0), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.5,0.2,0.5>}, \frac{m_4}{<0.2,0.3,0.6>} \right\} \right), \\ \left(((\mu_3, s, 0), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.4>} \right\} \right), \end{array} \right\}$$

Definition 3.23. If (ξ_1, A_1) and (ξ_2, A_2) are two single valued neutrosophic hypersoft expert sets over $\coprod$ then (ξ_1, A_1) OR (ξ_2, A_2) denoted by $(\xi_1, A_1) \vee (\xi_2, A_2)$ is defined by
$(\xi_1, A_1) \vee (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,
while $\xi_3(\beta, \gamma) = \xi_1(\beta) \cup \xi_2(\gamma), \forall (\beta, \gamma) \in A_1 \times A_2$.

Example 3.24. Taking into consideration the concept of example 3.3, suppose the following sets

$$A_1 = \left\{ (\mu_1, s, 1), (\mu_1, t, 1), (\mu_3, s, 0) \right\}$$

$$A_2 = \left\{ (\mu_1, s, 0), (\mu_3, s, 1) \right\}$$

Suppose (ξ_1, A_1) and (ξ_2, A_2) over $\coprod$ are two single valued neutrosophic hypersoft expert sets such that

$$(\xi_1, A_1) = \left\{ \begin{array}{l} \left((\mu_1, s, 1), \left\{ \frac{m_1}{<0.1,0.6,0.4>}, \frac{m_2}{<0.6,0.4,0.5>}, \frac{m_3}{<0.4,0.5,0.6>}, \frac{m_4}{<0.1,0.8,0.7>} \right\} \right), \\ \left((\mu_1, t, 1), \left\{ \frac{m_1}{<0.3,0.4,0.8>}, \frac{m_2}{<0.6,0.3,0.5>}, \frac{m_3}{<0.2,0.5,0.6>}, \frac{m_4}{<0.1,0.6,0.7>} \right\} \right), \\ \left((\mu_3, s, 0), \left\{ \frac{m_1}{<0.1,0.6,0.9>}, \frac{m_2}{<0.3,0.6,0.7>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.3>} \right\} \right) \end{array} \right\}$$

$$(\xi_2, A_2) = \left\{ \begin{array}{l} \left((\mu_1, s, 0), \left\{ \frac{m_1}{<0.2,0.1,0.3>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.5,0.2,0.5>}, \frac{m_4}{<0.2,0.3,0.6>} \right\} \right), \\ \left((\mu_3, s, 1), \left\{ \frac{m_1}{<0.1,0.5,0.6>}, \frac{m_2}{<0.4,0.2,0.5>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \end{array} \right\}$$

Then $(\xi_3, A_3) \vee (\xi_2, A_2) = (\xi_3, A_1 \times A_2)$,

$$(\xi_3, A_1 \times A_2) = \left\{ \begin{array}{l} \left(((\mu_1, s, 1), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.2,0.1,0.3>}, \frac{m_2}{0<0.7,0.2,0.4>}, \frac{m_3}{<0.5,0.2,0.5>}, \frac{m_4}{<0.2,0.3,0.6>} \right\} \right), \\ \left(((\mu_1, t, 1), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.3,0.1,0.3>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.5,0.2,0.5>}, \frac{m_4}{<0.2,0.3,0.6>} \right\} \right), \\ \left(((\mu_1, t, 1), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.3,0.4,0.6>}, \frac{m_2}{<0.6,0.2,0.5>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \\ \left(((\mu_1, s, 1), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.1,0.5,0.4>}, \frac{m_2}{<0.6,0.2,0.5>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right), \\ \left(((\mu_3, s, 0), (\mu_1, s, 0)), \left\{ \frac{m_1}{<0.2,0.1,0.3>}, \frac{m_2}{<0.7,0.2,0.4>}, \frac{m_3}{<0.6,0.1,0.2>}, \frac{m_4}{<0.7,0.2,0.3>} \right\} \right), \\ \left(((\mu_3, s, 0), (\mu_3, s, 1)), \left\{ \frac{m_1}{<0.1,0.5,0.6>}, \frac{m_2}{<0.4,0.2,0.5>}, \frac{m_3}{<0.7,0.1,0.2>}, \frac{m_4}{<0.8,0.1,0.4>} \right\} \right) \end{array} \right\}$$

Proposition 3.25. *If $(\xi_1, A_1), (\xi_2, A_2)$ and (ξ_3, A_3) are three single valued neutrosophic hypersoft expert sets over $\coprod$, then*

(1). $((\xi_1, A_1) \wedge (\xi_2, A_2))^c = ((\xi_1, A_1))^c \vee ((\xi_2, A_2))^c$
(2). $((\xi_1, A_1) \vee (\xi_2, A_2))^c = ((\xi_1, A_1))^c \wedge ((\xi_2, A_2))^c$

Proposition 3.26. *If $(\xi_1, A_1), (\xi_2, A_2)$ and (ξ_3, A_3) are three single valued neutrosophic hypersoft expert sets over $\coprod$, then*

(1). $((\xi_1, A_1) \wedge (\xi_2, A_2)) \wedge (\xi_3, A_3) = (\xi_1, A_1) \wedge ((\xi_2, A_2) \wedge (\xi_3, A_3))$
(2). $((\xi_1, A_1) \vee (\xi_2, A_2)) \vee (\xi_3, A_3) = (\xi_1, A_1) \vee ((\xi_2, A_2) \vee (\xi_3, A_3))$
(3). $(\xi_1, A_1) \vee ((\xi_2, A_2) \wedge (\xi_3, A_3)) = ((\xi_1, A_1) \vee ((\xi_2, A_2)) \wedge ((\xi_1, A_1) \vee (\xi_3, A_3))$
(4). $(\xi_1, A_1) \wedge ((\xi_2, A_2) \vee (\xi_3, A_3)) = ((\xi_1, A_1) \wedge ((\xi_2, A_2)) \vee ((\xi_1, A_1) \wedge (\xi_3, A_3))$

4. An Application to Single valued Neutrosophic Hypersoft expert set

In this section, an application of single valued neutrosophic hypersoft expert set theory in a decision making problem, is presented.

Statement of the problem

Mr. John wants to purchase a mobile from a mobile market for his personal use. He takes help from his some friends (Stephen, Thomas and Umar) who have expertise in mobile purchase.

Proposed Algorithm

The following algorithm is adopted for this selection (purchase).

(1). Construct SVNHSES (ξ, K),

(2). Determine the values of $\mu(c_i) - \nu(c_i) - \omega(c_i)$ for each $c_i \in \coprod$ where $\mu(c_i)$ is a membership function, $\nu(c_i)$ indeterminacy function and $\omega(c_i)$ is a non membership function for each element of $\coprod$.

(3). Calculate the the highest numerical grade for the agree-SVNHSES and disagree-SVNHSES,

(4). Determine the score of each element $c_i \in \coprod$ by taking the sum of the products of the numerical grade of each element for the agree- SVNHSES and disagree SVNHSES,denoted by G_i and H_i respectively

(5). Determine $j_i = G_i - H_i$ for each element $c_i \in \coprod$,

(6). Compute n, for which $M = \max j_i$. Then the decision is to choose element as the optimal or best solution to the problem.

Step-1

Let four categories of mobile are there which form the universe of discourse $coprod = \{c_1, c_2, c_3, c_4\}$ and X $= \{E_1 = Stephen, E_2 = Thomas, E_3 = Umar\}$ be a set of experts for this purchase. The following are the attribute-valued sets for prescribed attributes:

$L_1 = Brand = \{X = l_1, Y = l_2\}$

$L_2 = Price = \{20,000 = l_3, 15,000 = l_4\}$

$L_3 = Colour = \{White = l_5, Blue = l_6\}$

$L_4 = Memory = \{6GB = l_7, 4GB = l_8\}$

$L_5 = Resolution(size) = \{5inch = l_9, 6inch = l_{10}\}$

and then

$L = L_1 \times L_2 \times L_3 \times L_4 \times L_5$

Muhammad Ihsan, Atiqe Ur Rahman, Muhammad Saeed, Single Valued Neutrosophic Hypersoft Expert Set with Application in Decision Making

$$L = \left\{ \begin{array}{l} (l_1,l_3,l_5,l_7,l_9), (l_1,l_3,l_5,l_7,l_{10}), (l_1,l_3,l_5,l_8,l_9), (l_1,l_3,l_5,l_8,l_{10}), (l_1,l_3,l_6,l_7,l_9), \\ (l_1,l_3,l_6,l_7,l_{10}), (l_1,l_3,l_6,l_8,l_9), (l_1,l_3,l_6,l_8,l_{10}), (l_1,l_4,l_5,l_7,l_9), (l_1,l_4,l_5,l_7,l_{10}), \\ (l_1,l_4,l_5,l_8,l_9), (l_1,l_4,l_5,l_8,l_{10}), (l_1,l_4,l_6,l_7,l_9), (l_1,l_4,l_6,l_7,l_{10}), (l_1,l_4,l_6,l_8,l_9), \\ (l_1,l_4,l_6,l_8,l_{10}), (l_2,l_3,l_5,l_7,l_9), (l_2,l_3,l_5,l_7,l_{10}), (l_2,l_3,l_5,l_8,l_9), (l_2,l_3,l_5,l_8,l_{10}), \\ (l_2,l_3,l_6,l_7,l_9), (l_2,l_3,l_6,l_7,l_{10}), (l_2,l_3,l_6,l_8,l_9), (l_2,l_3,l_6,l_8,l_{10}), (l_2,l_4,l_5,l_7,l_9), \\ (l_2,l_4,l_5,l_7,l_{10}), (l_2,l_4,l_5,l_8,l_9), (l_2,l_4,l_5,l_8,l_{10}), (l_2,l_4,l_6,l_7,l_9), (l_2,l_4,l_6,l_7,l_{10}), \\ (l_2,l_4,l_6,l_8,l_9), (l_2,l_4,l_6,l_8,l_{10}) \end{array} \right\}$$

Now take $K \subseteq L$ as

$K = \{k_1 = (l_1,l_3,l_5,l_7,l_9), k_2 = (l_1,l_3,l_6,l_7,l_{10}), k_3 = (l_1,l_4,l_6,l_8,l_9), k_4 = (l_2,l_3,l_6,l_8,l_9), k_5 = (l_2,l_4,l_6,l_7,l_{10})\}$

$$(\xi, A)_1 = \left\{ \begin{array}{l} \left((k_1,E_1,1), \left\{ \frac{c_1}{<0.9,0.1,0.7>}, \frac{c_2}{<0.3,0.4,0.5>}, \frac{c_3}{<0.6,0.2,0.4>}, \frac{c_4}{<0.3,0.7,0.2>} \right\}\right), \\ \left((k_1,E_2,1), \left\{ \frac{c_1}{<0.8,0.2,0.7>}, \frac{c_2}{<0.1,0.2,0.3>}, \frac{c_3}{<0.6,0.2,0.8>}, \frac{c_4}{<0.3,0.6,0.5>} \right\}\right), \\ \left((k_1,E_3,1), \left\{ \frac{c_1}{<0.7,0.3,0.1>}, \frac{c_2}{<0.3,0.7,0.2>}, \frac{c_3}{<0.3,0.1,0.4>}, \frac{c_4}{<0.3,0.6,0.7>} \right\}\right), \\ \left((k_2,E_1,1), \left\{ \frac{c_1}{<0.6,0.4,0.8>}, \frac{c_2}{<0.4,0.2,0.1>}, \frac{c_3}{<0.7,0.1,0.6>}, \frac{c_4}{<0.5,0.2,0.6>} \right\}\right), \\ \left((k_2,E_2,1), \left\{ \frac{c_1}{<0.5,0.2,0.3>}, \frac{c_2}{<0.6,0.4,0.5>}, \frac{c_3}{<0.3,0.6,0.7>}, \frac{c_4}{<0.4,0.2,0.9>} \right\}\right), \\ \left((k_2,E_3,1), \left\{ \frac{c_1}{<0.4,0.3,0.1>}, \frac{c_2}{<0.3,0.2,0.4>}, \frac{c_3}{<0.3,0.2,0.5>}, \frac{c_4}{<0.8,0.2,0.7>} \right\}\right), \\ \left((k_3,E_1,1), \left\{ \frac{c_1}{<0.2,0.4,0.9>}, \frac{c_2}{<0.5,0.2,0.6>}, \frac{c_3}{<0.6,0.2,0.7>}, \frac{c_4}{<0.7,0.2,0.6>} \right\}\right), \\ \left((k_3,E_2,1), \left\{ \frac{c_1}{<0.2,0.3,0.4>}, \frac{c_2}{<0.4,0.2,0.5>}, \frac{c_3}{<0.6,0.2,0.4>}, \frac{c_4}{<0.3,0.2,0.3>} \right\}\right), \\ \left((k_3,E_3,1), \left\{ \frac{c_1}{<0.3,0.4,0.1>}, \frac{c_2}{<0.6,0.3,0.2>}, \frac{c_3}{<0.4,0.2,0.3>}, \frac{c_4}{<0.3,0.6,0.4>} \right\}\right), \\ \left((k_4,E_1,1), \left\{ \frac{c_1}{<0.9,0.1,0.7>}, \frac{c_2}{<0.1,0.3,0.8>}, \frac{c_3}{<0.5,0.1,0.7>}, \frac{c_4}{<0.4,0.3,0.2>} \right\}\right), \\ \left((k_4,E_2,1), \left\{ \frac{c_1}{<0.8,0.1,0.4>}, \frac{c_2}{<0.4,0.3,0.5>}, \frac{c_3}{<0.2,0.7,0.6>}, \frac{c_4}{<0.7,0.2,0.7>} \right\}\right), \\ \left((k_4,E_3,1), \left\{ \frac{c_1}{<0.6,0.2,0.4>}, \frac{c_2}{<0.1,0.3,0.2>}, \frac{c_3}{<0.3,0.5,0.9>}, \frac{c_4}{<0.6,0.1,0.3>} \right\}\right), \\ \left((k_5,E_1,1), \left\{ \frac{c_1}{<0.6,0.3,0.2>}, \frac{c_2}{<0.2,0.8,0.4>}, \frac{c_3}{<0.1,0.2,.03>}, \frac{c_4}{<0.1,0.7,0.4>} \right\}\right), \\ \left((k_5,E_2,1), \left\{ \frac{c_1}{<0.5,0.3,0.4>}, \frac{c_2}{<0.6,0.2,0.4>}, \frac{c_3}{<0.4,0.3,0.5>}, \frac{c_4}{<0.6,0.1,0.2>} \right\}\right), \\ \left((k_5,E_3,1), \left\{ \frac{c_1}{<0.4,0.3,0.1>}, \frac{c_2}{<0.6,0.1,0.2>}, \frac{c_3}{<0.3,0.5,0.6>}, \frac{c_4}{<0.5,0.4,0.2>} \right\}\right), \end{array} \right\}$$

and

$$(\xi, \mathrm{K})_0 = \left\{ \begin{array}{l} \left((k_1, E_1, 0), \left\{ \frac{c_1}{<0.4,0.3,0.1>}, \frac{c_2}{<0.9,0.1,0.2>}, \frac{c_3}{<0.8,0.2,0.3>}, \frac{c_4}{<0.6,0.1,0.8>} \right\}\right), \\ \left((k_1, E_2, 0) \left\{ \frac{c_1}{<0.7,0.2,0.3>}, \frac{c_2}{<0.6,0.4,0.2>}, \frac{c_3}{<0.2,0.7,0.1>}, \frac{c_4}{<0.8,0.2,0.7>} \right\}\right), \\ \left((k_1, E_3, 0), \left\{ \frac{c_1}{<0.2,0.3,0.4>}, \frac{c_2}{<0.6,0.2,0.5>}, \frac{c_3}{<0.1,0.7,0.2>}, \frac{c_4}{<0.1,0.4,0.5>} \right\}\right), \\ \left((k_2, E_1, 0), \left\{ \frac{c_1}{<0.1,0.4,0.5>}, \frac{c_2}{<0.4,0.3,0.7>}, \frac{c_3}{<0.2,0.7,0.4>}, \frac{c_4}{<0.3,0.2,0.6>} \right\}\right), \\ \left((k_2, E_2, 0), \left\{ \frac{c_1}{<0.2,0.5,0.6>}, \frac{c_2}{<0.7,0.2,0.3>}, \frac{c_3}{<0.6,0.3,0.2>}, \frac{c_4}{<0.5,0.3,0.4>} \right\}\right), \\ \left((k_2, E_3, 0), \left\{ \frac{c_1}{<0.7,0.2,0.1>}, \frac{c_2}{<0.3,0.5,0.2>}, \frac{c_3}{<0.1,0.7,0.8>}, \frac{c_4}{<0.3,0.4,0.3>} \right\}\right), \\ \left((k_3, E_1, 0), \left\{ \frac{c_1}{<0.9,0.1,0.6>}, \frac{c_2}{<0.4,0.2,0.7>}, \frac{c_3}{<0.5,0.1,0.9>}, \frac{c_4}{<0.3,0.2,0.1>} \right\}\right), \\ \left((k_3, E_2, 0), \left\{ \frac{c_1}{<0.8,0.2,0.1>}, \frac{c_2}{<0.2,0.7,0.4>}, \frac{c_3}{<0.3,0.6,0.7>}, \frac{c_4}{<0.5,0.2,0.6>} \right\}\right), \\ \left((k_3, E_3, 0), \left\{ \frac{c_1}{<0.6,0.2,0.4>}, \frac{c_2}{<0.6,0.1,0.3>}, \frac{c_3}{<0.5,0.4,0.9>}, \frac{c_4}{<0.4,0.5,0.2>} \right\}\right), \\ \left((k_4, E_1, 0), \left\{ \frac{c_1}{<0.6,0.3,0.2>}, \frac{c_2}{<0.3,0.5,0.6>}, \frac{c_3}{<0.2,0.6,0.5>}, \frac{c_4}{<0.3,0.4,0.6>} \right\}\right), \\ \left((k_4, E_2, 0), \left\{ \frac{c_1}{<0.5,0.4,0.3>}, \frac{c_2}{<0.5,0.2,0.1>}, \frac{c_3}{<0.3,0.1,0.2>}, \frac{c_4}{<0.6,0.2,0.1>} \right\}\right), \\ \left((k_4, E_3, 0), \left\{ \frac{c_1}{<0.4,0.5,0.3>}, \frac{c_2}{<0.7,0.2,0.1>}, \frac{c_3}{<0.6,0.1,0.4>}, \frac{c_4}{<0.9,0.1,0.6>} \right\}\right), \\ \left((k_5, E_1, 0), \left\{ \frac{c_1}{<0.2,0.5,0.7>}, \frac{c_2}{<0.6,0.2,0.8>}, \frac{c_3}{<0.9,0.1,0.3>}, \frac{c_4}{<0.9,0.2,0.5>} \right\}\right), \\ \left((k_5, E_2, 0), \left\{ \frac{c_1}{<0.3,0.6,0.1>}, \frac{c_2}{<0.2,0.8,0.3>}, \frac{c_3}{<0.3,0.6,0.2>}, \frac{c_4}{<0.4,0.3,0.2>} \right\}\right), \\ \left((k_5, E_3, 0), \left\{ \frac{c_1}{<0.1,0.7,0.3>}, \frac{c_2}{<0.5,0.2,0.4>}, \frac{c_3}{<0.5,0.1,0.7>}, \frac{c_4}{<0.4,0.6,0.3>} \right\}\right) \end{array} \right\}$$

are single valued neutrosophic hypersoft expert sets.

Step-2

Table 1 represents the values of $\mu(c_i)$-$\nu(c_i)$-$\omega(c_i)$

Step-(2-5)

Table 2 and table 3 represent the grade values of agree and disagree single valued neutrosophic hypersoft expert set respectively. Table 4 depicts the difference of scores of agree and disagree SVNHSES. The scores for agree SVNHSES are :

$S(c_1) = 0.6$, $S(c_2) = 0.5$, $S(c_3) = 0.4$ and $S(c_4) = 1.5$

whereas scores for disagree SVNHSES are:

$S(c_1) = 1.4$, $S(c_2) = 0.7$, $S(c_3) = 0.5$ and $S(c_4) = -0.2$.

Step-6; Decision

As j_4 is maximum, so category c_4 is preferred to be best.

TABLE 1. Agree-single valued neutrosophic hypersoft expert set

C	c_1	c_2	c_3	c_4	C	c_1	c_2	c_3	c_4
$(k_1, E_1, 1)$	0.1	-0.6	0.0	-0.6	$(k_1, E_1, 0)$	0.0	0.6	0.3	-0.3
$(k_1, E_2, 1)$	-0.1	-0.4	-0.4	0.2	$(k_1, E_2, 0)$	0.2	0.0	-0.6	-0.1
$(k_1, E_3, 1)$	0.3	-0.2	-0.2	0.4	$(k_1, E_3, 0)$	-0.5	-0.1	-0.8	-0.8
$(k_2, E_1, 1)$	-0.6	0.1	0.0	-0.3	$(k_2, E_1, 0)$	-0.8	-0.6	-0.9	-0.5
$(k_2, E_2, 1)$	0.0	-0.3	0.4	-0.7	$(k_2, E_2, 0)$	-0.9	0.2	0.1	-0.2
$(k_2, E_3, 1)$	0.0	-0.3	-0.4	-0.1	$(k_2, E_3, 0)$	0.4	-0.4	-1.4	-0.4
$(k_3, E_1, 1)$	-1.1	-0.3	-0.3	-0.1	$(k_3, E_1, 0)$	0.2	-0.5	-0.5	0.0
$(k_3, E_2, 1)$	-0.5	-0.3	0.0	-0.2	$(k_3, E_2, 0)$	0.5	-0.9	-1.0	-0.3
$(k_3, E_3, 1)$	-0.2	0.1	-0.1	-0.7	$(k_3, E_3, 0)$	0.0	0.2	-0.8	-0.3
$(k_4, E_1, 1)$	0.1	-1.0	-0.3	-0.1	$(k_4, E_1, 0)$	0.1	-0.8	-0.9	-0.7
$(k_4, E_2, 1)$	0.3	-0.6	-1.1	-0.2	$(k_4, E_2, 0)$	-0.1	0.2	0.0	0.3
$(k_4, E_3, 1)$	0.0	-0.4	-1.1	0.2	$(k_4, E_3, 0)$	-1.4	0.4	0.1	0.2
$(k_5, E_1, 1)$	0.1	-0.7	-0.4	-1.0	$(k_5, E_1, 0)$	-1.0	-0.4	0.5	0.2
$(k_5, E_2, 1)$	-0.2	0.0	-0.4	0.3	$(k_5, E_2, 0)$	-0.4	-0.9	-0.5	0.1
$(k_5, E_3, 1)$	0.0	0.3	-0.8	-0.1	$(k_5, E_3, 0)$	-0.9	-0.1	-0.3	-0.5

TABLE 2. Numerical Grades of agree SVNHSES

	c_i	Highest Numerical Grade
$(k_1, E_1, 1)$	c_1	0.1
$(k_1, E_2, 1)$	c_4	0.2
$(k_1, E_3, 1)$	c_4	0.4
$(k_2, E_1, 1)$	c_2	0.1
$(k_2, E_2, 1)$	c_3	0.4
$(k_2, E_3, 1)$	c_4	0.6
$(k_3, E_1, 1)$	c_1	0.0
$(k_3, E_2, 1)$	c_3	0.0
$(k_3, E_3, 1)$	c_2	0.1
$(k_4, E_1, 1)$	c_1	0.1
$(k_4, E_2, 1)$	c_1	0.3
$(k_4, E_3, 1)$	c_4	0.2
$(k_5, E_1, 1)$	c_1	0.1
$(k_5, E_2, 1)$	c_4	0.3
$(k_5, E_3, 1)$	c_2	0.3

5. Conclusions

In this paper, the fundamentals of single valued neutrosophic hypersoft expert set are established and some basic properties, laws and operations are generalized. A decision-making

TABLE 3. Numerical Grades of disagree SVNHSES

	c_i	Highest Numerical Grade
$(k_1, E_1, 0)$	c_2	0.6
$(k_1, E_2, 0)$	c_1	0.2
$(k_1, E_3, 0)$	c_2	-0.1
$(k_2, E_1, 0)$	c_4	-0.5
$(k_2, E_2, 0)$	c_2	0.2
$(k_2, E_3, 0)$	c_1	0.4
$(k_3, E_1, 0)$	c_1	0.2
$(k_3, E_2, 0)$	c_1	0.5
$(k_3, E_3, 0)$	c_2	0.2
$(k_4, E_1, 0)$	c_1	0.1
$(k_4, E_2, 0)$	c_4	0.3
$(k_4, E_3, 0)$	c_2	0.4
$(k_5, E_1, 0)$	c_3	0.5
$(k_5, E_2, 0)$	c_2	0.1
$(k_5, E_3, 0)$	c_2	-0.1

TABLE 4. Numerical values of $j_i = G_i - H_i$

G_i	H_i	$j_i = G_i - H_i$
$S(c_1) = 0.6$	$S(c_1) = 1.4$	-1.8
$S(c_2) = 0.5$	$S(c_2) = 0.7$	-0.2
$S(c_3) = 0.4$	$S(c_3) = 0.5$	-0.1
$S(c_4) = 1.5$	$S(c_4) = -0.2$	1.7

application regarding the selection of the best product is presented with the help of proposed algorithm. Future work may include the extension of the presented work for other single valued neutrosophic hypersoft-like hybrids.

Conflicts of Interest: "The authors declare no conflict of interest."

References

1. Smarandache, F. (2005). A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic. Rehoboth: American Research Press.
2. Smarandache, F. (2005). Neutrosophic set, a generialization of the intuituionistics fuzzy sets. Inter. J. Pure Appl. Math., 24, 287 297.
3. Smarandache, F. (2013). Introduction to neutrosophic measure, neutrosophic measure neutrosophic integral, and neutrosophic propability. http://fs.gallup.unm.edu/eBooks-otherformats.htm EAN: 9781599732534.
4. Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338-353.
5. Atanassov, K. T. (1986). Intuitionistic Fuzzy Sets. Fuzzy Sets and Systems, 20(1), 87-96.

6. Wang, H., Smarandache, F., Zhang, Y., & Sunderraman, R. (2010). Single valued Neutrosophic Sets. Multisspace and Multistructure, 4, 410-413.

7. Broumi, S., & Smarandache, F. (2015). Single valued neutrosophic soft expert sets and their application in decision making. Journal of New Theory, 3, 67-88.

8. Molodtsov, D. (1999). Soft set theoryfirst results. Computers and Mathematics with Applications, 37(4-5), 19-31.

9. Maji, P. K., Roy, A. R., & Biswas, R. (2001). Fuzzy Soft Sets. Journal of Fuzzy Mathematics, 9, 589-602.

10. Maji, P. K., Roy, A. R., & Biswas, R. (2003). Soft set theory. Computers and Mathematics with Applications. 45(4-5), 555-562 45(4-5), 555-562.

11. Maji, P. K., Roy, A. R., & Biswas, R. (2002). An application of soft sets in a decision making problem. Computers & Mathematics with Applications, 44(8-9), 1077-1083.

12. Ali, M. I., Feng, F., Liu, X., Min, W. K., & Shabir, M. (2009). On some new operations in soft set theory. Computers & Mathematics with Applications, 57(9), 1547-1553.

13. Babitha, K. V., & Sunil, J. (2010). Soft set relations and functions. Computers & Mathematics with Applications, 60(7), 1840-1849.

14. Babitha, K. V., & Sunil, J. J. (2011). Transitive closures and orderings on soft sets. Computers & Mathematics with Applications, 62(5), 2235-2239.

15. Ge, X., & Yang, S. (2011). Investigations on some operations of soft sets. World Academy of Science, Engineering and Technology, 51, 1112-1115.

16. Alkhazaleh, S.,& Salleh, A. R. (2011). Soft Expert Sets. Adv. Decis. Sci., 2011, 757868-1.

17. Alkhazaleh, S., & Salleh, A. R. (2014). Fuzzy soft expert set and its application. Applied Mathematics, 2014, 5, 1349-1368.

18. Broumi, S., & Smarandache, F. (2015). Intuitionistic fuzzy soft expert sets and its application in decision making. Journal of new theory, 1, 89-105.

19. Smarandache, F. (2018). Extension of soft set to hypersoft set, and then to plithogenic hypersoft set. Neutrosophic Sets and Systems, 22, 168-170.

20. Saeed, M., Rahman, A. U., Ahsan, M., & Smarandache, F. (2021). An inclusive study on fundamentals of hypersoft set. Theory and Application of Hypersoft Set, (pp.1-23). Pons Publication House.

21. Abbas, M., Murtaza, G., & Smarandache, F. (2020). Basic operations on hypersoft sets and hypersoft point. Neutrosophic Sets and Systems, 35, 407-421.

22. Rahman, A. U., Saeed, M., Smarandache, F., & Ahmad, M. R. (2020). Development of Hybrids of Hypersoft Set with Complex Fuzzy Set, Complex Intuitionistic Fuzzy set and Complex Neutrosophic Set. Neutrosophic Sets and Systems, 38, 335-354.

23. Rahman, A. U., Saeed, M., & Smarandache, F. (2020). Convex and concave hypersoft sets with some properties, Neutrosophic Sets and Systems, 38, 497-508.

24. Rahman, A. U., Saeed, M., & Dhital, A. (2021). Decision making application based on neutrosophic parameterized hypersoft set theory. Neutrosophic Sets and Systems, 41, 1-14.

25. Rahman, A. U., Saeed, M., & Hafeez, A. (2021). Theory of Bijective Hypersoft Set with Application in Decision Making. Punjab University Journal of Mathematics, 53(7), 511-526.

26. Rahman, A. U., Saeed, M., Alodhaibi, S. S., & Khalifa, H. A. E. W. (2021). Decision Making Algorithmic Approaches Based on Parameterization of Neutrosophic Set under Hypersoft Set Environment with Fuzzy, Intuitionistic Fuzzy and Neutrosophic Settings. CMES-Computer Modeling in Engineering & Sciences, 128(2), 743-777.

27. Rahman, A. U., Hafeez, A., Saeed, M., Ahmad, M. R., & Farwa, U. (2021). Development of rough hypersoft set with application in decision making for the best choice of chemical material. In Theory and Application of Hypersoft Set, (pp. 192-202). Pons Publication House.

28. Rahman, A. U., Saeed, M., & Zahid, S. (2021). Application in decision making based on fuzzy parameterized hypersoft set theory. Asia Mathematika, 5(1), 19-27.

29. Rahman, A. U., Saeed, M., Arshad, M., & Dhital, A. (2021). A Novel Approach to Neutrosophic Hypersoft Graphs with Properties. Neutrosophic Sets and Systems, 46, 336-355.

30. Rahman, A. U., Saeed, M., Khalid, A., Ahmad, M. R.,& Ayaz, S. (2021). Decision-Making Application Based on Aggregations of Complex Fuzzy Hypersoft Set and Development of Interval-Valued Complex Fuzzy Hypersoft Set. Neutrosophic Sets and Systems, 46, 300-317.

31. Saeed, M., Ahsan, M., & Abdeljawad, T. (2021). A development of complex multi-fuzzy hypersoft set with application in MCDM based on entropy and similarity measure. IEEE Access, 9, 60026-60042.

32. Saeed, M., Ahsan, M., & Rahman, A. U. (2021). A novel approach to mappings on hypersoft classes with application. In Theory and Application of Hypersoft Set (pp. 175-191). Pons Publication House.

33. Saeed, M., Rahman, A. U., & Arshad, M. (2021). A study on some operations and products of neutrosophic hypersoft graphs. Journal of Applied Mathematics and Computing, 1-28.

34. Saeed, M., Ahsan, M., Saeed, M. H., Mehmood, A., & Abdeljawad, T. (2021). An Application of Neutrosophic Hypersoft Mapping to Diagnose Hepatitis and Propose Appropriate Treatment. IEEE Access, 9, 70455-70471.

35. Saeed, M., Siddique, M. K., Ahsan, M., Ahmad, M. R., & Rahman, A. U. A Novel Approach to the Rudiments of Hypersoft Graphs. In Theory and Application of Hypersoft Set, (pp.203-214). Pons Publication House.

36. Saeed, M., Ahsan, M., Rahman, A. U., Saeed, M. H., & Mehmood, A. (2021). An application of neutrosophic hypersoft mapping to diagnose brain tumor and propose appropriate treatment. Journal of Intelligent & Fuzzy Systems, 41(1), 677-1699.

37. Ihsan, M., Rahman, A. U., & Saeed, M. (2021). Hypersoft Expert Set With Application in Decision Making for Recruitment Process. Neutrosophic Sets and Systems, 42, 191-207.

38. Ihsan, M., Rahman, A. U., Saeed, M., (2021). Fuzzy Hypersoft Expert Set with Application in Decision Making for the Best Selection of Product. Neutrosophic Sets and Systems, 46, 318-335.

39. Saqlain, M., Saeed, M., Ahmad, M. R., & Smarandache, F. (2019). Generalization of TOPSIS for Neutrosophic Hypersoft set using Accuracy Function and its Application. Neutrosophic Sets and Systems, 27, 131-137.

40. Zulqarnain, R. M., Xin, X. L., & Saeed, M. (2020). Extension of TOPSIS method under intuitionistic fuzzy hypersoft environment based on correlation coefficient and aggregation operators to solve decision making problem. AIMS Mathematics, 6(3), 2732-2755.

41. Saqlain, M., Riaz, M., Saleem, M. A., & Yang, M. S. (2021). Distance and Similarity Measures for Neutrosophic HyperSoft Set (NHSS) With Construction of NHSS-TOPSIS and Applications. IEEE Access, 9, 30803-30816.

42. Farooq, M. U., & Saqlain, M. (2021). The Application of Neutrosophic Hypersoft Set TOPSIS (NHSS-TOPSIS) in the Selection of Carbon Nano Tube based Field Effective Transistors CNTFETs. Neutrosophic Sets and Systems, 43, 212-224.

43. Saqlain, M., Jafar, N., Moin, S., Saeed, M., & Broumi, S. (2020). Single and multi-valued neutrosophic hypersoft set and tangent similarity measure of single valued neutrosophic hypersoft sets. Neutrosophic Sets and Systems, 32, 317-329.