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Optimal Supplier Selection Via Decision-Making Algorithmic Technique Based on Single-Valued Neutrosophic Fuzzy Hypersoft Set

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Abstract. Hypersoft set, an extension of soft set, is more flexible and useful as it tackles the limitation of soft set for dealing with scenarios where distinct attributes are further classified into disjoint attribute-valued sets. It replaces single-argument approximate function of soft set with multi-argument approximate function. The main goal of this research is to align existing literature on single-valued neutrosophic fuzzy soft sets with the need for such a multi-argument function. Firstly, the novel notions of single-valued neutrosophic fuzzy hypersoft sets are characterized. Some of its essential basic properties and set theoretic operations are discussed with illustrated numerical examples. Secondly, fuzzy decision-making algorithm based on single-valued neutrosophic fuzzy hypersoft set matrix is proposed. Explicatory application is presented which depicts the structural validity of proposed structure for successful application to the problems involving vagueness and uncertainties. Lastly, a comparison of the proposed structure with existing structures, is made under appropriate indicators.

Keywords: Fuzzy set; Neutrosophic set; Single-valued neutrosophic set; Single-valued neutrosophic soft set; Single-valued neutrosophic fuzzy soft set; Hypersoft set.

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1. **Introduction**

In different mathematical disciplines, fuzzy sets theory (FS-Theory) [1] and intuitionistic fuzzy set theory (IFS-Theory) [2] are considered apt mathematical modes to tackle several intricate problems involving various uncertainties. The former emphasizes on a certain object's degree of true belongingness from the initial sample space, while the latter emphasizes degree of true membership and degree of non-membership with the state of their interdependence. These theories portray some kind of inadequacy in terms of providing due status to a degree of indeterminacy. The implementation of neutrosophic set theory (NS-Theory) [3, 4] overcomes this impediment by taking into account not only the proper status of degree of indeterminacy but also the state of dependence. This theory is more adaptable and suitable for dealing with inconsistent data. Wang et el [5] conceptualized single-valued neutrosophic set in which truth membership degree, indeterminacy degree and falsity degree are restricted within unit closed interval. Many researchers [6]- [14] have been drawn to NS-Theory for further application in statistics, topological spaces, and the construction of some neutrosophic-like blended structures with other existing models for useful applications in decision making. Edalatpanah [15] studied a system of neutrosophic linear equations (SNLE) based on the embedding approach. He used (α, β, γ)-cut for transformation of SNLE into a crisp linear system. Kumar et al. [16] exhibited a novel linear programming approach for finding the neutrosophic shortest path problem (NSSPP) considering Gaussian valued neutrosophic number.

FS-Theory, IFS-Theory and NS-Theory have some kind of complexities which restrain them to solve problem involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory (SS-Theory) [17] which is a new parameterized family of subsets of the universe of discourse. The researchers [18]- [27] studied and investigated some elementary properties, operations, laws and hybrids of SS-Theory with applications in decision making. The gluing concept of NS-Theory and SS-Theory, is studied in [28] to make the NS-Theory adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SS-Theory is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HS-Theory) [29] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. HS-Theory is an extension of SS-Theory as it transforms the single argument function into a multi-argument function. Certain elementary properties, aggregation operations, laws, relations and functions of HS-Theory, are investigated by [30]- [32] for proper understanding and further utilization in different fields. The applications of HS-Theory in decision making is studied by [33]- [36] and the intermingling study of HS-Theory with complex sets, convex and concave sets is studied by [37, 38]. Deli [39] characterized hybrid set structures under uncertainly parameterized hypersoft sets with theory and applications. Gayen et al. [40] analyzed some essential aspects of plithogenic hypersoft algebraic structures. They also investigated the notions and basic properties of plithogenic hypersoft subgroups ie plithogenic fuzzy hypersoft subgroup, plithogenic intuitionistic fuzzy hypersoft subgroup, plithogenic neutrosophic hypersoft subgroup. Saeed et al. [41, 42] discussed decision making techniques for neutrosophic hypersoft mapping and complex multi-fuzzy hypersoft set. Rahman et al. [43–45] studied decision making applications based on neutrosophic parameterized hypersoft Set, fuzzy parameterized hypersoft set and rough hypersoft set. Ihsan et al. [46] investigated hypersoft expert set with application in decision making for the best selection of product.

1.1. *Motivation*

The attributes must be further partitioned into attribute values in a variety of real-world applications for a more vivid understanding. As a generalization of soft set, hypersoft set overcomes this restriction and emphasizes the disjoint attribute-valued sets for distinct attributes. This generalization shows that using the hypersoft set with neutrosophic, intuitionistic, and fuzzy set theory to build a relation between alternatives and attributes would be extremely useful. In addition, the hypersoft set will reduce the case study's complexity. It's important to note that the hypersoft theory can be applied to every decision-making challenge, regardless of the decision-makers' ability to choose values. Multi-criteria decision making (MCDM), Multi-criteria group decision making (MCGDM), shortest path selection, employee selection, e-learning, graph theory, medical diagnosis, and other applications may all benefit from this theory. The current literature on soft sets should be adequate with regard to the presence and consideration of attribute-valued sets, so the main goal of this research is to extend the concept presented in [48, 49] and to develop novel theory of single-valued neutrosophic fuzzy hypersoft set (an embedded structure of fuzzy set, single-valued neutrosophic set and hypersoft set). After characterizing its basic essential properties and operations, decision-making based algorithm is proposed to solve a real life problem relating to the purchase of most suitable and appropriate supplier with the help of single-valued neutrosophic fuzzy hypersoft set matrix.

1.2. *Paper Organization*

The remaining paper is systemized as:

Section 2 : Some essential definitions and terminologies are recalled.

Section 3 : Theory of single-valued neutrosophic fuzzy hypersoft set is developed with suitable examples.

Section 4 : Decision-making based algorithm and application of single-valued neutrosophic fuzzy hypersoft set are presented.

Section 5 : Comparison Analysis of proposed structures is discussed.

Section 6 : Paper is summarized with future directions.

2. Preliminaries

In the following, we present a short survey of definitions which are necessary to this paper. Here some basic terms are recalled from existing literature to support the proposed work. Throughout the paper, $\hat{\mathcal{V}}$ and $\mathbb{I}$ will denote the universe of discourse and closed unit interval respectively. In this work, algorithmic approach is followed from decision making method stated in [49].

Definition 2.1. [1]
A *fuzzy set* $\mathcal{F}$ defined as $\mathcal{F} = \{(\hat{u}, A_{\mathcal{F}}(\hat{u}))|\hat{u} \in \hat{\mathcal{V}}\}$ such that $A_{\mathcal{F}} : \hat{\mathcal{V}} \to \mathbb{I}$ where $A_{\mathcal{F}}(\hat{u})$ denotes the belonging value of $\hat{u} \in \mathcal{F}$.

Definition 2.2. [2]
An *intuitionistic fuzzy set* $\mathcal{Y}$ defined as $\mathcal{Y} = \{(\hat{u}, < A_{\mathcal{Y}}(\hat{u}), B_{\mathcal{Y}}(\hat{u}) >)|\hat{u} \in \hat{\mathcal{V}}\}$ such that $A_{\mathcal{Y}} : \ddot{\mathcal{U}} \to \mathbb{I}$ and $B_{\mathcal{Y}} : \ddot{\mathcal{U}} \to \mathbb{I}$, where $A_{\mathcal{Y}}(\hat{u})$ and $B_{\mathcal{Y}}(\hat{u})$ denote the belonging value and not-belonging value of $\hat{u} \in \mathcal{Y}$ with condition of $0 \leq A_{\mathcal{Y}}(\hat{u}) + B_{\mathcal{Y}}(\hat{u}) \leq 1$.

Definition 2.3. [3]
A *neutrosophic set* $\mathcal{Z}$ defined as
$\mathcal{Z} = \{(\hat{u}, < \mathcal{A}_{\mathcal{Z}}(\hat{u}), \mathcal{B}_{\mathcal{Z}}(\hat{u}), \mathcal{C}_{\mathcal{Z}}(\hat{u}) >)|\hat{u} \in \ddot{\mathcal{U}}\}$
such that $\mathcal{A}_{\mathcal{Z}}(\hat{u}), \mathcal{B}_{\mathcal{Z}}(\hat{u}), \mathcal{C}_{\mathcal{Z}}(\hat{u}) : \hat{\mathcal{V}} \to (^-0, 1^+)$,
where $\mathcal{A}_{\mathcal{Z}}(\hat{u}), \mathcal{B}_{\mathcal{Z}}(\hat{u})$ and $\mathcal{C}_{\mathcal{Z}}(\hat{u})$ denote the degrees of membership, indeterminacy and non-membership of $\hat{u} \in \mathcal{Z}$ with condition of $^-0 \leq \mathcal{A}_{\mathcal{Z}}(\hat{u}) + \mathcal{B}_{\mathcal{Z}}(\hat{u}) + \mathcal{C}_{\mathcal{Z}}(\hat{u}) \leq 3^+$.

Note: If $\mathcal{A}_{\mathcal{Z}}(\hat{u}) \in \mathbb{I}, \mathcal{B}_{\mathcal{Z}}(\hat{u}) \in \mathbb{I}$ and $\mathcal{C}_{\mathcal{Z}}(\hat{u}) \in \mathbb{I}$, with

$$0 \leq \mathcal{A}_{\mathcal{Z}}(\hat{u}) + \mathcal{B}_{\mathcal{Z}}(\hat{u}) + \mathcal{C}_{\mathcal{Z}}(\hat{u}) \leq 3,$$

then neutrosophic set $\mathcal{Z}$ is called single-valued neutrosophic set [5].

Definition 2.4. [17]
A pair $(\mathfrak{F}_S, \Lambda)$ is called a *soft set* over $\hat{\mathcal{V}}$, where $F_S : \Lambda \to \mathbb{P}(\hat{\mathcal{V}})$ and Λ be a subset of a set of attributes $\mathfrak{E}$.

For more detail on soft set, see [18, 19].

Definition 2.5. [29]
The pair $(\mathcal{W}, \mathcal{H})$ is called a hypersoft set over $\hat{\mathcal{V}}$, where $\mathcal{H}$ is the cartesian product of n disjoint sets $\mathcal{H}_1, \mathcal{H}_2, \mathcal{H}_3, ..., \mathcal{H}_n$ having attribute values of n distinct attributes $\hat{h}_1, \hat{h}_2, \hat{h}_3, ..., \hat{h}_n$ respectively and $\mathcal{W} : \mathcal{H} \to \mathbb{P}(\hat{\mathcal{V}})$.

Note: If $\mathbb{P}(\hat{\mathcal{V}})$ is replaced with $\mathbb{N}(\hat{\mathcal{V}})$(A collection of neutrosophic subsets) in Definition 2.5, then it becomes neutrosophic hypersoft set [28]. For more definitions and operations of hypersoft set, see [30–32].

3. Single-Valued Neutrosophic Fuzzy Hypersoft Set(SV-NFHS-Set)

Definition 3.1. The pair $(\hat{\Psi}_{HS}, \hat{\mathcal{W}})$ is said to be *single-valued neutrosophic fuzzy hypersoft set* (SV-NFHS-Set) over $\hat{\mathcal{V}}$, denoted by $(SVNFHS)^{\hat{\mathcal{V}}\hat{\mathcal{W}}}$, in $(HS)^{\hat{\mathcal{V}}\hat{\mathcal{W}}}$ if

$$\hat{\Psi}_{(\hat{w})} : \hat{\mathcal{W}} \rightarrow SVNF(\hat{\mathcal{V}})$$

defined by

$$\hat{\Psi}_{(\hat{w})} = \left\{ \begin{array}{l} \left. \dfrac{\left(\mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \xi(\hat{v})\right)}{\hat{v}} \right| \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}}, \\ 0 \leq \mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq 3 \end{array} \right\}$$

where

(i) $\hat{\mathcal{W}} = \hat{\mathcal{E}}_1 \times \hat{\mathcal{E}}_2 \times \hat{\mathcal{E}}_3 \times .... \times \hat{\mathcal{E}}_n$ for n disjoint attribute-valued sets $\hat{\mathcal{E}}_i, i = 1, 2, ..., n$ corresponding to n distinct attributes $\hat{\varepsilon}_j, j = 1, 2, ..., n$ from set of attributes $\hat{\mathcal{E}}$,

(ii) $SVNF(\hat{\mathcal{V}})$ is the collection of all single-valued neutrosophic fuzzy subsets over $\hat{\mathcal{V}}$,

(iii) All components i.e. $\mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})$, $\mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})$, $\mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})$ and $\xi(\hat{v})$ belong to $\mathbb{I}_{\bullet}^{\hat{\mathcal{V}}}$,

(iv) $(HS)^{\hat{\mathcal{V}}\hat{\mathcal{W}}}$ is a hypersoft universe.

Note: The collection of all $(SVNFHS)^{\hat{\mathcal{V}}\hat{\mathcal{W}}}$ over $(HS)^{\hat{\mathcal{V}}\hat{\mathcal{W}}}$ is denoted by $\biguplus_{svnfhs}$.

Example 3.2. Consider $\hat{\mathcal{V}} = \{\hat{v}_1, \hat{v}_2, \hat{v}_3\}$ consists of three kinds of mobile and $\hat{\mathcal{E}} = \{\hat{\varepsilon}_1, \hat{\varepsilon}_2, \hat{\varepsilon}_3\}$ be the set of attributes, where $\hat{\varepsilon}_1$ is storage(GB), $\hat{\varepsilon}_2$ is camera resolution (pixels) and $\hat{\varepsilon}_3$ is battery power(mAh). Their corresponding attribute-valued sets are $\hat{\mathcal{E}}_1 = \{j_{11} = 32, j_{12} = 64\}$, $\hat{\mathcal{E}}_2 = \{j_{21} = 8, j_{22} = 16\}$ and $\hat{\mathcal{E}}_3 = \{j_{31} = 4000\}$. Now $\hat{\mathcal{W}} = \hat{\mathcal{E}}_1 \times \hat{\mathcal{E}}_2 \times \hat{\mathcal{E}}_3$,
$\hat{\mathcal{W}} = \{(\hat{w}_{11}, \hat{w}_{21}, \hat{w}_{31}), (\hat{w}_{11}, \hat{w}_{22}, \hat{w}_{31}), (\hat{w}_{12}, \hat{w}_{21}, \hat{w}_{31}), (\hat{w}_{12}, \hat{w}_{22}, \hat{w}_{31})\}$.
or
$\hat{\mathcal{W}} = \{\hat{w}_1, \hat{w}_2, \hat{w}_3, \hat{w}_4\}$
Then for $\xi \in \mathbb{I}_{\bullet}^{\hat{\mathcal{V}}}$, $\hat{\Psi}_{(\hat{w}_i)} \in \biguplus_{svnfhs}$ are defined as follows $(i = 1, 2, 3, 4)$:

$$\hat{\Psi}_{(\hat{w}_1)} = \left\{ \frac{(0.2, 0.6, 0.4, 0.4)}{\hat{v}_1}, \frac{(0.2, 0.8, 0.6, 0.4)}{\hat{v}_2}, \frac{(0.3, 0.6, 0.7, 0.4)}{\hat{v}_3} \right\},$$

$$\hat{\Psi}_{(\hat{w}_2)} = \left\{ \frac{(0.6, 0.8, 0.7, 0.5)}{\hat{v}_1}, \frac{(0.3, 0.6, 0.6, 0.4)}{\hat{v}_2}, \frac{(0.7, 0.9, 0.8, 0.3)}{\hat{v}_3} \right\},$$

$$\hat{\Psi}_{(\hat{w}_3)} = \left\{ \frac{(0.1, 0.5, 0.5, 0.4)}{\hat{v}_1}, \frac{(0.4, 0.6, 0.7, 0.5)}{\hat{v}_2}, \frac{(0.3, 0.6, 0.5, 0.2)}{\hat{v}_3} \right\},$$

$$\hat{\Psi}_{(\hat{w}_4)} = \left\{ \frac{(0.3, 0.6, 0.4, 0.2)}{\hat{v}_1}, \frac{(0.2, 0.5, 0.7, 0.4)}{\hat{v}_2}, \frac{(0.6, 0.4, 0.7, 0.4)}{\hat{v}_3} \right\}.$$

It can be represented in matrix form as

$$\hat{\Psi} = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.2, 0.6, 0.4, 0.4) & (0.2, 0.8, 0.6, 0.4) & (0.3, 0.6, 0.7, 0.4) \\ \hat{w}_2 & (0.6, 0.8, 0.7, 0.5) & (0.3, 0.6, 0.6, 0.4) & (0.7, 0.9, 0.8, 0.3) \\ \hat{w}_3 & (0.1, 0.5, 0.5, 0.4) & (0.4, 0.6, 0.7, 0.5) & (0.3, 0.6, 0.5, 0.2) \\ \hat{w}_4 & (0.3, 0.6, 0.4, 0.2) & (0.2, 0.5, 0.7, 0.4) & (0.6, 0.4, 0.7, 0.4) \end{pmatrix}$$

Definition 3.3. Let $\hat{\Psi}^1_{(\hat{w})}$, $\hat{\Psi}^2_{(\hat{w})} \in \biguplus_{svnfhs}$ and $\xi^1, \xi^2 \in \mathbb{I}_{\bullet}^{\hat{\mathcal{V}}}$, where

$$\hat{\Psi}^1_{(\hat{w})} = \left\{ \begin{array}{l} \frac{\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \xi^1(\hat{v})}{\hat{v}} \Bigg| \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}}, \\ 0 \leq \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq 3 \end{array} \right\}$$

and

$$\hat{\Psi}^2_{(\hat{w})} = \left\{ \begin{array}{l} \frac{\mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \xi^2(\hat{v})}{\hat{v}} \Bigg| \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}}, \\ 0 \leq \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq 3 \end{array} \right\}.$$

Then $\hat{\Psi}^1_{(\hat{w})}$ is said to be a SV-NFHS- subset of $\hat{\Psi}^2_{(\hat{w})}$, expressed by $\hat{\Psi}^1_{(\hat{w})} \subseteq \hat{\Psi}^2_{(\hat{w})}$, if

(i) $\xi^1 \leq \xi^2$,

(ii) $\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \geq \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \geq \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})$.

Example 3.4. Considering data from Example 3.2, let another $\hat{\Psi}'_{(\hat{w})} \in \biguplus_{svnfhs}$ be defined as below:

$$\hat{\Psi}' = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.4, 0.5, 0.3, 0.4) & (0.3, 0.6, 0.5, 0.5) & (0.4, 0.5, 0.4, 0.6) \\ \hat{w}_2 & (0.7, 0.7, 0.6, 0.6) & (0.5, 0.4, 0.5, 0.6) & (0.8, 0.8, 0.7, 0.5) \\ \hat{w}_3 & (0.2, 0.5, 0.3, 0.7) & (0.7, 0.5, 0.4, 0.7) & (0.4, 0.5, 0.4, 0.3) \\ \hat{w}_4 & (0.5, 0.4, 0.3, 0.6) & (0.4, 0.4, 0.6, 0.6) & (0.7, 0.3, 0.3, 0.5) \end{pmatrix}$$

Thus, $\hat{\Psi}_{(\hat{w})} \subseteq \hat{\Psi}'_{(\hat{w})}$ for all $\hat{w}_i \in \hat{\mathcal{W}}, i = 1, 2, 3, 4$

Definition 3.5. Assuming $\hat{\Psi}^1_{(\hat{w})}$, $\hat{\Psi}^2_{(\hat{w})} \in \biguplus_{svnfhs}$ and $\xi^1, \xi^2 \in \mathbb{I}_{\bullet}^{\hat{\mathcal{V}}}$, as stated in Definition 3.3, then $\hat{\Psi}^1_{(\hat{w})}$ and $\hat{\Psi}^2_{(\hat{w})}$ are said to be SV-NFHS-equal sets, denoted by $\hat{\Psi}^1_{(\hat{w})} = \hat{\Psi}^2_{(\hat{w})}$, if $\hat{\Psi}^1_{(\hat{w})} \subset \hat{\Psi}^2_{(\hat{w})}$ and $\hat{\Psi}^2_{(\hat{w})} \subset \hat{\Psi}^1_{(\hat{w})}$.

Definition 3.6. Let $\hat{\Psi}_{(\hat{w})} \in \biguplus_{svnfhs}$ and $\xi \in \mathbb{I}_{\bullet}^{\hat{\mathcal{V}}}$, then

(i) $\hat{\Psi}_{(\hat{w})}$ is known to be a SV-NFHS-null set, represented by $\hat{\emptyset}_{(\hat{w})}$, if

$$\mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 0, \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 1, \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 1, \xi(\hat{v}) = 0$$

.

(ii) $\hat{\Psi}_{(\hat{w})}$ is known to be a SV-NFHS-universal set, denoted by $\hat{\mathcal{V}}_{(\hat{w})}$, if

$$\mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 1, \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 0, \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) = 0, \xi(\hat{v}) = 1.$$

Definition 3.7. Let us suppose $\hat{\Psi}^1_{(\hat{w})}$, $\hat{\Psi}^2_{(\hat{w})} \in \biguplus_{svnfhs}$ and $\xi^1, \xi^2 \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$, as described in Definition 3.3, then,

(i) the union of $\hat{\Psi}^1_{(\hat{w})}$ and $\hat{\Psi}^2_{(\hat{w})}$, denoted by $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^2_{(\hat{w})}$, is a SV-NFHS-set $\hat{\Omega}^1_{(\hat{w})}$ such that

$$\hat{\Omega}^1_{(\hat{w})} = \left\{ \left. \begin{array}{l} \dfrac{\left(\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \xi^1(\hat{v}) \circledcirc \xi^2(\hat{v})\right)}{\hat{v}} \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}} \end{array} \right| \right\}.$$

(ii) the intersection of $\hat{\Psi}^1_{(\hat{w})}$ and $\hat{\Psi}^2_{(\hat{w})}$, denoted by $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^2_{(\hat{w})}$, is a SV-NFHS-set $\hat{\Omega}^2_{(\hat{w})}$ such that

$$\hat{\Omega}^2_{(\hat{w})} = \left\{ \left. \begin{array}{l} \dfrac{\left(\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}),\ \xi^1(\hat{v}) \circledast \xi^2(\hat{v})\right)}{\hat{v}} \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}} \end{array} \right| \right\}.$$

In above definitions, $\circledast$ and $\circledcirc$ denote the t-norm ($\wedge$) and t-conorm ($\vee$) respectively. If $a, b \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$, then $a \circledast b = a \wedge b$ and $a \circledcirc b = a \vee b$.

Example 3.8. Supposing the matrix representations of SV-NFHS-sets as determined in Example 3.2 and Example 3.4, then we have

$$\hat{\Omega}^1_{(\hat{w})} = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.4, 0.5, 0.3, 0.4) & (0.3, 0.6, 0.5, 0.5) & (0.4, 0.5, 0.4, 0.6) \\ \hat{w}_2 & (0.7, 0.7, 0.6, 0.6) & (0.5, 0.4, 0.5, 0.6) & (0.8, 0.8, 0.7, 0.5) \\ \hat{w}_3 & (0.2, 0.5, 0.3, 0.7) & (0.7, 0.5, 0.4, 0.7) & (0.4, 0.5, 0.4, 0.3) \\ \hat{w}_4 & (0.5, 0.4, 0.3, 0.6) & (0.4, 0.4, 0.6, 0.6) & (0.7, 0.3, 0.3, 0.5) \end{pmatrix}$$

and

$$\hat{\Omega}^2_{(\hat{w})} = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.2, 0.6, 0.4, 0.4) & (0.2, 0.8, 0.6, 0.4) & (0.3, 0.6, 0.7, 0.4) \\ \hat{w}_2 & (0.6, 0.8, 0.7, 0.5) & (0.3, 0.6, 0.6, 0.4) & (0.7, 0.9, 0.8, 0.3) \\ \hat{w}_3 & (0.1, 0.5, 0.5, 0.4) & (0.4, 0.6, 0.7, 0.5) & (0.3, 0.6, 0.5, 0.2) \\ \hat{w}_4 & (0.3, 0.6, 0.4, 0.2) & (0.2, 0.5, 0.7, 0.4) & (0.6, 0.4, 0.7, 0.4) \end{pmatrix}$$

Proposition 3.9. *Let $\hat{\emptyset}_{(\hat{w})}, \hat{\mathcal{V}}_{(\hat{w})}, \hat{\Psi}_{(\hat{w})} \in \biguplus_{svnfhs}$ and $\xi \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$. Then the following properties hold:*

(i) $\hat{\Psi}_{(\hat{w})} \hat{\sqcup} \hat{\Psi}_{(\hat{w})} = \hat{\Psi}_{(\hat{w})}$,

(ii) $\hat{\Psi}_{(\hat{w})} \hat{\sqcap} \hat{\Psi}_{(\hat{w})} = \hat{\Psi}_{(\hat{w})}$,

(iii) $\hat{\Psi}_{(\hat{w})} \hat{\sqcup} \hat{\emptyset}_{(\hat{w})} = \hat{\Psi}_{(\hat{w})}$,

(iv) $\hat{\Psi}_{(\hat{w})} \hat{\sqcap} \hat{\emptyset}_{(\hat{w})} = \hat{\emptyset}_{(\hat{w})}$,
(v) $\hat{\Psi}_{(\hat{w})} \hat{\sqcup} \hat{\mathcal{V}}_{(\hat{w})} = \hat{\mathcal{V}}_{(\hat{w})}$,
(vi) $\hat{\Psi}_{(\hat{w})} \hat{\sqcap} \hat{\mathcal{V}}_{(\hat{w})} = \hat{\Psi}_{(\hat{w})}$.

Proof. The above properties (i)-(vi) can easily be proved with the help of Definition 3.6 and Definition 3.7. □

Proposition 3.10. *Let* $\hat{\Psi}^1_{(\hat{w})}, \hat{\Psi}^2_{(\hat{w})}, \hat{\Psi}^3_{(\hat{w})} \in \biguplus_{svnfhs}$ *and* $\xi^1, \xi^2, \xi^3 \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$. *Then the results (given below) hold:*

(i) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^2_{(\hat{w})} = \hat{\Psi}^2_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^1_{(\hat{w})}$,
(ii) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^2_{(\hat{w})} = \hat{\Psi}^2_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^1_{(\hat{w})}$,
(iii) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \left(\hat{\Psi}^2_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^3_{(\hat{w})}\right) = \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^2_{(\hat{w})}\right) \hat{\sqcup} \hat{\Psi}^3_{(\hat{w})}$,
(iv) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \left(\hat{\Psi}^2_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^3_{(\hat{w})}\right) = \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^2_{(\hat{w})}\right) \hat{\sqcap} \hat{\Psi}^3_{(\hat{w})}$,
(v) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \left(\hat{\Psi}^2_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^3_{(\hat{w})}\right) = \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^2_{(\hat{w})}\right) \hat{\sqcup} \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^3_{(\hat{w})}\right)$,
(vi) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \left(\hat{\Psi}^2_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^3_{(\hat{w})}\right) = \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^2_{(\hat{w})}\right) \hat{\sqcap} \left(\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^3_{(\hat{w})}\right)$.

Proof. Proofs of (i)-(vi) are straightforward by following the concept of Definition 3.7. □

Proposition 3.11. *Let* $\hat{\Psi}^1_{(\hat{w})}, \hat{\Psi}^2_{(\hat{w})} \in \biguplus_{svnfhs}$ *and* $\xi^1, \xi^2 \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$ *and* $\hat{\Psi}^1_{(\hat{w})} \subseteq \hat{\Psi}^2_{(\hat{w})}$, *then the below given results hold:*

(i) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcup} \hat{\Psi}^2_{(\hat{w})} = \hat{\Psi}^2_{(\hat{w})}$,
(ii) $\hat{\Psi}^1_{(\hat{w})} \hat{\sqcap} \hat{\Psi}^2_{(\hat{w})} = \hat{\Psi}^1_{(\hat{w})}$.

Proof. By following the concept of Definition 3.3 and Definition 3.7, proofs are straightforward. □

Definition 3.12. Let $\hat{\Psi}_{(\hat{w})} \in \biguplus_{svnfhs}, \xi \in \mathbb{I}^{\hat{\mathcal{V}}}_{\bullet}$, (as described in Definition 3.1) then, its complement $\hat{\Psi}^c_{(\hat{w})}$ is stated as

$$\hat{\Psi}^c_{(\hat{w})} = \left\{ \begin{array}{c|} \frac{\left(\mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi(\hat{v})\right)}{\hat{v}} \\ \hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \check{\mathcal{V}} \end{array} \right\}.$$

Example 3.13. The complement of SV-NFHS-set $\hat{\Psi}_{(\hat{w})}$ (as determined in Example 3.2) is calculated as

$$\hat{\Psi}^c_{(\hat{w})} = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.3, 0.5, 0.4, 0.6) & (0.6, 0.2, 0.2, 0.6) & (0.4, 0.5, 0.4, 0.6) \\ \hat{w}_2 & (0.7, 0.2, 0.6, 0.5) & (0.6, 0.4, 0.3, 0.6) & (0.8, 0.1, 0.7, 0.7) \\ \hat{w}_3 & (0.5, 0.5, 0.1, 0.6) & (0.7, 0.4, 0.4, 0.5) & (0.5, 0.4, 0.3, 0.8) \\ \hat{w}_4 & (0.4, 0.4, 0.3, 0.8) & (0.7, 0.5, 0.2, 0.6) & (0.7, 0.6, 0.6, 0.6) \end{pmatrix}.$$

Proposition 3.14. *Let* $\hat{\emptyset}_{(\hat{w})}, \hat{\mathcal{V}}_{(\hat{w})}, \hat{\Psi}_{(\hat{w})} \in \biguplus_{svnfhs}$ *and* $\xi \in \mathbb{I}_\bullet^{\hat{\mathcal{V}}}$. *Then, we have the following results:*

(i) $\hat{\emptyset}^c_{(\hat{w})} = \hat{\mathcal{V}}_{(\hat{w})}$,
(ii) $\hat{\mathcal{V}}^c_{(\hat{w})} = \hat{\emptyset}_{(\hat{w})}$,
(iii) $\left(\hat{\Psi}^c_{(\hat{w})}\right)^c = \hat{\Psi}_{(\hat{w})}$.

Proof. These results can easily be proved with the help of concepts stated in Definition 3.6 and Definition 3.12. □

Remark 3.15. In general, the following results are not hold.

(i) $\hat{\Psi}_{(\hat{w})} \hat{\cup} \hat{\Psi}^c_{(\hat{w})} = \hat{\mathcal{V}}_{(\hat{w})}$,
(ii) $\hat{\Psi}_{(\hat{w})} \hat{\cap} \hat{\Psi}^c_{(\hat{w})} = \hat{\emptyset}_{(\hat{w})}$.

Example 3.16. The Remark 3.15 can easily be verified by considering SV-NFHS-set $\hat{\Psi}_{(\hat{w})}$ from Example 3.2 and its complement $\hat{\Psi}^c_{(\hat{w})}$ from Example 3.13. So we have

$$\hat{\Psi} \hat{\cup} \hat{\Psi}^c = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.3, 0.5, 0.4, 0.6) & (0.6, 0.2, 0.2, 0.6) & (0.4, 0.5, 0.4, 0.6) \\ \hat{w}_2 & (0.7, 0.2, 0.6, 0.5) & (0.6, 0.4, 0.3, 0.6) & (0.8, 0.1, 0.7, 0.7) \\ \hat{w}_3 & (0.5, 0.5, 0.1, 0.6) & (0.7, 0.4, 0.4, 0.5) & (0.5, 0.4, 0.3, 0.8) \\ \hat{w}_4 & (0.4, 0.4, 0.3, 0.8) & (0.7, 0.5, 0.2, 0.6) & (0.7, 0.6, 0.6, 0.6) \end{pmatrix}.$$

And

$$\hat{\Psi} \hat{\cap} \hat{\Psi}^c = \begin{pmatrix} \hat{\mathcal{W}} & \hat{v}_1 & \hat{v}_2 & \hat{v}_3 \\ \hat{w}_1 & (0.2, 0.6, 0.4, 0.4) & (0.2, 0.8, 0.6, 0.4) & (0.3, 0.6, 0.7, 0.4) \\ \hat{w}_2 & (0.6, 0.8, 0.7, 0.5) & (0.3, 0.6, 0.6, 0.4) & (0.7, 0.9, 0.8, 0.3) \\ \hat{w}_3 & (0.1, 0.5, 0.5, 0.4) & (0.4, 0.6, 0.7, 0.5) & (0.3, 0.6, 0.5, 0.2) \\ \hat{w}_4 & (0.3, 0.6, 0.4, 0.2) & (0.2, 0.5, 0.7, 0.4) & (0.6, 0.4, 0.7, 0.4) \end{pmatrix}.$$

Hence the Remark 3.15 is verified.

Proposition 3.17. *Let* $\hat{\Psi}^1_{(\hat{w})}, \hat{\Psi}^2_{(\hat{w})} \in \biguplus_{svnfhs}$ *and* $\xi^1, \xi^2 \in \mathbb{I}_\bullet^{\hat{\mathcal{V}}}$. *Then, the following results hold:*

(i) $\left(\hat{\Psi}^1_{(\hat{w})} \hat{\cup} \hat{\Psi}^2_{(\hat{w})}\right)^c = \left(\hat{\Psi}^1_{(\hat{w})}\right)^c \hat{\cap} \left(\hat{\Psi}^2_{(\hat{w})}\right)^c$,

(ii) $\left(\hat{\Psi}^1_{(\hat{w})} \hat{\cap} \hat{\Psi}^2_{(\hat{w})}\right)^c = \left(\hat{\Psi}^1_{(\hat{w})}\right)^c \hat{\cup} \left(\hat{\Psi}^2_{(\hat{w})}\right)^c$.

Proof. Consider the concept stated in Definition 3.7, for $\hat{w} \in \hat{\mathcal{W}}, \hat{v} \in \hat{\mathcal{V}}$, we have

(i). Since $\left(\hat{\Psi}^1_{(\hat{w})} \hat{\cup} \hat{\Psi}^2_{(\hat{w})}\right)^c (\hat{v})$

$$= \left(\left\{ \frac{\left(\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \xi^1(\hat{v}) \circledast \xi^2(\hat{v})\right)}{\hat{v}} \right\}\right)^c$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})\right), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\xi^1(\hat{v}) \circledast \xi^2(\hat{v})\right)\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})\right), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\xi^1(\hat{v}) \vee \xi^2(\hat{v})\right)\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v}) \wedge 1-\xi^2(\hat{v})\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v}) \circledcirc 1-\xi^2(\hat{v})\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v})}{\hat{v}} \right\} \hat{\cap} \left\{ \frac{\mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^2(\hat{v})}{\hat{v}} \right\}$$

$$= \left(\hat{\Psi}^1_{(\hat{w})}\right)^c \hat{\cap} \left(\hat{\Psi}^2_{(\hat{w})}\right)^c.$$

(ii). Since $\left(\hat{\Psi}^1_{(\hat{w})} \hat{\cap} \hat{\Psi}^2_{(\hat{w})}\right)^c$

$$= \left(\left\{ \frac{\left(\mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \xi^1(\hat{v}) \circledcirc \xi^2(\hat{v})\right)}{\hat{v}} \right\}\right)^c$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})\right), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\xi^1(\hat{v}) \circledcirc \xi^2(\hat{v})\right)\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee \mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v})\right), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\left(\xi^1(\hat{v}) \wedge \xi^2(\hat{v})\right)\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \vee \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \wedge \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v}) \vee 1-\xi^2(\hat{v})\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\left(\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledast \mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \circledcirc \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v}) \circledast 1-\xi^2(\hat{v})\right)}{\hat{v}} \right\}$$

$$= \left\{ \frac{\mathcal{R}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^1{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^1(\hat{v})}{\hat{v}} \right\} \hat{\cup} \left\{ \frac{\mathcal{R}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\mathcal{Q}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), \mathcal{P}^2{}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}), 1-\xi^2(\hat{v})}{\hat{v}} \right\}$$

$$= \left(\hat{\Psi}^1_{(\hat{w})}\right)^c \hat{\cup} \left(\hat{\Psi}^2_{(\hat{w})}\right)^c. \square$$

4. **Application of SV-NFHS-Set in Decision-Making**

In this section, a decision-making algorithm is proposed and is elaborated by daily life decision-making problem.

Algorithm 4.1. Step 1: Input the SV-NFHS- set $\hat{\Phi}_{(\hat{w})} \in \biguplus_{svnfhs}$ as follows:

$$\hat{\Phi}_{(\hat{w})} = \left\{ \begin{array}{l} \frac{\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}), \mathcal{B}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}), \mathcal{C}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}), \xi(\hat{u})\right)}{\hat{u}} \Bigg| \\ \hat{w} \in \hat{\mathcal{W}}, \hat{u} \in \hat{\mathcal{V}}, \\ 0 \leq \mathcal{A}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}) + \mathcal{B}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}) + \mathcal{C}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}) \leq 3 \end{array} \right\},$$

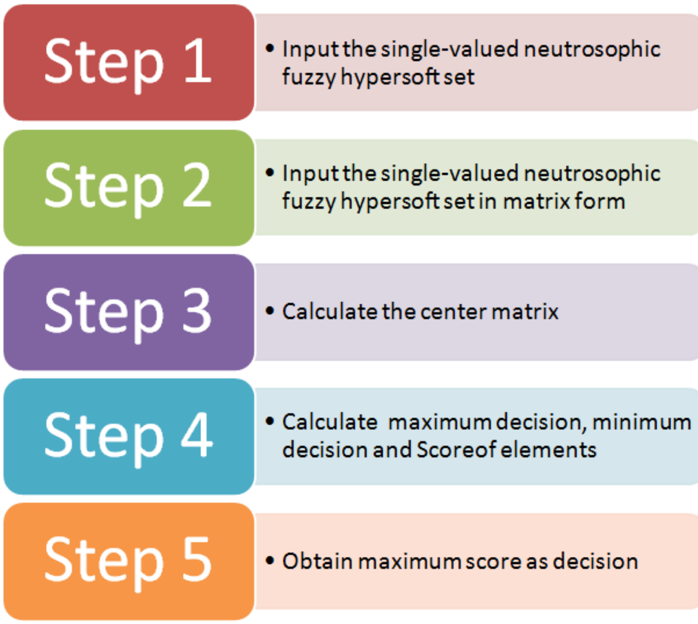

FIGURE 1. Optimal Decision Based on a SV-NFHS-Set matrix

to be evaluated by a group of experts n to element x on parameter i.

Step 2: Input the SV-NFHS- set in matrix form (written as $\mathcal{M}_{l\times k}, k, l \in N$).

$$\mathcal{M}_{l\times k} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1k} \\ m_{21} & m_{22} & \cdots & m_{2k} \\ m_{31} & m_{32} & \cdots & m_{3k} \\ \vdots & \vdots & \ddots & \vdots \\ m_{l1} & m_{l2} & \cdots & m_{lk} \end{bmatrix}$$

where

$$m_{11} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_1), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_1), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_1), \xi(\hat{u}_1)\right)$$

$$m_{12} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_2), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_2), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_2), \xi(\hat{u}_2)\right)$$

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$$m_{1k} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_k), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_k), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_k), \xi(\hat{u}_k)\right)$$

$$m_{21} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_1), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_1), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_1), \xi(\hat{u}_1)\right)$$

$$m_{22} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_2), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_2), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_2), \xi(\hat{u}_2)\right)$$

....

....

....

$$m_{2k} = \left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_k), \mathcal{B}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_k), \mathcal{C}_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_k), \xi(\hat{u}_k)\right)$$

$m_{31}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_1),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_1),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_1),\xi(\hat{u}_1)\right)$

$m_{32}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_2),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_2),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_2),\xi(\hat{u}_2)\right)$

....

....

....

$m_{3k}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_k),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_k),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_3)}}(\hat{u}_k),\xi(\hat{u}_k)\right)$

....

....

....

$m_{l1}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_1),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_1),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_1),\xi(\hat{u}_1)\right)$

$m_{l2}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_2),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_2),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_2),\xi(\hat{u}_2)\right)$

....

....

....

$m_{lk}=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_k),\mathcal{B}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_k),\mathcal{C}_{\hat{\Phi}_{(\hat{w}_l)}}(\hat{u}_k),\xi(\hat{u}_k)\right)$

Step 3: Determine the core matrix (i.e.,
$\delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_k)=\left(\mathcal{A}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_k)+\mathcal{B}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_k)+\mathcal{C}_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_k)\right)-\xi(\hat{u}_k))$:

$$G_{l\times k}=\begin{pmatrix} \delta_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_1) & \delta_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_2) & \dots & \delta_{\hat{\Phi}_{(\hat{w}_1)}}(\hat{u}_k) \\ \delta_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_1) & \delta_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_2) & \dots & \delta_{\hat{\Phi}_{(\hat{w}_2)}}(\hat{u}_k) \\ \vdots & \vdots & \ddots & \vdots \\ \delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_1) & \delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_2) & \dots & \delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_k) \end{pmatrix}.$$

Step 4: Calculate the $\mathfrak{D}^{\max}(\hat{u}_j)$ (Max. decision), $\mathfrak{D}^{\min}(\hat{u}_j)$ (Min.decision), and $\mathfrak{S}(\hat{u}_j)$ (score) of all $\hat{u}_j$ (j = 1,2,..., k):

$$\mathfrak{D}^{\max}(\hat{u}_j)=\sum_{i=1}^{l}\left(1-\delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_j)\right)^2,\mathfrak{D}^{\min}(\hat{u}_j)=\sum_{i=1}^{l}\left(\delta_{\hat{\Phi}_{(\hat{w})}}(\hat{u}_j)\right)^2$$

$$\mathfrak{S}(\hat{u}_j)=\mathfrak{D}^{\max}(\hat{u}_j)+\mathfrak{D}^{\min}(\hat{u}_j).$$

(to understand the motivation behind this method, let ρ be the Euclidean metric on R^l, $0=(0,...,0)^T\in R^l, 1=(1,...,1)^T\in R^l$, and $\theta_j=\left(\theta_{1,\hat{u}_j},\theta_{2,\hat{u}_j},...,\theta_{l,\hat{u}_j}\right)^T\in R^l$. Thus $\mathfrak{S}(\hat{u}_j)=[\rho(\theta_j,1)]^2+[\rho(\theta_j,0)]^2\,(j=1,2,...,k)$.

Step 5: Find optimal decision ρ with the help of following criterion

$$\hat{u}_k=Max\{\mathfrak{S}(\hat{u}_1),\mathfrak{S}(\hat{u}_2),...,\mathfrak{S}(\hat{u}_k)\}.$$

Now Algorithm 4.1 is elaborated with the help of following example.

Example 4.2. A manufacturing company is wants to select a supplier for one of its manufactures products. Consider a SV-NFHS-set which illustrates the suitability of suppliers and enables the company to select the most suitable supplier for its product. There are five alternatives suppliers and these form the discourse of universe $\mathcal{Z} = \{z_1, z_2, z_3, z_4, z_5\}$. Company constitutes a committee (i.e. Decision-Makers)consisting of its procurement supervisor (chairman of committee) and buyers for the evaluation of these suppliers. With mutual consultation, the chairman develops a mutual consensus and designs a set of parameters $\mathcal{H} = \{h_1, h_2, h_3, h_4\}$, for this evaluation, where h_1, h_2, h_3 and h_4 represent quality and reliability, cost, service and process and design capabilities respectively. These attributes are further classified into attribute-valued sets $\mathcal{J}_1, \mathcal{J}_2, \mathcal{J}_3$ and $\mathcal{J}_4$ respectively on the basis of their respective categorical criteria, where
$\mathcal{J}_1 = \{j^{11}, j^{12}\}$
$\mathcal{J}_2 = \{j^{21}, j^{22}\}$
$\mathcal{J}_3 = \{j^{31}, j^{32}\}$
and
$\mathcal{J}_4 = \{j^{41}, j^{42}\}$.
Therefore,
$\mathcal{K} = \mathcal{J}_1 \times \mathcal{J}_2 \times \mathcal{J}_3 \times \mathcal{J}_4 = \{k^1, k^2, ..., k^{16}\}$, where each $k^i, i = 1, 2, ..., 16$ is a $4 - tuple$ element of $\mathcal{K}$. For the sake of convenience, k^1, k^3, k^5 and k^{14} are given preference during the evaluation process by the committee. The evaluation process is completed with the help of proposed Algorithm 4.1.

Step 1 The whole scenario is interpreted in the form of SV-NFHS-set $\hat{\Phi}_{(k^i)} \in \biguplus_{svnfhs}$ and is given below

$\mathcal{K}$	z_1	z_2	z_3	z_4	z_5
k^1	$(0.8, 0.4, 0.3, 0.2)$	$(0.7, 0.5, 0.4, 0.3)$	$(0.6, 0.6, 0.5, 0.4)$	$(0.5, 0.7, 0.6, 03)$	$(0.4, 0.8, 0.7, 0.2)$
k^3	$(0.7, 0.5, 0.4, 0.3)$	$(0.6, 0.6, 0.5, 0.4)$	$(0.5, 0.7, 0.6, 0.5)$	$(0.4, 0.8, 0.7, 0.4)$	$(0.3, 0.9, 0.9, 0.4)$
k^5	$(0.6, 0.4, 0.5, 0.4)$	$(0.5, 0.7, 0.6, 0.5)$	$(0.4, 0.8, 0.7, 0.6)$	$(0.3, 0.9, 0.8, 0.5)$	$(0.2, 0.1, 0.1, 0.2)$
k^{14}	$(0.5, 0.5, 0.5, 0.5)$	$(0.4, 0.8, 0.7, 0.6)$	$(0.3, 0.9, 0.8, 0.7)$	$(0.2, 0.3, 0.9, 0.6)$	$(0.1, 0.2, 0.2, 0.3)$

Step 2

$$\mathcal{M}_{5\times 4} = \begin{pmatrix} (0.8,0.4,0.3,0.2) & (0.7,0.5,0.4,0.3) & (0.6,0.4,0.5,0.4) & (0.5,0.5,0.5,0.5) \\ (0.7,0.5,0.4,0.3) & (0.6,0.6,0.5,0.4) & (0.5,0.7,0.6,0.5) & (0.4,0.8,0.7,0.6) \\ (0.6,0.6,0.5,0.4) & (0.5,0.7,0.6,0.5) & (0.4,0.8,0.7,0.6) & (0.3,0.9,0.8,0.7) \\ (0.5,0.7,0.6,03) & (0.4,0.8,0.7,0.4) & (0.3,0.9,0.8,0.5) & (0.2,0.3,0.9,0.6) \\ (0.4,0.8,0.7,0.2) & (0.3,0.9,0.9,0.4) & (0.2,0.1,0.1,0.2) & (0.1,0.2,0.2,0.3) \end{pmatrix}.$$

Step 3

$$C_{3\times 5} = \begin{pmatrix} 1.3 & 1.3 & 1.1 & 1.0 \\ 1.3 & 1.3 & 1.3 & 1.3 \\ 1.3 & 1.3 & 1.3 & 1.3 \\ 1.5 & 1.5 & 1.5 & 0.8 \\ 1.7 & 1.7 & 0.2 & 0.2 \end{pmatrix}.$$

Step 4 The values of $\mathfrak{D}^{\max}(z_j)$, $\mathfrak{D}^{\min}(z_j)$ and $S(z_j)$ for $j = 1, 2, 3, 4, 5$ are given in the below table

	z_1	z_2	z_3	z_4	z_5
$\mathfrak{D}^{\max}(z_j)$	0.19	0.36	0.36	0.79	2.26
$\mathfrak{D}^{\min}(z_j)$	5.59	6.76	6.76	7.39	5.86
$S(z_j)$	5.78	7.12	7.12	8.18	8.12

Step 5 It is vivid that z_5 is selected as the best decision due to its highest score.

5. Comparison Analysis

There are many cases where consideration of only attributes is not sufficient, all available distinct attributes are further partitioned into their respective disjoint attribute-valued sets. Decision making techniques based on most of existing models are inadequate for such cases. Therefore, our proposed model not only emphasizes on the due status of such partitioning of attributes but also facilitates the decision makers to deal daily life problems with great ease. Table 1 and Table 2 present a vivid comparison of our proposed model with some existing relevant models under the evaluating indicators MD (Membership Degree), NMD (Non-membership Degree), ID (Indeterminacy Degree), SAAF (Single Argument Approximate Function) and MAAF (Multi Argument Approximate Function).

5.1. *Discussion*

In this subsection, we present the generalization of our proposed structure. Our proposed structure SV-NFHS-set is very useful for tackling many decisive systems and it is the most generalized structure (see Figure 2) as it transforms to:

(i) Single-Valued Neutrosophic Hypersoft Set (SV-NHS-set) if fuzzy valued degree is omitted.

(ii) Neutrosophic Hypersoft Fuzzy Set (SV-NFHS-set) if

$$0 \leq \mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq 3$$

is replaced with

$$^{-}0 \leq \mathcal{P}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{Q}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) + \mathcal{R}_{\hat{\Psi}_{(\hat{w})}}(\hat{v}) \leq 3^{+}$$

.

TABLE 1. Comparison with existing models

Authors	Structure	Remarks
Broumi et al. [10]	Intuitionistic NS-Set	(i) Single set of parameters is used with intuitionistic fuzzy values, (ii) Approximate function is the subset of universal set
Das et al. [48]	Neutrosophic fuzzy set	(i) Single set of parameters is used with intuitionistic fuzzy values, (ii) Approximate function is the subset of intuitionistic fuzzy set
Khalil et al. [49]	Single-valued NFS-Set	(i) Single set of parameters is used with neutrosophic values, (ii) Approximate function is the subset of universal set
Proposed Model	Single-valued NFHS-Set	(i) Single set of parameters is further classified into disjoint attribute-valued sets used with intuitionistic fuzzy values, (ii) Approximate function is the subset of neutrosophic set

TABLE 2. Comparison with existing models under appropriate features

Authors	Structure	MD	NMD	ID	SAAF	MAAF
Broumi et al. [10]	Intuitionistic NS-Set	✓	✓	✓	✓	×
Das et al. [48]	Neutrosophic fuzzy set	✓	✓	✓	×	×
Khalil et al. [49]	Single-valued NFS-Set	✓	✓	✓	✓	×
Proposed Model	Single-valued NFHS-Set	✓	✓	✓	✓	✓

(iii) Intuitionistic Fuzzy Hypersoft Set (IFHS-set) if indeterminacy degree and fuzzy valued degree are ignored and remaining uncertain components be restricted within closed unit interval in approximate function of SV-NFHS-set.

(iv) Intuitionistic Fuzzy Soft Set (IFS-set) if
- attribute-valued sets are replaced with only attributes,
- indeterminacy degree and fuzzy valued degree are ignored,
- remaining two uncertain components are made interdependent within closed unit interval in approximate function of SV-NFHS-set.

(v) Fuzzy Hypersoft Set (FHS-set) if
- only fuzzy membership is focussed,

- falsity, indeterminacy degree and fuzzy valued degree are ignored.

(vi) Hypersoft Set (HS-set) if all uncertain components membership, non-membership, indeterminacy and fuzzy valued degrees are ignored.

(vii) Soft Set (S-set) if

- attribute-valued sets are replaced with only attributes,
- all uncertain components membership, non-membership, indeterminacy and fuzzy valued degrees are ignored.

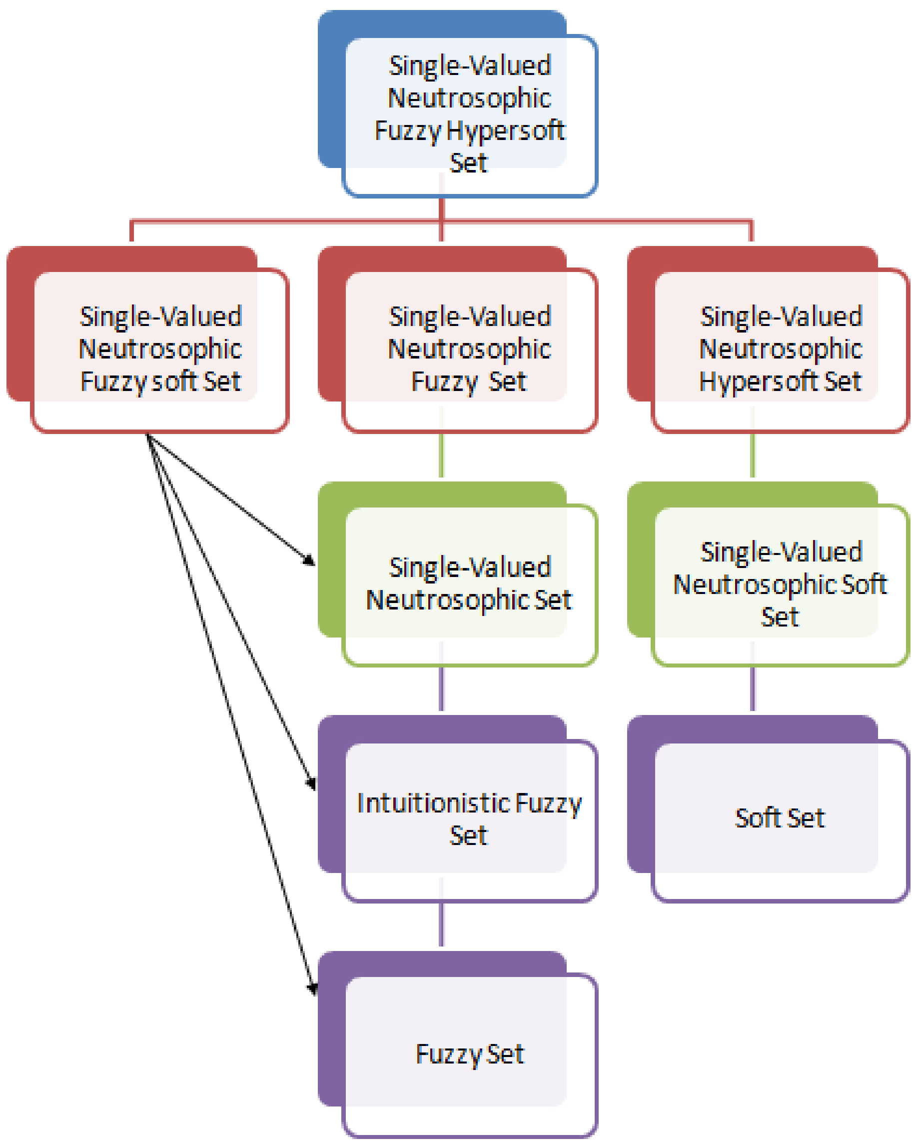

Figure 2. Generalization of IV-NFHS-Set

6. Conclusions

In this study, single-valued neutrosophic fuzzy hypersoft set is conceptualized with some of its elementary properties and theoretic operations. Novel algorithm is proposed for decision making and is validated with the help of illustrative examples for appropriate choosing of suitable supplier. Future work may include the extension of this work for:

- The development of algebraic structures i.e. topological spaces, vector spaces etc.,
- Dealing with decision making problems with multi-criteria decision making techniques,
- Applying in medical diagnosis and optimization for agricultural yield,
- Investigating and determining similarity, distance, dissimilarity measures and entropies between the proposed structures.

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