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Properties on Topologized Domination in Neutrosophic Graphs

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Abstract: In this paper, the new concept Topologized domination on Neutrosophic Graphs is introduced. The idea of N-Top domination is discussed in cycle, path, complete graph, star graph. The basic properties of N-Top dom set, N-Top minimum dom set, N-Top minimal dom set are introduced and N-Top dom number is also established with some necessary examples.

Keywords: N-Top dom set, N-Top minimum dom set, N-Top minimal dom set, N-Top dom number.

1 Introduction

The concept of topologized graph was introduced by Antoine Vella in 2005 [1]. Antoine Vella extended topology to the topologized graph by the S_1 space and the boundary of every vertex and edges of a graph G. The space is called S_1 space if every singleton in the topological space either open or closed. Chang [5] introduced the concept of the notion of fuzzy topology.In 2017,topologizd graph extended to Topologized bipartitie graph Topologized Hamiltonian and complete graph by vimala.s et al [13,14]. Ore [9] introduced the concept of theory of domination of graph.In 1997 T.Heynes , S. Hedetniemi and P. Slater published the book, ”Fundamentals of domination in graphs” [6]. After this publication there has been a rapid growth of research on this area and a wide variety of domination parameters have been introduced.

Bhuvaneswari et. al [4] handled the concept of topologized domination in graph and explained some of its properties.Smarandache[10] was first person introduced the idea of neutrosophic theory. He discussed some types of neutrosophic sets like Over, Under/Off sets etc.,[12]. He extended work on HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-) HyperAlgebra[11].Smarandache has introduced in 2020 the n-SuperHyperGraph, with super-vertices [that are groups of vertices] and hyper-edges defined on power-set of power-set... that is the most general form of graph as today, and n-HyperAlgebra. A SuperHyperGraph, is a HyperGraph (where a group of Edges form a HyperEdge) such that a group of vertices are united all together into a SuperVertex like a group of people (=vertices) that are united all together into an organization (=SuperVertex) ;and further on the n-SuperHyperGraph where many groups (=SuperVertices) are united all together to form a group-of-groups (called 2-SuperVertex, or Type-2 SuperVertex), then a group of Type-2 SuperVertices forms a Type-3 SuperVertex, . . . , and so on up to Type-n SuperVertex, for any n 1, which better reflects our reality. Later Narmada Devi[7,8] worked on new type of neutrosophic off graph and minimal domination via neutrosophic over graph. In this article, the novel of topologized domination of N-graphs are developed and some of its interesting properties are established.

2 Preliminiaries

Definition 2.1. **[4]** A topologized graph is a topological space $\mathscr{H}$ such that

(i) every singleton is open or closed

(ii) $\forall h \in \mathscr{H}$, $|\eth(h)| \leq 2$, since $\eth(h)$ is denoted by the boundary of a point h.

Definition 2.2. **[8]** A set $\mathscr{S}$ of vertices of $\mathscr{G}$ is said to be a top domination set $\mathscr{S}$ if $\mathscr{G}$ is a top graph and every vertex in $\mathscr{V}(\mathscr{G})) - \mathscr{S}$ is adjacent to atleast one vertex of in $\mathscr{S}$.

Definition 2.3. **[8]** The minimum cardinality among all the top dom set of $\mathscr{G}$ is called the top dom number of G and it is denoted by $\tau\gamma(\mathscr{G})$.

Definition 2.4. **[8]** A Ngraph is a pair $\mathscr{G} = (P, Q)$ of a crisp graph $\mathscr{G}^* = (V, E)$ where P is Nvertex set in V and Q is a Nedge set in E such that

(i) $\mathscr{T}_Q\left(m_i m_j\right) \leq \mathscr{T}_P\left(m_i\right) \wedge \mathscr{T}_P\left(m_j\right)$

(ii) $\mathscr{I}_Q\left(m_i m_j\right) \leq \mathscr{I}_P\left(m_i\right) \wedge \mathscr{I}_P\left(m_j\right)$

(iii) $\mathscr{F}_Q\left(m_i m_j\right) \geq \mathscr{T}_P\left(m_i\right) \vee \mathscr{T}_P\left(m_j\right)$ $(m_i, m_j) \in E$

3 Neutrosophic Topologized Domination Graphs

An important concept of N-Top dom in graphs with suitable examples are discussion this section. Throughout this paper $\mathscr{G}^* = (V, E)$ denotes a crisp graph and $\mathscr{G} = (P, Q)$ a Ngraph.

Definition 3.1. A Ngraph $\mathscr{G}$ is called N-Top graph if $\mathscr{G}^*$ satisfy the following condition

(i) every singleton is open or closed

(ii) $\forall h \in \mathscr{H}$, $|\eth(h)| \leq 2$, since $\eth(h)$ is denoted by the boundary of a point h.

Definition 3.2. A set $\mathscr{S}$ of vertices of $\mathscr{G}$ is said to be N-Top dom set in $\mathscr{G}$ if $\mathscr{G}$ is a N-Top graph and every vertex in $\mathscr{V}(\mathscr{G})) - \mathscr{S}$ is adjacent to atleast one vertex in $\mathscr{S}$at the degree of truth, indeterminacy and falsity-membership belongs to $[0, 1]$ such that $0 \leq \mathscr{T}_P(m) + \mathscr{I}_P(m) + \mathscr{F}_P(m) \leq 3,\ \forall m \in V$

Example 3.1.

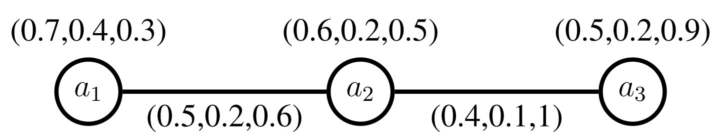


Figure 1: S_3-star graph

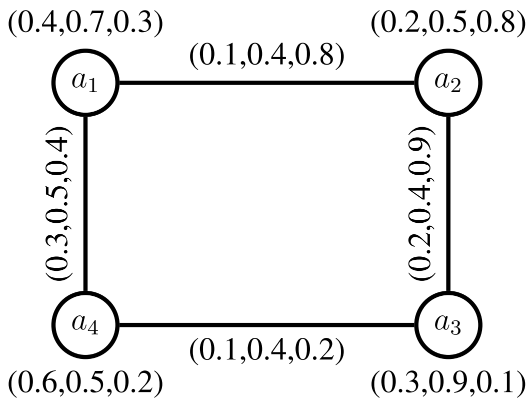
Let $\mathscr{H} = \{a_1, a_2, a_3, (0.5, 0.2, 0.6), (0.4, 0.1, 1)\}$ be a topological space defined by the topology $\tau = \{\mathscr{H}, \emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\}$. Here for every $\{h\} \in \mathscr{H}$ is open or closed and $|\eth(h)| \leq 2$. By the definition of 3.1, $\mathscr{G} = (P, Q)$ is N-Top graph. Also N-top dom sets by $\mathscr{D}_1 = \{a_2\}$ and $\mathscr{D}_2 = \{a_1, a_3\}$.

Theorem 3.1. Let $\mathscr{G}$ be a N-Top graph with atmost degree two. If $\mathscr{S}$ is a top dom set of $\mathscr{G}$, then it is a N-Top dom set of $\mathscr{G}$.

Proof:

Let $\mathscr{H}$ be a topological space with topology τ defined by $V \cup E$. Since every singleton set is open or closed and $\mathscr{G}$ is a N-graph where the truth, indeterminacy and falsity membership function with unit interval $[0, 1]$ such that $0 \leq \mathscr{T}_P(m) + \mathscr{I}_P(m) + \mathscr{F}_P(m) \leq 3$, $\forall m \in V$ and atmost degree two. Hence $|\eth(h)| \leq 2$. This implies that $\mathscr{G}$ is N-Top graph. Let $\mathscr{S}$ be top dom set then every vertex in $\mathscr{V}(\mathscr{G})) - \mathscr{S}$ is adjacent to atleast one vertex of $\mathscr{S}$ thus implies that $\mathscr{S}$ is a N-Top dom set of $\mathscr{G}$.

Example 3.2.



Let a_1, a_2, a_3 and a_4 denote the vertices and $(0.1, 0.4, 0.8)$, $(0.2, 0.4, 0.9)$, $(0.1, 0.4, 0.2)$, $(0.3, 0.5, 0.4)$ denote the edges which are labelled $f(0.1, 0.4, 0.8) = \{a_1, a_2\}$, $f(0.2, 0.4, 0.9) = \{a_2, a_3\}$, $f(0.1, 0.4, 0.2) = \{a_3, a_4\}$, $f(0.3, 0.5, 0.4) = \{a_4, a_1\}$,

Let a_1, a_2, a_3 and a_4 denote the vertices and $(0.1, 0.4, 0.8)$, $(0.2, 0.4, 0.9)$, $(0.1, 0.4, 0.2)$, $(0.3, 0.5, 0.4)$ denote the edges.

Let $\mathscr{H} = \{a_1, a_2, a_3, a_4, (0.1, 0.4, 0.2), (0.3, 0.5, 0.4), (0.1, 0.4, 0.8), (0.2, 0.4, 0.9)\}$ be a topological space defined by the topology

$$\tau = \Big\{\mathscr{H}, \emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_3, a_4\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}\Big\}$$

Here for every $\{h\} \in \mathscr{H}$ is open and $|\partial(h)| \leq 2$. We have $\partial(a_1) = \{a_2, a_4\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_2, a_4\}$ and $\partial(a_4) = \{a_1, a_3\}$ with $|\eth(h_i)| = 2$ where $i = 1, 2, 3, 4$.

Hence this graph is a N-Top graph.

Then $D = \{a_1, a_3\}$ and $\{a_2, a_4\}$ is a N-Top dom set in V whose maximum and minimum degrees of $\mathscr{T}, \mathscr{I}, \mathscr{F}$ respectively.

	$\mathscr{T}_A$	$\mathscr{I}_A$	$\mathscr{F}_A$
a_1	0.4	0.7	0.3
a_2	0.2	0.5	0.8
a_3	0.3	0.9	0.1
a_4	0.6	0.5	0.2

	a_1a_2	a_2a_3	a_3a_4	a_4a_1
$\mathscr{T}_{m_i \cup m_j} = \max \langle m_i, m_j \rangle$	0.4	0.3	0.6	0.6
$\mathscr{I}_{m_i \cup m_j} = \max \langle m_i, m_j \rangle$	0.7	0.9	0.9	0.7
$\mathscr{F}_{m_i \cap m_j} = \min \langle m_i, m_j \rangle$	0.3	0.1	0.1	0.2

	a_1a_2	a_2a_3	a_3a_4	a_4a_1
$\mathscr{T}_{m_i \cap m_j} = \text{min}\langle m_i, m_j\rangle$	0.2	0.2	0.3	0.4
$\mathscr{I}_{m_i \cap m_j} = \text{min}\langle m_i, m_j\rangle$	0.5	0.5	0.5	0.5
$\mathscr{F}_{m_i \cup m_j} = \text{max}\langle m_i, m_j\rangle$	0.8	0.8	0.2	0.3

Therefore this graph is N-TOP dom graph.

Definition 3.3. A dominating set $\mathscr{S}$ of the N-Top graph $\mathscr{G}$ is said to be a minimal N-Top dom set if for every vertex v in $\mathscr{S}$, $\mathscr{S} - \{v\}$ is not a of $\mathscr{S}$ is a N-Top dom set. i.e., no proper subset of $\mathscr{S}$ is a N-Top dom set.

Example 3.3. From Example 1, $\{a_1, a_3\}$ is N-Top minimal dom set but which is not a N-Top dom set.

Theorem 3.2. Let $\mathscr{G}$ be a N-Top graph with atmost degree two. If $\mathscr{S}$ is a N-TOP minimum dom set, then D is a N-Top minimal dom set.

Proof:

Let $\mathscr{H}$ be a topological space with topology τ defined by $V \cup E$.

Since every singleton set is open or closed and $\mathscr{G}$ is a N-graph where the truth, indeterminacy and falsity membership function with unit interval $[0, 1]$ such that $0 \leq \mathscr{T}_P(m) + \mathscr{I}_P(m) + \mathscr{F}_P(m) \leq 3$, $\forall m \in V$ and atmost degree two. Hence $|\eth(h)| \leq 2$. This implies that $\mathscr{G}$ is N-Top graph.

Let $\mathscr{S}$ be a top minimum dom set. Then every $v \in \mathscr{S}, \mathscr{S} - \{v\}$ is not a top dom set which implies that $\mathscr{S}$ is a N-Top minimal dom set.

Remark 3.1. The converse of the above theorem need not by true. Since every graph need not be a N-Top graph. Consider the following example.

Example 3.4. Let $\mathscr{G}$ be a N complete graph K_4 with 4 vertices.

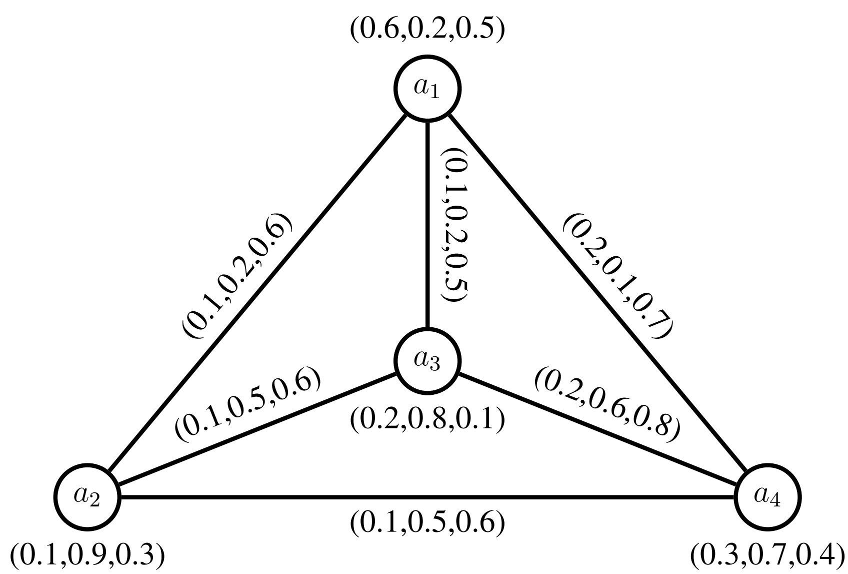
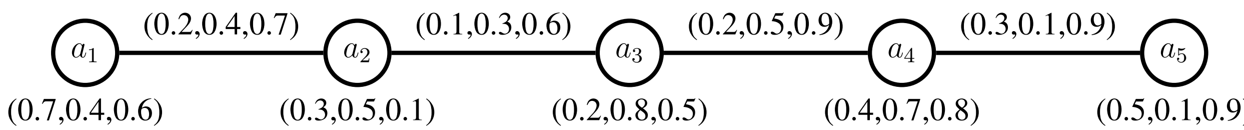


Figure 2: K_4 N-complete graph

Here every singleton sets are minimum dominating sets. Clearly the complete graph K_4 is not a N-Top graph, since $n \geq 4$. Then the a N-Top dom set does exits.Then the dom sets need not be a N-Top dom set.

Lemma .1. Let P_n be a N-path with n vertices which is a N-Top graph. Then the N-Top dom number is $\tau_\gamma(P_n) \geq \lceil n/3 \rceil$.

Example 3.5.



Let a_1, a_2, a_3, a_4 and a_5 denote the vertices and $(0.2, 0.4, 0.7), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9), (0.3, 0.1, 0.9)$ denote the edges which are labelled $f(0.2, 0.4, 0.7) = \{a_1, a_2\}$, $f(0.1, 0.3, 0.6) = \{a_2, a_3\}$, $f(0.2, 0.5, 0.9) = \{a_3, a_4\}$, $f(0.3, 0.1, 0.9) = \{a_4, a_5\}$,

Let $\mathscr{H} = \{a_1, a_2, a_3, a_4, a_5, (0.2, 0.4, 0.7), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9), (0.3, 0.1, 0.9)\}$ be a topological space defined by the topology

$$\tau = \Big\{ \mathscr{H}, \emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_2, a_5\}, \{a_3, a_4\}, \{a_3, a_5\}, \{a_4, a_5\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_1, a_2, a_5\}, \{a_2, a_3, a_4\}, \{a_2, a_3, a_5\}, \{a_3, a_4, a_5\}, \{a_1, a_2, a_3, a_4\}, \{a_1, a_2, a_3, a_5\} \Big\}$$

Here for every $\{h\} \in \mathscr{H}$ is open and $|\eth(h)| \leq 2$. By the definition 3.1,it is a N-Top graph.
Then $\mathscr{S} = \{a_2, a_5\}$ is a N-Top dom set in V whose maximum and minimum degrees of $\mathscr{T}, \mathscr{I}, \mathscr{F}$ respectively.

	a_1	a_2	a_3	a_4	a_5
$\mathscr{T}_A$	0.7	0.3	0.2	0.4	0.5
$\mathscr{I}_A$	0.4	0.5	0.8	0.7	0.1
$\mathscr{F}_A$	0.6	0.1	0.5	0.8	0.9

	a_1a_2	a_2a_3	a_3a_4	a_4a_5
$\mathscr{T}_B$(min)	0.2	0.1	0.2	0.3
$\mathscr{I}_B$(min)	0.4	0.3	0.5	0.1
$\mathscr{F}_B$(max)	0.7	0.6	0.9	0.9

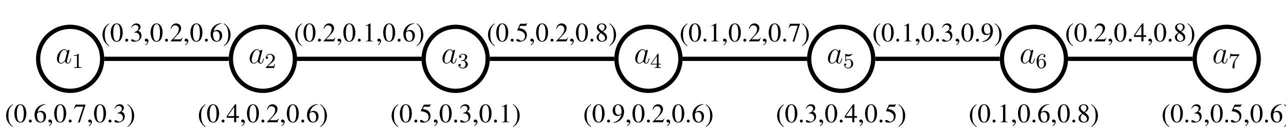
	a_1a_2	a_2a_3	a_3a_4	a_4a_5
$\mathscr{T}_B$(max)	0.7	0.3	0.4	0.5
$\mathscr{I}_B$(max)	0.5	0.8	0.8	0.7
$\mathscr{F}_B$(min)	0.6	0.5	0.8	0.9

The N-Top dom set is $\mathscr{S} = \{a_2, a_5\}$.

$$\tau_\gamma(P_5) = \lceil 5/3 \rceil = \lceil 1.666 \rceil = 2.$$

Therefore this graph is N-Top dom graph.

Example 3.6.



Let $a_1, a_2, a_3, a_4, a_5, a_6$ and a_7 denote the vertices and $(0.3, 0.2, 0.6)$, $(0.2, 0.1, 0.6)$, $(0.5, 0.2, 0.8)$, $(0.1, 0.2, 0.7)$, $(0.1, 0.3, 0.9)$, $(0.2, 0.4, 0.8)$ denote the edges which are labelled $f(0.3, 0.2, 0.6) = \{a_1, a_2\}$, $f(0.2, 0.1, 0.6 = \{a_2, a_3\}$, $f(0.5, 0.2, 0.8) = \{a_3, a_4\}$, $f(0.1, 0.2, 0.7) = \{a_4, a_5\}$, $f(0.1, 0.3, 0.9) = \{a_5, a_6\}$, $f(0.2, 0.4, 0.8) = \{a_6, a_7\}$,

Let $\mathscr{H} = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, (0.3, 0.2, 0.6), (0.2, 0.1, 0.6), (0.5, 0.2, 0.8), (0.1, 0.2, 0.7), (0.1, 0.3, 0.9), (0.2, 0.4, 0.8)\}$ be a topological space defined by the topology

$$\begin{aligned}\tau = \Big\{&\mathscr{H}, \emptyset, \{a_1\}, \{a_2, a_3\}, \{a_4\}, \{a_5, a_6\}, \{a_7\}, \{a_1, a_2, a_3\}, \{a_1, a_4\}, \{a_1, a_5, a_6\}, \{a_1, a_7\},\\ &\{a_2, a_3, a_4\}, \{a_2, a_3, a_5, a_6\}, \{a_2, a_3, a_7\}, \{a_4, a_5, a_6\}, \{a_4, a_7\}, \{a_5, a_6, a_7\}, \{a_1, a_2, a_3, a_4\},\\ &\{a_1, a_2, a_3, a_5, a_6\}, \{a_1, a_2, a_3, a_7\}, \{a_1, a_2, a_3, a_5, a_6\}, \{a_1, a_2, a_3, a_4, a_5, a_6\}, \{a_1, a_2, a_3, a_4, a_7\},\\ &\{a_1, a_2, a_3, a_5, a_6, a_7\}, \{a_1, a_4, a_5, a_6\}, \{a_1, a_4, a_7\}, \{a_1, a_4, a_7\}, \{a_1, a_4, a_5, a_6, a_7\},\\ &\{a_1, a_2, a_3, a_4, a_5, a_6\}, \{a_1, a_2, a_3, a_4, a_7\}, \{a_2, a_3, a_4, a_5, a_6, a_7\}, \{a_2, a_3, a_4, a_7\},\\ &\{a_1, a_2, a_3, a_5, a_6, a_7\}, \{a_4, a_5, a_6, a_7\}, \{a_2, a_3, a_5, a_6, a_7\}, \{a_2, a_3, a_4, a_7\}, \{a_2, a_3, a_4, a_5, a_6\},\\ &\{a_1, a_5, a_6, a_7\}, \{a_1, a_4, a_7\}, \{a_1, a_4, a_5, a_6\}, \{a_1, a_3, a_4, a_5, a_6, a_7\}, \{a_1, a_2, a_4, a_5, a_6, a_7\},\\ &\{a_1, a_2, a_3, a_4, a_6, a_7\}, \{a_1, a_2, a_3, a_4, a_5, a_7\}, \Big\}\end{aligned}$$

Here for every $\{h\} \in \mathscr{H}$ is open or closed and $|\eth(h)| \leq 2$. By the definition of 3.1,it is a N-Top graph.

Then $\mathscr{S} = \{a_2, a_5, a_7\}$ is a N-Top dom set in V whose maximum and minimum degrees of $\mathscr{T}, \mathscr{I}, \mathscr{F}$ respectively.

	a_1	a_2	a_3	a_4	a_5	a_6	a_7
$\mathscr{T}_P$	0.6	0.4	0.5	0.9	0.3	0.1	0.3
$\mathscr{I}_P$	0.7	0.2	0.3	0.2	0.4	0.6	0.5
$\mathscr{F}_P$	0.3	0.6	0.1	0.6	0.5	0.8	0.6

	a_1a_2	a_2a_3	a_3a_4	a_4a_5	a_5a_6	a_6a_7
$\mathscr{T}_Q$(max)	0.6	0.5	0.9	0.9	0.3	0.3
$\mathscr{I}_Q$(max)	0.7	0.3	0.3	0.4	0.6	0.6
$\mathscr{F}_Q$(min)	0.3	0.1	0.1	0.5	0.5	0.6

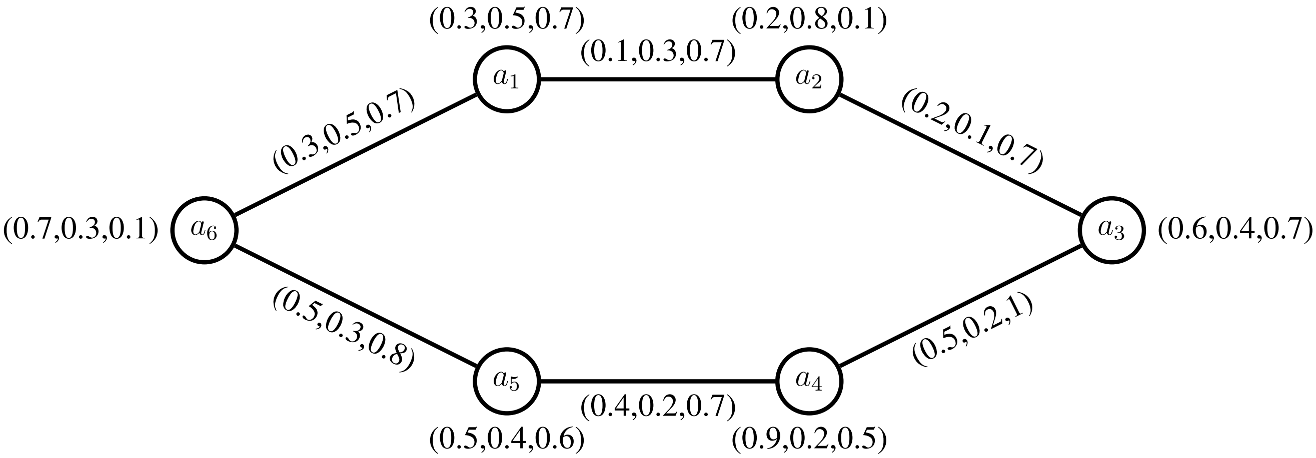
	a_1a_2	a_2a_3	a_3a_4	a_4a_5	a_5a_6	a_6a_7
$\mathscr{T}_Q$(min)	0.4	0.3	0.5	0.3	0.1	0.1
$\mathscr{I}_Q$(min)	0.2	0.1	0.2	0.2	0.4	0.5
$\mathscr{F}_Q$(max)	0.6	0.7	0.8	0.6	0.8	0.9

Thererfore N-top dom set is $\mathscr{S} = \{a_2, a_5, a_7\}$.

$$\tau_\gamma(P_7) = \lceil 7/3 \rceil = \lceil 2.333 \rceil > 2.$$

Lemma .2. Let $\mathscr{C}_n$ be a N-cycle with $n-$vertices which is a N-Top graph. Then the N-Top dom number $\tau_\gamma(\mathscr{C}_n) \geq \lceil n/3 \rceil$.

Example 3.7.



Let a_1, a_2, a_3, a_4, a_5 and a_6 denote the vertices and $(0.1, 0.3, 0.9)$, $(0.2, 0.1, 0.7)$, $(0.5, 0.2, 0.1)$, $(0.4, 0.2, 0.7)$, $(0.5, 0.3, 0.8)$, $(0.3, 0.5, 0.7)$ denote the edges which are labelled $f(0.1, 0.3, 0.9) = \{a_1, a_2\}$, $f(0.2, 0.1, 0.7) = \{a_2, a_3\}$, $f(0.5, 0.2, 0.1) = \{a_3, a_4\}$, $f(0.4, 0.2, 0.7) = \{a_4, a_5\}$, $f(0.5, 0.3, 0.8) = \{a_5, a_6\}$, $f(0.3, 0.5, 0.7) = \{a_6, a_1\}$,

Let $\mathscr{H} = \{a_1, a_2, a_3, a_4, a_5, a_6, (0.1, 0.3, 0.9), (0.2, 0.1, 0.7), (0.5, 0.2, 0.1), (0.4, 0.2, 0.7), (0.5, 0.3, 0.8), (0.3, 0.5, 0.7)\}$ be a topological space defined by the topology

$$\begin{aligned}\tau = \Big\{&\mathscr{H}, \emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_5\}, \{a_6\}, \\ &\{a_1, a_2\}, \{a_1, a_3\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_1, a_6\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_2, a_5\}, \{a_2, a_6\}, \{a_3, a_4\}, \\ &\{a_3, a_5\}, \{a_3, a_6\}, \{a_4, a_5\}, \{a_4, a_6\}, \{a_5, a_6\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_1, a_2, a_5\}, \{a_1, a_2, a_6\}, \\ &\{a_1, a_2, a_3, a_4\}, \{a_1, a_2, a_3, a_5\}, \{a_1, a_2, a_3, a_6\}, \{a_1, a_2, a_3, a_4, a_5\}, \{a_1, a_2, a_3, a_4, a_6\}, \\ &\{a_2, a_3, a_4\}, \{a_2, a_3, a_5\}, \{a_2, a_3, a_6\}, \{a_3, a_4, a_5\}, \{a_3, a_4, a_6\}, \{a_4, a_5, a_6\}, \{a_2, a_3, a_4, a_5\}, \\ &\{a_2, a_3, a_4, a_6\}, \{a_2, a_3, a_4, a_5, a_6\}, \{a_1, a_3, a_4, a_5, a_6\}, \{a_1, a_2, a_4, a_5, a_6\}, \{a_1, a_2, a_3, a_5, a_6\}, \\ &\{a_1, a_2, a_3, a_4, a_5\}, \{a_3, a_4, a_5, a_6\}, \{a_2, a_4, a_5, a_6\}, \{a_2, a_3, a_4, a_6\}, \{a_2, a_3, a_4, a_5\}, \{a_3, a_5, a_6\}\Big\}\end{aligned}$$

Here for every $\{h\} \in \mathscr{H}$ is open and $|\eth(h)| \leq 2$. By the definition of 3.1, it is a N-Top graph.

Then $\mathscr{S} = \{a_3, a_6\}$ is a N-Top dom set in V whose maximum and minimum degrees of $\mathscr{T}, \mathscr{I}, \mathscr{F}$ respectively.

	a_1	a_2	a_3	a_4	a_5	a_6
$\mathscr{T}_P$	0.3	0.2	0.6	0.9	0.5	0.7
$\mathscr{I}_P$	0.5	0.8	0.4	0.2	0.4	0.3
$\mathscr{F}_P$	0.7	0.1	0.7	0.5	0.6	0.1

	a_1a_2	a_2a_3	a_3a_4	a_4a_5	a_5a_6	a_6a_1
$\mathscr{T}_Q$(max)	0.3	0.6	0.9	0.9	0.8	0.7
$\mathscr{I}_Q$(max)	0.8	0.8	0.3	0.5	0.4	0.5
$\mathscr{F}_Q$(min)	0.1	0.1	0.5	0.5	0.1	0.1

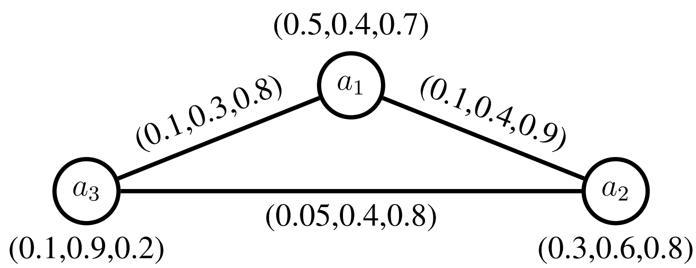
	a_1a_2	a_2a_3	a_3a_4	a_4a_5	a_5a_6	a_6a_1
$\mathscr{T}_Q$(min)	0.1	0.2	0.5	0.4	0.5	0.3
$\mathscr{I}_Q$(min)	0.3	0.1	0.2	0.2	0.3	0.5
$\mathscr{F}_Q$(max)	0.7	0.7	0.8	0.7	0.8	0.7

Therefore N-Top dom set is $\mathscr{S} = \{a_3, a_6\}$.

$$\tau_\gamma(C_6) = \lceil 6/3 \rceil = 2.$$

Lemma .3. Let $(\mathscr{K}_n)$ be a N-complete graph with $n-$vertices $(n = 2, 3)$ which is a N-Top graph. Then the N-Top dom number $\tau_\gamma(\mathscr{K}_n) = 1$.

Example 3.8.



Let a_1, a_2 and a_3 denote the vertices and $(0.1, 0.4, 0.9), (0.05, 0.4, 0.8), (0.1, 0.3, 0.8)$ denote the edges which are labelled $f(0.1, 0.4, 0.9) = \{a_1, a_2\}$, $f(0.05, 0.4, 0.8) = \{a_2, a_3\}$, $f(0.1, 0.3, 0.8) = \{a_3, a_1\}$,

Let $\mathscr{H} = \{a_1, a_2, a_3, (0.1, 0.4, 0.9), (0.05, 0.4, 0.8), (0.1, 0.3, 0.8)\}$ be a topological space defined by the topology

$$\tau = \left\{ \mathscr{H}, \emptyset, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\} \right\}$$

Here for every $\{h\} \in \mathscr{H}$ is open and $|\eth(h)| \leq 2$. By the definition 3.1, it is a N-Top graph.

Then $\mathscr{S} = \{a_1\}$ and $\{a_2a_3\}$ is a N-Top dom set in V whose maximum and minimum degrees of $\mathscr{T}, \mathscr{I}, \mathscr{F}$ respectively.

	a_1	a_2	a_3
T_A	0.5	0.3	0.1
I_A	0.4	0.6	0.9
F_A	0.7	0.8	0.2

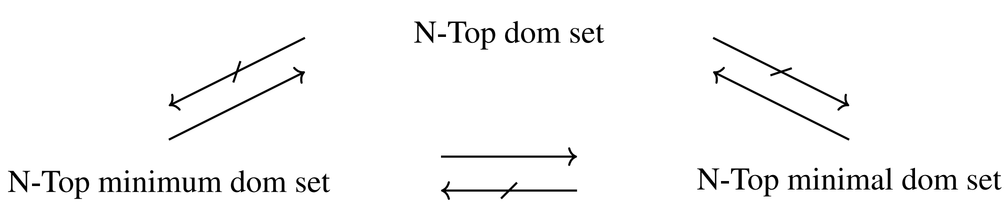
	a_1a_2	a_2a_3	a_3a_1
$\mathscr{T}_P$(max)	0.5	0.3	0.5
$\mathscr{I}_P$(max)	0.6	0.9	0.9
$\mathscr{F}_P$(min)	0.7	0.2	0.2

	a_1a_2	a_2a_3	a_3a_1
$\mathscr{T}_Q$(min)	0.3	0.1	0.1
$\mathscr{I}_Q$(min)	0.4	0.6	0.4
$\mathscr{F}_Q$(max)	0.8	0.8	0.7

Therefore N-Top dom set is $\mathscr{S} = \{a_1\}$, whose top dom number is given by

$$\tau_\gamma(\mathscr{K}_3) = 1.$$

Remark 3.2. The interrelationship among N-Top dom set as given below



4 Conclusion

This paper has focused on calculating the dominating number of N-Top graph G by using top domination conditions.The Top dom condition is introduced in new method to find the domination number. The N-Top domination for some standard N-graphs such as a path,cycle are specified.The future study can be continued by forming different types of N-Top domination set with various applications.

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