Neutrosophic Sets and Systems, Vol. 47, 2021
University of New Mexico

Generalized Neutrosophic Semirings

Muhammad Gulistan[1], Inayatur Rehman[2,*], Muhammad Shahzad[3], Shah Nawaz[4] and Salma Khan[5]

[1,3,4,5]Department of Mathematics, Hazar University Mansehra, Pakistan
[1]gulistan21@yahoo.com, [3]stanolipk@gmail.com, [4]shahnawazawan82@gmail.com and [5]salmakhan359@gmail.com
[*2] Department of Mathematics and Sciences, College of Arts and Applied Sciences, Dhofar University, Salalah, Oman
[*2]irehman@du.edu.om

* Correspondence: irehman@du.edu.om; Tel.: (+96890126747)

Abstract: In this paper we are going to give the idea of Neutrosophic Semirings and study their algebraic structure. To understand Neutrosophic Semirings, we provide number of examples. Through ideals and congruence relations, we discuss the structural behavior and properties of this notion by establishing number of results.

Keywords: Neutrosophic Sets; Neutrosophic Semirings; Neutrosophic Ideals and Congruence Relations.

1. Introduction

Different researchers have defined algebraic structures which were based on the crisp set. But actually, the real-life problems could not be solved by crisp set theory. The crisp set deals with yes or no only and it never tells about in between yes and no. In 1965, Zadeh [1] introduced a fuzzy set theory to address the vagueness of various real-life problems. The fuzzy sets deal with membership in between 0 and 1. Later, in 1986, Atanassov [2], initiated the idea of intuitionistic fuzzy set. In intuitionistic fuzzy set focused on the degree of membership and non-membership. In fact, these theories have remained unsuccessful in finding a solution to many real-life mathematical challenges.

In 1999, Smarandache [3] gave the notion of Neutrosophic set. Neutrosophic set is a generalized form of fuzzy set as well as intuitionistic fuzzy set. Nowadays, Neutrosophic set attains more attention of researchers due to its characteristic behavior to solve the indeterminate situations in the different fields of life. In 2006, Smarandache et al., [4] were the first ones who applied the concept of Neutrosophic sets on some algebraic structure and in their work, they introduced the Neutrosophic rings. Later, in 2011 Agboola et al., [5], discussed Neutrosophic Rings-I. Neutrosophic groups and Neutrosophic sub-groups were introduced in 2012 by Agboola et al., [6]. Ali et al., [7,8,9,10] have used Neutrosophic set approach for different algebraic structures. In 2016, Khan et al., [11] briefly discussed the characterization of Neutrosophic left almost semigroups. On the other hand, the application of Neutrosophic sets is getting more attention of researchers. For more study about the application aspect of Neutrosophic sets, the readers are referred to [12,13,14,15,16,17,18,19].

In 1992, Golan [20], has done a worth full work and defined a new structure named semiring. A detailed discussion has been done in this paper. We used Neutrosophic set approach to semiring and gave the idea of Neutrosophic semiring. We discussed the characteristic properties of Neutrosophic semiring and its substructures. For that we have given various examples. We talked about the ideals of Neutrosophic semirings. In last section, congruence relations are characterized in the Neutrosophic semirings. We established number of results. For instance: Suppose $\{J_k: k \in K\}$ is a collection of left ideals in neutrosophic semiring N. Then $\bigcup J_k$ is a left ideal if $J_k \subseteq J_{k+1}$ for all $k \in K$. Another main result: A relation ρ on a neutrosophic semiring N is congruence relation if and only if it is left congruence and right as well.

2. Neutrosophic Semirings

We generalize the theory of semrings to neutrosophic semirings. We also discuss about ideals and congruences of neutrosophic semirings with some properties.

Definition 2.1. An algebraic structure $(S \cup I, *_1, *_2)$ is called neutrosophic semiring if $*_1$ and $*_2$ are the closed and associative binary operations and $*_2$ is distributive over $*_1$ where S is semiring with respect to $*_1$ and $*_2$ and I is the neutrosophic element ($I = I^2$) and $< S \cup I > = \{a + bI ; a, b \in S \}$.

Let us give some examples to understand this notion completely.

Example 2.2. The set $S = \{\overline{0}, \overline{1}, \overline{2}\}$ is the semiring under the operation of addition $*_1$ and multiplication $*_2$(mod3) which can be verified.

Then $< S \cup I > = \{0 + 0I, 0 + 1I, 0 + 2I, 1 + 0I, 1 + 1I, 1 + 2I, 2 + 0I, 2 + 1I, 2 + 2I\}$ is the neutrosophic semiring under the same operations as that of S i. e for all $a + bI, c + dI \in S \cup I$,

$(a + bI) *_1 (c + dI) = (a + c) + (b + d)I$, $(a + bI) *_2 (c + dI) = (ac) + (bd)I$.

Closure property: Let $(a + bI)$ and $(c + dI) \in S \cup I$ then $(a + bI) *_1 (c + dI) = (a + c) + (b + d)I \in S \cup I$ and $(a + bI) *_2 (c + dI) = (ac) + (bd)I \in S \cup I$. Which shows that closure property is satisfied.

Distributive law: Let $(a + bI)$, $(c + dI)$ and $(e + f)I \in S \cup I$ then $(a + bI) *_2 [(c + dI) *_1 (e + f)I]$
$= (a + bI) *_2 ((c + e) + (d + f)I) = (ac + ae) + (bd + bf)I$.

and $[(a + bI) *_2 (c + dI)] *_1 [(a + bI) *_2 (e + f)I] = ((ac) + (bd)I) *_1 ((ae) + (bf)I)$
$= (ac + ae) + (bd + bf)I$.

Associative law: Let $(a + bI)$, $(c + dI)$ and $(e + f)I \in S \cup I$. Then

$[(a + bI) *_1 (c + dI)] *_1 (e + f)I = ((a + c) + (b + d)I) *_1 (e + f)I = (a + c + e) + (b + d + f)I$.

$(a + bI) *_1 [(c + dI) *_1 (e + f)I] = (a + bI) *_1 ((c + e) + (d + f)I) = (a + c + e) + (b + d + f)I$.

Again

$[(a + bI) *_2 (c + dI)] *_2 (e + f)I = ((ac) + (bd)I) *_2 (e + f)I = (ace) + (bdf)I.$

$(a + bI) *_2 [(c + dI) *_2 (e + f)I] = (a + bI) *_2 ((ce) + (df)I) = (ace) + (bdf)I.$

Example 2.3. The structure $(Z, *_1, *_2)$ is the semiring under the operations of usual multiplication and addition where Z is the set of integers and $*_1$ and $*_2$ represent the usual multiplication and addition. Then $< Z\cup I > = \{a + bI; a, b \in Z\}$ is the neutrosophic semiring and for all $a + bI, c + dI \in Z\cup I$ $(a + bI) *_1 (c + dI) = (ac) + (bd)I$ and $(a + bI) *_2 (c + dI) = (a + c) + (b + d)I$.

Definition 2.4. A proper subset P of a neutrosophic semiring $(S\cup I, *_1, *_2)$ is called neutrosophic subsemiring if P itself is a neutrosophic semiring under the same binary operation as that of $S\cup I$.

Let us give some examples of neutrosophic subsemiring.

Example 2.5. Let Wand N are the sets of whole and natural numbers respectively then $W\cup I = \{a + bI; a, b \in W\}$ is the neutrosophic semiring under $*_1$ and $*_2$ are of example number **2.3** Then $N\cup I = \{ a + bI; a, b \in N \}$ is the subset of $W\cup I$ and hence under the operations of $W\cup I$ the subset $N\cup I$ becomes the neutrosophic subsemiring. One can check it easily.

Example 2.6. Let $N = \{1, 2, 3,\ldots\}$ and $S = \{ 2, 3, 4,\ldots\}$ then algebraic structure $(N\cup I, *_1, *_2)$ is the neutrosophic semiring where $*_1$ and $*_2$ are the operations of example 3. Then $S\cup I$ is obviously subset of $N\cup I$ and hence is the neutrosophic subsemiring of $N\cup I$ under the operations of $N\cup I$.

Lemma 2.7. A subset P of neutrosophic semiring $(S\cup I, *_1, *_2)$ is neutrosophic subsemiring if and only if it satisfies the following:

(i) $(a + bI) *_1 (c + dI) \in P$ and

(ii) $(a + bI) *_2 (c + dI) \in P \quad \forall (a + bI), (c + dI) \in P$.

Proof. Let $(P, *_1, *_2)$ is a neutrosophic subsemiring then by closure property

$\forall (a + bI), (c + dI) \in P$, $(a + bI) *_1 (c + dI) \in P$ and $(a + bI) *_2 (c + dI) \in P$.

Conversely,

let $\forall (a + bI), (c + dI) \in P$,

(i) $(a + bI) *_1 (c + dI) \in P$

(ii) $(a + bI) *_2 (c + dI) \in P$.

Then from (i) and (ii) closure property is satisfied. And since elements of P are of neutrosophic semiring $(S\cup I, *_1, *_2)$ and operations are also same as that of $S\cup I$ and since associative and distributive laws are satisfied therein so both laws are satisfied for P with respect to $*_1$ and $*_2$. Hence this makes $(P, *_1, *_2)$ into neutrosophic subsemiring.

Proposition 2.8. The intersection of any number of neutrosophic subsemiring is either empty or neutrosophic subsemiring.

Proof. Let $\{B_i: i\in J\}$ is the collection of neutrosophic subsemirings of $(S\cup I, *_1, *_2)$. Let $\cap B_i \neq \phi$ then we show that the subset $\cap B_i$ of $S\cup I$ is the neutrosophic subsemiring. Let $(a + bI)$ and $(c + dI) \in \cap B_i \Rightarrow (a + bI)$ and $(c + dI) \in B_i \quad \forall i\in J$. This implies $(a + bI) *_1 (c + dI)$ and $(a + bI) *_2 (c + dI) \in B_i \quad \forall i\in J$ because each B_i is neutrosophic subsemiring of $(S\cup I, *_1, *_2)$. So, this implies $(a + bI) *_1 (c + dI)$ and $(a + bI) *_2 (c + dI) \in\cap B_i \quad \forall i\in J$. Thus by the **Lemma 2.7** $\cap B_i$ is the neutrosophic subsemiring.

Remark 2.9. Union of neutrosophic subsemirings may or may not be the neutrosophic subsemiring.

Example 2.10. If X = {a, b, c, d, e} then (P(X), ∪, ∩) is a semiring. Now let P(X)=S, then <S∪I> = {A + BI : A , B ∈ P(X)} is the neutrosophic semiring. Suppose S_1 = {{}, {a}, {a, b}, X} and S_2 = {{}, {c}, {b, c}, X} are the subsets of S. Then < S_1∪I > = {C + DI; C, D ∈ S_1} and < S_2∪I > = {E + FI: E, F ∈ S_2}. In tabular form: < S_1∪I > = {{} + { }I , { } + {a}I, { } + {a, b }I, { } + X I , { a } + { }I , { a} + {a}I, { a} + {a, b }I, {a } + X I, { a, b} + {a, b }I , {a, b } + {a}I, {a, b } + {a, b }I, {a, b } + X I, X + { }I , X + {a}I, X + {a, b }I, X + X I} and < S_2∪I > = {{ } + { }I ,{ } + {c}I, { } + {b, c }I, { } + X I , { c } + { }I , { c} + {c}I, { c} + {b, c }I, {c } + X I, { b, c} + {b, c }I , {b, c } + {c}I, {b, c } + {b, c }I, {b, c } + X I, X + { }I , X + {c}I, X + {b, c }I, X + X I } are the neutrosophic semirings but their union < S_1∪I > ∪ < S_2∪I > = {{ } + { }I , { } + {a}I, { } + {a, b }I, { } + X I , { a } + { }I , { a} + {a}I, { a} + {a, b }I, {a } + X I, { a, b} + {a, b }I , {a, b } + {a}I, {a, b } + {a, b }I, {a, b } + X I, X + { }I , X + {a}I, X + {a, b }I, X + X I , { } + { }I , { } + {c}I, { } + {b, c }I, { } + X I , { c } + { }I , { c} + {c}I, { c} + {b, c }I, {c } + X I, { b, c} + {b, c }I , {b, c } + {c}I, {b, c } + {b, c }I, {b, c } + X I, X + { }I , X + {c}I, X + {b, c }I, X + X I } is not the neutrosophic semiring because the operation of union is not closed i.e. { a, b} + {a, b }I ∪{ b, c} + {b, c }I = ({ a, b}∪{ b, c}) + ({ a, b}∪{ b, c}) I = {a, b, c} + {a, b, c}I ∉ < S_1∪I > ∪ < S_2∪I >.

Now we will show that under what condition union of neutrosophic subsemiring is again a neutrosophic subsemiring.

Proposition 2.11. Let {B_k: k ∈ K} be the family of neutrosophic subsemiring. If $B_k \subseteq B_{k+1}$ then ⋃ B_k is neutrosophic subsemiring.

Proof. Let a + bI, c + dI ∈ ⋃B_k then this implies that there exist some p and q∈ K such that a + bI ∈ B_p and c + dI ∈ B_q. Let q > p then a + bI, c + dI ∈ B_q. Now since B_q is neutrosophic subsemiring so (a + bI) $*_1$ (c + dI) and (a + bI) $*_2$ (c + dI) ∈ B_q so, this implies that (a + bI) $*_1$ (c + dI) and (a + bI) $*_2$ (c + dI) ∈⋃B_k thus by the **Lemma 2.7** ⋃B_k is neutrosophic subsemiring.

Corollary 2.12. Union of two neutrosophic subsemirings N_1 and N_2 of N is again neutrosophic subsemiring of N if either $N_1 \subseteq N_2$ or $N_2 \subseteq N_1$.

Proof. Let $N_1 \subseteq N_2$ then obviously $N_1 \cup N_2 = N_2$. It follows that $N_1 \cup N_2$ is a neutrosophic subsemiring. Again let $N_2 \subseteq N_1$ then clearly $N_1 \cup N_2 = N_1$. Now as N_1 is neutrosophic subsemiring so, $N_1 \cup N_2$ is by transitive property.

We discuss the ideals of neutrosophic semirings and describe its properties.

Definition 2.13. Let J be the nonempty subset of neutrosophic semiring S, then J is called left ideal of S if ∀ ($a_1 + b_1I$), ($a_2 + b_2I$) ∈ J, ($a_1 + b_1I$) $*_1$ ($a_2 + b_2I$) ∈ J and (a + bI) $*_2$ (c + dI) ∈ J ∀ (a + bI) ∈ S and (c + dI) ∈ J.

Example 2.14. Let N = {1, 2, 3,…} is the set of natural numbers and $*_1$, $*_2$ represent the operations of multiplication and addition as defined in **Example 2.3** then N∪I ={a + bI: a, b∈ N}becomes the neutrosophic semiring and for set S = {2, 3, 4,…} the structure S∪I ={c + dI: c, d ∈ S } becomes the left ideal because ∀ e + f I ∈ N , a + bI ∈ S and c + dI ∈ S, (a + bI) $*_1$ (c + dI) ∈ S and (e + f I) $*_2$ (c + dI) ∈ S .

Theorem 2.15. The intersection of any number of left ideals of neutrosophic semiring N is again a left ideal.

Proof. We suppose that $\{J_k: k \in K\}$ be the family of left ideals of neutrosophic semiring N. We have to show that $\bigcap J_k$ is the left ideal of N. Let (a + bI), (c + dI) $\in \bigcap J_k$ and (e + f I) ∈ N this implies that (a + bI), (c + dI) $\in J_k$ ∀ k ∈ K, and (e + f I) ∈ N this implies that (a + bI) $*_1$ (c + dI) $\in J_k$, and (e + f I) $*_2$ (a + bI) $\in J_k$ ∀ k∈ K because each J_k is the left ideal so, this shows that (e + f I) $*_2$ (a + bI) $\in \bigcap J_k$ ∀ k∈ K so from hence we say that $\bigcap J_k$ is the left ideal.

Note: The case is same for right ideals and ideals of neutrosophic semiring.

Remark 2.16. Union of left ideals of neutrosophic semiring may or may not be the left ideal again.

In order to be more clear with the preceding remark we give an example.

Example 2.17. The set N∪I ={a + bI :a, b ∈N } under the operations (a + bI) $*_1$ (c + dI) = gcd (a, c) + (gcd (b, d))I and (a + bI) $*_2$ (c + dI) = lcm (a , c) + (lcm (b, d))I is the neutrosophic semiring and 2N∪I = { a + bI : a , b ∈ 2N } and 3N∪I = { a + bI : a , b ∈ 3N }are the left ideals of N∪I but (2N∪I) ∪ (3N∪I) is not the left ideal of N∪I because 2 + 2I and 9 + 9I ∈ (2N∪I) ∪ (3N∪I) and (2 + 2I) $*_1$ (9 + 9I) = gcd(2, 9) + (gcd(2, 9))I = 1 + 1I which does not belong to (2N∪I) ∪ (3N∪I), hence because of failure of closure property (2N∪I) ∪ (3N∪I) does not become ideal again.

Now we write the condition for union of left ideals of neutrosophic semiring to become ideal so, here we go.

Theorem 2.18. Let $\{J_k: k \in K\}$ be the family of left ideals of neutrosophic semiring N. Then $\bigcup J_k$ is left ideal if $J_k \subseteq J_{k+1}$ for all k ∈ K.

Proof. Let (a + bI), (c + dI) $\in \bigcup J_k$ and (e + f I) ∈ N then there exist p and q ∈ K such that (a + bI) $\in J_p$ and (c + dI) $\in J_q$. Let q > p then (a + bI), (c + dI) $\in J_q$. Now as J_q is an ideal so it follows that (a + bI) $*_1$ (c + dI) $\in J_q$ and (e + f I) $*_2$ (a + bI) $\in J_q$. Thus (e + f I) $*_2$ (a + bI) $\in \bigcup J_k$. so it follows that $\bigcup J_k$ is the left ideal. In similar fashion we can show that $\bigcup J_k$ is right ideal. This completes the proof.

Corollary 2.19. Union of two left ideals J_1 and J_2 of neutrosophic semiring N is an ideal if $J_1 \subseteq J_2$ or $J_2 \subseteq J_1$.

Proof. If $J_1 \subseteq J_2$ then $J_1 \bigcup J_2 = J_2$ and result is obvious. Again if $J_2 \subseteq J_1$ then $J_1 \bigcup J_2 = J_1$ and since J_1 is the left ideal of neutrosophic semiring N. So, by transitive property $J_1 \bigcup J_2$ is an ideal.

Theorem 2.20. Every left ideal of a neutrosophic semiring N is neutrosophic subsemiring of N.

Proof. Let J be the left ideal of N and for all (a + bI), (c + dI) ∈ J, (a + bI) $*_1$ (c + dI) ∈ J. Since J⊆ N, so (a + bI) ∈ N. Now as J is the left ideal of N so, (a + bI) $*_2$ (c + dI) ∈ J and associative and distributive laws are surely satisfied because J⊆ N and N is neutrosophic semiring. Similar result can be proved for every right ideal.

Remark.2.21. Converse of the above theorem is not true in general and it can be seen from following example.

Example 2.22. Let N = {1, 2, 3, …} be the set of natural numbers. Then N∪I ={a + bI: a, b ∈ N } under the operations defined as (a + bI) $*_1$ (c + dI) = min(a , c) + (min(b , d))I and (a + bI) $*_2$ (c + dI) = max(a , c) + (max(b, d))I, becomes the neutrosophic semiring and 2N∪I ={a + bI :a, b ∈ 2N }being the subset of N∪I becomes the neutrosophic subsemiring. But 2N∪I ={a+ bI: a, b ∈ 2N } is not a left ideal because if we take 5 + 5I from N∪I and 4 + 4I from 2N∪I and apply operation $*_2$ upon the selected members, we will see that resultant element is not of 2N∪I i.e. (5+5I) $*_2$ (4 + 4I) = max(5, 4) + (max(5, 4))I = 5 + 5I ∉ 2N∪I. Hence it is clear now that the converse part of above theorem is not true in general.

3. Congruence Relations in Neutrosophic Semirings

We now study the concept of congruences in neutrosophic semirings.

Definition 3.1. Let N∪I={a + bI : a, b ∈ N } be the neutrosophic semiring under the operations $*_1$ and $*_2$. A relation ρ on N∪I is called left compatible if for all (a + bI), (c + dI) and (e + f I) ∈ N∪I and ((a + bI), (c + dI)) ∈ ρ, ((e + f I) $*_1$ (a + bI), (e + f I) $*_1$ (c + dI)) ∈ ρ and ((e + f I) $*_2$ (a + bI), (e + f I) $*_2$ (c + dI)) ∈ ρ .
The relation is called right compatible if for all (a + bI), (c + dI) and (e + f I) ∈ N∪I and ((a+bI),(c+dI))∈ρ,((a+bI)$*_1$(e+fI),(c+dI)$*_1$(e+fI))∈ρ
and ((a + bI) $*_2$ (e + f I), (c + dI) $*_2$ (e + f I)) ∈ ρ.
It is called compatible relation if for all (a + bI), (c + dI), (e + f I) and (g + hI) ∈ N∪I and ((a + bI), (c + dI)) and ((e + f I), (g + hI)) ∈ ρ, ((a + bI) $*_1$ (e + f I), (c + dI) $*_1$ (g + hI)) ∈ ρ and ((a + bI) $*_2$ (e + f I), (c + dI) $*_2$ (g + hI)) ∈ ρ.
A left (right) compatible equivalence relation is called left (right) congruence relation. A compatible equivalence relation is called congruence relation.
To understand the above concept, we consider the following example.
Example 3.2. Consider the neutrosophic semiring (Z_5∪ I, $*_1$, $*_2$) where for all (a + bI), (c + dI) ∈ Z_5∪I, (a + bI)$*_1$(c + dI) = min ((a + bI), (c + dI)) and (a + bI) $*_2$ (c + dI) = max ((a + bI), (c + dI)) and consider ρ = {((a + bI), (c + dI)): (a + bI) = (c + dI))} then this relation is left compatible (left congruence), right compatible (right congruence) and compatible(congruence) and one can verify it easily.
Proposition 3.3. A relation ρ on a neutrosophic semiring N is congruence relation if and only if it is both left and right congruence relation.
Proof. Let be a congruence relation on a neutrosophic semiring N. Let (a + bI), (c + dI) and (e + f I)∈N and ((a + bI), (c + dI))∈ρ**.** Then ((e + f I)$*_1$(a + bI), (e + f I)$*_1$(c + dI)) and ((e + f I)$*_2$(a + bI), (e + f I)$*_2$(c + dI))∈ρ. This shows that ρ is a left congruence relation and similarly it can be shown that ρ is a right congruence relation.
Conversely, assume that ρ is both left and right congruence relation. Let (a + bI), (c + dI), (e + f I) and (g + hI)∈N such that ((a + bI), (c + dI)) and ((e + f I), (g + hI)) ∈ ρ. Therefore ((a + bI)$*_1$(e + f I), (c + dI)$*_1$(e + f I))∈ρ and ((a + bI)$*_2$(e + f I), (c + dI)$*_2$(e + f I))∈ρ as ρ is a right compatible. Also ((c + dI)$*_1$(e + f I), (c + dI) $*_1$ (g + hI))∈ρ and ((c + dI) $*_2$ (e + f I), (c + dI)$*_2$(g + hI))∈ρ because ρ is a left compatible relation. Therefore ((a + bI)$*_1$(e + f I), (c + dI)$*_1$(g + hI))∈ρ and ((a + bI)$*_2$(e + f I), (c + dI)$*_2$(g + hI))∈ρ. Thus ρ is a congruence relation.

4. Conclusions

In this article, we presented the theme of Neutrosophic Semiring to analyze its algebraic structural behavior. We studied the characteristic properties of Neutrosophic Semirings through their ideals and congruence relations. In the luminosity of our findings, we may conclude that our study is going to be a good addition to the list of algebraic structures based on Neutrosophic sets. Further we are planning to work out the structural study of Neutrosophic Semiring by extending it to Neutrosophic bi-Semiring and Neutrosophic N-semiring and also some theoretical applications of fuzzy sets and soft sets can be studied in these Neutrosophic algebraic structures.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Zadeh, L. Fuzzy sets. *Information and Control* **1965**, 338-353.
2. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **1986**, 20, 87-96.
3. Smarandache, F. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability. *American Reserch Press, Rehoboth, NM,* **1999.**
4. Kandasamy, W. B. V.; Smarandache, F. Neutrosophic rings. *Phoenix, Arizona* ***2006.***
5. Agboola, A. A. A.; Akinola, A. D.; Oyebola, O. Y. Neutrosophic rings I. *International Journal of Mathematical Combinatorics* **2011,** 4, 1-14.
6. Agboola, A. A. A.; Akinola, A. D.; Oyebola, O. Y. Neutrosophic Groups and Neutrosophic Subgroups. *International Journal of Mathematical Combinatorics* **2012,** 3, 1-9.
7. Ali, M.; Smarandache, F.; Shabir, M.; Vladareanu, L. Generalization of Neutrosophic rings and Neutrosophic fields. *Neutrosophic Sets and Systems* **2014**, 5, 9-14.
8. Ali, M.; Shabir, M.; Naz, M.; Smarandache, F. Neutrosophic left almost semigroup. *Neutrosophic Sets and Systems* **2014**, 3, 18-28.
9. Ali, M.; Smarandache, F.; Shabir, M.; Naz, M. Neutrosophic bi-LA-semi group and Neutosophic N-LA-semi group. *Neutrosophic Sets and Systems* **2014**, 4, 19-24.
10. Ali, M.; Shabir, M.; Smarandache, F.; Vladareanu, L. Neutrosophic LA-semigroup rings. *Neutrosophic Sets and Systems* **2015**, 7, 81-88.
11. Khan, M.; Smarandache, F.; Afzal, S. Neutrosophic set approach for characterizations of left almost semi groups. *Neutrosophic Sets and Systems* **2016**, 11, 79-94.
12. Hashim, R. M.; Gulistan, M.; Rehman, I.; Hassan, N.; Nasruddin, A. M. Neutrosophic Bipolar Fuzzy Set and its Application in Medicines Preparation. *Neutrosophic Sets and Systems* **2020**, DOI. 10.5281/zenodo.3639213.
13. Hashim, R. M.; Gulistan, M.; Smarandache, F. Applications of Neutrosophic Bipolar fuzzy Sets in HOPE Foundation for Planning to Build a children Hospital with Different Types of similarity measures. Symmetry **2018**, 10-31.
14. Abdel-Basset, M. A novel and powerful framework based on Neutrosophic sets to aid patients with cancer. *Future Generation Computer Systems* **2019**, 98, 144-153.
15. Abdel-Basset, M.; Manogaran, G.; Gamal, A.; Smarandache, F. A group decision making framework based on Neutrosophic TOPSIS approach for smart medical device selection. *Journal of Medical Systems* **2019**, 43, 1-13.
16. Akram, M.; Ishfaq, N.; Smarandache, F.; Broumi, S. Application of Bipolar Neutrosophic sets to Incidence Graphs. *Neutrosophic Sets and Systems* **2019**, 27, pp. 180.
17. Hernandez, B.; Ruilova, M.; Cueva, and Mazacón, N. Prospective analysis of public management scenarios modeled by the Fuzzy Delphi method. *Neutrosophic Sets and Systems* **2019,** vol. 26, no. 1, pp. 17.

18. Shigui Du, Rui Yong and Jun Ye, Subtraction Operational Aggregation Operators of Simplified Neutrosophic Numbers and Their Multi-Attribute Decision Making Approach. *Neutrosophic Sets and Systems* **2020,** vol. 33, pp. 158.
19. Huseyin, K. Neutrosophic Cubic Hamacher Aggregation Operators and Their Applications in Decision Making. *Neutrosophic Sets and Systems* **2020,** vol. 33, pp. 235.
20. Golan, J. S. The theory of semirings with application in mathematics and theoretical computer science. *longman scientific and technical, U.K* **1992.**

Received: Aug 14, 2021. Accepted: Dec 5, 2021