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Interval-Valued Intuitionistic Hypersoft Sets and Their Algorithmic Approach in Multi-criteria Decision Making

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Abstract: Hypersoft set(HSS) is one of the recent topics developed by Smarandache in 2018 to be presented by replacing the single attribute function, used in soft set(SS), with a multi attribute function i.e a function can be further bifurcated using HSS. So, HSS provides more options to the decision-makers than SS to make precise and valid decisions. Also, the interval-valued intuitionistic fuzzy set (IVIFS) is developed to counter a kind of uncertain complex decision-making problem where the membership and non-membership values of a certain element are not precise. Basically, the IVIFS is the generalization of a fuzzy set(FS), interval-valued fuzzy set(IVFS), and intuitionistic fuzzy set(IFS). Therefore, the mixture of HSS and IVIFS will surely give a new field of study for the decision-makers to enhance their critical thinking ability to make a conclusive decisions. The main aim of the paper is to present the notion of interval-valued intuitionistic fuzzy hypersoft sets (IVIFHSSs) and study some fundamental operations on them which are worthy in critical decision making. The IVIFHSSs can be viewed as a hybrid structure that can be formed by combining interval-valued intuitionistic fuzzy sets (IVIFSs) and hypersoft sets (HSSs). On the idea of IVIFHSSs and their kinds, different operators such as complement, union, intersection, OR, AND etc have been introduced, and by using these operators we can encounter real-life-based problems that contain incomplete and parameterized information or data. A new algorithm based on IVIFHSSs has been initiated. Finally, a numerical example is employed to check the reliability and validity of the algorithm. In the future, we use the proposed concept practically in medical diagnosis, personality selection, weather forecasting, data clustering, parameter reduction, decision making, etc.
Keywords: Interval-valued intuitionistic fuzzy set; Hypersoft set; Interval-valued intuitionistic hypersoft set; Decision making.

1. Introduction

In most real-life problems, there is an existence of a considerable amount of ambiguity and it is due to the uncertainty involved in the information. That is uncertainty arises when the information is not precise and accurate. The classical mathematical tools can't measure such kinds of data. So, there is a serious need to introduce a powerful tool that is capable to measure uncertainty without any fail. Finally, the invention of the fuzzy set(FS) by Zadeh[1] in 1965 helped us to deal with uncertainty in a structured manner. In FS, each element of the universe has a membership degree $\mu_A(x) \in [0,1]$. After the introduction of FS, it has been developed rapidly and it has an extensive application in various fields of knowledge. Some of the recent works and applications associated with FS are discussed in the literature given in [2-5]. The FS theory captures the attention of the researchers over the decades and they are motivated a lot which gives rise to other mathematical tools namely rough set[6], fuzzy logic[7], vague set[8], etc. We know that hesitancy is an integral part of human thinking and FS theory does not measure hesitancy due to its inbuilt difficulties. Such difficulties were removed with the creation of an intuitionistic fuzzy set(IFS) by Atanassov[9]. The IFS is formed by adding a non-membership degree to the FS in such a manner that the sum of the membership and the non-membership degree can't exceed one. Decision-making is a scientific approach to select the best alternative among the set of attributes and the approach of the decision-making process by the decision-makers depends upon the nature of the fuzzy environment. This leads to the introduction of the interval-valued fuzzy set(IVFS)[10], interval-valued intuitionistic fuzzy set(IVIFS)[11], hesitant fuzzy set(HFS)[12], picture fuzzy set(PFS)[13], Pythagorean fuzzy set(PFS)[14], etc.

To work under fuzzy environment, there is always a challenge to construct membership function and there exist some issues in real-world that can't be solved with an aid of membership function because recently we are encountered the kind of data that are parametric and there is an inadequacy in FSs and their variants to parameterize data. To overcome such difficulties, in 1999, Molodtsov[15] introduced the soft set(SS) theory. There is a lot of instances in a real-life situation where SS theory proved to be more functional than FS theory to describe uncertain parametric information without any effort. The SS theory removes the difficulty of constructing membership function in each event. So, we claim that SS is a more functional general framework than FS to model uncertainty without assigning membership function. Later on, Maji et al.[16] presented several assertions on SS, Cagman et al.[17] used SS in decision-making, Ali et al.[18] introduced some new operations on SS etc. An amalgamation of two or more concepts provides more information to the decision-makers to make their decisions more vulnerable. Because of this, some new hybrid structures such as fuzzy soft sets (FSSs), intuitionistic fuzzy soft sets(IFSSs),interval-valued fuzzy soft sets(IVFSSs), interval-valued intuitionistic fuzzy soft sets(IVIFSSs),etc. are introduced. Some of the works related to these are the following: Agarwal et al.[19] introduced generalized IFSSs and their applications in decision-making, FSS theory and its application given in[20], Cagman et al.[21] applied IFSS in decision-making, Chetia et al.[22] presented an application based on IVFSS, Jiang et al.[23] discussed IVIFSSs and their related properties, entropy on IFSSs and IVFSSs are proposed in [24], Ma et al.[25] introduced the parameter reduction of IVFSSs and its related algorithms, Majumder et al.[26] presented generalized FSSs, Maji et al.[27] initiated more on IFSSs, algorithms for IVFSSs in emergency decision-making shown in [28], a complete model for evaluation system based on IVFSS given in [29], Roy et al.[30] introduced an FSS theoretic approach to decision-making problems, Tripathy et al.[31] given a new approach to IVFSSs and its application in decision-making, Yang et al.[32] studied combination of IVFS and SS, a novel approach to IVIFSS is initiated by Zhang et al. in [33].

In 2018, Smarandache[34] has extended SS to the hypersoft set(HSS) and pilthogenic hypersoft set(PHSS). The HSS is introduced by transforming the single attribute function F to a multi attribute function $F_1 \times F_2 \times ....\times F_n$, where each attribute has some preference values such that $F_i \cap F_j = \varnothing, \, for \; i \neq j$. However, HSS provides more options to the decision-makers than SS to make their decisions more constructive and meaningful. Some recent works based on HSSs are given in [35-40]. In HSS, the belongingness of an element is denoted by 1 and non- belongingness is denoted by 0 i.e. values are crisp. To deal with uncertainty under the hypersoft environment, a fuzzy hypersoft set (FHSS)[41,42] is introduced, and to handle hesitancy under the hypersoft environment, an intuitionistic fuzzy hypersoft set(IFHSS)[43] is introduced. Some more recent works based on HSSs are given in [44-51].

In 2010, Jiang et al.[23] introduced IVIFSSs and their properties and in 2021, Yolcu et al.[43] introduced IFHSS. In IVIFSS, there is only one attribute function, but there is some urgency to solve certain types of problems where there is more than one attribute or an attribute is further bifurcated. To address such issues there is a demand to introduce IVIFHSSs. On the other hand, in IFHSS, the membership and non-membership values are precise, but in real-life decision-making problems, we find the existence of the environment where the membership and non-membership degrees are uncertain i.e they are subjective. This situation also IVIFHSSs solve the purpose which cannot be handled by IFHSS. Therefore, there are two aspects of introducing IVIFHSS in the proposed study. Moreover, the following diagrammatic illustration will give an insight into the proposed study:

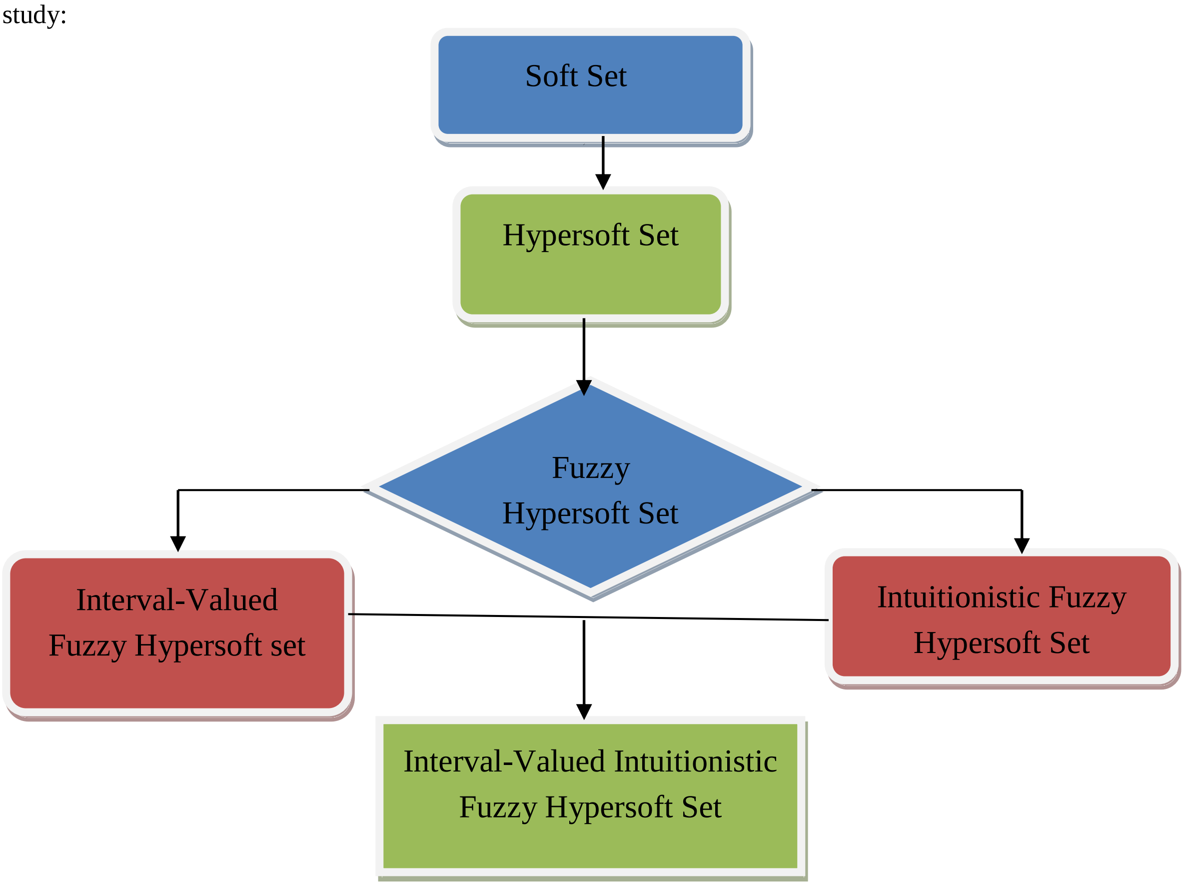

Fig 1 Diagrammatic representation of the soft set and its generalization for the proposed study

There is no research work yet to be done on IVIFHSS. This gives us the motivation to present the paper. The rest of the paper is organized as follows:

Section 2 provides an overview of the earlier research works that are useful for the present study. In section 3, we establish the IVIFHSSs and obtain some properties and important results on them. In section 4, an algorithm is being constructed for multi-criteria decision-making problems using the notion of membership score function, non-membership score function, and the total score function under the IVIFHSS environment. Conclusion and future work are added in section 5.

2. Literature Review

In this section, we give some basic definitions and results that are useful for the rest of the paper.

Definition 2.1 [11] An interval-valued intuitionistic fuzzy set(IVIFS) H over the universe of discourse X is an object of the form $\left\{\left\langle x, \mu_H(x), \gamma_H(x)\right\rangle : x \in X\right\}$ where, μ_H , $\gamma_H : X \rightarrow Int([0,1])$, where $Int([0,1])$ stands for the set of all close subintervals of $[0,1]$ satisfying the following condition:

$\forall x \in X, \sup(\mu_H(x)) + \sup(\gamma_H(x)) \leq 1$. Further, $\mu_H(x)$ and $\gamma_H(x)$ can be written as $\mu_H(x) = \left[\mu^l{}_H(x), \mu^u{}_H(x)\right]$ and $\gamma_H(x) = \left[\gamma^l{}_H(x), \gamma^u{}_H(x)\right]$. The class of all IVIFS is denoted by $IVIFS(X)$.

Definition 2.2 [15] Let U be an initial universe and E be a set of parameters. Also, $P(U)$ denotes the power set of U and $A \subseteq E$. Then the pair (F, A) where $F : A \rightarrow P(U)$ is called the soft set over U . A SS is a parameterized family of subsets over the universe U .

Definition 2.3 [23] Let U be the universe of discourse and E be the set of parameters and $IVIFS(U)$ denote the set of all IVIFSs over U . Also, let $A \subseteq E$. Then the pair (F, A) is called an IVIFSS over U where $F : A \rightarrow IVIFS(U)$.

Example 2.3.1 Let $U = \{c_1, c_2, c_3, c_4, c_5\}$ be a set of cars under consideration and $E = \{e_1 = size, e_2 = color, e_3 = \textit{fuel efficiency}, e_4 = \exp ensive, e_5 = style, e_6 = comfortable\}$ be a set of parameters and $A = \{e_1, e_2, e_3, e_6\} \subseteq E$. Under the advice of a decision-maker, Mr. X wants to purchase a car. The IVIFSS is denoted by (F, A) which describes the "attractiveness of the cars" to the decision-maker. Then, the tabular representation of (F, A) is given in Table 1.

(F,A)	c_1	c_2	c_3	c_4	c_5
e_1	$\langle[0.4,0.6],[0.2,0.3]\rangle$	$\langle[0.5,0.6],[0.1,0.2]\rangle$	$\langle[0.3,0.5],[0.2,0.5]\rangle$	$\langle[0.3,0.4],[0.4,0.5]\rangle$	$\langle[0.2,0.4],[0.3,0.5]\rangle$
e_2	$\langle[0.5,0.6],[0.2,0.3]\rangle$	$\langle[0.2,0.3],[0.3,0.5]\rangle$	$\langle[0.4,0.5],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.1,0.2]\rangle$	$\langle[0.1,0.3],[0.6,0.7]\rangle$
e_3	$\langle[0.2,0.3],[0.4,0.5]\rangle$	$\langle[0.2,0.3],[0.4,0.5]\rangle$	$\langle[0.1,0.3],[0.5,0.6]\rangle$	$\langle[0.3,0.6],[0.3,0.4]\rangle$	$\langle[0.5,0.6],[0.3,0.4]\rangle$
e_6	$\langle[0.4,0.5],[0.3,0.4]\rangle$	$\langle[0.2,0.4],[0.3,0.5]\rangle$	$\langle[0.7,0.8],[0.1,0.2]\rangle$	$\langle[0.5,0.7],[0.1,0.2]\rangle$	$\langle[0.4,0.5],[0.2,0.3]\rangle$

Table1. Tabular representation of IVIFSS (F,A)

The above representation is very useful for storage such big data in a computer as it consumes less memory and it is handy for numerical calculation which solves the purpose of the decision maker to make a precise decision.

Definition 2.4 [34, 39] Let S denotes the set of the universe and $P(S)$ is the power set of S. Let $e_1, e_2, ..., e_n$ be n distinct attributes, where $n \geq 1$, whose corresponding attribute values are respectively the sets $E_1, E_2, ..., E_n$ with $E_i \cap E_j = \varnothing, i \neq j$ and $i, j \in N$. Then the hypersoft set(HSS) is denoted by $(\Omega, E_1 \times E_2 \times .... \times E_n)$ where $\Omega : E_1 \times E_2 \times .... \times E_n \rightarrow P(S)$. For simplicity, we represent the HSS by (Ω, E) where $E = E_1 \times E_2 \times .... \times E_n$. Thus, in HSS, the attribute function can be further split until it is not suitable for the decision-maker in a certain environment. Therefore, HSS qualitatively enhanced the decision-making process.

Example 2.4.1 Let $S = \{\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4\}$ be a set of journals and the set of attributes are E_1 =citation style, E_2 =indexing and abstracting, E_3 =article processing charge(APC), E_4 = impact factor(IF) and their respective attribute values are given by:

E_1 =citation style= $\{APA(e_1), MLA(e_2), Har\,\mathrm{var}\,d(e_3), Bibtex(e_4)\}$, E_2 =indexing and abstracting= $\{SCI(e_5), Scopus(e_6), Web\ of\ Science(e_7), Taylor\ and\ Francis(e_8)\}$, E_3 =article processing charge= $\{Nil(e_9)\}$, E_4 = impact factor= $\{high(e_{10}), low(e_{11}), medium(e_{12})\}$.

Let $A_i \subseteq E_i, where\ i = 1,2,3,4$ and suppose $A_1 = \{e_1, e_4\}$, $A_2 = \{e_5, e_6, e_7\}$, $A_3 = \{e_9\}$ and $A_3 = \{e_{10}, e_{12}\}$. Then the HSS $(\Omega, A_1 \times A_2 \times A_3 \times A_4)$ defined as

$$(\Omega, A_1 \times A_2 \times A_3 \times A_4) =$$

$$\left\{\begin{array}{l} \langle(e_1,e_5,e_9,e_{10}),\{\varsigma_1,\varsigma_3\}\rangle, \langle(e_1,e_5,e_9,e_{12}),\{\varsigma_1,\varsigma_2\}\rangle, \langle(e_1,e_6,e_9,e_{10}),\{\varsigma_1,\varsigma_3,\varsigma_4\}\rangle, \\ \langle(e_1,e_6,e_9,e_{12}),\{\varsigma_2,\varsigma_4\}\rangle, \langle(e_1,e_7,e_9,e_{10}),\{\varsigma_1,\varsigma_2\}\rangle, \langle(e_1,e_7,e_9,e_{12}),\{\varsigma_3,\varsigma_4\}\rangle, \\ \langle(e_4,e_5,e_9,e_{10}),\{\varsigma_1,\varsigma_3,\varsigma_4\}\rangle, \langle(e_4,e_5,e_9,e_{12}),\{\varsigma_2,\varsigma_4\}\rangle, \langle(e_4,e_6,e_9,e_{10}),\{\varsigma_1,\varsigma_2,\varsigma_3\}\rangle, \\ \langle(e_4,e_6,e_9,e_{12}),\{\varsigma_3,\varsigma_4\}\rangle, \langle(e_4,e_7,e_9,e_{10}),\{\varsigma_2,\varsigma_3\}\rangle, \langle(e_4,e_7,e_9,e_{12}),\{\varsigma_3,\varsigma_4\}\rangle \end{array}\right\}$$

Therefore, the HSS $(\Omega, A_1 \times A_2 \times A_3 \times A_4)$ is not a normal set, it is a multiple parameterized family of sets over S. It is a new scientific approach of representing bifurcated parametric representation and it's a very useful model that provides sufficient information to the decision-maker to make their decisions elegantly. It is a more powerful and sophisticated tool than SS to deal with a wide range of problems related to various fields.

Definition 2.5 [41, 42] Let F^S be the set of all fuzzy subsets of the universe set S and let $E_1 \times E_2 \times .... \times E_n$ be the set of parameters where $E_i \cap E_j = \varnothing, i \neq j$,and $i, j \in N$. For every $\varepsilon \in E_1 \times E_2 \times .... \times E_n$, the pair $(\Omega, E_1 \times E_2 \times ...... \times E_n)$ is called the fuzzy hypersoft set(FHSS) over S, where $\Omega : E_1 \times E_2 \times .... \times E_n \to F^S$ and the FHSS defined as

$$(\Omega, E_1 \times E_2 \times ...... \times E_n) = \left\{ \left\langle \varepsilon, \left(\frac{x}{\left(\mu_{\Omega(\varepsilon)}(x)\right)} \right) \right\rangle : \varepsilon \in E_1 \times E_2 \times ...... \times E_n\ and\ x \in S \right\},$$ where

$\mu_{\Omega(\varepsilon)}(x)$ denotes the membership value such that $\mu_{\Omega(\varepsilon)}(x) \in [0,1]$.

Example 2.5.1 If we take the same sets of attributes proposed in **example 2.4.1**, then the representation of FHSS $(\Omega, A_1 \times A_2 \times A_3 \times A_4)$ in the following form:

$(\Omega, A_1 \times A_2 \times A_3 \times A_4)$ =

$$\left\{\begin{array}{l} \left\langle (e_1,e_5,e_9,e_{10}),\left\{\frac{\varsigma_1}{0.4},\frac{\varsigma_3}{0.6}\right\}\right\rangle, \left\langle (e_1,e_5,e_9,e_{12}),\left\{\frac{\varsigma_1}{0.5},\frac{\varsigma_2}{0.8}\right\}\right\rangle, \left\langle (e_1,e_6,e_9,e_{10}),\left\{\frac{\varsigma_1}{0.6},\frac{\varsigma_3}{0.7},\frac{\varsigma_4}{0.8}\right\}\right\rangle, \\ \left\langle (e_1,e_6,e_9,e_{12}),\left\{\frac{\varsigma_2}{0.2},\frac{\varsigma_4}{0.4}\right\}\right\rangle, \left\langle (e_1,e_7,e_9,e_{10}),\left\{\frac{\varsigma_1}{0.8},\frac{\varsigma_2}{0.3}\right\}\right\rangle, \left\langle (e_1,e_7,e_9,e_{12}),\left\{\frac{\varsigma_3}{0.3},\frac{\varsigma_4}{0.4}\right\}\right\rangle, \\ \left\langle (e_4,e_5,e_9,e_{10}),\left\{\frac{\varsigma_1}{0.4},\frac{\varsigma_3}{0.3},\frac{\varsigma_4}{0.6}\right\}\right\rangle, \left\langle (e_4,e_5,e_9,e_{12}),\left\{\frac{\varsigma_2}{0.8},\frac{\varsigma_4}{0.9}\right\}\right\rangle, \left\langle (e_4,e_6,e_9,e_{10}),\left\{\frac{\varsigma_1}{0.3},\frac{\varsigma_2}{0.5},\frac{\varsigma_3}{0.6}\right\}\right\rangle, \\ \left\langle (e_4,e_6,e_9,e_{12}),\left\{\frac{\varsigma_3}{0.5},\frac{\varsigma_4}{0.6}\right\}\right\rangle, \left\langle (e_4,e_7,e_9,e_{10}),\left\{\frac{\varsigma_2}{0.4},\frac{\varsigma_3}{0.5}\right\}\right\rangle, \left\langle (e_4,e_7,e_9,e_{12}),\left\{\frac{\varsigma_3}{0.4},\frac{\varsigma_4}{0.8}\right\}\right\rangle \end{array}\right\}$$

By FHSS we represent the multi-parameterized family of uncertain data.

Definition 2.6 [43] Let IF^S be the set of all intuitionistic fuzzy subsets of the universe set S and let $E_1 \times E_2 \times .... \times E_n$ be the set of parameters where $E_i \cap E_j = \varnothing, i \neq j$,and $i, j \in N$. For every $\varepsilon \in E_1 \times E_2 \times .... \times E_n$, the pair $(\Omega, E_1 \times E_2 \times ...... \times E_n)$ is called the intuitionistic fuzzy hypersoft set(IFHSS) over S , where $\Omega : E_1 \times E_2 \times .... \times E_n \rightarrow IF^S$ and the FHSS defined as,

$$(\Omega, E_1 \times E_2 \times ...... \times E_n) = \left\{ \left\langle \varepsilon, \left(\frac{x}{\langle \mu_{\Omega(\varepsilon)}(x), \gamma_{\Omega(\varepsilon)}(x) \rangle} \right) \right\rangle : \varepsilon \in E_1 \times E_2 \times ...... \times E_n \text{ and } x \in S \right\}$$,where $\mu_{\Omega(\varepsilon)}(x)$ and $\gamma_{\Omega(\varepsilon)}(x)$ respectively denotes the membership and the non-membership values where $\mu_{\Omega(\varepsilon)}(x), \gamma_{\Omega(\varepsilon)}(x) \in [0,1]$ such that $0 \leq \mu_{\Omega(\varepsilon)}(x) + \gamma_{\Omega(\varepsilon)}(x) \leq 1$. The hesitancy is determined by $\Pi_{\Omega(\varepsilon)}(x) = 1 - \mu_{\Omega(\varepsilon)}(x) - \gamma_{\Omega(\varepsilon)}(x)$.

Example 2.6.1 Revisiting **example 2.4.1**, we address the IFHSS $(\Omega, A_1 \times A_2 \times A_3 \times A_4)$ in the following manner:

$$\left(\Omega, A_1 \times A_2 \times A_3 \times A_4\right) =$$

$$\left\{\begin{array}{l}
\left\langle \left(e_1, e_5, e_9, e_{10}\right), \left\{\frac{\varsigma_1}{\langle 0.3, 0.6\rangle}, \frac{\varsigma_3}{\langle 0.2, 0.5\rangle}\right\}\right\rangle, \left\langle \left(e_1, e_5, e_9, e_{12}\right), \left\{\frac{\varsigma_1}{\langle 0.4, 0.6\rangle}, \frac{\varsigma_2}{\langle 0.4, 0.5\rangle}\right\}\right\rangle, \\
\left\langle \left(e_1, e_6, e_9, e_{10}\right), \left\{\frac{\varsigma_1}{\langle 0.2, 0.3\rangle}, \frac{\varsigma_3}{\langle 0.4, 0.5\rangle}, \frac{\varsigma_4}{\langle 0.6, 0.2\rangle}\right\}\right\rangle, \left\langle \left(e_1, e_6, e_9, e_{12}\right), \left\{\frac{\varsigma_2}{\langle 0.2, 0.4\rangle}, \frac{\varsigma_4}{\langle 0.4, 0.7\rangle}\right\}\right\rangle, \\
\left\langle \left(e_1, e_7, e_9, e_{10}\right), \left\{\frac{\varsigma_1}{\langle 0.7, 0.2\rangle}, \frac{\varsigma_2}{\langle 0.1, 0.4\rangle}\right\}\right\rangle, \left\langle \left(e_1, e_7, e_9, e_{12}\right), \left\{\frac{\varsigma_3}{\langle 0.3, 0.7\rangle}, \frac{\varsigma_4}{\langle 0.4, 0.5\rangle}\right\}\right\rangle, \\
\left\langle \left(e_4, e_5, e_9, e_{10}\right), \left\{\frac{\varsigma_1}{\langle 0.4, 0.5\rangle}, \frac{\varsigma_3}{\langle 0.8, 0.1\rangle}, \frac{\varsigma_4}{\langle 0.6, 0.4\rangle}\right\}\right\rangle, \left\langle \left(e_4, e_5, e_9, e_{12}\right), \left\{\frac{\varsigma_2}{\langle 0.2, 0.3\rangle}, \frac{\varsigma_4}{\langle 0.5, 0.4\rangle}\right\}\right\rangle, \\
\left\langle \left(e_4, e_6, e_9, e_{10}\right), \left\{\frac{\varsigma_1}{\langle 0.4, 0.5\rangle}, \frac{\varsigma_2}{\langle 0.4, 0.8\rangle}, \frac{\varsigma_3}{\langle 0.2, 0.8\rangle}\right\}\right\rangle, \left\langle \left(e_4, e_6, e_9, e_{12}\right), \left\{\frac{\varsigma_3}{\langle 0.1, 0.5\rangle}, \frac{\varsigma_4}{\langle 0.6, 0.3\rangle}\right\}\right\rangle, \\
\left\langle \left(e_4, e_7, e_9, e_{10}\right), \left\{\frac{\varsigma_2}{\langle 0.2, 0.7\rangle}, \frac{\varsigma_3}{\langle 0.5, 0.4\rangle}\right\}\right\rangle, \left\langle \left(e_4, e_7, e_9, e_{12}\right), \left\{\frac{\varsigma_3}{\langle 0.5, 0.3\rangle}, \frac{\varsigma_4}{\langle 0.6, 0.3\rangle}\right\}\right\rangle
\end{array}\right\}$$

By IFHSS, we represent the multi-parameterized family of hesitant data.

3. Interval-Valued Intuitionistic Fuzzy Hypersoft Sets (IVIFHSSs)

In this section, first, we give the basic definition of IVIFHSS and its associated sets with real-life-based examples. Then, we defined different operators such as union, intersection, complement, OR, AND,etc on IVIFHSSs and investigated their properties.

Definition 3.1 Let X be the universal set and $IVIF^X$ denote the collection of all interval-valued intuitionistic fuzzy (IVIF) subsets of X. Again, let $C_1, C_2, ..., C_n$ for $n \geq 1$ be n well-defined attributes, whose corresponding preferences are denoted by the sets $P_1, P_2, ..., P_n$ with $P_i \cap P_j = \varnothing, i \neq j$ and $i, j \in N$. Let P_i be non-empty subsets of C_i for each $i \in N$. Then the IVIFHSS is denoted as the pair $\left(Y, P_1 \times P_2 \times .... \times P_n\right)$ where $Y : P_1 \times P_2 \times .... \times P_n \to IVIF^X$ and defined as

$$Y\left(P_1 \times P_2 \times .... \times P_n\right) = \left\{\left\langle \eta, \frac{x}{\left\langle \varphi_{Y(\eta)}(x), \psi_{Y(\eta)}(x)\right\rangle}\right\rangle : x \in X, \eta \in P_1 \times P_2 \times .... \times P_n \subseteq C_1 \times C_2 \times .... \times C_n\right\}$$

, where

$\varphi_{Y(\eta)}(x)=\left[\varphi^{l}{}_{Y(\eta)}(x),\varphi^{u}{}_{Y(\eta)}(x)\right]$, and $\psi_{Y(\eta)}(x)=\left[\psi^{l}{}_{Y(\eta)}(x),\psi^{u}{}_{Y(\eta)}(x)\right]$ are the membership and non-membership intervals such that $0\le\varphi^{u}{}_{Y(\eta)}(x)+\psi^{u}{}_{Y(\eta)}(x)\le 1$ and $\varphi_{Y(\eta)},\psi_{Y(\eta)}:X\to D([0,1])$. Here $D([0,1])$ denotes the set of all closed subintervals of $[0,1]$.

For simplification, we write the symbol Σ for $C_1\times C_2\times....\times C_n$, Γ for $P_1\times P_2\times....\times P_n$ and η for any element of the set Γ.

Thus, $(Y,P_1\times P_2\times....\times P_n)$ represent a multi-parameterized family whose universe of discourse is $IVIF^{X}$.

Example 3.1.1 Let $X=\{x_1,x_2,x_3\}$ be the set of cars under consideration and the sets of attribute are

$C_1=Quality=\{good(c_1), very\ good(c_2), excellent(c_3)\}$,

$C_2=\mathrm{Re}liability=\{satisfactory(c_4)\}$,

$C_3=Color=\{\mathrm{red}(c_5),\mathrm{black}(c_6),blue(c_7),yellow(c_8)\},C_4=Fuel\ Efficiency=\{economical(c_9),high(c_{10})\}$

Suppose, P_i and Q_i are subsets of $C_i\ (i=1,2,3,4)$ such that

$P_1=\{c_2,c_3\},P_2=\{c_4\},P_3=\{c_5,c_7\},P_4=\{c_9\}$, and $Q_1=\{c_3\},Q_2=\{c_4\},Q_3=\{c_5,c_6\},Q_4=\{c_9\}$

Then the IVIFHSSs with respect to P_i and Q_i denoted as (Y,Γ_1), and (Z,Γ_2) respectively and defined in the following:

$$(Y,\Gamma_1)=\left\{\begin{array}{l}\left\langle (c_2,c_4,c_5,c_9),\left\{\dfrac{x_1}{\langle[0.3,0.4],[0.5,0.6]\rangle},\dfrac{x_3}{\langle[0.5,0.7],[0.1,0.2]\rangle}\right\}\right\rangle,\\ \left\langle (c_2,c_4,c_7,c_9),\left\{\dfrac{x_1}{\langle[0.4,0.6],[0.3,0.4]\rangle},\dfrac{x_2}{\langle[0.7,0.8],[0.1,0.2]\rangle}\right\}\right\rangle\\ \left\langle (c_3,c_4,c_5,c_9),\left\{\dfrac{x_2}{\langle[0.45,0.55],[0.23,0.35]\rangle},\dfrac{x_3}{\langle[0.35,0.55],[0.25,0.4]\rangle}\right\}\right\rangle,\\ \left\langle (c_2,c_4,c_7,c_9),\left\{\dfrac{x_1}{\langle[0.6,0.8],[0.1,0.2]\rangle},\dfrac{x_2}{\langle[0.3,0.5],[0.25,0.45]\rangle}\right\}\right\rangle\end{array}\right\}$$

$$\left(\mathrm{Z},\Gamma_2\right)=\left\{\begin{array}{l}\left\langle\left(c_3,c_4,c_5,c_9\right),\left\{\frac{x_2}{\left\langle[0.6,0.7],[0.1,0.3]\right\rangle},\frac{x_3}{\left\langle[0.35,0.45],[0.45,0.5]\right\rangle}\right\}\right\rangle,\\ \left\langle\left(c_2,c_4,c_6,c_9\right),\left\{\frac{x_1}{\left\langle[0.2,0.3],[0.3,0.4]\right\rangle},\frac{x_2}{\left\langle[0.6,0.7],[0.1,0.2]\right\rangle}\right\}\right\rangle\end{array}\right\}$$

Note: By adding HSS with IVIFS, a new structure called IVIFHSS is introduced and it is viable to increase the features of selecting an object which a decision-maker could not imagine with an aid of multi-attribute function. Every IVIFHSS is IVIFSS and in IVIFHSS, when the lower and upper membership and lower and upper non-membership coincide then IVIFHSS is reduced to IFHSS. However, the IVIFHSS is preferable for the environment where there is no precise membership and non-membership value i.e vagueness involved in assigning the membership and non-membership values. For example, suppose if we want to describe the attractiveness of a house by IVIFHSS, then by membership interval, we can measure its minimum attractiveness and maximum attractiveness, on the other hand, the non-membership interval tells the minimum non-attractiveness and maximum non-attractiveness. So, IVIFHSS is more functional than IFHSS to represent parameterized hesitant information.

Definition 3.2 An IVIFHSS $\left(Y,\Sigma\right)$ over X is said to be null IVIFHSS if for all $x\in X$ and $\eta\in\Sigma$, $\varphi_{Y(\eta)}\left(x\right)=\left[0,0\right]$ and $\psi_{Y(\eta)}\left(x\right)=\left[1,1\right]$ and it is denoted by $\Phi_{(Y,\Sigma)}$.

On the other hand, $\left(Y,\Sigma\right)$ over X is called a universal IVIFHSS if for all $x\in X$ and $\eta\in\Sigma$ $\varphi_{Y(\eta)}\left(x\right)=\left[1,1\right]$ and $\psi_{Y(\eta)}\left(x\right)=\left[0,0\right]$ and it is denoted by $\Lambda_{(Y,\Sigma)}$.

Definition 3.3 Let X be the set of universe and $\left(Y,\Gamma_1\right)$ and $\left(Z,\Gamma_2\right)$ be two IVIFHSSs over X. Then, we say that $\left(Y,\Gamma_1\right)$ is an IVIFHS subset of $\left(Z,\Gamma_2\right)$ if

(i) $\Gamma_1\subseteq\Gamma_2$

(ii) For any $\eta\in\Gamma_1, Y\left(\eta\right)\subseteq Z\left(\eta\right)$ and it is denoted by and denoted by $\left(Y,\Gamma_1\right)\tilde{\subseteq}\left(Z,\Gamma_2\right)$.

That is, for all $x\in X$ and $\eta\in\Gamma_1$,

$$\varphi^l_{Y(\eta)}\left(x\right)\le\varphi^l_{Z(\eta)}\left(x\right),\varphi^u_{Y(\eta)}\left(x\right)\le\varphi^u_{Z(\eta)}\left(x\right),\text{and}\,\psi^l_{Y(\eta)}\left(x\right)\ge\psi^l_{Z(\eta)}\left(x\right),\psi^u_{Y(\eta)}\left(x\right)\ge\psi^u_{Z(\eta)}\left(x\right).$$

Definition 3.4 Let X be the set of universe and (Y,Γ_1) and (Z,Γ_2) be two IVIFHSSs over X. Then, we say that (Y,Γ_1) is said to be equal to (Z,Γ_2) if (Y,Γ_1) is an IVIFHS subset of (Z,Γ_2) and conversely and it is denoted by $(Y,\Gamma_1)\widetilde{=}(Z,\Gamma_2)$.

Otherwise, (Y,Γ_1) and (Z,Γ_2) are said to be equal if for all $x \in X$,and $\eta \in \Sigma$,

$\varphi_{Y(\eta)}(x)=\varphi_{Z(\eta)}(x) \Rightarrow \varphi^{l}{}_{Y(\eta)}(x)=\varphi^{l}{}_{Z(\eta)}(x)$,and $\varphi^{u}{}_{Y(\eta)}(x)=\varphi^{u}{}_{Z(\eta)}(x)$

And $\psi_{Y(\eta)}(x)=\psi_{Z(\eta)}(x) \Rightarrow \psi^{l}{}_{Y(\eta)}(x)=\psi^{l}{}_{Z(\eta)}(x)$ and $\psi^{u}{}_{Y(\eta)}(x)=\psi^{u}{}_{Z(\eta)}(x)$

Theorem 3.5 Let X be an initial universe and (Y,Γ_1) , (W,Γ_2) and (Z,Γ_3) be three IVIHSSs over X and $\Gamma_1,\Gamma_2,\Gamma_3 \subseteq \Sigma$. Then

(i) $(Y,\Gamma_1)\widetilde{\subseteq}\Lambda_{(Y,\Sigma)}$

(ii) $\Phi_{(Y,\Sigma)}\widetilde{\subseteq}(Y,\Gamma_1)$

(iii) $(Y,\Gamma_1)\widetilde{\subseteq}(W,\Gamma_2)$,and $(W,\Gamma_2)\widetilde{\subseteq}(Z,\Gamma_3) \Rightarrow (Y,\Gamma_1)\widetilde{\subseteq}(Z,\Gamma_3)$.

Proof: All proofs are straightforward.

Definition 3.6 The Complement of IVIFHSS (Y,Σ) over X is denoted by $(Y,\Sigma)^c$ and defined as $(Y,\Sigma)^c=(Y^c,\neg\Sigma)$, where $Y^c:\neg\Sigma \to IVIF^X$ and the set-theoretic presentation is given by

$$(Y,\Sigma)^c=\left\{\left\langle \eta,\left(\frac{x}{\left\langle\left[\psi^{l}_{Y(\eta)}(x),\psi^{u}_{Y(\eta)}(x)\right],\left[\varphi^{l}_{Y(\eta)}(x),\varphi^{u}_{Y(\eta)}(x)\right]\right\rangle}\right)\right\rangle : x\in X,\eta\in\Sigma\right\}$$

Theorem 3.7 Let (Y,Σ) be any IVIFHSS over the initial universe X. Then

(i) $\left((Y,\Sigma)^c\right)^c=(Y,\Sigma)$

(ii) $\Phi^c_{(Y,\Sigma)}=\Lambda_{(Y,\Sigma)}$

(iii) $\Lambda^{c}_{(Y,\Sigma)} = \Phi_{(Y,\Sigma)}$

Proof: Proofs are obvious.

Definition 3.8 If (Y,Γ_1) and (Z,Γ_2) be two IVIFHSSs over a common universe X, then "(Y,Γ_1) **AND** (Z,Γ_2)" is denoted by $(Y,\Gamma_1)\tilde{\wedge}(\mathrm{Z},\Gamma_2)$ and is defined by $(Y,\Gamma_1)\tilde{\wedge}(\mathrm{Z},\Gamma_2) = (W,\Gamma_1\times\Gamma_2)$, where $W(\alpha,\beta) = Y(\alpha)\cap Z(\beta), \forall(\alpha,\beta)\in\Gamma_1\times\Gamma_2$ such a way that,

$$W(\alpha,\beta)(x) = \left(\left\langle \begin{array}{l} \left[\inf\left(\varphi^{l}_{Y(\alpha)}(x),\varphi^{l}_{Z(\beta)}(x)\right),\inf\left(\varphi^{u}_{Y(\alpha)}(x),\varphi^{u}_{Z(\beta)}(x)\right)\right], \\ \left[\sup\left(\psi^{l}_{Y(\alpha)}(x),\psi^{l}_{Z(\beta)}(x)\right),\sup\left(\psi^{u}_{Y(\alpha)}(x),\psi^{u}_{Z(\beta)}(x)\right)\right] \end{array} \right\rangle : \forall(\alpha,\beta)\in\Gamma_1\times\Gamma_2, x\in X \right)$$

Definition 3.9 If (Y,Γ_1) and (Z,Γ_2) be two IVIFHSSs over a common universe X, then "(Y,Γ_1) **OR** (Z,Γ_2)" is denoted by $(Y,\Gamma_1)\tilde{\vee}(\mathrm{Z},\Gamma_2)$ and is defined by $(Y,\Gamma_1)\tilde{\vee}(\mathrm{Z},\Gamma_2) = (W,\Gamma_1\times\Gamma_2)$, where $W(\alpha,\beta) = Y(\alpha)\cup Z(\beta), \forall(\alpha,\beta)\in\Gamma_1\times\Gamma_2$ such a way that,

$$W(\alpha,\beta)(x) = \left(\left\langle \begin{array}{l} \left[\sup\left(\varphi^{l}_{Y(\alpha)}(x),\varphi^{l}_{Z(\beta)}(x)\right),\sup\left(\varphi^{u}_{Y(\alpha)}(x),\varphi^{u}_{Z(\beta)}(x)\right)\right], \\ \left[\inf\left(\psi^{l}_{Y(\alpha)}(x),\psi^{l}_{Z(\beta)}(x)\right),\inf\left(\psi^{u}_{Y(\alpha)}(x),\psi^{u}_{Z(\beta)}(x)\right)\right] \end{array} \right\rangle : \forall(\alpha,\beta)\in\Gamma_1\times\Gamma_2, x\in X \right)$$

Theorem 3.10 Let $(Y,\Gamma_1),(\mathrm{Z},\Gamma_2)$ and (W,Γ_3) be three IVIFHSSs over X, then we have the following:

(i) $\left((Y,\Gamma_1)\tilde{\wedge}(\mathrm{Z},\Gamma_2)\right)^{c} = (Y,\Gamma_1)^{c}\,\tilde{\vee}(\mathrm{Z},\Gamma_2)^{c}$

(ii) $\left((Y,\Gamma_1)\tilde{\vee}(\mathrm{Z},\Gamma_2)\right)^{c} = (Y,\Gamma_1)^{c}\,\tilde{\wedge}(\mathrm{Z},\Gamma_2)^{c}$

(iii) $(Y,\Gamma_1)\tilde{\wedge}\left((Z,\Gamma_2)\tilde{\wedge}(W,\Gamma_3)\right) = \left((Y,\Gamma_1)\tilde{\wedge}(Z,\Gamma_2)\right)\tilde{\wedge}(W,\Gamma_3)$

(iv) $(Y,\Gamma_1)\tilde{\vee}\left((Z,\Gamma_2)\tilde{\vee}(W,\Gamma_3)\right)=\left((Y,\Gamma_1)\tilde{\vee}(Z,\Gamma_2)\right)\tilde{\vee}(W,\Gamma_3)$

Proof: (i) $(Y,\Gamma_1)\tilde{\wedge}(Z,\Gamma_2)=(W,\Gamma_1\times\Gamma_2)$

Then, $\left((Y,\Gamma_1)\tilde{\wedge}(Z,\Gamma_2)\right)^c=(W,\Gamma_1\times\Gamma_2)^c=\left(W^c,\neg(\Gamma_1\times\Gamma_2)\right)$

Similarly, $(Y,\Gamma_1)^c=(Y^c,\neg\Gamma_1)$ and $(Z,\Gamma_2)^c=(Z^c,\neg\Gamma_2)$

Then, $(Y,\Gamma_1)^c\tilde{\vee}(Z,\Gamma_2)^c=(Y^c,\neg\Gamma_1)\tilde{\vee}(Z^c,\neg\Gamma_2)=(H,\neg\Gamma_1\times\neg\Gamma_2)=\left(H,\neg(\Gamma_1\times\Gamma_2)\right)$ where $\forall(\neg\alpha,\neg\beta)\in\neg\Gamma_1\times\neg\Gamma_2,\ x\in X$. Then we have,

$$\varphi_{H(\neg\alpha,\neg\beta)}(x)=\left[\sup\left(\varphi^l_{Y^c(\neg\alpha)}(x),\varphi^l_{Z^c(\neg\beta)}(x)\right),\sup\left(\varphi^u_{Y^c(\neg\alpha)}(x),\varphi^u_{Z^c(\neg\beta)}(x)\right)\right]$$

$$\psi_{H(\neg\alpha,\neg\beta)}(x)=\left[\inf\left(\psi^l_{Y^c(\neg\alpha)}(x),\psi^l_{Z^c(\neg\beta)}(x)\right),\inf\left(\psi^u_{Y^c(\neg\alpha)}(x),\psi^u_{Z^c(\neg\beta)}(x)\right)\right]$$

Now, $Y^c(\neg\alpha)=\left\langle x,\psi_{Y(\alpha)}(x),\varphi_{Y(\alpha)}(x)\right\rangle$, $Z^c(\neg\beta)=\left\langle x,\psi_{Z(\beta)}(x),\varphi_{Z(\beta)}(x)\right\rangle$.

Then,

$$\varphi^l_{Y^c(\neg\alpha)}(x)=\psi^l_{Y(\alpha)}(x),\varphi^l_{Z^c(\neg\beta)}(x)=\psi^l_{Z(\beta)}(x),\varphi^u_{Y^c(\neg\alpha)}(x)=\psi^u_{Y(\alpha)}(x),\varphi^u_{Z^c(\neg\beta)}(x)=\psi^u_{Z(\beta)}(x)$$

Therefore, we have the following,

$$\varphi_{H(\neg\alpha,\neg\beta)}(x)=\left[\sup\left(\psi^l_{Y(\alpha)}(x),\psi^l_{Z(\beta)}(x)\right),\sup\left(\psi^u_{Y(\alpha)}(x),\psi^u_{Z(\beta)}(x)\right)\right]$$

$$\psi_{H(\neg\alpha,\neg\beta)}(x)=\left[\inf\left(\varphi^l_{Y(\alpha)}(x),\varphi^l_{Z(\beta)}(x)\right),\sup\left(\varphi^u_{Y(\alpha)}(x),\varphi^u_{Z(\beta)}(x)\right)\right]$$

We have, $(\neg\alpha,\neg\beta)\in\neg(\Gamma_1\times\Gamma_2)$. Since, $(W,\Gamma_1\times\Gamma_2)^c=\left(W^c,\neg(\Gamma_1\times\Gamma_2)\right)$, then we can write

$W^c(\neg\alpha,\neg\beta)=\left\langle x,\psi_{W(\alpha,\beta)}(x),\varphi_{W(\alpha,\beta)}(x)\right\rangle$.

Thus, $\varphi_{W^c(\neg\alpha,\neg\beta)}(x)=\psi_{W(\alpha,\beta)}(x)$ and $\psi_{W^c(\neg\alpha,\neg\beta)}(x)=\varphi_{W(\alpha,\beta)}(x)$.

Since, $(Y,\Gamma_1)\tilde{\wedge}(Z,\Gamma_2)=(W,\Gamma_1\times\Gamma_2)$, then

$$W_{(\alpha,\beta)}(x)=\left\langle \begin{array}{l} \left[Inf\left(\varphi^{l}_{Y(\alpha)}(x),\varphi^{l}_{Z(\beta)}(x)\right),\inf\left(\varphi^{u}_{Y(\alpha)}(x),\varphi^{u}_{Z(\beta)}(x)\right)\right], \\ \left[\sup\left(\psi^{l}_{Y(\alpha)}(x),\psi^{l}_{Z(\beta)}(x)\right),\sup\left(\psi^{u}_{Y(\alpha)}(x),\psi^{u}_{Z(\beta)}(x)\right)\right] \end{array}\right\rangle$$

Thus, $\varphi_{H(\alpha,\beta)}(x)=\left[Inf\left(\varphi^{l}_{Y(\alpha)}(x),\varphi^{l}_{Z(\beta)}(x)\right),\inf\left(\varphi^{u}_{Y(\alpha)}(x),\varphi^{u}_{Z(\beta)}(x)\right)\right]$ and

$\psi_{H(\alpha,\beta)}(x)=\left[\sup\left(\psi^{l}_{Y(\alpha)}(x),\psi^{l}_{Z(\beta)}(x)\right),\sup\left(\psi^{u}_{Y(\alpha)}(x),\psi^{u}_{Z(\beta)}(x)\right)\right]$

So, we can say that operators W^{c} and H are same.

Therefore, $\left(\left(Y,\Gamma_1\right)\tilde{\wedge}\left(\mathrm{Z},\Gamma_2\right)\right)^{c}=\left(Y,\Gamma_1\right)^{c}\tilde{\vee}\left(\mathrm{Z},\Gamma_2\right)^{c}$

Proofs of (ii) to (iv) are left as an exercise for the readers.

Definition 3.11 Let X be the universe of discourse and $\Gamma_1,\Gamma_2\subseteq\Sigma$. Let $\left(Y,\Gamma_1\right)$ and $\left(\mathrm{Z},\Gamma_2\right)$ be two IVIFHSSs over X. Then the union(relative) of $\left(Y,\Gamma_1\right)$ and $\left(\mathrm{Z},\Gamma_2\right)$ is denoted by $\left(Y,\Gamma_1\right)\tilde{\cup}\left(\mathrm{Z},\Gamma_2\right)=\left(W,\Gamma_3\right)$ where $\Gamma_3=\Gamma_1\bigcup\Gamma_2$ and defined as follows:

$$\varphi_{W(\eta)}(x)=\begin{cases} \varphi_{Y(\eta)}(x),\, if\, \eta\in\Gamma_1-\Gamma_2 \\ \varphi_{Z(\eta)}(x),\, if\, \eta\in\Gamma_2-\Gamma_1 \\ \left[\sup\left(\varphi^{l}_{Y(\eta)}(x),\varphi^{l}_{Z(\eta)}(x)\right),\sup\left(\varphi^{u}_{Y(\eta)}(x),\varphi^{u}_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\bigcap\Gamma_2, x\in X \end{cases}$$

$$\psi_{W(\eta)}(x)=\begin{cases} \psi_{Y(\eta)}(x),\, if\, \eta\in\Gamma_1-\Gamma_2 \\ \psi_{Z(\eta)}(x),\, if\, \eta\in\Gamma_2-\Gamma_1 \\ \left[\inf\left(\psi^{l}_{Y(\eta)}(x),\psi^{l}_{Z(\eta)}(x)\right),\inf\left(\psi^{u}_{Y(\eta)}(x),\psi^{u}_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\bigcap\Gamma_2, x\in X \end{cases}$$

Definition 3.12 Let X be the universe of discourse and $\Gamma_1,\Gamma_2\subseteq\Sigma$. Let $\left(Y,\Gamma_1\right)$ and $\left(\mathrm{Z},\Gamma_2\right)$ be two IVIFHSSs over X. Then the intersection of $\left(Y,\Gamma_1\right)$ and $\left(\mathrm{Z},\Gamma_2\right)$ is denoted by $\left(Y,\Gamma_1\right)\tilde{\cap}\left(\mathrm{Z},\Gamma_2\right)=\left(W,\Gamma_3\right)$ where $\Gamma_3=\Gamma_1\bigcup\Gamma_2$ and defined as follows:

$$\varphi_{W(\eta)}(x)=\begin{cases}\varphi_{Y(\eta)}(x), if\, \eta\in\Gamma_1-\Gamma_2\\ \varphi_{Z(\eta)}(x), if\, \eta\in\Gamma_2-\Gamma_1\\ \left[\inf\left(\varphi^l_{Y(\eta)}(x),\varphi^l_{Z(\eta)}(x)\right),\inf\left(\varphi^u_{Y(\eta)}(x),\varphi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\cap\Gamma_2, x\in X\end{cases}$$

$$\psi_{W(\eta)}(x)=\begin{cases}\psi_{Y(\eta)}(x), if\, \eta\in\Gamma_1-\Gamma_2\\ \psi_{Z(\eta)}(x), if\, \eta\in\Gamma_2-\Gamma_1\\ \left[\sup\left(\psi^l_{Y(\eta)}(x),\psi^l_{Z(\eta)}(x)\right),\sup\left(\psi^u_{Y(\eta)}(x),\psi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\cap\Gamma_2, x\in X\end{cases}$$

Theorem 3.13 Let X be the set of the universe and $\Gamma_1,\Gamma_2,\Gamma_3\subseteq\Sigma$. Let (Y,Γ_1), (Z,Γ_2) and (W,Γ_3) be three IVIFHSSs over X, then we have the following properties

(i) $(Y,\Gamma_1)\tilde{\cup}(Y,\Gamma_1)=(Y,\Gamma_1)$

(ii) $\Phi_{(Y,\Sigma)}\tilde{\cup}(Y,\Gamma_1)=(Y,\Gamma_1)$

(iii) $(Y,\Gamma_1)\tilde{\cup}\Lambda_{(Y,\Sigma)}=\Lambda_{(Y,\Sigma)}$

(iv) $(Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)=(Z,\Gamma_2)\tilde{\cup}(Y,\Gamma_1)$

(v) $\left((Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)\right)\tilde{\cup}(W,\Gamma_3)=(Y,\Gamma_1)\tilde{\cup}\left((Z,\Gamma_2)\tilde{\cup}(W,\Gamma_3)\right)$

Proof: Proofs are obvious.

Theorem 3.14 Let X be the set of the universe and $\Gamma_1,\Gamma_2,\Gamma_3\subseteq\Sigma$. Let (Y,Γ_1), (Z,Γ_2) and (W,Γ_3) be three IVIFHSSs over X, then we have the following properties

(i) $(Y,\Gamma_1)\tilde{\cap}(Y,\Gamma_1)=(Y,\Gamma_1)$

(ii) $\Phi_{(Y,\Sigma)}\tilde{\cap}(Y,\Gamma_1)=\Phi_{(Y,\Sigma)}$

(iii) $(Y,\Gamma_1)\tilde{\cap}\Lambda_{(Y,\Sigma)}=(Y,\Gamma_1)$

(iv) $(Y,\Gamma_1)\tilde{\cap}(Z,\Gamma_2)=(Z,\Gamma_2)\tilde{\cap}(Y,\Gamma_1)$

(v) $\left((Y,\Gamma_1)\tilde{\cap}(Z,\Gamma_2)\right)\tilde{\cap}(W,\Gamma_3)=(Y,\Gamma_1)\tilde{\cap}\left((Z,\Gamma_2)\tilde{\cap}(W,\Gamma_3)\right)$

(vi) $(Y,\Gamma_1)\tilde{\cap}\left((Z,\Gamma_2)\tilde{\cup}(W,\Gamma_3)\right)=\left((Y,\Gamma_1)\tilde{\cap}(Z,\Gamma_2)\right)\tilde{\cup}\left((Y,\Gamma_1)\tilde{\cap}(W,\Gamma_3)\right)$

(vii) $(Y,\Gamma_1)\tilde{\cup}\left((Z,\Gamma_2)\tilde{\cap}(W,\Gamma_3)\right)=\left((Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)\right)\tilde{\cap}\left((Y,\Gamma_1)\tilde{\cup}(W,\Gamma_3)\right)$

Proof: All are straightforward.

Definition 3.15 Let X be the universal set, $\Gamma_1,\Gamma_2\subseteq\Sigma$ and (Y,Γ_1), (Z,Γ_2) be two IVIFHSSs over X. Then, the difference between (Y,Γ_1) and (Z,Γ_2) is denoted by $(Y,\Gamma_1)\,\square\,(Z,\Gamma_2)=(W,\Gamma_3)$ where $(Y,\Gamma_1)\tilde{\cap}(Z,\Gamma_2)^c=(Z,\Gamma_2)=(W,\Gamma_3)$.

Theorem 3.16 Let X be the universal set, $\Gamma_1,\Gamma_2\subseteq\Sigma$ and (Y,Γ_1) and (Z,Γ_2) be two IVIFHSSs over X. Then we have the following properties:

(i) $\left((Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)\right)^c=(Y,\Gamma_1)^c\,\tilde{\cap}(Z,\Gamma_2)^c$

(ii) $\left((Y,\Gamma_1)\tilde{\cap}(Z,\Gamma_2)\right)^c=(Y,\Gamma_1)^c\,\tilde{\cup}(Z,\Gamma_2)^c$

Proof: (1) Let $(Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)=(W,\Gamma_3)$, where $\Gamma_3=\Gamma_1\bigcup\Gamma_2$ and for all $\eta\in\Gamma_3$, we have the following

$$\varphi_{W(\eta)}(x)=\begin{cases}\varphi_{Y(\eta)}(x),if\,\eta\in\Gamma_1-\Gamma_2\\ \varphi_{Z(\eta)}(x),if\,\eta\in\Gamma_2-\Gamma_1\\ \left[\sup\left(\varphi^l_{Y(\eta)}(x),\varphi^l_{Z(\eta)}(x)\right),\sup\left(\varphi^u_{Y(\eta)}(x),\varphi^u_{Z(\eta)}(x)\right)\right],if\ \eta\in\Gamma_1\bigcap\Gamma_2,x\in X\end{cases}$$

$$\psi_{W(\eta)}(x)=\begin{cases}\psi_{Y(\eta)}(x), if\, \eta\in\Gamma_1-\Gamma_2\\ \psi_{Z(\eta)}(x), if\, \eta\in\Gamma_2-\Gamma_1\\ \left[\inf\left(\psi^l_{Y(\eta)}(x),\psi^l_{Z(\eta)}(x)\right),\inf\left(\psi^u_{Y(\eta)}(x),\psi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\bigcap\Gamma_2, x\in X\end{cases}$$

Now, $\left((Y,\Gamma_1)\tilde{\cup}(Z,\Gamma_2)\right)^c$ = $(W,\Gamma_3)^c=\left(W^c,\neg\Gamma_3\right)$, where $W^c(\neg\eta)=\left\langle x,\psi_{W(\eta)}(x),\varphi_{W(\eta)}(x)\right\rangle$ for all $x\in X$ and $\neg\eta\in\neg\Gamma_3=\neg\left(\Gamma_1\bigcup\Gamma_2\right)=\neg\Gamma_1\bigcup\neg\Gamma_2$. Then we have,

$$\varphi^c{}_{W(\neg\eta)}(x)=\begin{cases}\psi_{Y(\eta)}(x), if\, \eta\in\Gamma_1-\Gamma_2\\ \psi_{Z(\eta)}(x), if\, \eta\in\Gamma_2-\Gamma_1\\ \left[\inf\left(\psi^l_{Y(\eta)}(x),\psi^l_{Z(\eta)}(x)\right),\inf\left(\psi^u_{Y(\eta)}(x),\psi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\bigcap\Gamma_2, x\in X\end{cases}$$

$$\psi^c{}_{W(\neg\eta)}(x)=\begin{cases}\varphi_{Y(\eta)}(x), if\, \eta\in\Gamma_1-\Gamma_2\\ \varphi_{Z(\eta)}(x), if\, \eta\in\Gamma_2-\Gamma_1\\ \left[\sup\left(\varphi^l_{Y(\eta)}(x),\varphi^l_{Z(\eta)}(x)\right),\inf\left(\varphi^u_{Y(\eta)}(x),\varphi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\bigcap\Gamma_2, x\in X\end{cases}$$

Since, $(Y,\Gamma_1)^c=\left(Y^c,\neg\Gamma_1\right)$ and $(Z,\Gamma_2)^c=\left(Z^c,\neg\Gamma_2\right)$ then

$(Y,\Gamma_1)^c\tilde{\cap}(Z,\Gamma_2)^c=\left(Y^c,\neg\Gamma_1\right)\tilde{\cap}\left(Z^c,\neg\Gamma_2\right)=(H,\Gamma_4)$ (say), where $\Gamma_4=\neg\Gamma_3=\neg\Gamma_1\cup\neg\Gamma_2$ and for all $\neg\eta\in\Gamma_4$,

$$\varphi_{H(\neg\eta)}(x)=\begin{cases}\varphi_{Y^c(\neg\eta)}(x), if \neg\eta\in\neg\Gamma_1-\neg\Gamma_2\\ \varphi_{Z^c(\neg\eta)}(x), if \neg\eta\in\neg\Gamma_2-\neg\Gamma_1\\ \left[\inf\left(\varphi^l_{Y^c(\neg\eta)}(x),\varphi^l_{Z^c(\neg\eta)}(x)\right),\inf\left(\varphi^u_{Y^c(\neg\eta)}(x),\varphi^u_{Z^c(\neg\eta)}(x)\right)\right], if \neg\eta\in\neg\Gamma_1\cap\neg\Gamma_2, x\in X\end{cases}$$

$$=\begin{cases}\psi_{Y(\eta)}(x), if\,\eta\in\Gamma_1-\Gamma_2\\ \psi_{Z(\eta)}(x), if\,\eta\in\Gamma_2-\Gamma_1\\ \left[\inf\left(\psi^l_{Y(\eta)}(x),\psi^l_{Z(\eta)}(x)\right),\inf\left(\psi^u_{Y(\eta)}(x),\psi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\cap\Gamma_2, x\in X\end{cases}$$

$$\psi_{H(\neg\eta)}(x)=\begin{cases}\psi_{Y^c(\neg\eta)}(x), if \neg\eta\in\neg\Gamma_1-\neg\Gamma_2\\ \psi_{Z^c(\neg\eta)}(x), if \neg\eta\in\neg\Gamma_2-\neg\Gamma_1\\ \left[\sup\left(\psi^l_{Y^c(\neg\eta)}(x),\varphi^l_{Z^c(\neg\eta)}(x)\right),\inf\left(\psi^u_{Y^c(\neg\eta)}(x),\psi^u_{Z^c(\neg\eta)}(x)\right)\right], if \neg\eta\in\neg\Gamma_1\cap\neg\Gamma_2, x\in X\end{cases}$$

$$=\begin{cases}\varphi_{Y(\eta)}(x), if\,\eta\in\Gamma_1-\Gamma_2\\ \varphi_{Z(\eta)}(x), if\,\eta\in\Gamma_2-\Gamma_1\\ \left[\sup\left(\varphi^l_{Y(\eta)}(x),\varphi^l_{Z(\eta)}(x)\right),\sup\left(\varphi^u_{Y(\eta)}(x),\varphi^u_{Z(\eta)}(x)\right)\right], if\ \eta\in\Gamma_1\cap\Gamma_2, x\in X\end{cases}$$

Therefore, W^c and H are the same operators. Thus, $\left((Y,\Gamma_1)\tilde{\cup}(\mathrm{Z},\Gamma_2)\right)^c=(Y,\Gamma_1)^c\tilde{\cap}(\mathrm{Z},\Gamma_2)^c$.

(ii) Similar to that of (i)

4. An Algorithmic Approach for Multi-criteria Decision Making Based on IVIFHSSs

A variety of real-based decision-making problems in different fields such as engineering, social science, economics, weather forecasting, risk management, medical science, etc. contains imprecise fuzzy data and it is due to diverse types of uncertainties present in the system. Day to day the problem becomes more and more complicated. There is a requirement to introduce another new tool that can handle a large amount of imprecision involved in a system. The introduction of IVIFHSSs is capable enough to encounter such problems. So, we present an algorithm to handle fuzzy decision-making problems based on IVIFHSSs, which is very much helpful for the decision-makers to obtain the optimal choice. Firstly, we give some definitions that are related to the proposed algorithm in the following:

Definition 4.1 Let (Y,Σ) be an IVIFHSS over the set of the universe $X=\{x_1,x_2,....,x_n\}$ where $\Sigma=E_1\times E_2\times......\times E_n$. For any $\eta\in\Sigma$, $\varphi_{Y(\eta)}(x_i)=\left\langle\left[\varphi^l{}_{Y(\eta)}(x_i),\varphi^u{}_{Y(\eta)}(x_i)\right]\right\rangle$ denotes the degree of membership of an element x_i via $Y(\eta)$. Then the score of membership degree of x_i for each η is denoted and defined as

$$S^M_{Y(\eta)}(x_i)=\sum_{k=1}^n\left[\left(\varphi^l_{Y(\eta)}(x_i)+\varphi^u_{Y(\eta)}(x_i)\right)-\left(\varphi^l_{Y(\eta)}(x_k)+\varphi^u_{Y(\eta)}(x_k)\right)\right]$$

Definition 4.2 Let (Y,Σ) be an IVIFHSS over the set of the universe $X=\{x_1,x_2,....,x_n\}$ where $\Sigma=E_1\times E_2\times......\times E_m$. For any $\eta_j\in\Sigma$, $\psi_{Y(\eta_j)}(x_i)=\left\langle\left[\varphi^l{}_{Y(\eta_j)}(x_i),\varphi^u{}_{Y(\eta_j)}(x_i)\right]\right\rangle$ denotes the degree of non-membership of an element x_i via $Y\left(\eta_j\right)$. Then the score of non-membership degree of x_i for each η_j is denoted and defined as

$$S^N_{Y(\eta_j)}(x_i)=-\sum_{k=1}^n\left[\left(\psi^l_{Y(\eta_j)}(x_i)+\psi^u_{Y(\eta_j)}(x_i)\right)-\left(\psi^l_{Y(\eta_j)}(x_k)+\psi^u_{Y(\eta_j)}(x_k)\right)\right]$$

Definition 4.3 Let (Y,Σ) be an IVIFHSS over the set of the universe $X=\{x_1,x_2,....,x_n\}$ where $\Sigma=E_1\times E_2\times......\times E_n$. For any $\eta_j\in\Sigma$, the score of the membership and non-membership degree of each x_i denoted by $S^M_{Y(\eta_j)}(x_i)$ and $S^N_{Y(\eta_j)}(x_i)$ respectively. Then the total score of x_i is denoted by $T_{Y(\eta_j)}(x_i)$ and is defined as

$$T_{Y(\eta_j)}(x_i)=S^M_{Y(\eta_j)}(x_i)+S^N_{Y(\eta_j)}(x_i)$$

The steps of the algorithms, based on these definitions are discussed below:

Algorithm:

Step1: Input an IVIFHSS (Y, Σ) over X

Step2: Compute the score of membership degrees $S^{M}_{Y(\eta j)}(x_i)$ and the score of non-membership degrees $S^{N}_{Y(\eta j)}(x_i)$ for every $\eta \in \Sigma$.

Step3: Compute the total score $T_{Y(\eta j)}(x_i)$.

Step4: Obtain λ, for which $T_\lambda = \max_{x_i \in X}\left(T_{Y(\eta j)}(x_i)\right)$. Thus, $x_\lambda \in X$ is the optimal choice for the decision-maker.

Example 4.4 Considering example **3.1.1,** we have

Step1:

$$(Y,\Gamma_1)=\left\{\begin{array}{l}\left\langle (c_2,c_4,c_5,c_9),\left\{\dfrac{x_1}{\langle[0.3,0.4],[0.5,0.6]\rangle},\dfrac{x_3}{\langle[0.5,0.7],[0.1,0.2]\rangle}\right\}\right\rangle,\\ \left\langle (c_2,c_4,c_7,c_9),\left\{\dfrac{x_1}{\langle[0.4,0.6],[0.3,0.4]\rangle},\dfrac{x_2}{\langle[0.7,0.8],[0.1,0.2]\rangle}\right\}\right\rangle\\ \left\langle (c_3,c_4,c_5,c_9),\left\{\dfrac{x_2}{\langle[0.45,0.55],[0.23,0.35]\rangle},\dfrac{x_3}{\langle[0.35,0.55],[0.25,0.4]\rangle}\right\}\right\rangle,\\ \left\langle (c_2,c_4,c_7,c_9),\left\{\dfrac{x_1}{\langle[0.6,0.8],[0.1,0.2]\rangle},\dfrac{x_2}{\langle[0.3,0.5],[0.25,0.45]\rangle}\right\}\right\rangle\end{array}\right\}$$

Step2:

$$S^{M}_{Y(\eta_j)}(x_1)=\left[(0.3+0.4)-(0.5+0.7)\right]+\left[(0.4+0.6)-(0.7+0.8)\right]+\left[(0.6+0.8)-(0.3+0.5)\right]$$
$$=(-0.5-0.5+0.6)=-0.4$$

$$S^{M}_{Y(\eta_j)}(x_2)=\left[(0.7+0.8)-(0.4+0.6)\right]+\left[(0.45+0.55)-(0.35+0.55)\right]+\left[(0.3+0.5)-(0.6+0.8)\right]$$
$$=(0.5+0.1-0.6)=0.0$$

$$S^{M}_{Y(\eta_j)}(x_3)=\left[(0.5+0.7)-(0.3+0.4)\right]+\left[(0.35+0.55)-(0.45+0.55)\right]$$
$$=(0.5-0.1)=0.4$$

$$S^N_{Y(\eta_j)}(x_1) = -\{[(0.5+0.6)-(0.1+0.2)]+[(0.3+0.4)-(0.1+0.2)]+[(0.1+0.2)-(0.25+0.45)]\}$$
$$= -(0.8+0.4-0.4) = -0.8$$
$$S^N_{Y(\eta_j)}(x_2) = -\{[(0.1+0.2)-(0.3+0.4)]+[(0.23+0.35)-(0.25+0.4)]+[(0.25+0.45)-(0.1+0.2)]\}$$
$$= -(-0.4-0.07+0.4) = 0.07$$
$$S^N_{Y(\eta_j)}(x_3) = -\{[(0.1+0.2)-(0.5+0.6)]+[(0.25+0.4)-(0.23+0.35)]\}$$
$$= -(-0.8+0.07) = 0.73$$

Step3:

$$T_{Y(\eta_j)}(x_1) = -1.2, T_{Y(\eta_j)}(x_2) = 0.07, T_{Y(\eta_j)}(x_3) = 1.13$$

Step4:

T_λ =Max $\{-1.2, 0.07, 1.13\}$ =1.13

Thus, x_3 is the optimal choice for the decision-maker. If there is a tie, then we reassess all the attributes and repeat all the steps.

5. Conclusion and Future Scope

In this work, a new mathematical model called IVIFHSSs has been introduced. The IVIFHSSs are the extensions of IVIFSSs, HSSs, FHSSs, IFHSSs etc. We also studied some basic operations such as union, intersection, complement, difference, AND, OR on them. Further, some properties of IVIFHSSs are investigated. We present an algorithm based on IVIFHSSs to solve real-world problems. In the end, to check the feasibility of the proposed algorithm a numerical example is employed.

For future direction, there is a scope to introduce parameterized reduction method, TOPSIS method, Similarity measures, weight operators, entropy method, cluster analysis method,etc. on IVIFHSSs to solve vivid types of decision making problems.

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