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# Neutrosophic Design of the Exponential Model with Applications

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**Abstract:** Operations on neutrosophic numbers generalize operations on crisp numbers. In this way, the neutrosophic approach quantifies data ambiguity and enables the generalization of the existing statistical model. This study presents an extension of the conventional exponential distribution in a neutrosophic context. Neutrosophic generalization is restricted to characterize the properties of the neutrosophic exponential distribution (NED); however, related results can to other stochastic models for handling the situations involving uncertainties or vagueness in processing data. All essential features of the proposed NED, such as neutrosophic moments, neutrosophic distribution function, and other related quantities, are explored. The mathematical results in this work lay the groundwork for using the exponential distribution to produce drivers for other generalized models. The neutrosophic logic of the proposed model is illustrated with examples. The estimation technique for treating the imprecision in the unknown parameter is established. The performance of the estimator neutrosophic estimator has been evaluated through Monte Carlo simulation. Simulation findings reveal that a larger sample size provides reliable estimation results.

**Keywords:** Neutrosophic probability; neutrosophic distribution; exponential model; estimation

## 1. Introduction

Probability distributions are now an essential part of every scientific research. Several real-world random events are described by these probability models [1]. A basic statistical probability model is commonly applicable to problems encountered by researchers. One of the most common continuous distributions is the exponential distribution [2]. The exponential model is considerably connected with the Poisson distribution [3]. It is commonly utilized as a model to measure the time between events occurrence. Some examples of its application include measuring the time associated with obtaining a faulty component on an assembling line in an engineering framework, predicting the risk of a portfolio of financial assets on next default and calculating radioactive decay in physics [4]. It is also used to estimate the probability of a certain number of defaults occurring during a particular time period [5]. The exponential distribution is an adequate failure model for describing the failure patterns of many components and devices with constant hazard rates in reliability analysis [6]. In hydrology, the exponential distribution is frequently used to examine extreme values of yearly or monthly maximum river flow and total rainfall [7]. A DNA strand length between mutations or the distance between roads fatalities are examples of situations where exponential variables may also be used to describe the likelihood of events occurring at a constant rate per unit distance [8-9].

In this study, a novel generalization of the NED has been described with the primary goal of incorporating vague information about the study variables. The exponential distribution is considered a neutrosophic version because it is a versatile model that can reflect a wide range of distribution forms. This extension provides a broader and clear analysis of the studied variables under consideration. The neutrosophic extension of the exponential model paves the path for working with other classical probability models established for the precisely described datasets. This study presents the NED in a way that the conventional logic of the exponential model cannot handle the many applied data problems. This generalization is based on the notion of neutrosophy presented by Smarandache [10]. The analysis of false or true statements, but indeterminate, neutral, inconsistent, or something in between, is oriented by Neutrosophic logic [11]. Every area has its neutrosophic component, namely the indeterminacy part, on the mathematical side. Smarandache made the first effort to use the neutrosophic approach in statistics, precalculus, and calculus to cope with imprecision in study variables [12]. As a result, neutrosophic statistics have given rise to research topics that deal with the effect of indeterminacy in statistical modeling. Some recent literature has recently made the first step toward describing the neutrosophic principle of statistical modeling [13-16]. Neutrosophic measures probability and descriptive statistical are discussed in [17]. Neutrosophic decision-making applications in quality control seem to be very efficient [18]. Alhabib et al. first looked at the neutrosophic algebraic structures of probability distributions [19]. Some recent work on neutrosophical probabability distributions can be seen in [20-23]. Nevertheless, works focusing on neutrosophic statistics have always relied on the applications side of the neutrosophic logic, and algebraic structures of probability distributions have rarely been addressed.

The work is structured as follows: The NED and algebraic framework of the neutrosophic numbers are given in section 2. Mathematical properties of the proposed NED are provided in section 3. Section 4 demonstrates some examples of the NED. The estimation approach for the imprecise parameter of NED is established in section 5. A simulation study for demonstrating the performance of the NML estimator is carried out in section 6. A real application of the proposed model is given in section 7. Lastly, section 8 summarizes the research findings.

**2. Preliminaries**

All essential features of the proposed NED, such as moments, shape coefficients, and the moment generating function, are based on the algebraic framework of the neutrosophic numbers. Let $\mathrm{M} = (\mathrm{t_m}, \mathrm{i_m}, \mathrm{f_m})$ and $\mathrm{N} = (\mathrm{t_n}, \mathrm{i_n}, \mathrm{f_n})$ are two single-valued neutrosophic numbers with $\mathrm{t_m}, \mathrm{t_n}, \mathrm{i_m}, \mathrm{i_n}, \mathrm{f_m}, \mathrm{f_n} \in [0,1]$, $0 \le \mathrm{t_m}, \mathrm{i_m}, \mathrm{f_m} \le 3$ and $0 \le \mathrm{t_n}, \mathrm{i_n}, \mathrm{f_n} \le 3$ then the following operation are commonly employed in the framework of the neutrosophic algebra [16]:

$$\mathrm{M} \oplus \mathrm{N} = (\mathrm{t_m} + \mathrm{t_n} - \mathrm{t_m}\mathrm{t_n}, \mathrm{i_m}\mathrm{i_n} + \mathrm{f_m}\,\mathrm{f_n}) \quad (1)$$

$$\mathrm{M} \otimes \mathrm{N} = (\mathrm{t_m}\mathrm{t_n}, \mathrm{i_m} + \mathrm{i_n} - \mathrm{t_n}, \mathrm{f_m} + \mathrm{f_n} - \mathrm{f_m}\,\mathrm{f_n}) \quad (2)$$

$$\omega\mathrm{M} = (1 - (1 - \mathrm{t_m})^{\omega}, \mathrm{t_m}^{\omega}, \mathrm{f_m}^{\omega}); \quad (3)$$

$$\mathrm{M}^{\omega} = (\mathrm{t_m}^{\omega} - 1 - (1 - \mathrm{i_m})^{\omega}, 1 - (1 - \mathrm{f_m})^{\omega}), \quad (4)$$

where the scalar $\omega > 0, and\ \omega \in R$.

Equations (1), (2), (3) and (4) represent neutrosophic summation, neutrosophic multiplication, scalar multiplication and neutrosophic power respectively. Likewise, the single-valued neutrosophic operations can be extended to neutrosophic sets.

***Definition 2.1*** Neutrosophic data extends the classic data that contain some imprecise, vague or indeterminacy in some or all values. In general terms, it can be represented as:
$x = \text{constant} + I$,
where $I \in [u, l]$; for example, $7 + I$ where $I \in [3, 3.5]$.
***Definition 2.2*** The neutrosophic random variable W, which equals the distance between successive events in a Poisson process, follows the NED model with the following neutrosophic density function ($PDF_N$).

$$\varphi_N(w) = \theta_N \exp(-w\theta_N); \; w > 0, \; \text{and} \; z > 0, \tag{5}$$

where $\theta_N \in \{\theta_l, \theta_u\}$. Figure 1 shows the form of the distribution with neutrosophic parameter $\theta_N = \{0.25, 0.50\}, \{1.00, 1.50\}$ and $\{2.00, 2.50\}$ if the data are believed to be NED.

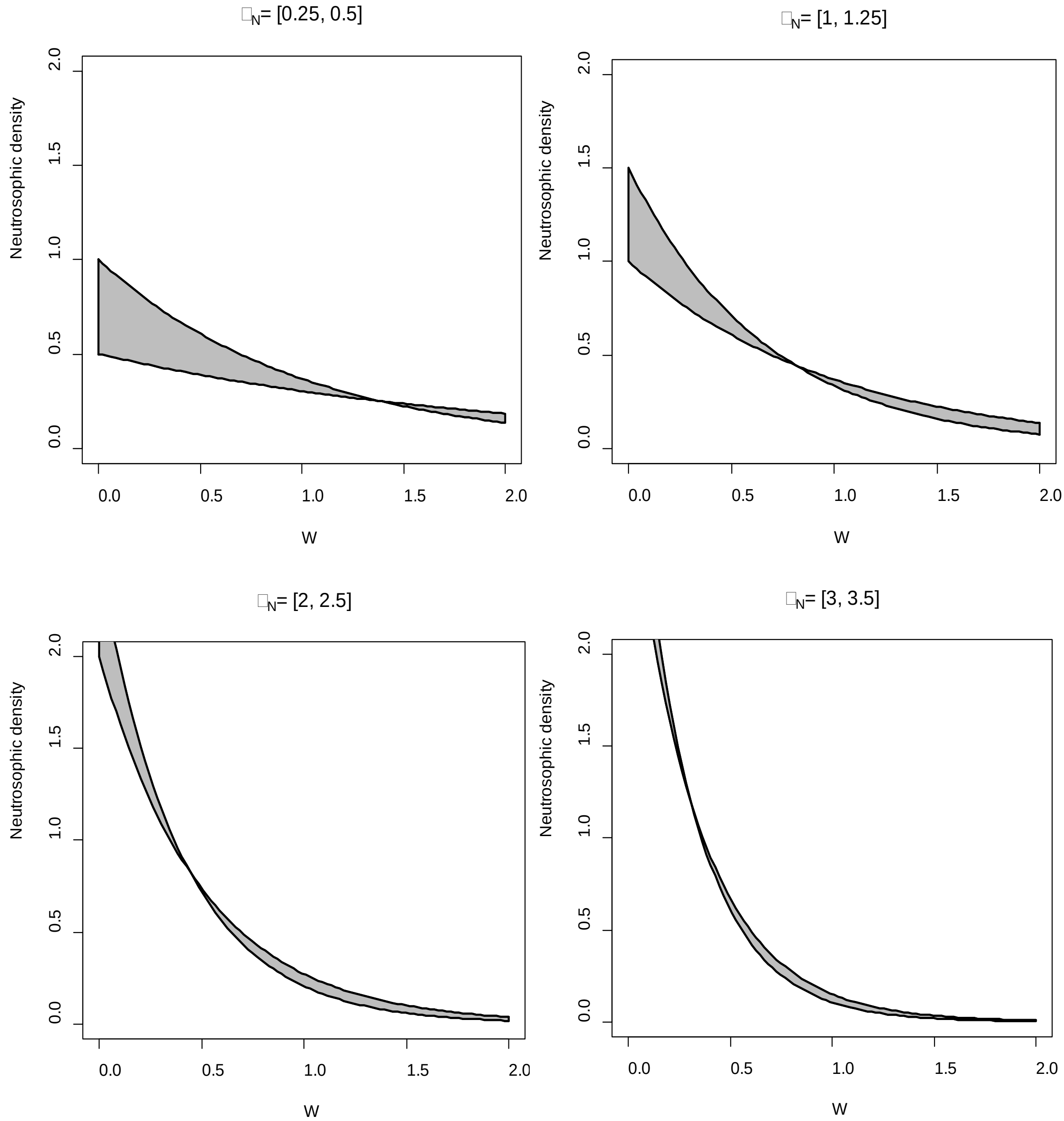


**Figure 1** Neutrosophic density graph of the NED

Figure 1 shows the neutrosophic area because of the indeterminate value of the failure rate parameter $\theta_N$. It is clearly demonstrated from Figure 1 that parameter settings may be changed to create a variety of neutrosophic exponential curves.

### 3. Some useful functions of the proposed NED

In this section, some widely used properties of the NED can be established in the form of the following theorems:

**Theorem 1**. Show that $r^{th}$ moment of the NED is $\frac{\Gamma(r+1)}{\theta_N^r}$

**Proof** By definition the $r^{th}$ moment of the NED can define as:

$$\mu_{rN}^{'} = \int_0^\infty w^r \theta_N \exp(-w\theta_N) dw$$
$$= \int_0^\infty w^r [\theta_l \exp(-w\theta_l), \quad \theta_u \exp(-w\theta_u)] dw$$
$$= \left[\int_0^\infty w^r \theta_l \exp(-w\theta_l) dw, \ \int_0^\infty w^r \theta_u \exp(-w\theta_u) dw\right] \tag{6}$$

By substituting $y = w\theta_N$, we get from (6)

$$\int_0^\infty w^r \theta_l \exp(-w\theta_l) dw = \frac{\Gamma(r+1)}{\theta_l^r}$$
$$\int_0^\infty w^r \theta_l \exp(-w\theta_l) dw = \frac{\Gamma(r+1)}{\theta_u^r}$$

Thus (6) provides

$$= \left[\frac{\Gamma(r+1)}{\theta_l^r}, \quad \frac{\Gamma(r+1)}{\theta_u^r}\right]$$

Hence,

$$\mu_{rn}^{'} = \frac{\Gamma(r+1)}{\theta_N^r} \qquad \text{where } r = 1, 2, 3, \tag{7}$$

Thus first four raw moments can be derived as:

$\mu_{1N}^{'} = \frac{1}{\theta_N}$, $\mu_{2N}^{'} = \frac{1}{2\theta_N^2}$, $\mu_{3N}^{'} = \frac{1}{6\theta_N^3}$ and $\mu_{4N}^{'} = \frac{1}{24\theta_N^4}$

**Theorem 2.** The distribution function $\Phi_N(w)$ of the NED is $1 - \exp(-w\theta_N)$.

**Proof** The result of the distribution function is obtained by solving the following expression:

$$\Phi_N(w) = \int_0^w \varphi_N(w) dw$$
$$= 1 - \exp(-w\theta_N) \tag{8}$$

Sketch of the CDF function of the proposed NED with neutrosophic parameter $\theta_N = \{1.5, 2\}$ is displayed in Figure 2.

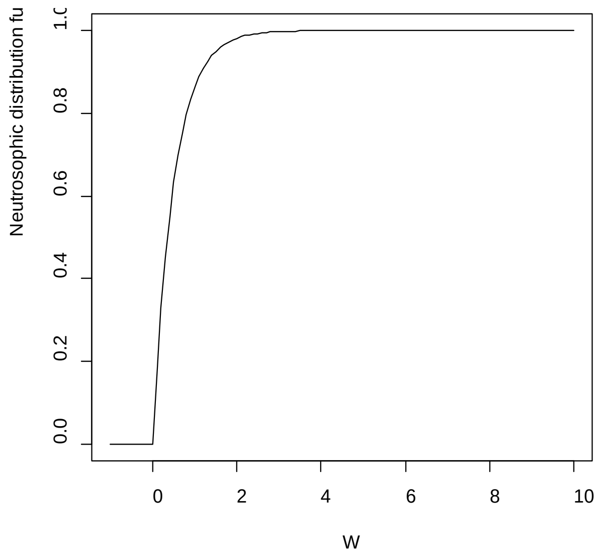


**Figure 2** CDF curve of the NED with $\theta_N = \{1.5, 2\}$

**Theorem 3.** The median of the NED is $\left[\frac{\ln(2)}{\theta_l}, \frac{\ln(2)}{\theta_u}\right]$.

**Proof** Neutrosophic median ($M_N$) is the solution of the following expression:

$$\int_0^{M_N} \Phi_N(w)dw = \left[\frac{1}{2}, \quad \frac{1}{2}\right]$$

$$\left[\int_0^{M_N} \Phi_l(w)dw, \quad \int_0^{M_N} \Phi_u(w)d\right] = \left[\frac{1}{2}, \frac{1}{2}\right] \tag{9}$$

where $\Phi_l(w) = 1 - \exp(w\theta_u)$ and $\Phi_u(w) = 1 - \exp(w\theta_u)$.

Analytical simplification of (9) implies:

$M_N \theta_l = \ln(2)$

$M_N \theta_u = \ln(2)$

Implying thereby $M_N = \left[\frac{\ln(2)}{\theta_u}, \frac{\ln(2)}{\theta_l}\right]$.

Theorem 4. First quantile ($Q_{IN}$) and the third quantile ($Q_{3N}$) of the NED are $\left[\frac{\ln(\frac{4}{3})}{\theta_u}, \frac{\ln(\frac{4}{3})}{\theta_l}\right]$ and $\left[\frac{\ln(4)}{\theta_u}, \frac{\ln(4)}{\theta_l}\right]$ respectively.

Proof The $Q_{IN}$ and $Q_{3N}$ by definition are corresponded to solutions such as:

$$\int_{0}^{Q_{IN}} \Phi_N(w)dw = \left[\frac{1}{4}, \quad \frac{1}{4}\right]$$

$$\int_{0}^{Q_{3N}} \Phi_N(w)dw = \left[\frac{3}{4}, \quad \frac{3}{4}\right]$$

Therefore following theorem 3, we can write:

$Q_{IN} = \left[\frac{\ln(4)}{\theta_u}, \frac{\ln(4)}{\theta_l}\right]$ and $Q_{3N} = \left[\frac{\ln(\frac{4}{3})}{\theta_u}, \frac{\ln(\frac{4}{3})}{\theta_l}\right]$.

Theorem 5 The mean of the NED is $\frac{1}{\theta_N}$

Proof The neutrosophic mean of the NED is determined as:

$$\mu_N = \int_{0}^{\infty} \omega_N(w)dw$$

$$= \int_{0}^{\infty} [\omega_l(w), \omega_u(w)]dw$$

$$= \left[\int_{0}^{\infty} \exp(-w\theta_l)dw, \quad \int_{0}^{\infty} \exp(-w\theta_u)dw\right]$$

$$= \left[\frac{1}{\theta_u}, \quad \frac{1}{\theta_l}\right]$$

$$= \frac{1}{\theta_N}. \quad (10)$$

Theorem 6. The variance of the NED is $\frac{1}{\theta^2{}_N}$

Proof By definition variance is

$$\sigma_N{}^2(W) = E(W^2) - (\mu_N)^2 \quad (11)$$

where $\sigma_N{}^2(W)$ stands for neutrosophic variance

Now $E(W^2) = \int_0^\infty w^2 \varphi_N(w)dw$ (12)

Since $\varphi_N(w) = -\omega_N'(w)$

It follows:

$$E(W^2) = \frac{2}{\theta_N}\int_0^\infty \omega_N(w)dw$$

$$= \frac{2}{\theta_N}\int_0^\infty [\omega_l(w), \omega_u(w)]dw$$

$$= \frac{2}{\theta_N}\left[\int_0^\infty \exp(-w\theta_l)dw, \quad \int_0^\infty \exp(-w\theta_u)dw\right]$$

$$= \frac{2}{\theta_N}\left[\frac{1}{\theta_u}, \quad \frac{1}{\theta_l}\right]$$

$$= \left[\frac{2}{\theta^2{}_u}, \quad \frac{2}{\theta^2{}_l}\right]$$

Thus (11) yields

$$\sigma_N{}^2(W) = \left[\frac{2}{\theta^2{}_u}, \frac{2}{\theta^2{}_l}\right] - \left(\left[\frac{1}{\theta_u}, \frac{1}{\theta_l}\right]\right)^2 \quad (13)$$

Simplifying (13) provides

$$\sigma_N{}^2(W) = \left[\frac{1}{\theta^2{}_u}, \frac{1}{\theta^2{}_l}\right] \quad (14)$$

Likewise, the other properties of the NED can be established in a neutrosophic environment. Some applications of the proposed model are presented to understand the initial concepts derived for the NED.

**4. Illustrative Examples**

In this section the notion of the NED has been described with a series of examples in the area of applied statistics.

**Example1** Hits to certain website follow a Poisson process with an average of {2,4} hits per hour in a day. Let the time between two hits is denoted by the random variable W. Find the probability that waiting time is less than an hour.

**Solution** Poisson distribution is connected with the exponential distribution. The waiting time between Poisson events occurring follows the exponential distribution.

Using theorem 2 we can write:

$P(W < 1) = \Phi_N(1)$

$= 1 - \exp(-w\{2,4\})$

$= \{0.86, 0.98\}$

Thus chance to hit the website less than an hour is $\{86, 98\}\%$.

**Example 2** Failure mechanism of the alternators used in automobiles follows the NED for an average lifespan of $[8, 12]$ years. Mr. Adnan buys a six years old car with a functioning alternator to keep it for eight years. Determine the probability of the alternator failing during his possession.

**Solution** Let W denote the neutrosophic random variable that follows NED.

Given that $\mu_N = \left[\frac{1}{\theta_u}, \frac{1}{\theta_l}\right] = [8, 12]$ years

This implies $[\theta_l, \theta_u] = [0.083, 0.125]$

Now the required probability:

$P[W < 8] = \Phi_N(8)$

$= [0.079, 0.117]$

Thus the chance that the alternator fails during his ownership is approximated by $[8, 12]\%$.

**Example 3** Let an electrical device has a certain component whose failure time (in months) is determined by the random variable W that is nicely modelled by the NED with average time to failure equal to $\{5, 6\}$. What is the probability that the component would still be functional after 4 months?

**Solution** Using (1) we can write:

$$P(W > 6) = \int_4^\infty \{5, 6\}\exp(-w\{5, 6\})dw$$

$$= 1 - \int_0^4 \{5, 6\}\exp(-w\{5, 6\})dw$$

Using the result given in the theorem 2 we can write:

$$= 1 - \Phi_N(4)$$

$$= \{0.48, 0.55\}$$

**5. Sample Estimation**

The method for estimating the parameter of the NED namely neutrosophic maximum likelihood estimation (NML) estimation has been introduced. Let we haves n sample $\{X_i, i = 1,2,..n\}$ values are taken from the NED. The question is, which value of the neutrosophic parameter should be used for the observed sample?. This value can be determined by the likelihood function of the neutrosophic model. As neutrosophy exist in the parameter of the NED, therefore NML function of the NED is given by:

$$\varpi_N(w, \theta_N) = n\log\theta_N - \theta_N \sum_i^n w_i \quad (15)$$

The NML estimates namely $\hat{\theta}_L$ and $\hat{\theta}_U$ can be obtained by solving the following expression:

$$= \frac{\delta\varpi_N(w, \theta_N)}{\delta\theta_N}$$

Using the neutrosophic calculus [12], yielded

$$= \left[\frac{\delta\varpi_L(w,\theta_l)}{\delta\theta_u}, \frac{\delta\varpi_U(w,\theta_u)}{\delta\theta_l}\right] \quad (16)$$

where $\varpi_L(w, \theta_l) = n\log\theta_l - \theta_L \sum_i^n w_i$ and $\varpi_N(w, \theta_u) = n\log\theta_u - \theta_u \sum_i^n w_i$

Simplification of (15) provides:

$$\frac{\delta\varpi_N(w,\theta_N)}{\delta\theta_N} = \left[\frac{n}{\theta_l} - \sum_i^n w_i, \frac{n}{\theta_u} - \sum_i^n w_i\right] \quad (17)$$

Setting (17) equating to $[0, 0]$ provides:

$\hat{\theta}_l = \frac{n}{\sum_i^n w_i}$ and $\hat{\theta}_u = \frac{n}{\sum_i^n w_i}$

Thus

$\hat{\theta}_N = [\hat{\theta}_l, \hat{\theta}_u] = \frac{n}{\sum_i^n w_i}$ which is a single crisp value and coincides with the classical MLE.

However, if imprecision in the observed data ($\tilde{z}$) is considered then NML of the neutrosophic parameter would be modified as:

$$\hat{\theta}_N = [\hat{\theta}_l, \hat{\theta}_u] = \left[\frac{n}{A}, \frac{n}{B}\right] \quad (18)$$

where

$A = \min_{\tilde{w}} = \text{sum ofthe minmum values of the neutrosophic dataset}$

$B = \max_{\tilde{w}} = \text{sum of the maximum values of the neutrosophic dataset}$

**6 Simulation Analysis**

In this part, the performance of the NML estimator has been assessed in terms of the neutrosophic average biased ($AB_N$) and neutrosophic root mean square error ($RMS_N$) as defined below [21]:

$$AB_N = \frac{\sum_{j=1}^{N}(\hat{\theta}_{Nj} - \theta_N)}{N}$$

$$RMSE_N = \sqrt{\frac{\sum_{j=1}^{N}(\hat{\theta}_{Nj} - \theta_N)^2}{N}}$$

A Monte Carlo simulation is run in R software with various sample sizes and fixed value of the neutrosophic parameter $\theta_N = [0.5,\ 1.5]$. An imprecise dataset is generated using the NED with $\theta_N = [0.5,\ 1.5]$ and simulation analysis is replicated for a total of $N = 10000$ times with sample sizes of $n = 5, 15, 30,$ and 60, respectively. The performance measures of the NML estimator are then computed and given in Table 1.

Performance of NML estimate of the NED for simulated neutrosophic data

| | $AB_N$ | $RMSE_N$ |
|---|---|---|
| 5 | [0.124, 0.373] | [0.384, 1.143] |
| 15 | [0.035, 0.106] | [0.152, 0.457] |
| 30 | [0.017, 0.051] | [0.098, 0.296] |
| 60 | [0.008, 0.025] | [0.067, 0.201] |
| 150 | [0.003, 0.009] | [0.041, 0.125] |
| 300 | [0.002, 0.005] | [0.029, 0.087] |

It can be seen from the results, as the sample size $n$ goes up, the biases $AB_N$ and $RMSE_N$ decrease. Thus, the study concluded that the NML estimator provides reliable estimation with a larger sample size.

## 7 Real Application

In this section, real data has been used to illustrate the application of the proposed model. The data used for analysis is taken from the source [24]. Data contains the lifetime failures (in hours) of air conditioning instrument used in 720-Boeing planes. To check the adequacy of exponential model, an informal procedure of some necessary graphs have been used. The graphical diagnostic of the exponential model along with other candidate models to failure time data is displayed in Figure 3.

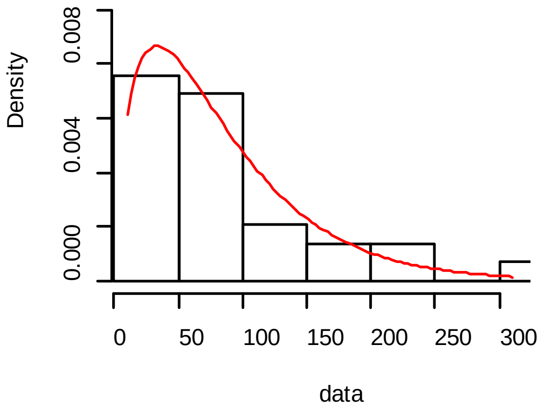


**Figure 3** Model fitting to failure time data using the candidate exponential family models

Figure 1 emphasizes the adequacy of the exponential distribution on life failures data. Theoretical lines in these necessary graphs from the exponential are shown with colored lines. Theoretical fits show the appropriateness of the exponential model among the predefined set of candidate probability models for the observed variable. Figure 1 describes that the exponential good fits the data at both tails and center of the empirical distribution. It has been assumed that all data values are not précised defined, and some values involve uncertainties and are given in the form of intervals. These uncertain observations are intentionally created according to the methodology defined in [25]. The indeterminate failure times data is given in Table 2.

**Table 2** The lifetime failures of air conditioning instrument used in 720-Boeing planes

| Failure times (in hours) | | | | |
|---|---|---|---|---|
| [89.40, 90.80] | [ 9.90, 10.02] | [59.12, 60.66] | [185.71, 186.66] | [ 60.80, 61.95] |
| [48.25, 49.21] | [13.05, 14.71] | [23.17, 24.45] | [55.36, 56.80] | [19.44, 20.25] |
| [78.29, 79.10] | [ 83.91, 84.18] | [ 43.33, 44.11] | [58.43, 59.11] | [28.28, 29.24] |
| [117.22, 118.90] | [24.12, 25.00] | [155.83, 156.07] | [309.10, 310.47] | [75.511, 76.43] |
| [ 25.51, 26.19] | [ 43.99, 44.70] | [22.82, 23.96] | [ 61.87, 62.64] | [129.82, 130.38] |
| [207.23, 208.68] | [ 69.28, 70.63] | [100.07, 101.48] | [207.97, 208.16] | |

The conventional exponential model cannot be used to analyze such data, as shown in Table 2. The values in Table 2 are provided in intervals because indeterminacies or exact values failure times are not recorded perfectly. On the contrary, the proposed exponential distribution can easily analyze such data. A descriptive summary of the failure times data rooted in the proposed model is shown in Table 3.

**Table 3** Descriptive summary of failure times data using the proposed model

| Descriptive Summary | |
|---|---|
| Estimated Rate parameter | [0.011, 0.012] |
| Estimated Mean | [82, 84] |
| Estimated Variance | [6888, 7051] |

The estimated values for rate, mean, and variance are in intervals due to indeterminacies in the observed data. Thus, the proposed model analyzes data more efficiently than the conventional model.

## 8 Conclusions

A new generalization of the classical exponential model, namely NED, has been presented in this work. The notion of neutrosophic theory has been utilized in order to quantify ambiguity in the absence of accurate distribution theory for analyzing data. The mathematical form of the proposed NED in a neutrosophic environment is thoroughly presented. The analytical expressions for the key properties of the proposed model, including neutrosophic moments, neutrosophic distribution function, and other related quantities, are derived. Some applicability examples of the NED mainly for the processing indeterminacies in data have been provided. An estimation approach of the maximum likelihood to estimate the parameter of the NED for dealing with imprecise data values is developed. To validate the performance of the neutrosophic estimator, a simulation study has been carried out. The simulation results demonstrate that indeterminate sample data with a larger size may be used to accurately estimate the unknown parameter of the proposed model.

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