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Vaccine distribution technique under QSVN environment using different aggregation operator

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Abstract. In this article some Dombi operations on Quadripartitioned single valued neutrosophic (QSVN) set are studied. Later on some QSVN weighted Dombi operators i.e. QSVNWDA and QSVNWDG operators are introduced and their properties are studied. Finally a vaccine distribution technique is solved with the help of QSVNWDA operator and QSVNWDG operator.

Keywords:QSVN set; Dombi operation, QSVN weighted Dombi arithmetic (QSVNWDA) operator; QSVN weighted Dombi geometric (QSVNWDG) operator; MADM.

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1. **Introduction**

To deal with the inconsistent and uncertain data in a more powerful way, Smarandache introduced Neutrosophic set (NS) theory [2]. Gradually many developments on NS structure have been made by a couple of researchers and applied it to different branches of science [3–12]. An extension of SVN set i.e. QSVN set was further restudied in [13]. Based on QSVN set R.Chatterjee et. al introduced the idea of $\mathcal{QNN}$ number in 2009 [16]. On contrary Dombi [1] presented the operations of Dombi T-norms (DT) and T-conorms (DTC) in 1982. Both norms has a huge operational flexibility as a parameter. Many researchers extended the idea of Dombi norms to IFS [15], NS [14] theories and applied to different MADM problems [17–21]. In this paper we have applied Dombi norms on $\mathcal{QNN}$. Vaccine distribution in India will be a very difficult task for Government of India in the upcoming years. To overcome this difficulty we have defined a model method of vaccine distribution under QSVN environment using different aggregation operator. In Section 2 we have discussed some preliminary theories which will be used throughout the rest of the article. We have defined some order relations on $\mathcal{QNN}$ in Section 3. In Section 4 QSVNWDA and QSVNWDG operators are defined and their properties are studied. A MADM problem is solved on the basis of QSVNWDA and QSVNWDG operators in Section 5. Section 6 winds up the article.

2. **Some basics**

Definition 2.1. [13] A QSVN set A over a set $X \neq \phi$ characterizes each element x in X by a truth-membership function A_t, a contradiction membership function A_c, an ignorance-membership function A_u and a falsity membership function A_f s.t. for each $x \in X$, $A_t(x), A_c(x), A_u(x), A_f(x) \in [0,1]$ and $0 \leq A_t(x) + A_c(x) + A_u(x) + A_f(x) \leq 4$.

Definition 2.2. [16] An element $\beta = \langle \beta_t, \beta_c, \beta_u, \beta_f \rangle \in [0,1]^4$ is said to be a QNN number. We express the collection of QNN numbers as $\mathcal{QNN}$.

Definition 2.3. [16] Consider $\mu, \nu, \omega \in \mathcal{QNN}$ and $i, j, k \in \mathbb{N}$. Then the following basic operations hold on $\mathcal{QNN}$:

(i) $\mu \bigoplus \nu = \langle \mu_t + \nu_t - \mu_t\nu_t, \mu_c + \nu_c - \mu_c\nu_c, \mu_u\nu_u, \mu_f\nu_f \rangle$,
(ii) $\mu \bigodot \nu = \langle \mu_t\nu_t, \mu_c\nu_c, \mu_u + \nu_u - \mu_u\nu_u, \mu_f + \nu_f - \mu_f\nu_f \rangle$,
(iii) $(\mu)^k = \langle (\mu_t)^k, (\mu_c)^k, 1 - (1 - \mu_u)^k, 1 - (1 - \mu_f)^k \rangle$,
(iv) $k\mu = \langle 1 - (1 - \mu_t)^k, 1 - (1 - \mu_c)^k, (\mu_u)^k, (\mu_f)^k \rangle$,

Both the above operations are commutative and associative on $\mathcal{QNN}$.

2.1. *DT and DTC*

Definition 2.4. [1] Suppose $r, s \in \mathbb{R}$. Then DT ($D(r,s)$) and DTC ($\overline{D}(r,s)$) between r and s are defined respectively as below:

$$D(r,s) = \frac{1}{1 + \{(\frac{1-r}{r})^{\varrho} + (\frac{1-s}{s})^{\varrho}\}^{\frac{1}{\varrho}}}$$
$$\overline{D}(r,s) = \frac{1}{1 + \{(\frac{r}{1-r})^{\varrho} + (\frac{s}{1-s})^{\varrho}\}^{\frac{1}{\varrho}}},$$

where $\varrho \geq 1$ and $(p,q) \in [0,1] \times [0,1]$.

3. **Order relations on $\mathcal{QNN}$**

In this section we will first define some order relations of QNN based on newly introduced score functions and accuracy functions on $\mathcal{QNN}$.

Definition 3.1. The score function $S(\beta) : \mathcal{QNN} \to \mathbb{R}$ of $\beta = \langle \beta_t, \beta_c, \beta_u, \beta_f \rangle \in \mathcal{QNN}$ is defined as

$$S(\beta) = \frac{2 + \beta_t + \beta_c - \beta_u - \beta_f}{4}$$

The corresponding accuracy functions $H_i : \mathcal{QNN} \to \mathbb{R}, i = 1, 2, 3$ are defined as follows:

$$\begin{aligned} H_1(\beta) &= (\beta_t + \beta_c) - (\beta_u + \beta_f) \\ H_2(\beta) &= \frac{\beta_t - \beta_c}{2} \\ H_3(\beta) &= \frac{\beta_u - \beta_f}{2}. \end{aligned}$$

Remark 3.2. Now for any $\beta \in \mathcal{QNN}$, it follows that

(i) $0 \leq S(\beta) \leq 1$.
(ii) $-2 \leq H_1(\beta) \leq 2$.
(iii) $-0.5 \leq H_2(\beta) \leq 0.5$.
(iv) $-0.5 \leq H_3(\beta) \leq 0.5$.

Definition 3.3. Suppose $\beta, \gamma \in \mathcal{QNN}$. We define the order relation between any two $\beta, \gamma \in \mathcal{QNN}$ as follows:

(i) If $S(\beta) < S(\gamma)$, then $\beta \leq \gamma$.
(ii) If $S(\beta) = S(\gamma)$, then
  (a) $H_1(\beta) < H_1(\gamma) \Rightarrow \beta \leq \gamma$ else if
  (b) $H_1(\beta) = H_1(\gamma)$ with $H_2(\beta) < H_2(\gamma) \Rightarrow \beta \leq \gamma$ else if
  (c) $H_1(\beta) = H_1(\gamma), H_2(\beta) = H_2(\gamma)$ with $H_3(\beta) < H_3(\gamma) \Rightarrow \beta \leq \gamma$ else if
  (d) $H_1(\beta) = H_1(\gamma), H_2(\beta) = H_2(\gamma)$ and $H_3(\beta) = H_3(\gamma) \Rightarrow \beta = \gamma$.

Here $\beta \leq \gamma$ denotes β proceeds γ.

3.1. *Some QSVN Dombi operations*

In this section we have discussed some QSVN Dombi operations [22].

Definition 3.4. Suppose $\alpha = \langle m_1, n_1, p_1, q_1 \rangle \in \mathcal{QNN}$ and $\beta = \langle m_2, n_2, p_2, q_1 \rangle \in \mathcal{QNN}$, $\varrho \geq 1$ and $k > 0$. Then the DT and DTC operations on $\mathcal{QNN}$ are defined as below:

(i)

$$\alpha \oplus \beta = \left\langle 1 - \frac{1}{1 + \left((\frac{m_1}{1-m_1})^{\varrho} + (\frac{m_2}{1-m_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{n_1}{1-n_1})^{\varrho} + (\frac{n_2}{1-n_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{1-p_1}{p_1})^{\varrho} + (\frac{1-p_2}{p_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{1-q_1}{q_1})^{\varrho} + (\frac{1-q_2}{q_2})^{\varrho} \right)^{\frac{1}{\varrho}}} \right\rangle$$

(ii)

$$\alpha \odot \beta = \left\langle 1 - \frac{1}{1 + \left((\frac{1-m_1}{m_1})^{\varrho} + (\frac{1-m_2}{m_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{1-n_1}{n_1})^{\varrho} + (\frac{1-n_2}{n_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{p_1}{1-p_1})^{\varrho} + (\frac{p_2}{1-p_2})^{\varrho} \right)^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left((\frac{q_1}{1-q_1})^{\varrho} + (\frac{q_2}{1-q_2})^{\varrho} \right)^{\frac{1}{\varrho}}} \right\rangle$$

(iii) $k\alpha=\left\langle 1-\frac{1}{1+\left(k(\frac{m_1}{1-m_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{n_1}{1-n_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{1-p_1}{p_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{1-q_1}{q_1})^{\varrho}\right)^{\frac{1}{\varrho}}}\right\rangle$,

(iv) $\alpha^k=\left\langle 1-\frac{1}{1+\left(k(\frac{1-m_1}{m_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{1-n_1}{n_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{p_1}{1-p_1})^{\varrho}\right)^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left(k(\frac{q_1}{1-q_1})^{\varrho}\right)^{\frac{1}{\varrho}}}\right\rangle$.

4. Dombi weighted aggregation operators on $\mathcal{QNN}$

Definition 4.1. Let $\beta_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. A QSVN weighted Dombi arithmetic (QSVNWDA) operator of dimension l is a function $f : \mathcal{QNN}^l \to \mathcal{QNN}$ defined by:

$$f(\beta_1, \beta_2, \ldots, \beta_l) = \bigoplus_{j=1}^{l} \omega_j\beta_j$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_l)^T$ is the weight vector, ω_j is attached with $\beta_j, j = 1, 2, \ldots, l$ with $0 \le \omega_j \le 1$ and $\sum_{j=1}^{l} \omega_j = 1$.

Theorem 4.2. *Suppose $\beta_j = \langle m_j, n_j, p_j, q_j\rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$ along weight vector ω. Then*

$$f(\beta_1, \beta_2, \ldots, \beta_l) = \bigoplus_{j=1}^{l} \omega_j\beta_j$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle.$$

Proof. Here $\omega_1 \in \omega$ and $\beta_1 \in \mathcal{QNN}$. Now we have $\omega_1\beta_1 = \left\langle 1-\frac{1}{1+\left\{\left(\frac{m_1}{1-m_1}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{n_1}{1-n_1}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{1-p_1}{p_1}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{1-q_1}{q_1}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle$. Hence the above equation trivially holds for $l = 1$. In a parallel way $\omega_2 \in \omega$ and $\beta_2 \in \mathcal{QNN}$ and we have $\omega_2\beta_2 = \left\langle 1-\frac{1}{1+\left\{\left(\frac{m_2}{1-m_2}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{n_2}{1-n_2}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{1-p_2}{p_2}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\left(\frac{1-q_2}{q_2}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle$.
Therefore

$$f(\beta_1, \beta_2) = \omega_1\beta_1 \bigoplus \omega_2\beta_2$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{2}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{2}\omega_j\left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{2}\omega_j\left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{2}\omega_j\left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle.$$

Hence the equation is valid for $l = 1, 2$. We assume that the equation is valid for $l = s$ i.e.

$$f(\beta_1, \beta_2, \ldots, \beta_s) = \bigoplus_{j=1}^{s} \omega_j\beta_j$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{s}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{s}\omega_j\left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{s}\omega_j\left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1-\frac{1}{1+\left\{\sum_{j=1}^{s}\omega_j\left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle.$$

Finally for $l = s+1$, one can easily see that

$$f(\beta_1, \beta_2, \ldots, \beta_s) = \bigoplus_{j=1}^{s} \omega_j \beta_j \bigoplus \omega_{s+1}\beta_{s+1}$$

$$= \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{s} \omega_j \left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s} \omega_j \left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s} \omega_j \left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s} \omega_j \left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}} \right\rangle \oplus \omega_{s+1}\beta_{s+1}$$

$$= \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{s+1} \omega_j \left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s+1} \omega_j \left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s+1} \omega_j \left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{s+1} \omega_j \left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}} \right\rangle.$$

Thus in general the equation

$$f(\beta_1, \beta_2, \ldots, \beta_l) = \bigoplus_{j=1}^{l} \omega_j \beta_j$$

$$= \left\langle 1 - \frac{1}{1+\left\{\sum_{j=1}^{l} \omega_j \left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{l} \omega_j \left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{l} \omega_j \left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1+\left\{\sum_{j=1}^{l} \omega_j \left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}} \right\rangle.$$

holds $\forall l \in \mathbb{N}$. □

Theorem 4.3. *The QSVNWDA operator f satisfies the following properties:*

(i) *Consistency:* $f(\beta_1, \beta_2, \ldots, \beta_l) \in \mathcal{QNN}$.

(ii) *Idempotency:* $f(\beta, l \text{ times} \ldots, \beta) = \beta$.

(iii) *Commutativity:* $f(\beta_1, \beta_2, \ldots, \beta_l) = f(\beta_l, \beta_{l-1}, \ldots, \beta_1)$.

(iv) $f(\beta_{\pi(1)}, \beta_{\pi(2)}, \ldots, \beta_{\pi(l)}) = f(\beta_1, \beta_2, \ldots, \beta_l)$ *where π is a permutation on* $\{1, 2, \ldots, l\}$.

Proof. The proof of consistency and commutativity properties of QSVNWDA operator is quite easy. We now proceed to prove the part (ii). Since $\sum_{j=1}^{l} \omega_j = 1$, thus

$$f(\beta, l \text{ times} \ldots, \beta) = \bigoplus_{j=1}^{l} \omega_j \beta_j = (\sum_{j=1}^{l} \omega_j)\beta = \beta.$$

Finally considerπ as a permutation on $\{1, 2, \ldots, l\}$. Now due to additive commutativity in $\mathcal{QNN}$

$$f(\beta_{\pi(1)}, \beta_{\pi(2)}, \ldots, \beta_{\pi(l)}) = \bigoplus_{j=1}^{l} \omega(\beta_{\pi(j)})\beta_{\pi(j)} = \bigoplus_{j=1}^{l} \omega(\beta_j)\beta_j \quad = f(\beta_1, \beta_2, \ldots, \beta_l).$$

Hence we are done. □

Theorem 4.4. *Consider $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l$ and $\gamma_j = \langle \widetilde{m_j}, \widetilde{n_j}, \widetilde{p_j}, \widetilde{q_j} \rangle, j = 1, 2, \ldots, l)$ are two collections on $\mathcal{QNN}$ such that $m_j \leq \widetilde{m_j}, n_j \leq \widetilde{n_j}, p_j \geq \widetilde{p_j}, q_j \geq \widetilde{q_j} \forall j$. Then $f(\beta_1, \beta_2, \ldots, \beta_l) \leq f(\gamma_1, \gamma_2, \ldots, \gamma_l)$.*

Proof. Here,

$$f(\beta_1,\beta_2,\dots,\beta_l)=\bigoplus_{j=1}^{l}\omega_j\beta_j$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle.$$

$$f(\gamma_1,\gamma_2,\dots,\gamma_l)=\bigoplus_{j=1}^{l}\omega_j\gamma_j$$

$$=\left\langle 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{n_j}}{1-\widetilde{n_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-\widetilde{p_j}}{\widetilde{p_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}},1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-\widetilde{q_j}}{\widetilde{q_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}\right\rangle.$$

Firstly we suppose that $m_j<\widetilde{m_j},n_j<\widetilde{n_j},p_j>\widetilde{p_j},q_j>\widetilde{q_j}\forall j\in\{1,\dots,l\}$. Then

$$1-m_j>1-\widetilde{m_j}\forall j\in\{1,\dots,l\}$$
$$\Rightarrow\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)<\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)$$
$$\Rightarrow 1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}<1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}$$
$$\Rightarrow\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}>\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}$$
$$\Rightarrow 1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{m_j}{1-m_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}<1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{m_j}}{1-\widetilde{m_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}.$$

In a similar way we have

$$1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{n_j}{1-n_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}<1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{\widetilde{n_j}}{1-\widetilde{n_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}.$$

Conversely we can easily see that

$$1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-p_j}{p_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}>1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-\widetilde{p_j}}{\widetilde{p_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}.$$

$$1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-q_j}{q_j}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}>1-\frac{1}{1+\left\{\sum_{j=1}^{l}\omega_j\left(\frac{1-\widetilde{q_j}}{\widetilde{q_j}}\right)^{\varrho}\right\}^{\frac{1}{\varrho}}}.$$

Combining all the above we get

$$S(f(\beta_1,\beta_2,\dots,\beta_l))<S(f(\gamma_1,\gamma_2,\dots,\gamma_l)).$$

Hence $f(\beta_1, \beta_2, \ldots, \beta_l) < f(\gamma_1, \gamma_2, \ldots, \gamma_l)$. Now if $m_j = \widetilde{m_j}, n_j = \widetilde{n_j}, p_j = \widetilde{p_j}, q_j = \widetilde{q_j}$ $\forall j \in \{1, \ldots, l\}$. Then all the equalities as well as the score functions become equal i.e. $S(f(\beta_1, \beta_2, \ldots, \beta_l)) = S(f(\gamma_1, \gamma_2, \ldots, \gamma_l))$. Finally $f(\beta_1, \beta_2, \ldots, \beta_l) \leq f(\gamma_1, \gamma_2, \ldots, \gamma_l)$. □

Theorem 4.5. *Consider a collection of $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l$ in $\mathcal{QNN}$. Then*

$$\underline{\beta} \leq f(\beta_1, \beta_2, \ldots, \beta_l) \leq \overline{\beta}, \textit{where}$$

$$\underline{\beta} = \langle \min_j(m_j), \min_j(n_j), \min_j(p_j), \min_j(q_j) \rangle = \langle \underline{m_j}, \underline{n_j}, \underline{p_j}, \underline{q_j} \rangle \textit{ and}$$

$$\overline{\beta} = \langle \max_j(m_j), \max_j(n_j), \max_j(p_j), \max_j(q_j) \rangle = \langle \overline{m_j}, \overline{n_j}, \overline{p_j}, \overline{q_j} \rangle.$$

Proof. From Definition of $\mathcal{QNN}$ we have $\forall j = \{1, 2, \ldots, l\}$,

$$\underline{m_j} \leq m_j, \underline{n_j} \leq n_j, \underline{p_j} \geq p_j, \underline{q_j} \geq q_j \text{ and}$$

$$m_j \leq \overline{m_j}, n_j \leq \overline{n_j}, p_j \geq \overline{p_j}, q_j \geq \overline{q_j} \text{ and}$$

Then

$$f(\underline{\beta}, l\ times, \underline{\beta}) \leq f(\beta_1, \beta_2, \ldots, \beta_l) \leq f(\overline{\beta}, l\ times, \overline{\beta}), i.e$$

$$\underline{\beta} \leq f(\beta_1, \beta_2, \ldots, \beta_l) \leq \overline{\beta}.$$

□

Definition 4.6. Suppose $\beta_j = \langle m_j, n_j, p_j, q_j \rangle$, $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. Then from Definition 4.1 a QSVNWDA operator f of dimension l can be written as follows

$$f(\beta_1, \beta_2, \ldots, \beta_l) = \bigoplus_{j=1}^{l} \omega_j \beta_j$$

Now if $\omega_j = \frac{1}{l}$ $\forall\, j \in \{1, 2, \ldots, l\}$ then

$$f(\beta_1, \beta_2, \ldots, \beta_l) = \frac{1}{l} \bigoplus_{j=1}^{l} \beta_j.$$

In that case $f(\beta_1, \beta_2, \ldots, \beta_l)$ is called average QSVNWDA operator i.e.QSVNWADA operator of $\beta_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$.

Definition 4.7. Let $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. A quadri-partioned single valued neutrosophic weighted Dombi geometric (QSVNWDG) operator of dimension l is a function $g : \mathcal{QNN}^l \to \mathcal{QNN}$ defined by:

$$g(\beta_1, \beta_2, \ldots, \beta_l) = \bigodot_{j=1}^{l} \beta_j^{\omega_j}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_l)^T$ is the weight vector, ω_j is attached with $\beta_j, j = 1, 2, \ldots, l$ with $0 \le \omega_j \le 1$ and $\sum_{j=1}^{l} \omega_j = 1$.

Theorem 4.8. *Suppose $\beta_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$ along weight vector ω. Then*

$$g(\beta_1, \beta_2, \ldots, \beta_l) = \bigoplus_{j=1}^{l} \beta_j^{\omega_j}$$

$$= \left\langle 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \omega_j \left(\frac{1-m_j}{m_j} \right)^{\varrho} \right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \omega_j \left(\frac{1-n_j}{n_j} \right)^{\varrho} \right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \omega_j \left(\frac{p_j}{1-p_j} \right)^{\varrho} \right\}^{\frac{1}{\varrho}}}, 1 - \frac{1}{1 + \left\{ \sum_{j=1}^{l} \omega_j \left(\frac{q_j}{1-q_j} \right)^{\varrho} \right\}^{\frac{1}{\varrho}}} \right\rangle.$$

Proof. We have omitted it due to similarity with Theorem 4.2. □

Theorem 4.9. *The QSVNWDG operator g satisfies properties as defined below:*

(i) *Consistency: $g(\beta_1, \beta_2, \ldots, \beta_l) \in \mathcal{QNN}$.*
(ii) *Idempotency: $g(\beta, l$ times $\ldots, \beta) = \beta$.*
(iii) *Commutativity: $g(\beta_1, \beta_2, \ldots, \beta_l) = g(\beta_l, \beta_{l-1}, \ldots, \beta_1)$.*
(iv) *$g(\beta_{\pi(1)}, \beta_{\pi(2)}, \ldots, \beta_{\pi(l)}) = g(\beta_1, \beta_2, \ldots, \beta_l)$ where π is a permutation on $\{1, 2, \ldots, l\}$.*

Proof. We have omitted it due to similarity with Theorem 4.3. □

Theorem 4.10. *Consider $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l)$ and $\gamma_j = \langle \widetilde{m_j}, \widetilde{n_j}, \widetilde{p_j}, \widetilde{q_j} \rangle$ $(j = 1, 2, \ldots, l)$ are two collections on $\mathcal{QNN}$ such that $m_j \le \widetilde{m_j}, n_j \le \widetilde{n_j}, p_j \ge \widetilde{p_j}, q_j \ge \widetilde{q_j} \forall j$. Then $g(\beta_1, \beta_2, \ldots, \beta_l) \le g(\gamma_1, \gamma_2, \ldots, \gamma_l)$.*

Proof. Here the proof is similar with Theorem 4.4, hence we have omitted it. □

Theorem 4.11. *Consider a collection of $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l$ in $\mathcal{QNN}$. Then*

$$\underline{\beta} \le g(\beta_1, \beta_2, \ldots, \beta_l) \le \overline{\beta}, \text{ where}$$

$$\underline{\beta} = \langle \min_j(m_j), \min_j(n_j), \min_j(p_j), \min_j(q_j) \rangle = \langle \underline{m_j}, \underline{n_j}, \underline{p_j}, \underline{q_j} \rangle \text{ and}$$

$$\overline{\beta} = \langle \max_j(m_j), \max_j(n_j), \max_j(p_j), \max_j(q_j) \rangle = \langle \overline{m_j}, \overline{n_j}, \overline{p_j}, \overline{q_j} \rangle.$$

Proof. Again proof is not done due to its similarity with Theorem 4.5. □

Definition 4.12. Suppose $\beta_j = \langle m_j, n_j, p_j, q_j \rangle, j = 1, 2, \ldots, l)$ be a collection on $\mathcal{QNN}$. Then from Definition 4.7 a QSVNWDG operator g of dimension l can be written as follows

$$g(\beta_1, \beta_2, \ldots, \beta_l) = \bigodot_{j=1}^{l} \beta_j^{\omega_j}$$

Now if $\omega_j = \frac{1}{l}\ \forall\ j \in \{1, 2, \ldots, l\}$ then

$$g(\beta_1, \beta_2, \ldots, \beta_l) = (\bigodot_{j=1}^{l} \beta_j)^{\frac{1}{l}}.$$

In that case $g(\beta_1, \beta_2, \ldots, \beta_l)$ is called average QSVNWDG operator i.e.QSVNWADG operator of $\beta_j = \langle m_j, n_j, p_j, q_j \rangle$ $(j = 1, 2, \ldots, l)$.

5. An application in MADM of QSVNWDA and QSVNWDG operator

Without an application in real life it is very tough to realize the utility of any operator. A reader can not get any interest if the operators cannot be used properly in MADM technique. For this reason we proposed a model with the help of QSVNWDA and QSVNWDG operator. Suppose Govt. of India want to distribute the Covid-19 vaccine in a smooth manner so that every Indian will get the vaccine. Now Govt of India has 4 vaccine $v_i, i = 1, 2, 3, 4$ in hand where v_1: the co-vaxin from Bharat Bio-tech, v_2: Sputnik-V from Russia, v_3: Astrazeneca vaccine from Oxford university, v_4: Pfizer vaccine from USA with equal storage. However there are four attributes $C_j, j =, 2, 3, 4$ which are to be considered for choosing a particular vaccine from the above list i.e. (C_1) : the cost of the vaccine, (C_2) : the effectiveness of a vaccine in human body, (C_3): the rate of production of a vaccine (C_4): the risk factor of a particular vaccine. In order to get a suitable vaccine V_i after consideration of all attributes C_j we have represented these MADM problems in the form of a decision making matrix $D(v_{ij})$ on $\mathcal{QNN}$ as following:

$$D(v_{ij}) = \begin{bmatrix} \langle 0.4, 0.5, 0.2, 0.6 \rangle & \langle 0.5, 0.5, 0.8, 0.1 \rangle & \langle 0.2, 0.6, 0.3, 0.2 \rangle & \langle 0.6, 0.5, 0.6, 0.7 \rangle \\ \langle 0.4, 0.2, 0.7, 0.6 \rangle & \langle 0.8, 0.5, 0.3, 0.4 \rangle & \langle 0.4, 0.1, 0.1, 0.1 \rangle & \langle 0.6, 0.6, 0.5, 0.5 \rangle \\ \langle 0.4, 0.4, 0.4, 0.5 \rangle & \langle 0.3, 0.6, 0.1, 0.4 \rangle & \langle 0.9, 0.2, 0.7, 0.3 \rangle & \langle 0.4, 0.2, 0.1, 0.1 \rangle \\ \langle 0.1, 0.1, 0.6, 0.3 \rangle & \langle 0.5, 0.3, 0.4, 0.2 \rangle & \langle 0.4, 0.8, 0.3, 0.2 \rangle & \langle 0.4, 0.5, 0.1, 0.5 \rangle \end{bmatrix}.$$

Here we consider the weight vector as $(0.25, 0.25, 0.25, 0.25)$ since every vaccine has equal stock.

Case-I: We now consider the QSVNWDA operator to face the MADM problem. In that case we consider $\varrho = 1$ and derive the collection of QSVNs say v_i to find suitable vaccine among $V_i (i = 1, 2, 3, 4)$ by the help of Definition 4.1 as follows:

$$v_1 = \langle 0.460, 0.529, 0.644, 0.779 \rangle$$

$$v_2 = \langle 0.630, 0.417, 0.761, 0.753 \rangle$$

$$v_3 = \langle 0.319, 0.400, 0.833, 0.775 \rangle$$

$$v_1 = \langle 0.164, 0.341, 0.192, 0.538 \rangle.$$

Based on the Definition 3.1 we have:

$$S(v_1) = 0.392, S(v_2) = 0.384, S(v_3) = 0.3801, S(v_4) = 0.3624.$$

Hence the priority order of vaccine is $v_1 > v_2 > v_3 > v_4$. .

Case-II: Now We consider the QSVNWDG operator to face our problem. We again consider $\varrho = 1$ and derive the collective QSVNs v_i with the help of Definition 4.7 as follows:

$$v_1 = \langle 0.5102, 0.4782, 0.607, 0.512 \rangle$$
$$v_2 = \langle 0.657, 0.785, 0.492, 0.4503 \rangle$$
$$v_3 = \langle 0.576, 0.718, 0.446, 0.355 \rangle$$
$$v_1 = \langle 0.739, 0.758, 0.403, 0.628 \rangle.$$

Again based on the Definition 3.1 we get:

$$S(v_1) = 0.935, S(v_2) = 0.625, S(v_3) = 0.6231, S(v_4) = 0.616.$$

Therefore the priority order of vaccine is $v_4 < v_3 < v_2 < v_1$. To find the more effect of the quantity ϱ in the QSVNWDA and QSVNWDG operator we take the value of ϱ in an increasing order starting from 0.2 to 1 with an increment 0.2. Our results are given in the following tables:

Table of QSVNWDA operator

ϱ	$S(v_1), S(v_2), S(v_3), S(v_4)$	*Order of priority*
0.2	0.627, 0.606, 0.598, 0.377	
0.4	0.635, 0.621, 0.612, 0.396	
0.6	0.676, 0.648, 0.639, 0.404	$v_4 < v_3 < v_2 < v_1$
0.8	0.695, 0.664, 0.652, 0.418	
1.0	0.692, 0.678, 0.664, 0.444	

Table of QSVNWDG operator

ϱ	$S(v_4), S(v_3), S(v_2), S(v_1)$	*Order of priority*
0.2	0.429, 0.492, 0.541, 0.568	
0.4	0.417, 0.476, 0.525, 0.549	
0.6	0.411, 0.449, 0.484, 0.502	$v_4 < v_3 < v_2 < v_1$
0.8	0.426, 0.442, 0.474, 0.491	
1.0	0.394, 0.410, 0.439, 0.462	

In both of the above cases we have seen that in respect of the values of ϱ, the order of priority of the vaccines remains always same for an individual operator. Thus the MADM of finding suitable vaccine using the QSVNWDA operator as well as QSVNWDG operator gives us a flexibility of choosing the value of ϱ. Thus the Govt of India will choose the vaccine v_1 in topmost priority.The above procedure help our Govt to choose a multi-solution based on the current situation at that time.

6. Conclusion

In this article two aggregation operators i.e. QSVNWDA and QSVNWDG operator based on Dombi operations on $\mathcal{QNN}$ sets are introduced. We have studied the properties of QSVNWDA and QSVNWDG operators. Finally we have solved a MADM problems using QSVNWDA and QSVNWDG operators. In solving MADM problems we have utilized the score functions of $\mathcal{QNN}$ to finding the order of priority of different parameters. Also we have seen that different large values of ϱ may effect the score functions. In future we will develop more advanced type of QSVNWDA operator and QSVNWDG operator on $\mathcal{QNN}$ and will apply them to real life MADM problems.

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