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(α, β) Neutrosophic Subbisemiring of Bisemiring

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Abstract. We introduce the notion of neutrosophic subbisemiring(shortly NSBS), level sets of NSBS and neutrosophic normal subbisemiring(NNSBS) of a bisemiring. The concept of neutrosophic subbisemiring is a new generalization of fuzzy subbisemiring over bisemiring. We interact the theory for (α, β) NSBS and NNSBS over bisemiring. Let A be the neutrosophic subset in $\mathbb{S}$, we show that $\tilde{\varpi} = (\varpi_A^T, \varpi_A^I, \varpi_A^F)$ is an NSBS of $\mathbb{S}$ if and only if all non empty level set $\tilde{\varpi}^{(t,s)}$ is a subbisemiring of $\mathbb{S}$ for $t, s \in [0, 1]$. Let A be the NSBS of a bisemiring $\mathbb{S}$ and V be the strongest neutrosophic relation of $\mathbb{S}$, we observe that A is an NSBS of $\mathbb{S}$ if and only if V is an NSBS of $\mathbb{S} \times \mathbb{S}$. Let $A_1, A_2, ..., A_n$ be the family of $NSBS^s$ of $\mathbb{S}_1, \mathbb{S}_2, ..., \mathbb{S}_n$ respectively. We show that $A_1 \times A_2 \times ... \times A_n$ is an NSBS of $\mathbb{S}_1 \times \mathbb{S}_2 \times ... \times \mathbb{S}_n$. The homomorphic image of NSBS is an NSBS. The homomorphic preimage of NSBS is an NSBS. Examples are provided to illustrate our results.

Keywords: Neutrosophic subbisemiring; Neutrosophic bisemiring; Homomorphism; Normal.

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1. Introduction

The study of semirings was opened by the Dedekind in interaction with ideals of commutative rings. In 1934, semiring was studied by Vandever. It was basically the generalization of rings and distributive lattices. In 1950, However the developments of the theory in semirings had been taking place. The classic article of 1965, Zadeh proposed fuzzy set theory [15]. According to this definition a fuzzy set is a function described by a membership value . It takes degrees in real unit interval. But, later it has been seen that this definition is inadequate by considering not only the degree of membership but also the degree of non-membership. Neutrosophic set is a generalization of the fuzzy set and intuitionistic fuzzy set, where the truth-membership, indeterminacy-membership, and falsity-membership are represented independently. Atanassov [4] described a set that is called an intuitionistic fuzzy set to handle mentioned ambiguity. Since this set has some problems in applications, Smarandache [14] introduced neutrosophy to deal with the problems that involves indeterminate and inconsistent information. Arulmozhi interact the theory for various algebraic structures such semirings and ternary semirings [2,3]. A semiring $(S,+,\cdot)$ is a non-empty set in which $(S,+)$ and $(S,\cdot)$ are semigroups such that " $\cdot$ " is distributive over " $+$ " [6]. In 1993, J. Ahsan, K. Saifullah, and F. Khan [1] introduced the notion of fuzzy semirings. In 2001, M.K Sen and S. Ghosh were introduced in bisemirings. A bisemiring $(\mathbb{S},+,\circ,\times)$ is an algebraic structure in which $(\mathbb{S},+,\circ)$ and $(\mathbb{S},\circ,\times)$ are semirings in which $(\mathbb{S},+),(\mathbb{S},\circ)$ and $(\mathbb{S},\times)$ are semigroups such that (i) $x\circ(y+z)=(x\circ y)+(x\circ z)$, (ii) $(y+z)\circ x=(y\circ x)+(z\circ x)$ (iii) $x\times(y\circ z)=(x\times y)\circ(x\times z)$ and (iv) $(y\circ z)\times x=(y\times x)\circ(z\times x),\ \forall\ x,y,z\in\mathbb{S}$ [13]. A non-empty subset A of a bisemiring $(\mathbb{S},+,\circ,\times)$ is a subbisemiring if and only if $x+y\in A,\ \ x\circ y\in A$ and $x\times y\in A$ for all $x,y\in A$ [5]. Palanikumar et al. discussed various ideal structure of subbisemiring theory [7]- [12].

2. Preliminaries

Definition 2.1. [14] A neutrosophic set A in a universe U is an object having the form $A=\left\{\left\langle x,\varpi_A^T(x),\varpi_A^I(x),\varpi_A^F(x)\right\rangle : x\in X\right\}$, where $\varpi_A^T(x),\varpi_A^I(x),\varpi_A^F(x) : X\rightarrow[0,1]$ represents the truth-membership function , the indeterminacy membership function and the falsity-membership function respectively. For simplicity the symbol $\left\langle\varpi_A^T,\varpi_A^I,\varpi_A^F\right\rangle$ is used for the neutrosophic set $A=\left\{\left\langle x,\varpi_A^T(x),\varpi_A^I(x),\varpi_A^F(x)\right\rangle : x\in X\right\}$.

Definition 2.2. [14] Let $A=\{x,\varpi_A^T(x),\varpi_A^I(x),\varpi_A^F(x)\}$ and $B=\{x,\varpi_B^T(x),\varpi_B^I(x),\varpi_B^F(x)\}$ be the two neutrosophic set of a set X. Then
(i) $A\cap B=\left\{\left(x,min\{\varpi_A^T(x),\varpi_B^T(x)\},min\{\varpi_A^I(x),\varpi_B^I(x)\},max\{\varpi_A^F(x),\varpi_B^F(x)\}\right)\Big|x\in X\right\}$.
(ii) $A\cup B=\left\{\left(x,max\{\varpi_A^T(x),\varpi_B^T(x)\},max\{\varpi_A^I(x),\varpi_B^I(x)\},min\{\varpi_A^F(x),\varpi_B^F(x)\}\right)\Big|x\in X\right\}$.

Definition 2.3. [14] For any neutrosophic set $A=\{x,\varpi_A^T(x),\varpi_A^I(x),\varpi_A^F(x)\}$ of a set X, we defined a (α,β)-cut of as the crisp subset $\{x\in X|\varpi_A^T(x)\geq\alpha,\varpi_A^I(x)\geq\alpha,\varpi_A^F(x)\leq\beta\}$ of X.

Definition 2.4. [14] Let A and B be be two neutrosophic subsets of S. The Cartesian product of A and B denoted by $A\times B$ is defined as $A\times B = \{\varpi_{A\times B}^T(x,y),\varpi_{A\times B}^I(x,y),\varpi_{A\times B}^F(x,y)|\ for\ all\ x,y\in S\}$, where

$$\left\{\begin{array}{l}\varpi_{A\times B}^T(x,y)=\min\{\varpi_A^T(x),\varpi_B^T(y)\}\\ \varpi_{A\times B}^I(x,y)=\frac{\varpi_A^I(x)+\varpi_B^I(y)}{2}\\ \varpi_{A\times B}^F(x,y)=\max\{\varpi_A^F(x),\varpi_B^F(y)\}\end{array}\right\}.$$

Definition 2.5. [8] A fuzzy subset A of a bisemiring $(S,\diamond_1,\diamond_2,\diamond_3)$ is said to be a fuzzy subbisemiring of S if

$$\left\{\begin{array}{l}\varpi_A(x\diamond_1 y)\geq\min\{\varpi_A(x),\varpi_A(y)\}\\ \varpi_A(x\diamond_2 y)\geq\min\{\varpi_A(x),\varpi_A(y)\}\\ \varpi_A(x\diamond_3 y)\geq\min\{\varpi_A(x),\varpi_A(y)\}\end{array}\right\}$$

for all $x,y\in S$.

Definition 2.6. [8] A fuzzy subset A of a bisemiring $(S, \diamond_1, \diamond_2, \diamond_3)$ is said to be a fuzzy normal subbisemiring of S if it satisfies the following conditions:

$$\begin{Bmatrix} \varpi_A(x \diamond_1 y) = \varpi_A(y \diamond_1 x) \\ \varpi_A(x \diamond_2 y) = \varpi_A(y \diamond_2 x) \\ \varpi_A(x \diamond_3 y) = \varpi_A(y \diamond_3 x) \end{Bmatrix}$$

for all $x, y \in S$.

Definition 2.7. [5] Let $(S, +, \cdot, \times)$ and $(T, \boxplus, \circ, \otimes)$ be two bisemirings. A function $\phi : S \to T$ is said to be a homomorphism if it satisfies the following conditions:

$$\begin{Bmatrix} \phi(x + y) = \phi(x) \boxplus \phi(y) \\ \phi(x \cdot y) = \phi(x) \circ \phi(y) \\ \phi(x \times y) = \phi(x) \otimes \phi(y) \end{Bmatrix}$$

for all $x, y \in S$.

3. Neutrosophic Subbisemiring

In what follows, let $\mathbb{S}$ denote a bisemiring unless otherwise stated. Here NSBS stands for neutrosophic subbisemiring.

Definition 3.1. A neutrosophic subset A of $\mathbb{S}$ is said to be an NSBS of $\mathbb{S}$ if it satisfies the following conditions:

$$\begin{Bmatrix} \varpi^T_A(x \diamond_1 y) \geq \min\{\varpi^T_A(x), \varpi^T_A(y)\} \\ \varpi^T_A(x \diamond_2 y) \geq \min\{\varpi^T_A(x), \varpi^T_A(y)\} \\ \varpi^T_A(x \diamond_3 y) \geq \min\{\varpi^T_A(x), \varpi^T_A(y)\} \end{Bmatrix} \quad \begin{Bmatrix} \varpi^I_A(x \diamond_1 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2} \\ OR \\ \varpi^I_A(x \diamond_2 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2} \\ OR \\ \varpi^I_A(x \diamond_3 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2} \end{Bmatrix}$$

$$\begin{Bmatrix} \varpi^F_A(x \diamond_1 y) \leq \max\{\varpi^F_A(x), \varpi^F_A(y)\} \\ \varpi^F_A(x \diamond_2 y) \leq \max\{\varpi^F_A(x), \varpi^F_A(y)\} \\ \varpi^F_A(x \diamond_3 y) \leq \max\{\varpi^F_A(x), \varpi^F_A(y)\} \end{Bmatrix}$$

for all $x, y \in \mathbb{S}$.

Example 3.2. Let $\mathbb{S} = \{n_1, n_2, n_3, n_4\}$ be the bisemiring with the following Cayley table:

$\diamond_1$	n_1	n_2	n_3	n_4
n_1	n_1	n_1	n_1	n_1
n_2	n_1	n_2	n_1	n_2
n_3	n_1	n_1	n_3	n_3
n_4	n_1	n_2	n_3	n_4

$\diamond_2$	n_1	n_2	n_3	n_4
n_1	n_1	n_2	n_3	n_4
n_2	n_2	n_2	n_4	n_4
n_3	n_3	n_4	n_3	n_4
n_4	n_4	n_4	n_4	n_4

$\diamond_3$	n_1	n_2	n_3	n_4
n_1	n_1	n_1	n_1	n_1
n_2	n_1	n_2	n_3	n_4
n_3	n_4	n_4	n_4	n_4
n_4	n_4	n_4	n_4	n_4

	$n = n_1$	$n = n_2$	$n = n_3$	$n = n_4$
$\varpi^T_A(n)$	0.7	0.6	0.3	0.5
$\varpi^I_A(n)$	0.4	0.3	0.1	0.2
$\varpi^F_A(n)$	0.5	0.6	0.9	0.8

Clearly, A is an NSBS of $\mathbb{S}$.

Theorem 3.3. *The intersection of a family of $NSBS^s$ of $\mathbb{S}$ is an NSBS of $\mathbb{S}$.*

Proof. Let $\{V_i : i \in I\}$ be a family of $NSBS^s$ of $\mathbb{S}$ and $A = \bigcap_{i\in I} V_i$.
Let x and y in $\mathbb{S}$. Then

$$\begin{aligned}\varpi^T_A(x \diamond_1 y) &= \inf_{i\in I} \varpi^T_{V_i}(x \diamond_1 y)\\ &\geq \inf_{i\in I} \min\{\varpi^T_{V_i}(x), \varpi^T_{V_i}(y)\}\\ &= \min\left\{\inf_{i\in I} \varpi^T_{V_i}(x), \inf_{i\in I} \varpi^T_{V_i}(y)\right\}\\ &= \min\{\varpi^T_A(x), \varpi^T_A(y)\}.\end{aligned}$$

Similarly, $\varpi^T_A(x \diamond_2 y) \geq \min\{\varpi^T_A(x), \varpi^T_A(y)\}$, $\varpi^T_A(x \diamond_3 y) \geq \min\{\varpi^T_A(x), \varpi^T_A(y)\}$. Now,

$$\begin{aligned}\varpi^I_A(x \diamond_1 y) &= \inf_{i\in I} \varpi^I_{V_i}(x \diamond_1 y)\\ &\geq \inf_{i\in I} \frac{\varpi^I_{V_i}(x) + \varpi^I_{V_i}(y)}{2}\\ &= \frac{\inf_{i\in I} \varpi^I_{V_i}(x) + \inf_{i\in I} \varpi^I_{V_i}(y)}{2}\\ &= \frac{\varpi^I_A(x) + \varpi^I_A(y)}{2}.\end{aligned}$$

Similarly, $\varpi^I_A(x \diamond_2 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2}$ and $\varpi^I_A(x \diamond_3 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2}$. Now,

$$\begin{aligned}\varpi^F_A(x \diamond_1 y) &= \sup_{i\in I} \varpi^F_{V_i}(x \diamond_1 y)\\ &\leq \sup_{i\in I} \max\{\varpi^F_{V_i}(x), \varpi^F_{V_i}(y)\}\\ &= \max\left\{\sup_{i\in I} \varpi^F_{V_i}(x), \sup_{i\in I} \varpi^F_{V_i}(y)\right\}\\ &= \max\{\varpi^F_A(x), \varpi^F_A(y)\}.\end{aligned}$$

Similarly, $\varpi^F_A(x \diamond_2 y) \leq \max\{\varpi^F_A(x), \varpi^F_A(y)\}$, $\varpi^F_A(x \diamond_3 y) \leq \max\{\varpi^F_A(x), \varpi^F_A(y)\}$. Hence A is an NSBS of $\mathbb{S}$.

Theorem 3.4. *If A and B are any two $NSBS^s$ of $\mathbb{S}_1$ and $\mathbb{S}_2$ respectively, then $A \times B$ is an NSBS of $\mathbb{S}_1 \times \mathbb{S}_2$.*

Proof. Let A and B be two $NSBS^s$ of $\mathbb{S}_1$ and $\mathbb{S}_2$ respectively. Let $x_1, x_2 \in \mathbb{S}_1$ and $y_1, y_2 \in \mathbb{S}_2$. Then (x_1, y_1) and (x_2, y_2) are in $\mathbb{S}_1 \times \mathbb{S}_2$. Now

$$\begin{aligned}
\varpi^T_{A\times B}[(x_1, y_1) \diamond_1 (x_2, y_2)] &= \varpi^T_{A\times B}(x_1 \diamond_1 x_2, y_1 \diamond_1 y_2) \\
&= \min\{\varpi^T_A(x_1 \diamond_1 x_2), \varpi^T_B(y_1 \diamond_1 y_2)\} \\
&\geq \min\{\min\{\varpi^T_A(x_1), \varpi^T_A(x_2)\}, \min\{\varpi^T_B(y_1), \varpi^T_B(y_2)\}\} \\
&= \min\{\min\{\varpi^T_A(x_1), \varpi^T_B(y_1)\}, \min\{\varpi^T_A(x_2), \varpi^T_B(y_2)\}\} \\
&= \min\{\varpi^T_{A\times B}(x_1, y_1), \varpi^T_{A\times B}(x_2, y_2)\}.
\end{aligned}$$

Also $\varpi^T_{A\times B}[(x_1, y_1) \diamond_2 (x_2, y_2)] \geq \min\{\varpi^T_{A\times B}(x_1, y_1), \varpi^T_{A\times B}(x_2, y_2)\}$, $\varpi^T_{A\times B}[(x_1, y_1) \diamond_3 (x_2, y_2)] \geq \min\{\varpi^T_{A\times B}(x_1, y_1), \varpi^T_{A\times B}(x_2, y_2)\}$. Now,

$$\begin{aligned}
\varpi^I_{A\times B}[(x_1, y_1) \diamond_1 (x_2, y_2)] &= \varpi^I_{A\times B}(x_1 \diamond_1 x_2, y_1 \diamond_1 y_2) \\
&= \frac{\varpi^I_A(x_1 \diamond_1 x_2) + \varpi^I_B(y_1 \diamond_1 y_2)}{2} \\
&\geq \frac{1}{2}\left[\frac{\varpi^I_A(x_1) + \varpi^I_A(x_2)}{2} + \frac{\varpi^I_B(y_1) + \varpi^I_B(y_2)}{2}\right] \\
&= \frac{1}{2}\left[\frac{\varpi^I_A(x_1) + \varpi^I_B(y_1)}{2} + \frac{\varpi^I_A(x_2) + \varpi^I_B(y_2)}{2}\right] \\
&= \frac{1}{2}\Big[\varpi^I_{A\times B}(x_1, y_1) + \varpi^I_{A\times B}(x_2, y_2)\Big].
\end{aligned}$$

Also $\varpi^I_{A\times B}[(x_1, y_1) \diamond_2 (x_2, y_2)] \geq \frac{1}{2}\Big[\varpi^I_{A\times B}(x_1, y_1) + \varpi^I_{A\times B}(x_2, y_2)\Big]$ and $\varpi^I_{A\times B}[(x_1, y_1) \diamond_3 (x_2, y_2)] \geq \frac{1}{2}\Big[\varpi^I_{A\times B}(x_1, y_1) + \varpi^I_{A\times B}(x_2, y_2)\Big]$. Now,

$$\begin{aligned}
\varpi^F_{A\times B}[(x_1, y_1) \diamond_1 (x_2, y_2)] &= \varpi^F_{A\times B}(x_1 \diamond_1 x_2, y_1 \diamond_1 y_2) \\
&= \max\{\varpi^F_A(x_1 \diamond_1 x_2), \varpi^F_B(y_1 \diamond_1 y_2)\} \\
&\leq \max\{\max\{\varpi^F_A(x_1), \varpi^F_A(x_2)\}, \max\{\varpi^F_B(y_1), \varpi^F_B(y_2)\}\} \\
&= \max\{\max\{\varpi^F_A(x_1), \varpi^F_B(y_1)\}, \max\{\varpi^F_A(x_2), \varpi^F_B(y_2)\}\} \\
&= \max\{\varpi^F_{A\times B}(x_1, y_1), \varpi^F_{A\times B}(x_2, y_2)\}.
\end{aligned}$$

Also $\varpi^F_{A\times B}[(x_1, y_1) \diamond_2 (x_2, y_2)] \leq \max\{\varpi^F_{A\times B}(x_1, y_1), \varpi^F_{A\times B}(x_2, y_2)\}$, $\varpi^F_{A\times B}[(x_1, y_1) \diamond_3 (x_2, y_2)] \leq \max\{\varpi^F_{A\times B}(x_1, y_1), \varpi^F_{A\times B}(x_2, y_2)\}$. Hence $A \times B$ is an NSBS of $\mathbb{S}$.

Corollary 3.5. *If $A_1, A_2, ..., A_n$ are the family of $NSBS^s$ of $\mathbb{S}_1, \mathbb{S}_2, ..., \mathbb{S}_n$ respectively, then $A_1 \times A_2 \times ... \times A_n$ is an NSBS of $\mathbb{S}_1 \times \mathbb{S}_2 \times ... \times \mathbb{S}_n$.*

Definition 3.6. Let A be a neutrosophic subset in $\mathbb{S}$, the strongest neutrosophic relation on $\mathbb{S}$, that is a neutrosophic relation on A is V given by

$$\left\{\begin{array}{l} \varpi_V^T(x,y) = min\{\varpi_A^T(x), \varpi_A^T(y)\} \\ \varpi_V^I(x,y) = \frac{\varpi_A^I(x)+\varpi_A^I(y)}{2} \\ \varpi_V^F(x,y) = max\{\varpi_A^F(x), \varpi_A^F(y)\} \end{array}\right\}.$$

Theorem 3.7. *Let A be the NSBS of $\mathbb{S}$ and V be the strongest neutrosophic relation of $\mathbb{S}$. Then A is an NSBS of $\mathbb{S}$ if and only if V is an NSBS of $\mathbb{S} \times \mathbb{S}$.*

Proof. Let A be the NSBS of $\mathbb{S}$ and V be the strongest neutrosophic relation of $\mathbb{S}$. Then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $\mathbb{S} \times \mathbb{S}$. We have

$$\begin{aligned} \varpi_V^T(x \diamond_1 y) &= \varpi_V^T[((x_1, x_2) \diamond_1 (y_1, y_2)] \\ &= \varpi_V^T(x_1 \diamond_1 y_1, x_2 \diamond_1 y_2) \\ &= \min\{\varpi_A^T(x_1 \diamond_1 y_1), \varpi_A^T(x_2 \diamond_1 y_2)\} \\ &\geq \min\{\min\{\varpi_A^T(x_1), \varpi_A^T(y_1)\}, \min\{\varpi_A^T(x_2), \varpi_A^T(y_2)\}\} \\ &= \min\{\min\{\varpi_A^T(x_1), \varpi_A^T(x_2)\}, \min\{\varpi_A^T(y_1), \varpi_A^T(y_2)\}\} \\ &= \min\{\varpi_V^T(x_1, x_2), \varpi_V^T(y_1, y_2)\} \\ &= \min\{\varpi_V^T(x), \varpi_V^T(y)\}. \end{aligned}$$

Also, $\varpi_V^T(x \diamond_2 y) \geq \min\{\varpi_V^T(x), \varpi_V^T(y)\}, \varpi_V^T(x \diamond_3 y) \geq \min\{\varpi_V^T(x), \varpi_V^T(y)\}$.
Now,

$$\begin{aligned} \varpi_V^I(x \diamond_1 y) &= \varpi_V^I[((x_1, x_2) \diamond_1 (y_1, y_2)] \\ &= \varpi_V^I(x_1 \diamond_1 y_1, x_2 \diamond_1 y_2) \\ &= \frac{\varpi_A^I(x_1 \diamond_1 y_1) + \varpi_A^I(x_2 \diamond_1 y_2)}{2} \\ &\geq \frac{1}{2}\left[\frac{\varpi_A^I(x_1) + \varpi_A^I(y_1)}{2} + \frac{\varpi_A^I(x_2) + \varpi_A^I(y_2)}{2}\right] \\ &= \frac{1}{2}\left[\frac{\varpi_A^I(x_1) + \varpi_A^I(x_2)}{2} + \frac{\varpi_A^I(y_1) + \varpi_A^I(y_2)}{2}\right] \\ &= \frac{\varpi_V^I(x_1, x_2) + \varpi_V^I(y_1, y_2)}{2} \\ &= \frac{\varpi_V^I(x) + \varpi_V^I(y)}{2}. \end{aligned}$$

Also, $\varpi_V^I(x \diamond_2 y) \geq \frac{\varpi_V^I(x)+\varpi_V^I(y)}{2}$ and $\varpi_V^I(x \diamond_3 y) \geq \frac{\varpi_V^I(x)+\varpi_V^I(y)}{2}$.
Similarly, $\varpi_V^F(x \diamond_1 y) \leq \max\{\varpi_V^F(x), \varpi_V^F(y)\}, \varpi_V^F(x \diamond_2 y) \leq \max\{\varpi_V^F(x), \varpi_V^F(y)\}$ and $\varpi_V^F(x \diamond_3 y) \leq \max\{\varpi_V^F(x), \varpi_V^F(y)\}$. Hence V is an NSBS of $\mathbb{S} \times \mathbb{S}$.

Conversely assume that V is an NSBS of $\mathbb{S} \times \mathbb{S}$, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in $\mathbb{S} \times \mathbb{S}$. We have

$$\begin{aligned}
\min\{\varpi_A^T(x_1 \diamond_1 y_1), \varpi_A^T(x_2 \diamond_1 y_2)\} &= \varpi_V^T(x_1 \diamond_1 y_1, x_2 \diamond_1 y_2) \\
&= \varpi_V^T[(x_1, x_2) \diamond_1 (y_1, y_2)] \\
&= \varpi_V^T(x \diamond_1 y) \\
&\geq \min\{\varpi_V^T(x), \varpi_V^T(y)\} \\
&= \min\{\varpi_V^T(x_1, x_2)\}, \varpi_V^T(y_1, y_2)\} \\
&= \min\{\min\{\varpi_A^T(x_1), \varpi_A^T(x_2)\}, \min\{\varpi_A^T(y_1), \varpi_A^T(y_2)\}\}.
\end{aligned}$$

If $\varpi_A^T(x_1 \diamond_1 y_1) \leq \varpi_A^T(x_2 \diamond_1 y_2)$, then $\varpi_A^T(x_1) \leq \varpi_A^T(x_2)$ and $\varpi_A^T(y_1) \leq \varpi_A^T(y_2)$. We get $\varpi_A^T(x_1 \diamond_1 y_1) \geq \min\{\varpi_A^T(x_1), \varpi_A^T(y_1)\}$ for all $x_1, y_1 \in \mathbb{S}$, and
$\min\{\varpi_A^T(x_1 \diamond_2 y_1), \varpi_A^T(x_2 \diamond_2 y_2)\} \geq \min\{\min\{\varpi_A^T(x_1), \varpi_A^T(x_2)\}, \min\{\varpi_A^T(y_1), \varpi_A^T(y_2)\}\}$
If $\varpi_A^T(x_1 \diamond_2 y_1) \leq \varpi_A^T(x_2 \diamond_2 y_2)$, then $\varpi_A^T(x_1 \diamond_2 y_1) \geq \min\{\varpi_A^T(x_1), \varpi_A^T(y_1)\}$.
$\min\{\varpi_A^T(x_1 \diamond_3 y_1), \varpi_A^T(x_2 \diamond_3 y_2)\} \geq \min\{\min\{\varpi_A^T(x_1), \varpi_A^T(x_2)\}, \min\{\varpi_A^T(y_1), \varpi_A^T(y_2)\}\}$.
If $\varpi_A^T(x_1 \diamond_3 y_1) \leq \varpi_A^T(x_2 \diamond_3 y_2)$, then $\varpi_A^T(x_1 \diamond_3 y_1) \geq \min\{\varpi_A^T(x_1), \varpi_A^T(y_1)\}$.
Now,

$$\begin{aligned}
\frac{1}{2}\Big[\varpi_A^I(x_1 \diamond_1 y_1) + \varpi_A^I(x_2 \diamond_1 y_2)\Big] &= \varpi_V^I(x_1 \diamond_1 y_1, x_2 \diamond_1 y_2) \\
&= \varpi_V^I[(x_1, x_2) \diamond_1 (y_1, y_2)] \\
&= \varpi_V^I(x \diamond_1 y) \\
&\geq \frac{\varpi_V^I(x) + \varpi_V^I(y)}{2} \\
&= \frac{\varpi_V^I(x_1, x_2) + \varpi_V^I(y_1, y_2)}{2} \\
&= \frac{1}{2}\left[\frac{\varpi_A^I(x_1) + \varpi_A^I(x_2)}{2} + \frac{\varpi_A^I(y_1) + \varpi_A^I(y_2)}{2}\right].
\end{aligned}$$

If $\varpi_A^I(x_1 \diamond_1 y_1) \leq \varpi_A^I(x_2 \diamond_1 y_2)$, then $\varpi_A^I(x_1) \leq \varpi_A^I(x_2)$ and $\varpi_A^I(y_1) \leq \varpi_A^I(y_2)$.
We get, $\varpi_A^I(x_1 \diamond_1 y_1) \geq \frac{\varpi_A^I(x_1)+\varpi_A^I(y_1)}{2}$.
Similarly, $\varpi_A^I(x_1 \diamond_2 y_1) \geq \frac{\varpi_A^I(x_1)+\varpi_A^I(y_1)}{2}$ and $\varpi_A^I(x_1 \diamond_3 y_1) \geq \frac{\varpi_A^I(x_1)+\varpi_A^I(y_1)}{2}$.
Similarly to prove that
$\max\{\varpi_A^F(x_1 \diamond_1 y_1), \varpi_A^F(x_2 \diamond_1 y_2)\} \leq \max\{\max\{\varpi_A^F(x_1), \varpi_A^F(x_2)\}, \max\{\varpi_A^F(y_1), \varpi_A^F(y_2)\}\}$.
If $\varpi_A^F(x_1 \diamond_1 y_1) \geq \varpi_A^F(x_2 \diamond_1 y_2)$, then $\varpi_A^F(x_1) \geq \varpi_A^F(x_2)$ and $\varpi_A^F(y_1) \geq \varpi_A^F(y_2)$.
We get, $\varpi_A^F(x_1 \diamond_1 y_1) \leq \max\{\varpi_A^F(x_1), \varpi_A^F(y_1)\}$.
$\max\{\varpi_A^F(x_1 \diamond_2 y_1), \varpi_A^F(x_2 \diamond_2 y_2)\} \leq \max\{\max\{\varpi_A^F(x_1), \varpi_A^F(x_2)\}, \max\{\varpi_A^F(y_1), \varpi_A^F(y_2)\}\}$.
If $\varpi_A^F(x_1 \diamond_2 y_1) \geq \varpi_A^F(x_2 \diamond_2 y_2)$, then $\varpi_A^F(x_1 \diamond_2 y_1) \leq \max\{\varpi_A^F(x_1), \varpi_A^F(y_1)\}$.
$\max\{\varpi_A^F(x_1 \diamond_3 y_1), \varpi_A^F(x_2 \diamond_3 y_2)\} \leq \max\{\max\{\varpi_A^F(x_1), \varpi_A^F(x_2)\}, \max\{\varpi_A^F(y_1), \varpi_A^F(y_2)\}\}$

If $\varpi_A^F(x_1 \diamond_3 y_1) \geq \varpi_A^F(x_2 \diamond_3 y_2)$, then $\varpi_A^F(x_1 \diamond_3 y_1) \leq \max\{\varpi_A^F(x_1), \varpi_A^F(y_1)\}$.
Hence A is an NSBS of $\mathbb{S}$.

Theorem 3.8. *Let A be a neutrosophic subset in $\mathbb{S}$. Then $\tilde{\varpi} = (\varpi_A^T, \varpi_A^I, \varpi_A^F)$ is an NSBS of $\mathbb{S}$ if and only if all non empty level set $\tilde{\varpi}^{(t,s)}$ is a subbisemiring of $\mathbb{S}$ for $t, s \in [0,1]$.*

Proof. Assume that $\tilde{\varpi}$ is an NSBS of $\mathbb{S}$. For each $t, s \in [0,1]$ and $a_1, a_2 \in \tilde{\varpi}^{(t,s)}$. We have $\varpi_A^T(a_1) \geq t, \varpi_A^T(a_2) \geq t$ and $\varpi_A^I(a_1) \geq t, \varpi_A^I(a_2) \geq t$ and $\varpi_A^F(a_1) \leq s, \varpi_A^F(a_2) \leq s$. Now, $\varpi_A^T(a_1 \diamond_1 a_2) \geq \min\{\varpi_A^T(a_1), \varpi_A^T(a_2)\} \geq t$ and $\varpi_A^I(a_1 \diamond_1 a_2) \geq \frac{\varpi_A^I(a_1)+\varpi_A^I(a_2)}{2} \geq \frac{t+t}{2} = t$ and $\varpi_A^F(a_1 \diamond_1 a_2) \leq \max\{\varpi_A^F(a_1), \varpi_A^F(a_2)\} \leq s$. This implies that $a_1 \diamond_1 a_2 \in \tilde{\varpi}^{(t,s)}$. Similarly, $a_1 \diamond_2 a_2 \in \tilde{\varpi}^{(t,s)}$ and $a_1 \diamond_3 a_2 \in \tilde{\varpi}^{(t,s)}$. Therefore $\tilde{\varpi}^{(t,s)}$ is a subbisemiring of $\mathbb{S}$ for each $t, s \in [0,1]$.

Conversely, assume that $\tilde{\varpi}^{(t,s)}$ is a subbisemiring of $\mathbb{S}$ for each $t, s \in [0,1]$. Suppose if there exist $a_1, a_2 \in \mathbb{S}$ such that $\varpi_A^T(a_1 \diamond_1 a_2) < \min\{\varpi_A^T(a_1), \varpi_A^T(a_2)\}$, $\varpi_A^I(a_1 \diamond_1 a_2) < \frac{\varpi_A^I(a_1)+\varpi_A^I(a_2)}{2}$ and $\varpi_A^F(a_1 \diamond_1 a_2) > \max\{\varpi_A^F(a_1), \varpi_A^F(a_2)\}$. Select $t, s \in [0,1]$ such that $\varpi_A^T(a_1 \diamond_1 a_2) < t \leq \min\{\varpi_A^T(a_1), \varpi_A^T(a_2)\}$ and $\varpi_A^I(a_1 \diamond_1 a_2) < t \leq \frac{\varpi_A^I(a_1)+\varpi_A^I(a_2)}{2}$ and $\varpi_A^F(a_1 \diamond_1 a_2) > s \geq \max\{\varpi_A^F(a_1), \varpi_A^F(a_2)\}$. Then $a_1, a_2 \in \tilde{\varpi}^{(t,s)}$, but $a_1 \diamond_1 a_2 \notin \tilde{\varpi}^{(t,s)}$. This contradicts to that $\tilde{\varpi}^{(t,s)}$ is a subbisemiring of $\mathbb{S}$. Hence $\varpi_A^T(a_1 \diamond_1 a_2) \geq \min\{\varpi_A^T(a_1), \varpi_A^T(a_2)\}$, $\varpi_A^I(a_1 \diamond_1 a_2) \geq \frac{\varpi_A^I(a_1)+\varpi_A^I(a_2)}{2}$ and $\varpi_A^F(a_1 \diamond_1 a_2) \leq \max\{\varpi_A^F(a_1), \varpi_A^F(a_2)\}$. Similarly, $\diamond_2$ and $\diamond_3$ cases. Hence $\tilde{\varpi} = (\varpi_A^T, \varpi_A^I, \varpi_A^F)$ is an NSBS of $\mathbb{S}$.

Definition 3.9. Let A be any NSBS of $\mathbb{S}$ and $a \in \mathbb{S}$. Then the pseudo neutrosophic coset $(aA)^p$ is defined by

$$\begin{Bmatrix} ((a\varpi_A^T)^p)(x) = p(a)\varpi_A^T(x) \\ ((a\varpi_A^I)^p)(x) = p(a)\varpi_A^I(x) \\ ((a\varpi_A^F)^p)(x) = p(a)\varpi_A^F(x) \end{Bmatrix}$$

for every $x \in \mathbb{S}$ and for some $p \in P$.

Theorem 3.10. *Let A be any NSBS of $\mathbb{S}$, then the pseudo neutrosophic coset $(aA)^p$ is an NSBS of $\mathbb{S}$, for every $a \in \mathbb{S}$.*

Proof. Let A be any NSBS of $\mathbb{S}$ and for every $x, y \in \mathbb{S}$. Now, $((a\varpi_A^T)^p)(x \diamond_1 y) = p(a)\, \varpi_A^T(x \diamond_1 y) \geq p(a)\, min\{\varpi_A^T(x), \varpi_A^T(y)\} = min\{p(a)\, \varpi_A^T(x), p(a)\, \varpi_A^T(y)\} = min\{((a\varpi_A^T)^p)(x), ((a\varpi_A^T)^p)(y)\}$. Thus, $((a\varpi_A^T)^p)(x \diamond_1 y) \geq \min\{((a\varpi_A^T)^p)(x), ((a\varpi_A^T)^p)(y)\}$. Now, $((a\varpi_A^I)^p)(x \diamond_1 y) = p(a)\, \varpi_A^I(x \diamond_1 y) \geq p(a)\left[\frac{\varpi_A^I(x)+\varpi_A^I(y)}{2}\right] = \frac{p(a)\, \varpi_A^I(x)+p(a)\, \varpi_A^I(y)}{2} = \frac{((a\varpi_A^I)^p)(x)+((a\varpi_A^I)^p)(y)}{2}$. Thus, $((a\varpi_A^I)^p)(x \diamond_1 y) \geq \frac{((a\varpi_A^I)^p)(x)+((a\varpi_A^I)^p)(y)}{2}$. Now, $((a\varpi_A^F)^p)(x \diamond_1 y) = p(a)\, \varpi_A^F(x \diamond_1 y) \leq p(a)\, \max\{\varpi_A^F(x), \varpi_A^F(y)\} = \max\{p(a)\, \varpi_A^F(x), p(a)\, \varpi_A^F(y)\} =$

$\max\{((a\varpi_A^F)^p)(x), ((a\varpi_A^F)^p)(y)\}$. Thus, $((a\varpi_A^F)^p)(x \diamond_1 y) \leq \max\{((a\varpi_A^F)^p)(x), ((a\varpi_A^F)^p)(y)\}$. Similarly, $\diamond_2$ and $\diamond_3$ cases. Hence $(aA)^p$ is an NSBS of $\mathbb{S}$.

Definition 3.11. Let $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ and $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ be any two bisemirings. Let $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ be any function and A be any NSBS in $\mathbb{S}_1$, V be any NSBS in $\Delta(\mathbb{S}_1) = \mathbb{S}_2$. If $\varpi_A = [\varpi_A^T, \varpi_A^I, \varpi_A^F]$ is a neutrosophic set in $\mathbb{S}_1$, then ϖ_V is a neutrosophic set in $\mathbb{S}_2$, defined by

$$\varpi_V^T(y) = \begin{cases} \sup \varpi_A^T(x) & if \ \ x \in \Delta^{-1}y \\ 0 & otherwise \end{cases} \qquad \varpi_V^I(y) = \begin{cases} \sup \varpi_A^I(x) & if \ \ x \in \Delta^{-1}y \\ 0 & otherwise \end{cases}$$

$$\varpi_V^F(y) = \begin{cases} \inf \varpi_A^F(x) & if \ \ x \in \Delta^{-1}y \\ 1 & otherwise \end{cases}$$

for all $x \in \mathbb{S}_1$ and $y \in \mathbb{S}_2$ is called the image of ϖ_A under Δ.
Similarly, If $\varpi_V = [\varpi_V^T, \varpi_V^I, \varpi_V^F]$ is a neutrosophic set in $\mathbb{S}_2$, then neutrosophic set $\varpi_A = \Delta \circ \varpi_V$ in $\mathbb{S}_1$ [ie, the neutrosophic set defined by $\varpi_A(x) = \varpi_V(\Delta(x))$] is called the preimage of ϖ_V under Δ.

Theorem 3.12. *Let* $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ *and* $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ *be any two bisemirings. The homomorphic image of NSBS of* $\mathbb{S}_1$ *is an NSBS of* $\mathbb{S}_2$.

Proof. Let $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ be any homomorphism. Then $\Delta(x \boxplus_1 y) = \Delta(x) \boxdot_1 \Delta(y), \Delta(x \boxplus_2 y) = \Delta(x) \boxdot_2 \Delta(y)$ and $\Delta(x \boxplus_3 y) = \Delta(x) \boxdot_3 \Delta(y)$ for all $x, y \in \mathbb{S}_1$. Let $V = \Delta(A)$, A is any NSBS of $\mathbb{S}_1$. Let $\Delta(x), \Delta(y) \in \mathbb{S}_2$. Let $x \in \Delta^{-1}(\Delta(x))$ and $y \in \Delta^{-1}(\Delta(y))$ be such that $\varpi_A^T(x) = \sup\limits_{z \in \Delta^{-1}(\Delta(x))} \varpi_A^T(z)$ and $\varpi_A^T(y) = \sup\limits_{z \in \Delta^{-1}(\Delta(y))} \varpi_A^T(z)$. Now,

$$\begin{aligned} \varpi_V^T(\Delta(x) \boxdot_1 \Delta(y)) &= \sup_{z' \in \Delta^{-1}(\Delta(x) \boxdot_1 \Delta(y))} \varpi_A^T(z') \\ &= \sup_{z' \in \Delta^{-1}(\Delta(x \boxplus_1 y)} \varpi_A^T(z') \\ &= \varpi_A^T(x \boxplus_1 y) \\ &\geq \min\{\varpi_A^T(x), \varpi_A^T(y)\} \\ &= \min\{\varpi_V^T \Delta(x), \varpi_V^T \Delta(y)\}. \end{aligned}$$

Thus, $\varpi_V^T(\Delta(x) \boxdot_1 \Delta(y)) \geq \min\{\varpi_V^T \Delta(x), \varpi_V^T \Delta(y)\}$.
Similarly, $\varpi_V^T(\Delta(x) \boxdot_2 \Delta(y)) \geq \min\{\varpi_V^T \Delta(x), \varpi_V^T \Delta(y)\}$ and
$\varpi_V^T(\Delta(x) \boxdot_3 \Delta(y)) \geq \min\{\varpi_V^T \Delta(x), \varpi_V^T \Delta(y)\}$.
Let $x \in \Delta^{-1}(\Delta(x))$ and $y \in \Delta^{-1}(\Delta(y))$ be such that $\varpi_A^I(x) = \sup\limits_{z \in \Delta^{-1}(\Delta(x))} \varpi_A^I(z)$ and

$\varpi^I_A(y) = \sup\limits_{z\in\Delta^{-1}(\Delta(y))} \varpi^I_A(z)$. Now,

$$\begin{aligned}\varpi^I_V(\Delta(x)\boxdot_1\Delta(y)) &= \sup_{z'\in\Delta^{-1}(\Delta(x)\boxdot_1\Delta(y))} \varpi^I_A(z')\\ &= \sup_{z'\in\Delta^{-1}(\Delta(x\boxplus_1 y)} \varpi^I_A(z')\\ &= \varpi^I_A(x\boxplus_1 y)\\ &\geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2}\\ &= \frac{\varpi^I_V\Delta(x)+\varpi^I_V\Delta(y)}{2}.\end{aligned}$$

Thus, $\varpi^I_V(\Delta(x)\boxdot_1\Delta(y)) \geq \frac{\varpi^I_V\Delta(x)+\varpi^I_V\Delta(y)}{2}$.
Similarly, $\varpi^I_V(\Delta(x)\boxdot_2\Delta(y)) \geq \frac{\varpi^I_V\Delta(x)+\varpi^I_V\Delta(y)}{2}$ and $\varpi^I_V(\Delta(x)\boxdot_3\Delta(y)) \geq \frac{\varpi^I_V\Delta(x)+\varpi^I_V\Delta(y)}{2}$.
Let $\Delta(x),\Delta(y)\in\mathbb{S}_2$. Let $x\in\Delta^{-1}(\Delta(x))$ and $y\in\Delta^{-1}(\Delta(y))$ be such that $\varpi^F_A(x) = \inf\limits_{z\in\Delta^{-1}(\Delta(x))} \varpi^F_A(z)$ and $\varpi^F_A(y) = \inf\limits_{z\in\Delta^{-1}(\Delta(y))} \varpi^F_A(z)$. Now,

$$\begin{aligned}\varpi^F_V(\Delta(x)\boxdot_1\Delta(y)) &= \inf_{z'\in\Delta^{-1}(\Delta(x)\boxdot_1\Delta(y))} \varpi^F_A(z')\\ &= \inf_{z'\in\Delta^{-1}(\Delta(x\boxplus_1 y)} \varpi^F_A(z')\\ &= \varpi^F_A(x\boxplus_1 y)\\ &\leq \max\{\varpi^F_A(x),\varpi^F_A(y)\}\\ &= \max\{\varpi^F_V\Delta(x),\varpi^F_V\Delta(y)\}.\end{aligned}$$

Thus, $\varpi^F_V(\Delta(x)\boxdot_1\Delta(y)) \leq \max\{\varpi^F_V\Delta(x),\varpi^F_V\Delta(y)\}$.
Similarly, $\varpi^F_V(\Delta(x)\boxdot_2\Delta(y)) \leq \max\{\varpi^F_V\Delta(x),\varpi^F_V\Delta(y)\}$ and
$\varpi^F_V(\Delta(x)\boxdot_3\Delta(y)) \leq \max\{\varpi^F_V\Delta(x),\varpi^F_V\Delta(y)\}$. Hence V is an NSBS of $\mathbb{S}_2$.

Theorem 3.13. *Let $(\mathbb{S}_1,\boxplus_1,\boxplus_2,\boxplus_3)$ and $(\mathbb{S}_2,\boxdot_1,\boxdot_2,\boxdot_3)$ be any two bisemirings. The homomorphic preimage of NSBS of $\mathbb{S}_2$ is an NSBS of $\mathbb{S}_1$.*

Proof. Let $\Delta:\mathbb{S}_1\to\mathbb{S}_2$ be any homomorphism. Then $\Delta(x\boxplus_1 y) = \Delta(x)\boxdot_1\Delta(y), \Delta(x\boxplus_2 y) = \Delta(x)\boxdot_2\Delta(y)$ and $\Delta(x\boxplus_3 y) = \Delta(x)\boxdot_3\Delta(y)$ for all $x,y\in\mathbb{S}_1$. Let $V=\Delta(A)$, where V is any NSBS of $\mathbb{S}_2$. Let $x,y\in\mathbb{S}_1$. Now, $\varpi^T_A(x\boxplus_1 y) = \varpi^T_V(\Delta(x\boxplus_1 y)) = \varpi^T_V(\Delta(x)\boxdot_1\Delta(y)) \geq \min\{\varpi^T_V\Delta(x),\varpi^T_V\Delta(y)\} = \min\{\varpi^T_A(x),\varpi^T_A(y)\}$. Thus, $\varpi^T_A(x\boxplus_1 y) \geq \min\{\varpi^T_A(x),\varpi^T_A(y)\}$. Now, $\varpi^I_A(x\boxplus_1 y) = \varpi^I_V(\Delta(x\boxplus_1 y)) = \varpi^I_V(\Delta(x)\boxdot_1\Delta(y)) \geq \frac{\varpi^I_V\Delta(x)+\varpi^I_V\Delta(y)}{2} = \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2}$. Thus, $\varpi^I_A(x\boxplus_1 y) \geq \frac{\varpi^I_A(x)+\varpi^I_A(y)}{2}$. Now, $\varpi^F_A(x\boxplus_1 y) = \varpi^F_V(\Delta(x\boxplus_1 y)) = \varpi^F_V(\Delta(x)\boxdot_1\Delta(y)) \leq \max\{\varpi^F_V\Delta(x),\varpi^F_V\Delta(y)\} = \max\{\varpi^F_A(x),\varpi^F_A(y)\}$. Thus, $\varpi^F_A(x\boxplus_1 y) \leq \max\{\varpi^F_A(x),\varpi^F_A(y)\}$. Similarly to prove two other operations, hence A is an NSBS of $\mathbb{S}_1$.

Theorem 3.14. *Let* $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ *and* $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ *be any two bisemirings. If* $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ *is a homomorphism, then* $\Delta(A_{(t,s)})$ *is a level subbisemiring of NSBS* V *of* $\mathbb{S}_2$.

Proof. Let $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ be any homomorphism. Then $\Delta(x \boxplus_1 y) = \Delta(x) \boxdot_1 \Delta(y), \Delta(x \boxplus_2 y) = \Delta(x) \boxdot_2 \Delta(y)$ and $\Delta(x \boxplus_3 y) = \Delta(x) \boxdot_3 \Delta(y)$ for all $x, y \in \mathbb{S}_1$. Let $V = \Delta(A)$, A is an NSBS of $\mathbb{S}_1$. By Theorem 3.12, V is an NSBS of $\mathbb{S}_2$. Let $A_{(t,s)}$ be any level subbisemiring of A. Suppose that $x, y \in A_{(t,s)}$. Then $\Delta(x \boxplus_1 y), \Delta(x \boxplus_2 y)$ and $\Delta(x \boxplus_3 y) \in A_{(t,s)}$. Now, $\varpi_V^T(\Delta(x)) = \varpi_A^T(x) \geq t, \varpi_V^T(\Delta(y)) = \varpi_A^T(y) \geq t$. Thus, $\varpi_V^T(\Delta(x) \boxdot_1 \Delta(y)) \geq \varpi_A^T(x \boxplus_1 y) \geq t$. Now, $\varpi_V^I(\Delta(x)) = \varpi_A^I(x) \geq t, \varpi_V^I(\Delta(y)) = \varpi_A^I(y) \geq t$. Thus, $\varpi_V^I(\Delta(x) \boxdot_1 \Delta(y)) \geq \varpi_A^I(x \boxplus_1 y) \geq t$. Now, $\varpi_V^F(\Delta(x)) = \varpi_A^F(x) \leq s, \varpi_V^F(\Delta(y)) = \varpi_A^F(y) \leq s$. Thus, $\varpi_V^F(\Delta(x) \boxdot_1 \Delta(y)) \leq \varpi_A^F(x \boxplus_1 y) \leq s$, for all $\Delta(x), \Delta(y) \in \mathbb{S}_2$. Similarly to prove other operations, hence $\Delta(A_{(t,s)})$ is a level subbisemiring of NSBS V of $\mathbb{S}_2$.

Theorem 3.15. *Let* $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ *and* $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ *be any two bisemirings. If* $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ *is any homomorphism, then* $A_{(t,s)}$ *is a level subbisemiring of NSBS* A *of* $\mathbb{S}_1$.

Proof. Let $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ be any homomorphism. Then $\Delta(x \boxplus_1 y) = \Delta(x) \boxdot_1 \Delta(y), \Delta(x \boxplus_2 y) = \Delta(x) \boxdot_2 \Delta(y)$ and $\Delta(x \boxplus_3 y) = \Delta(x) \boxdot_3 \Delta(y)$ for all $x, y \in \mathbb{S}_1$. Let $V = \Delta(A)$, V is an NSBS of $\mathbb{S}_2$. By Theorem 3.13, A is an NSBS of $\mathbb{S}_1$. Let $\Delta(A_{(t,s)})$ be a level subbisemiring of V. Suppose that $\Delta(x), \Delta(y) \in \Delta(A_{(t,s)})$. Then $\Delta(x \boxplus_1 y), \Delta(x \boxplus_2 y)$ and $\Delta(x \boxplus_3 y) \in \Delta(A_{(t,s)})$. Now, $\varpi_A^T(x) = \varpi_V^T(\Delta(x)) \geq t, \varpi_A^T(y) = \varpi_V^T(\Delta(y)) \geq t$. Thus, $\varpi_A^T(x \boxplus_1 y) \geq \min\{\varpi_A^T(x), \varpi_A^T(y)\} \geq t$. Now, $\varpi_A^I(x) = \varpi_V^I(\Delta(x)) \geq t, \varpi_A^I(y) = \varpi_V^I(\Delta(y)) \geq t$. Thus, $\varpi_A^I(x \boxplus_1 y) \geq \frac{\varpi_A^I(x)+\varpi_A^I(y)}{2} \geq t$. Now, $\varpi_A^F(x) = \varpi_V^F(\Delta(x)) \leq s, \varpi_A^F(y) = \varpi_V^F(\Delta(y)) \leq s$. Thus, $\varpi_A^F(x \boxplus_1 y) = \varpi_V^F(\Delta(x) \boxdot_1 \Delta(y)) \leq \max\{\varpi_A^F(x), \varpi_A^F(y)\} \leq s$, for all $x, y \in \mathbb{S}_1$. Similarly to prove other two operations, hence $A_{(t,s)}$ is a level subbisemiring of NSBS A of $\mathbb{S}_1$.

4. (α, β)- neutrosophic Subbisemiring

In this section, we discuss about (α, β)- neutrosophic subbisemiring. In what follows that, $(\alpha, \beta) \in [0, 1]$ be such that $0 \leq \alpha < \beta \leq 1$.

Definition 4.1. Let A be any neutrosophic subset of $\mathbb{S}$ is called a (α, β)- NSBS of $\mathbb{S}$ if it satisfies the following conditions:

$$\left\{\begin{matrix} \max\{\varpi_A^T(x \diamond_1 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\} \\ \max\{\varpi_A^T(x \diamond_2 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\} \\ \max\{\varpi_A^T(x \diamond_3 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\} \end{matrix}\right\}$$

$$\begin{Bmatrix} \max\{\varpi_A^I(x \diamond_1 y), \alpha\} \geq \min\left\{\frac{\varpi_A^I(x)+\varpi_A^I(y)}{2}, \beta\right\} \\ \text{OR} \\ \max\{\varpi_A^I(x \diamond_2 y), \alpha\} \geq \min\left\{\frac{\varpi_A^I(x)+\varpi_A^I(y)}{2}, \beta\right\} \\ \text{OR} \\ \max\{\varpi_A^I(x \diamond_3 y), \alpha\} \geq \min\left\{\frac{\varpi_A^I(x)+\varpi_A^I(y)}{2}, \beta\right\} \end{Bmatrix}$$

$$\begin{Bmatrix} \min\{\varpi_A^F(x \diamond_1 y), \alpha\} \leq \max\{\varpi_A^F(x), \varpi_A^F(y), \beta\} \\ \min\{\varpi_A^F(x \diamond_2 y), \alpha\} \leq \max\{\varpi_A^F(x), \varpi_A^F(y), \beta\} \\ \min\{\varpi_A^F(x \diamond_3 y), \alpha\} \leq \max\{\varpi_A^F(x), \varpi_A^F(y), \beta\} \end{Bmatrix}$$

for all $x, y \in \mathbb{S}$.

Example 4.2. By the Example 3.2,

	$n = n_1$	$n = n_2$	$n = n_3$	$n = n_4$
$\varpi_A^T(n)$	0.80	0.75	0.55	0.70
$\varpi_A^I(n)$	0.75	0.70	0.62	0.65
$\varpi_A^F(n)$	0.35	0.65	0.80	0.70

Clearly, A is a $(0.45, 0.60)$ NSBS of $\mathbb{S}$.

Theorem 4.3. *The intersection of a family of (α, β) $NSBS^s$ of $\mathbb{S}$ is a (α, β) $NSBS$ of $\mathbb{S}$.*

Proof. Let $\{V_i : i \in I\}$ be a family of (α, β) $NSBS^s$ of $\mathbb{S}$ and $A = \bigcap_{i \in I} V_i$.

Let x and y in $\mathbb{S}$. Now,

$$\begin{aligned} \max\{\varpi_A^T(x \diamond_1 y), \alpha\} &= \inf_{i\in I} \ \max\{\varpi_{V_i}^T(x \diamond_1 y), \alpha\} \\ &\geq \inf_{i\in I} \ \min\{\varpi_{V_i}^T(x), \varpi_{V_i}^T(y), \beta\} \\ &= \min\left\{\inf_{i\in I} \ \varpi_{V_i}^T(x), \inf_{i\in I} \ \varpi_{V_i}^T(y), \beta\right\} \\ &= \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\}. \end{aligned}$$

Similarly, $\max\{\varpi_A^T(x \diamond_2 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\}$ and
$\max\{\varpi_A^T(x \diamond_3 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\}$. Now,

$$\begin{aligned} \max\{\varpi_A^I(x \diamond_1 y), \alpha\} &= \inf_{i\in I} \ \max\{\varpi_{V_i}^I(x \diamond_1 y), \alpha\} \\ &\geq \inf_{i\in I} \ \min\left\{\frac{\varpi_{V_i}^I(x) + \varpi_{V_i}^I(y)}{2}, \beta\right\} \\ &= \min\left\{\frac{\inf_{i\in I} \ \varpi_{V_i}^I(x) + \inf_{i\in I} \ \varpi_{V_i}^I(y)}{2}, \beta\right\} \\ &= \min\left\{\frac{\varpi_A^I(x) + \varpi_A^I(y)}{2}, \beta\right\}. \end{aligned}$$

Similarly, $\max\{\varpi^I_A(x \diamond_2 y),\alpha\} \geq \min\left\{\frac{\varpi^I_A(x)+\varpi^I_A(y)}{2},\beta\right\}$ and $\max\{\varpi^I_A(x \diamond_3 y),\alpha\} \geq \min\left\{\frac{\varpi^I_A(x)+\varpi^I_A(y)}{2},\beta\right\}$. Now,

$$\begin{aligned}\min\{\varpi^F_A(x\diamond_1 y),\alpha\} &= \sup_{i\in I}\ \min\{\varpi^F_{V_i}(x\diamond_1 y),\alpha\}\\ &\leq \sup_{i\in I}\ \max\{\varpi^F_{V_i}(x),\varpi^F_{V_i}(y),\beta\}\\ &= \max\left\{\sup_{i\in I}\ \varpi^F_{V_i}(x),\sup_{i\in I}\ \varpi^F_{V_i}(y),\beta\right\}\\ &= \max\{\varpi^F_A(x),\varpi^F_A(y),\beta\}.\end{aligned}$$

Similarly, $\min\{\varpi^F_A(x\diamond_2 y),\alpha\} \leq \max\{\varpi^F_A(x),\varpi^F_A(y),\beta\}$ and $\min\{\varpi^F_A(x\diamond_3 y),\alpha\} \leq \max\{\varpi^F_A(x),\varpi^F_A(y),\beta\}$. Hence, A is a (α,β) NSBS of $\mathbb{S}$.

Theorem 4.4. *If A and B are any two (α,β) $NSBS^s$ of $\mathbb{S}_1$ and $\mathbb{S}_2$ respectively, then $A\times B$ is a (α,β) NSBS of $\mathbb{S}_1\times\mathbb{S}_2$.*

Proof. Let A and B be two (α,β) $NSBS^s$ of $\mathbb{S}_1$ and $\mathbb{S}_2$ respectively. Let $x_1,x_2\in\mathbb{S}_1$ and $y_1,y_2\in\mathbb{S}_2$. Then (x_1,y_1) and (x_2,y_2) are in $\mathbb{S}_1\times\mathbb{S}_2$. Now

$$\begin{aligned}\max\left\{\varpi^T_{A\times B}[(x_1,y_1)\diamond_1(x_2,y_2)],\alpha\right\} &= \max\left\{\varpi^T_{A\times B}(x_1\diamond_1 x_2,y_1\diamond_1 y_2),\alpha\right\}\\ &= \min\left\{\max\{\varpi^T_A(x_1\diamond_1 x_2),\alpha\},\max\{\varpi^T_B(y_1\diamond_1 y_2),\alpha\}\right\}\\ &\geq \min\left\{\min\{\varpi^T_A(x_1),\varpi^T_A(x_2),\beta\},\min\{\varpi^T_B(y_1),\varpi^T_B(y_2),\beta\}\right\}\\ &= \min\left\{\{\min\{\varpi^T_A(x_1),\varpi^T_B(y_1)\},\min\{\varpi^T_A(x_2),\varpi^T_B(y_2)\}\},\beta\right\}\\ &= \min\left\{\varpi^T_{A\times B}(x_1,y_1),\varpi^T_{A\times B}(x_2,y_2),\beta\right\}.\end{aligned}$$

Also, $\max\left\{\varpi^T_{A\times B}[(x_1,y_1)\diamond_2(x_2,y_2)],\alpha\right\} \geq \min\left\{\varpi^T_{A\times B}(x_1,y_1),\varpi^T_{A\times B}(x_2,y_2),\beta\right\}$ and $\max\left\{\varpi^T_{A\times B}[(x_1,y_1)\diamond_3(x_2,y_2)],\alpha\right\} \geq \min\left\{\varpi^T_{A\times B}(x_1,y_1),\varpi^T_{A\times B}(x_2,y_2),\beta\right\}$.

Now, $\max\left\{\varpi^I_{A\times B}[(x_1,y_1)\diamond_1(x_2,y_2)],\alpha\right\}$

$$\begin{aligned}&= \max\left\{\varpi^I_{A\times B}(x_1\diamond_1 x_2,y_1\diamond_1 y_2),\alpha\right\}\\ &= \min\left\{\frac{1}{2}\left[\max\left\{\varpi^I_A(x_1\diamond_1 x_2),\alpha\right\}+\max\left\{\varpi^I_B(y_1\diamond_1 y_2),\alpha\right\}\right]\right\}\\ &\geq \min\left\{\frac{1}{2}\left[\min\left\{\frac{\varpi^I_A(x_1)+\varpi^I_A(x_2)}{2},\beta\right\}+\min\left\{\frac{\varpi^I_B(y_1)+\varpi^I_B(y_2)}{2},\beta\right\}\right]\right\}\\ &= \min\left\{\frac{1}{2}\left[\frac{\varpi^I_A(x_1)+\varpi^I_B(y_1)}{2}+\frac{\varpi^I_A(x_2)+\varpi^I_B(y_2)}{2}\right],\beta\right\}\\ &= \min\left\{\frac{\varpi^I_{A\times B}(x_1,y_1)+\varpi^I_{A\times B}(x_2,y_2)}{2},\beta\right\}.\end{aligned}$$

Also, $\max\left\{\varpi^I_{A\times B}[(x_1,y_1)\diamond_2(x_2,y_2)],\alpha\right\}\geq\min\left\{\frac{\varpi^I_{A\times B}(x_1,y_1)+\varpi^I_{A\times B}(x_2,y_2)}{2},\beta\right\}$ and

$\max\left\{\varpi^I_{A\times B}[(x_1,y_1)\diamond_3(x_2,y_2)],\alpha\right\}\geq\min\left\{\frac{\varpi^I_{A\times B}(x_1,y_1)+\varpi^I_{A\times B}(x_2,y_2)}{2},\beta\right\}$.

Similarly,

$$\begin{aligned}\min\left\{\varpi^F_{A\times B}[(x_1,y_1)\diamond_1(x_2,y_2)],\alpha\right\}&=\min\left\{\varpi^F_{A\times B}(x_1\diamond_1 x_2,y_1\diamond_1 y_2),\alpha\right\}\\&=\max\left\{\min\{\varpi^F_A(x_1\diamond_1 x_2),\alpha\},\min\{\varpi^F_B(y_1\diamond_1 y_2),\alpha\}\right\}\\&\leq\max\left\{\max\{\varpi^F_A(x_1),\varpi^F_A(x_2),\beta\},\max\{\varpi^F_B(y_1),\varpi^F_B(y_2),\beta\}\right\}\\&=\max\left\{\{\max\{\varpi^F_A(x_1),\varpi^F_B(y_1)\},\max\{\varpi^F_A(x_2),\varpi^F_B(y_2)\}\},\beta\right\}\\&=\max\left\{\varpi^F_{A\times B}(x_1,y_1),\varpi^F_{A\times B}(x_2,y_2),\beta\right\}.\end{aligned}$$

Also, $\min\left\{\varpi^F_{A\times B}[(x_1,y_1)\diamond_2(x_2,y_2)],\alpha\right\}\leq\max\left\{\varpi^F_{A\times B}(x_1,y_1),\varpi^F_{A\times B}(x_2,y_2),\beta\right\}$,

$\min\left\{\varpi^F_{A\times B}[(x_1,y_1)\diamond_3(x_2,y_2)],\alpha\right\}\leq\max\left\{\varpi^F_{A\times B}(x_1,y_1),\varpi^F_{A\times B}(x_2,y_2),\beta\right\}$.

Hence $A\times B$ is a (α,β) NSBS of $\mathbb{S}_1\times\mathbb{S}_2$.

Corollary 4.5. *If $A_1,A_2,...,A_n$ are the family of (α,β) $NSBS^s$ of $\mathbb{S}_1,\mathbb{S}_2,...,\mathbb{S}_n$ respectively, then $A_1\times A_2\times...\times A_n$ is a (α,β) NSBS of $\mathbb{S}_1\times\mathbb{S}_2\times...\times\mathbb{S}_n$.*

Definition 4.6. Let A be a (α,β) neutrosophic subset in $\mathbb{S}$, the strongest (α,β) neutrosophic relation on $\mathbb{S}$, that is a (α,β) neutrosophic relation on A is V given by

$$\left\{\begin{array}{l}\max\{\varpi^T_V(x,y),\alpha\}=\min\{\varpi^T_A(x),\varpi^T_A(y),\beta\}\\\max\{\varpi^I_V(x,y),\alpha\}=\min\{\varpi^I_A(x),\varpi^I_A(y),\beta\}\\\min\{\varpi^F_V(x,y),\alpha\}=\max\{\varpi^F_A(x),\varpi^F_A(y),\beta\}\end{array}\right\}.$$

Theorem 4.7. *Let A be a (α,β) NSBS of $\mathbb{S}$ and V be the strongest (α,β) neutrosophic relation of $\mathbb{S}$. Then A is a (α,β) NSBS of $\mathbb{S}$ if and only if V is a (α,β) NSBS of $\mathbb{S}\times\mathbb{S}$.*

Theorem 4.8. *Let $(\mathbb{S}_1,\boxplus_1,\boxplus_2,\boxplus_3)$ and $(\mathbb{S}_2,\boxdot_1,\boxdot_2,\boxdot_3)$ be any two bisemirings. The homomorphic image of (α,β) NSBS of $\mathbb{S}_1$ is a (α,β) NSBS of $\mathbb{S}_2$.*

Proof. Let $\Delta:\mathbb{S}_1\to\mathbb{S}_2$ be any homomorphism. Then $\Delta(x\boxplus_1 y)=\Delta(x)\boxdot_1\Delta(y),\Delta(x\boxplus_2 y)=\Delta(x)\boxdot_2\Delta(y)$ and $\Delta(x\boxplus_3 y)=\Delta(x)\boxdot_3\Delta(y)$ for all $x,y\in\mathbb{S}_1$. Let $V=\Delta(A)$, A is any (α,β) NSBS of $\mathbb{S}_1$. Let $\Delta(x),\Delta(y)\in\mathbb{S}_2$. Let $x\in\Delta^{-1}(\Delta(x))$ and $y\in\Delta^{-1}(\Delta(y))$ be such that $\varpi^T_A(x)=\sup\limits_{z\in\Delta^{-1}(\Delta(x))}\varpi^T_A(z)$ and $\varpi^T_A(y)=\sup\limits_{z\in\Delta^{-1}(\Delta(y))}\varpi^T_A(z)$. Now,

$$\begin{aligned}\max\left[\varpi^T_V(\Delta(x)\boxdot_1\Delta(y))\ ,\alpha\right]&=\max\left[\sup_{z'\in\Delta^{-1}(\Delta(x)\boxdot_1\Delta(y))}\varpi^T_A(z')\ ,\alpha\right]\\&=\max\left[\sup_{z'\in\Delta^{-1}(\Delta(x\boxplus_1 y)}\varpi^T_A(z')\ ,\alpha\right]\end{aligned}$$

$$= \max\left[\varpi_A^T(x \boxplus_1 y), \alpha\right]$$
$$\geq \min\left\{\varpi_A^T(x), \varpi_A^T(y), \beta\right\}$$
$$= \min\left\{\varpi_V^T\Delta(x), \varpi_V^T\Delta(y), \beta\right\}.$$

Thus, $\max\left[\varpi_V^T(\Delta(x) \boxdot_1 \Delta(y))\ , \alpha\right] \geq \min\left\{\varpi_V^T\Delta(x), \varpi_V^T\Delta(y), \beta\right\}$.
Similarly, $\max\left[\varpi_V^T(\Delta(x) \boxdot_2 \Delta(y))\ , \alpha\right] \geq \min\left\{\varpi_V^T\Delta(x), \varpi_V^T\Delta(y), \beta\right\}$ and
$\max\left[\varpi_V^T(\Delta(x) \boxdot_3 \Delta(y))\ , \alpha\right] \geq \min\left\{\varpi_V^T\Delta(x), \varpi_V^T\Delta(y), \beta\right\}$.
Let $\Delta(x), \Delta(y) \in \mathbb{S}_2$. Let $x \in \Delta^{-1}(\Delta(x))$ and $y \in \Delta^{-1}(\Delta(y))$ be such that $\varpi_A^I(x) = \sup_{z\in\Delta^{-1}(\Delta(x))} \varpi_A^I(z)$ and $\varpi_A^I(y) = \sup_{z\in\Delta^{-1}(\Delta(y))} \varpi_A^I(z)$. Now,

$$\max\left[\varpi_V^I(\Delta(x) \boxdot_1 \Delta(y))\ , \alpha\right] = \max\left[\sup_{z^{'}\in\Delta^{-1}(\Delta(x)\boxdot_1\Delta(y))} \varpi_A^I(z^{'})\ , \alpha\right]$$
$$= \max\left[\sup_{z^{'}\in\Delta^{-1}(\Delta(x\boxplus_1 y)} \varpi_A^I(z^{'})\ , \alpha\right]$$
$$= \max\left[\varpi_A^I(x \boxplus_1 y), \alpha\right]$$
$$\geq \min\left\{\frac{\varpi_A^I(x) + \varpi_A^I(y)}{2}, \beta\right\}$$
$$= \min\left\{\frac{\varpi_V^I\Delta(x) + \varpi_V^I\Delta(y)}{2}, \beta\right\}$$

Thus, $\max\left[\varpi_V^I(\Delta(x) \boxdot_1 \Delta(y))\ , \alpha\right] \geq \min\left\{\frac{\varpi_V^I\Delta(x)+\varpi_V^I\Delta(y)}{2}, \beta\right\}$.
Similarly, $\max\left[\varpi_V^I(\Delta(x) \boxdot_2 \Delta(y))\ , \alpha\right] \geq \min\left\{\frac{\varpi_V^I\Delta(x)+\varpi_V^I\Delta(y)}{2}, \beta\right\}$ and
$\max\left[\varpi_V^I(\Delta(x) \boxdot_3 \Delta(y))\ , \alpha\right] \geq \min\left\{\frac{\varpi_V^I\Delta(x)+\varpi_V^I\Delta(y)}{2}, \beta\right\}$.
Let $x \in \Delta^{-1}(\Delta(x))$ and $y \in \Delta^{-1}(\Delta(y))$ be such that $\varpi_A^F(x) = \inf_{z\in\Delta^{-1}(\Delta(x))} \varpi_A^F(z)$ and $\varpi_A^F(y) = \inf_{z\in\Delta^{-1}(\Delta(y))} \varpi_A^F(z)$. Now,

$$\min\left[\varpi_V^F(\Delta(x) \boxdot_1 \Delta(y))\ , \alpha\right] = \min\left[\inf_{z^{'}\in\Delta^{-1}(\Delta(x)\boxdot_1\Delta(y))} \varpi_A^F(z^{'})\ , \alpha\right]$$
$$= \min\left[\inf_{z^{'}\in\Delta^{-1}(\Delta(x\boxplus_1 y)} \varpi_A^F(z^{'})\ , \alpha\right]$$
$$= \min\left[\varpi_A^F(x \boxplus_1 y), \alpha\right]$$
$$\leq \max\left\{\varpi_A^F(x), \varpi_A^F(y), \beta\right\}$$
$$= \max\left\{\varpi_V^F\Delta(x), \varpi_V^F\Delta(y), \beta\right\}.$$

Thus, $\min\left[\varpi_V^F(\Delta(x) \boxdot_1 \Delta(y))\ ,\alpha\right] \leq \max\left\{\varpi_V^F\Delta(x), \varpi_V^F\Delta(y), \beta\right\}$.
Similarly, $\min\left[\varpi_V^F(\Delta(x) \boxdot_2 \Delta(y))\ ,\alpha\right] \leq \max\left\{\varpi_V^F\Delta(x), \varpi_V^F\Delta(y), \beta\right\}$ and
$\min\left[\varpi_V^F(\Delta(x) \boxdot_3 \Delta(y))\ ,\alpha\right] \leq \max\left\{\varpi_V^F\Delta(x), \varpi_V^F\Delta(y), \beta\right\}$. Hence V is a (α,β) NSBS of $\mathbb{S}_2$.

Theorem 4.9. *Let $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ and $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ be any two bisemirings. The homomorphic preimage of (α,β) NSBS of $\mathbb{S}_2$ is a (α,β) NSBS of $\mathbb{S}_1$.*

Proof. Let $\Delta : \mathbb{S}_1 \to \mathbb{S}_2$ be any homomrphism. Then $\Delta(x \boxplus_1 y) = \Delta(x) \boxdot_1 \Delta(y), \Delta(x \boxplus_2 y) = \Delta(x) \boxdot_2 \Delta(y)$ and $\Delta(x \boxplus_3 y) = \Delta(x) \boxdot_3 \Delta(y)$ for all $x, y \in \mathbb{S}_1$. Let $V = \Delta(A)$, where V is any (α,β) NSBS of $\mathbb{S}_2$. Let $x, y \in \mathbb{S}_1$. Then $\max\{\varpi_A^T(x \boxplus_1 y), \alpha\} = \max\{\varpi_V^T(\Delta(x \boxplus_1 y)), \alpha\} = \max\{\varpi_V^T(\Delta(x) \boxdot_1 \Delta(y)), \alpha\} \geq \min\{\varpi_V^T\Delta(x), \varpi_V^T\Delta(y), \beta\} = \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\}$. Thus, $\max\{\varpi_A^T(x \boxplus_1 y), \alpha\} \geq \min\{\varpi_A^T(x), \varpi_A^T(y), \beta\}$. Now, $\max\{\varpi_A^I(x \boxplus_1 y), \alpha\} = \max\{\varpi_V^I(\Delta(x \boxplus_1 y)), \alpha\} = \max\{\varpi_V^I(\Delta(x) \boxdot_1 \Delta(y)), \alpha\} \geq \min\{\varpi_V^I\Delta(x), \varpi_V^I\Delta(y), \beta\} = \min\{\varpi_A^I(x), \varpi_A^I(y), \beta\}$. Thus, $\max\{\varpi_A^I(x \boxplus_1 y), \alpha\} \geq \min\{\varpi_A^I(x), \varpi_A^I(y), \beta\}$. Now, $\min\{\varpi_A^F(x \boxplus_1 y), \alpha\} = \min\{\varpi_V^F(\Delta(x \boxplus_1 y)), \alpha\} = \min\{\varpi_V^F(\Delta(x) \boxdot_1 \Delta(y)), \alpha\} \leq \max\{\varpi_V^F\Delta(x), \varpi_V^F\Delta(y), \beta\} = \max\{\varpi_A^F(x), \varpi_A^F(y), \beta\}$. Thus, $\min\{\varpi_A^F(x \boxplus_1 y), \alpha\} \leq \max\{\varpi_A^F(x), \varpi_A^F(y), \beta\}$. Similarly to prove other two operations, hence A is a (α,β) NSBS of $\mathbb{S}_1$.

5. (α,β) neutrosophic Normal Subbisemiring

In this section, we interact the theory for (α,β)- neutrosophic normal subbisemiring. Here NNSBS stands for neutrosophic normal subbisemiring.

Definition 5.1. Let A be any neutrosophic subset of $\mathbb{S}$ is said to be a NNSBS of $\mathbb{S}$ if it satisfies the following conditions:

$$\left\{\begin{matrix} \varpi_A^T(x \diamond_1 y) = \varpi_A^T(y \diamond_1 x) \\ \varpi_A^T(x \diamond_2 y) = \varpi_A^T(y \diamond_2 x) \\ \varpi_A^T(x \diamond_3 y) = \varpi_A^T(y \diamond_3 x) \end{matrix}\right\} \quad \left\{\begin{matrix} \varpi_A^I(x \diamond_1 y) = \varpi_A^I(y \diamond_1 x) \\ \text{OR} \\ \varpi_A^I(x \diamond_2 y) = \varpi_A^I(y \diamond_2 x) \\ \text{OR} \\ \varpi_A^I(x \diamond_3 y) = \varpi_A^I(y \diamond_3 x) \end{matrix}\right\}$$

$$\left\{\begin{matrix} \varpi_A^F(x \diamond_1 y) = \varpi_A^F(y \diamond_1 x) \\ \varpi_A^F(x \diamond_2 y) = \varpi_A^F(y \diamond_2 x) \\ \varpi_A^F(x \diamond_3 y) = \varpi_A^F(y \diamond_3 x) \end{matrix}\right\}$$

for all $x, y \in \mathbb{S}$.

Theorem 5.2. *(i) The intersection of a family of NNSBS s of $\mathbb{S}$ is a $NNSBS^s$ of $\mathbb{S}$.*
(ii) The intersection of a family of (α,β) NNSBS of $\mathbb{S}$ is a (α,β) NNSBS s of $\mathbb{S}$.

Proof. Proof follows from Theorem 3.3 and Theorem 4.3.

Theorem 5.3. *(i) If $A_1, A_2, ..., A_n$ are the family of $NNSBS^s$ of $\mathbb{S}_1, \mathbb{S}_2, ..., \mathbb{S}_n$ respectively, then $A_1 \times A_2 \times ... \times A_n$ is a NNSBS of $\mathbb{S}_1 \times \mathbb{S}_2 \times ... \times \mathbb{S}_n$.*
(ii) If $A_1, A_2, ..., A_n$ are the family of (α, β) $NNSBS^s$ of $\mathbb{S}_1, \mathbb{S}_2, ..., \mathbb{S}_n$ respectively, then $A_1 \times A_2 \times ... \times A_n$ is a (α, β) NNSBS of $\mathbb{S}_1 \times \mathbb{S}_2 \times ... \times \mathbb{S}_n$.

Proof. Proof follows from Theorem 3.4 and Theorem 4.4.

Theorem 5.4. *(i) Let A be any NNSBS of $\mathbb{S}$ and V be the strongest neutrosophic relation of $\mathbb{S}$. Then A is a NNSBS of $\mathbb{S}$ if and only if V is a NNSBS of $\mathbb{S} \times \mathbb{S}$.*
(ii) Let A be any (α, β) NNSBS of $\mathbb{S}$ and V be the strongest (α, β) neutrosophic relation of $\mathbb{S}$. Then A is a (α, β) NNSBS of $\mathbb{S}$ if and only if V is a (α, β) NNSBS of $\mathbb{S} \times \mathbb{S}$.

Proof. Proof follows from Theorem 3.7.

Theorem 5.5. *Let $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ and $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ be any two bisemirings.*
(i) The homomorphic image of any NNSBS of $\mathbb{S}_1$ is a NNSBS of $\mathbb{S}_2$.
(ii) The homomorphic image of any (α, β) NNSBS of $\mathbb{S}_1$ is a (α, β) NNSBS of $\mathbb{S}_2$.

Proof. Proof follows from Theorem 3.12 and Theorem 4.8.

Theorem 5.6. *Let $(\mathbb{S}_1, \boxplus_1, \boxplus_2, \boxplus_3)$ and $(\mathbb{S}_2, \boxdot_1, \boxdot_2, \boxdot_3)$ be any two bisemirings.*
(i) The homomorphic preimage of any NNSBS of $\mathbb{S}_2$ is a NNSBS of $\mathbb{S}_1$.
(ii) The homomorphic preimage of any (α, β) NNSBS of $\mathbb{S}_2$ is a (α, β) NNSBS of $\mathbb{S}_1$.

Proof. Proof follows from Theorem 3.13 and Theorem 4.9.

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