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# Neutrosophic Fuzzy Pairwise Local Function and Its Application

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**Abstract:** In this paper we introduce the notion of neutrosophic fuzzy bitopological ideals. The concept of neutrosophic fuzzy pairwise local function is also introduced here by utilizing the neutrosophic quasi-coincident neighbourhood (i.e. $Nq-nbd$) structure in a neutrosophic fuzzy topological space. As well as, the concepts of neutrosophic fuzzy bitopologies and several relations between different neutrosophic fuzzy bitopological ideals have been explored.

**Keywords:** Neutrosophic Fuzzy Bitopological Space; Neutrosophic Fuzzy Ideals; Neutrosophic Fuzzy Pairwise Local Function.

**1. Introduction:** The concept of neutrosophic fuzzy sets and neutrosophic fuzzy set operations was first introduced by Florentin [17]. Subsequently, Salama defined the notion of neutrosophic fuzzy topology [1]. Since then various aspects of bitopological spaces were investigated and carried out in neutrosophic fuzzy by several authors. The notions of neutrosophic fuzzy ideal and neutrosophic fuzzy local function were introduced and studied in [2-8]. Salama was the first researcher who initiated the study of neutrosophic fuzzy bitopological spaces where a neutrosophic fuzzy set equipped with two neutrosophic fuzzy topologies is called a neutrosophic fuzzy bitopological space. Concepts of the neutrosophic fuzzy ideals and the neutrosophic fuzzy local function were introduced and studied in [9-13]. The purpose of this paper is to suggest the neutrosophic fuzzy ideals in neutrosophic fuzzy bitopological spaces. The concept of neutrosophic fuzzy pairwise local function is also introduced here by utilizing the Nq-neighborhood structure [20], for more details of these concepts and other concepts, the readers can return to [14-19, 20,21].

**2. Preliminaries**

Throughout this paper, by (X, $\tau_1, \tau_2$ ) we mean a neutrosophic fuzzy bitopological space ($nfbts$ in short) in the sense of Salama [6]. A neutrosophic fuzzy point in X with support $x \in X$ and the value $\varepsilon_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ $(0 < \varepsilon \leq 1)$ is denoted by $x\varepsilon =< \varepsilon_1, \varepsilon_2, \varepsilon_3 >$,[9]. A neutrosophic fuzzy point $x\varepsilon$ is said to be contained in a neutrosophic fuzzy set $\mu =< \mu_1, \mu_2, \mu_3 > \in I^X$ iff $\varepsilon \leq \mu$ and this will be denoted by $x\varepsilon \; in \; \mu$ [ 9 ] . For a neutrosophic fuzzy set $\mu$ in a $nfbts\,(X, \tau_1, \tau_2)$, $\tau_i - Ncl(\mu)$, $\tau_i - NInt(\mu)$ , $i \in \{1,2\}$, and $\mu^c$ will respectively denote closure, interior and complement of $\mu$ The constant neutrosophic fuzzy sets that taking the values $0 \; and \; 1$ on $X$ are denoted by $0_N, 1_N$ respectively. A neutrosophic fuzzy set $\mu$ in $nfts$ is said to be neutrosophic quasi-coincident [9] with a neutrosophic fuzzy set $\eta =< \eta_1, \eta_2, \eta_3 >$, denoted by $\mu \, Nq \, \eta$, if there exists $x \; in \; X$ such that $\mu(x) + \eta(x) > 1$. A neutrosophic fuzzy set $v =< v_1, v_2, v_3 >$ in a $nfts\,(X, \tau)$ is called a $Nq - nbd$ [1,9] of a neutrosophic fuzzy point $x\varepsilon$ iff there exists a neutrosophic fuzzy open set μ such that $x\varepsilon \; Nq \; \mu \subseteq$ v we will denoted the set of all $Nq - nbd$ of $x\varepsilon$ in $(X, \tau)$ by $N\,(X, \tau)$. A nonempty collection of neutrosophic fuzzy sets $L$ of a set $X$ may be called neutrosophic fuzzy ideal [16,8,13] on $X$ iff

(i) $\mu \; in \; L$ and $\eta \subseteq \mu \Rightarrow \eta \; in \; L$ (heredity),

(ii) $\mu \; in \; L \; and \; \eta \; in \; L \Rightarrow \mu \vee \eta \; in \; L$ (Finite additivity).

The neutrosophic fuzzy local function [8] $\mu^* \in (L, \tau)$ of a neutrosophic fuzzy set $\mu$ may be the union of all neutrosophic fuzzy points $x\varepsilon$ such that if $v \; in \; N\,(x\varepsilon)$ and $\rho =< \rho_1, \rho_2, \rho_3 >$ in L then there is at least one $r \; in \; X$ for which $v(r) + \mu(r) - 1 > \rho(r)$. For a $nfts\,(X, \tau)$ with neutrosophic fuzzy ideal $L \; ncl^*(\mu) = \mu \vee \mu^{*^*}$ [8,16] for any neutrosophic fuzzy set $\mu \; of \; X$ and $\tau^*(L)$ be the neutrosophic fuzzy topology generated by $ncl^*$ [16].

**3. Neutrosophic Fuzzy Pairwise Local Functions.**

**Definition 3.1.** A neutrosophic fuzzy set $\mu =< \mu_1, \mu_2, \mu_3 >$ in a $nfbts\ (X, \tau_i), i\epsilon\{1,2\}$ is called neutrosophic Pairwise Quasi-coincident with a neutrosophic fuzzy set $\eta =< \eta_1, \eta_2, \eta_3 >$ and is denoted by $P(\mu\ Nq\ \eta)$, if there exists $x\epsilon X$ such that, either type 1 conditions satisfy, $\mu_1(x) + \eta_1(x) > 1$, $\mu_2(x) + \eta_2(x) > 1$, $\mu_3(x) + \eta_3(x) < 1$. Or type 2 conditions satisfied, $\mu_1(x) + \eta_1(x) > 1$, $\mu_2(x) + \eta_2(x) < 1$, $\mu_3(x) + \eta_3(x) < 1$.

It is obviously that for any two neutrosophic fuzzy sets $\mu$ and $\eta$, $NP(\mu\,Nq\,\eta)$ is identical to $NP(\eta\ Nq\ \mu)$.

**Definition 3. 2.** A neutrosophic fuzzy set $\mu =< \mu_1, \mu_2, \mu_3 >$ in a $nfbts\ (X, \tau_i), i\epsilon\{1,2\}$ is called neutrosophic pairwise quasi-neighborhood of the point $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ if and only if there exists a neutrosophic fuzzy $\tau_i$-open, $i\epsilon\{1,2\}$ set $\rho =< \rho_1, \rho_2, \rho_3 >$ such that $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}\ Nq\ \rho \subseteq \mu$. We will denote the set of all pairwise $Nq - nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}\ in\ (X, \tau_i), i\epsilon\{1,2\}$ by $P(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i), i\epsilon\{1,2\}$.

**Definition 3.3.** Let $(X, \tau_i), i\epsilon\{1,2\}$ be a $nfbts$ with neutrosophic fuzzy ideal L on X, and $\mu =< \mu_1, \mu_2, \mu_3 >$ in $1_N$. Then the neutrosophic fuzzy pairwise local function $NP\mu^*(L, \tau_i), i\epsilon\{1,2\}$ of $\mu =< \mu_1, \mu_2, \mu_3 >$ is the union of all neutrosophic fuzzy points $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ such that for $\rho =< \rho_1, \rho_2, \rho_3 >$ in $NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i), i\epsilon\{1,2\}$ and $\lambda$ in L then there is at least one r in X for which $\rho_1(r) + \mu_1(r) - 1 > \lambda(r)$, $\rho_2(r) + \mu_2(r) - 1 > \lambda(r)$, $\rho_3(r) + \mu_3(r) - 1 < \lambda(r)$ or $\rho_1(r) + \mu_1(r) - 1 > \lambda(r)$, $\rho_2(r) + \mu_2(r) - 1 < \lambda(r)$, $\rho_3(r) + \mu_3(r) - 1 < \lambda(r)$ where $NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i), i\epsilon\{1,2\}$ is the set of all $Nq - nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$. Therefore, any $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L, \tau_i), i\epsilon\{1, 2\}$(for any $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin \mu$ (any neutrosophic fuzzy set) implies hereafter, $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ maybe not contained in the neutrosophic fuzzy set $\mu$, i.e. $x < \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >> NP\mu^*(x)$, $\mu =< \mu_1, \mu_2, \mu_3 > (x)$ implies there is at least one $\rho$ in $NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i)$ such that for every r in X, $\rho_1(r) + \mu_1(r) - 1 \leq \lambda(r)$ , $\rho_2(r) + \mu_2(r) - 1 \leq \lambda(r)$, $\rho_3(r) + \mu_3(r) - 1 > \lambda(r)$, for some $\lambda$ in L. We will occasionally write $NP\mu^*$ or $NP\mu^*(L)$ for $NP\mu^*(L, \tau_i)$.We define $P^*$-neutrosophic fuzzy closure operator, denoted by $Npcl^*$ for fuzzy bitopology $\tau^*_i(L)$finer than $\tau_i$ as follows: $Npcl^*(\mu) = \mu \vee NP\mu^*$ for every fuzzy set $\mu =< \mu_1, \mu_2, \mu_3 >$ on X. When there is no ambiguity, we will simply write the symbols $NP\mu^*$and $\tau^*_i$ for $NP\mu^*(L, \tau_i)$and $\tau^{i^*}(L)$, respectively.

**Definition 3.4.** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a $nfbts$ with neutrosophic fuzzy ideal L on X, a neutrtosophic fuzzy pairwise local function $NP\mu^*(L, \tau_1 \vee \tau_2), i\epsilon\{1,2\}$ of $\mu =< \mu_1, \mu_2, \mu_3 >$ in $1_N$ is the union of all neutrosophic fuzzy points $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ such that for $\rho =< \rho_1 , \rho_2, \rho_3 >$ in $NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i)$ and $\lambda$ in L. Then there is at least one $r$ *in* $X$ may be for two types which:

type1, $\rho_1(r) + \mu_1(r) - 1 > \lambda(r),\ \rho_2(r) + \mu_2(r) - 1 > \lambda(r),\ \rho_3(r) + \mu_3(r) - 1 < \lambda(r)$,

type 2, $\rho_1(r) + \mu_1(r) - 1 < \lambda(r),\ \rho_2(r) + \mu_2(r) - 1 < \lambda(r),\ \rho_3(r) + \mu_3(r) - 1 > \lambda(r)$,

where $NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}, \tau_i)$ is the set of all $Nq - nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_1 \vee \tau_2$(where $\tau_1 \vee \tau_2$ is the neutrosophic fuzzy topology generated by $\tau_1, \tau_2$ .

**Example 3.1.** One may easily noticed

i- Consider $L = \{0_N\}$, then $NP\mu^*(L, \tau_i) = \tau_i - Ncl\big(\mu =< \mu_1, \mu_2 , \mu_3 >\big)$, for any $\mu =< \mu_1, \mu_2 , \mu_3 > \in 1_N, i\{1,2\}$.

ii- Consider $L = \{1_N\}$, then $NP\mu^*(L, \tau_i) = 0_N$, for any $\mu =< \mu_1, \mu_2 , \mu_3 > \in 1_N, i\{1,2\}$.

**Note 3.1.** In a $nfbts$ $(X, \tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, we will denote by $\sigma - Ncl(\mu =< \mu_1, \mu_2 , \mu_3 >)$ for the neutrosophic closure, and $\sigma - Nint(\mu)$ for the neutrosophic interior of a neutrosophic fuzzy subset $\mu =< \mu_1, \mu_2 , \mu_3 >$ in $1_N$ with respect to the neutrosophic fuzzy topology $\sigma = \tau_1 \vee \tau_2$.

The following theorems give some general properties of neutrosophic fuzzy pairwise-local function.

**Theorem 3.1.** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a $nfbts$ with neutrosophic fuzzy ideal L on $X, \mu =< \mu_1, \mu_2 , \mu_3 >$ $, \eta =< \eta_1, \eta_2, \eta_3 >$ in $1_N$. Then we have:

i- $NP\mu^*(L, \sigma) \subseteq NP\mu^*(L, \tau_i); i\epsilon\{1,2\}$.

ii- If $\mu =< \mu_1, \mu_2 , \mu_3 > \subseteq \eta =< \eta_1, \eta_2, \eta_3 >$ then $NP\mu^*(L, \sigma) \subseteq NP\eta^*(L, \tau_i); i\epsilon\{1,2\}$.

iii- $NP\mu^*(L, \sigma) \subseteq \sigma - Ncl(\mu) \subseteq \tau_i - Ncl(\mu)$.

iv- $NP\mu^{**}(L, \sigma) \subseteq NP\mu^*(L, \tau_i); i\epsilon\{1,2\}$.

**Proof**

i- Let $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L, \tau_i)$ i.e.$\varepsilon =< \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >> NP\mu^*(x)$so $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ is not contained in $NP\mu^*$, this implies there is at least one $\rho =< \rho_1 , \rho_2, \rho_3 > \in NPN(x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>})$ in $\tau_i$ such that for every r in X,

type 1, $\rho_1(r) + \mu_1(r) - 1 \leq \lambda(r)\ , \rho_2(r) + \mu_2(r) - 1 \leq \lambda(r)\ ,\ \rho_3(r) + \mu_3(r) - 1 > \lambda(r)$,

type 2, $\rho_1(r) + \mu_1(r) - 1 \leq \lambda(r)\ , \rho_2(r) + \mu_2(r) - 1 > \lambda(r)\ ,\ \rho_3(r) + \mu_3(r) - 1 > \lambda(r)$, for some $\lambda$ in $L$ . Hence $\rho$ in $NPN(x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}, \sigma)$ and so $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \notin NP\mu^*(L, \sigma)$ . Therefore $NP\mu^*(L, \sigma) \subseteq NP\mu^*(L, \tau_i); i\epsilon\{1,2\}$.

ii- Let $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in NP\eta^*(L, \tau_i); i\epsilon\{1,2\}$, . This implies there is at least one $Nq - nbd$ $\rho =< \rho_1, \rho_2, \rho_3 >$ in $NPN(x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}, \tau_i)$ such that every $r\epsilon X$, $\rho_1(r) + \eta_1(r) - 1 > \lambda(r)$, $\rho_2(r) + \eta_2(r) - 1 > \lambda(r)$, $\rho_3(r) + \eta_3(r) - 1 < \lambda(r)$ , or $\rho_1(r) + \eta_1(r) - 1 > \lambda(r)$ , $\rho_2(r) + \eta_2(r) - 1 < \lambda(r)$, $\rho_3(r) + \eta_3(r) - 1 < \lambda(r)$ , $\lambda$ in $L$. Hence $\rho =< \rho_1, \rho_2, \rho_3 >$ in $NPN(x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}, \sigma)$. Since $\mu =< \mu_1, \mu_2, \mu_3 > \subseteq \eta =< \eta_1, \eta_2, \eta_3 >$ , by the heredity property $\rho_1(r) + \mu_1(r) - 1 > \lambda(r)$, $\rho_2(r) + \mu_2(r) - 1 > \lambda(r)$, $\rho_3(r) + \mu_3(r) - 1 < \lambda(r)$ or $\rho_1(r) + \mu_1(r) - 1 > \lambda(r)$ , $\rho_2(r) + \mu_2(r) - 1 < \lambda(r)$ , $\rho_3(r) + \mu_3(r) - 1 < \lambda(r)$ . Therefore $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in NP\mu^*(L, \sigma)$.

iii- ,(iv)Obvious .

**Theorem 3.2.** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a $nfbts$ with neutrosophic fuzzy ideal $L$ $on$ $X$, $\mu =< \mu_1, \mu_2, \mu_3 >$ , $\eta =< \eta_1, \eta_2, \eta_3 >$ are two neutrosophic fuzzy sets, if $\tau_1 \subseteq \tau_2$, then

i- $NP\mu^*(L, \tau_2) \subseteq NP\mu^*(L, \tau_1)$, for every neutrosophic fuzzy set $\mu$ ,

ii- $\tau_1{}^* \subseteq \tau_2{}^*$.

**Proof.** i- Since every $Nq - nbd$ in $\tau_1$ of any neutrosophic fuzzy point $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ maybe also $Nq - nbd$ in $\tau_2$. Therefore, $NP\mu^*(L, \tau_2) \subseteq NP\mu^*(L, \tau_1)$ as there may be other $Nq - nbd$ in $\tau_2$ of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ where is the condition for $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ to be in $NP\mu^*(L, \tau_2)$ may be not hold true, although $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ in $NP\mu^*(L, \tau_1)$ .

ii- Clearly ,$\tau_1{}^* \subseteq \tau_2{}^*$ as $NP\mu^*(L, \tau_2) \subseteq NP\mu^*(L, \tau_1)$ .

**Theorem 3.3.** Let $(X, \tau_i), i\epsilon\{1,2\}$ be a nfbts and $L, J$ be two neutrosophic fuzzy ideals with neutrosophic fuzzy ideal $L$ on $X$. Then for any neutrosophic fuzzy sets $\mu =< \mu_1, \mu_2, \mu_3 >$ and $\rho =< \rho_1, \rho_2, \rho_3 >$. The following statements are satisfied:

i- $\mu =< \mu_1, \mu_2, \mu_3 > \subseteq \rho =< \rho_1, \rho_2, \rho_3 > \Longrightarrow NP\mu^*(L, \tau_i) \subseteq NP\rho^*(L, \tau_i), i\epsilon\{1,2\}$.

ii- $L \subseteq J \Longrightarrow NP\mu^*(L, \tau_i) \subseteq NP\mu^*(J, \tau_i), i\epsilon\{1,2\}$.

iii- $NP\mu^* = \tau_i - Ncl(NP\mu^*) \subseteq \tau_i - Ncl(\mu), i\epsilon\{1,2\}$.

iv- $NP\mu^{**}(L, \tau_i) \subseteq NP\mu^*(L, \tau_i), i\epsilon\{1,2\}$.

v- $NP(\mu \cup \rho)^*(L, \tau_i) = NP\mu^*(L, \tau_i) \cup \rho^*(L, \tau_i)$.

vi- $\rho =< \rho_1 , \rho_2, \rho_3 >$ in $L \Longrightarrow NP(\mu \cup \rho)^*(L,\tau_i) = NP\mu^*(L,\tau_i)$.

**Proof.**

i- Since $\mu \subseteq \rho$ implies $\mu \leq \rho$ for every x in X , therefore by Definition 3.1 $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $NP\mu^*(L,\tau_i)$ implies $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $NP\rho^*(L,\tau_i)$, which complete the proof of (i).

ii- Cleary, $L \subseteq J \Longrightarrow NP\mu^*(L,\tau_i) \subseteq NP\mu^*(J,\tau_i), i\epsilon\{1,2\}$ as there may be other neutrosophic fuzzy sets which belong toJso that for a neutrosophic fuzzy point $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $NP\mu^*(J,\tau_i)$ but $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ may be not contained $NP\mu^*(L,\tau_i), i\epsilon\{1,2\}$.

iii- Since $\{0_N\} \subseteq L$ for any neutrosophic fuzzy ideal L on X, Therefore by (ii) and Example 3.1, $NP\mu^*(L,\tau_i) \subseteq NP\mu^*(\{0_N\},\tau_i) = \tau_i - Ncl(\mu =< \mu_1, \mu_2 , \mu_3 >)$ for any neutrosophic fuzzy set $\mu =< \mu_1, \mu_2 , \mu_3 >$ of X. Suppose, $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i - Ncl(\mu =< \mu_1, \mu_2 , \mu_3 >^*)$, so there is at least one $r\epsilon X$ for which $NP\mu_1{}^* + \nu_1(r) - 1 > \lambda(r)$, $NP\mu_2{}^* + \nu_2(r) - 1 > \lambda(r)$, $NP\mu_3{}^* + \nu_3(r) - 1 < \lambda(r)$ or $NP\mu_1{}^* + \nu_1(r) - 1 > \lambda(r)$, $NP\mu_2{}^* + \nu_2(r) - 1 < \lambda(r)$, $NP\mu_3{}^* + \nu_3(r) - 1 < \lambda(r)$ , for each $Nq - nbd\ \nu =< \nu_1, \nu_2, \nu_3 >$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$. Hence $NP\mu^* \neq \{0_N\}$. Let $S = NP\mu^*(r)$. Cleary $r_{t=<t_1,t_2,t_3>}$ in $NP\mu^*(L,\tau_i)$ and $t_1 + \nu_1(r) > 1,\ t_2 + \nu_2(r) > 1,\ t_3 + \nu_3(r) < 1$ or $t_1 + \nu_1(r) > 1,\ t_2 + \nu_2(r) < 1,\ t_3 + \nu_3(r) < 1$ so there is $\nu =< \nu_1, \nu_2, \nu_3 >$ is also $Nq - nbd$ of $r_{t=<t_1,t_2,t_3>}$ in $\tau_i$. Now $r_{t=<t_1,t_2,t_3>}$ in $NP\mu^*(L,\tau_i)$ , so there may be at least one $r^/$ in X for which $\eta_1(r^/) + \mu_1(r^/) - 1 > \lambda(r^/)$, $\eta_2(r^/) + \mu_2(r^/) - 1 > \lambda(r^/)$, $\eta_3(r^/) + \mu_3(r^/) - 1 < \lambda(r^/)$ or $\mu_1(r^/) - 1 > \lambda(r^/)$, $\eta_2(r^/) + \mu_2(r^/) - 1 < \lambda(r^/)$, $\eta_3(r^/) + \mu_3(r^/) - 1 < \lambda(r^/)$ for each $Nq - nbd\ \eta$ of $r_{t=<t_1,t_2,t_3>}$ and $\lambda$ in L.This may be true for $\nu =< \nu_1, \nu_2, \nu_3 >$ so there is at least one $r^{//}$ in X such that $\nu_1(r^{//}) + \mu_1(r^{//}) - 1 > \lambda(r^{//})$, $\nu_2(r^{//}) + \mu_2(r^{//}) - 1 > \lambda(r^{//})$, $\nu_3(r^{//}) + \mu_3(r^{//}) - 1 < \lambda(r^{//})$ or $\nu_1(r^{//}) + \mu_1(r^{//}) - 1 > \lambda(r^{//})$, $\nu_2(r^{//}) + \mu_2(r^{//}) - 1 < \lambda(r^{//})$, $\nu_3(r^{//}) + \mu_3(r^{//}) - 1 < \lambda(r^{//})$ for each $\lambda$ in L. Since $\nu =< \nu_1, \nu_2, \nu_3 >$ may be an arbitrary $Nq - nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ therefore $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $NP\mu^*(L,\tau_i)$ hence $NP\mu^* = \tau_i - Ncl(NP\mu^*) \subseteq \tau_i - Ncl(\mu), i\epsilon\{1,2\}$,

iv- Clear

v- Suppose, $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L,\tau_i) \cup \rho^*(L,\tau_i)$ i.e. $\varepsilon = < \mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3 >, \varepsilon > (NP\mu^* \vee NP\rho^*)(x) = \max\{NP\mu^*(x), NP\rho^*\}$. So $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ is not contained in both $NP\mu^*$ and $NP\rho^*$. This implies that there is at least one $Nq - nbd\ \nu_1$ in $\tau_i$, of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ such that for every r in X, $\nu_1(r) + \mu_1(r) - 1 \leq \lambda_1(r)$,

$\nu_1(r)+\mu_2(r)-1\le\lambda_1(r)$, $\nu_1(r)+\mu_3(r)-1>\lambda_1(r)$, for some $\lambda_1$ in L and similarly, there is at least one $Nq-nbd$ $\nu_2$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that, for every $r$ in X, $\nu_2(r)+\rho_1(r)-1\le\lambda_2(r)$ , $\nu_2(r)+\rho_2(r)-1\le\lambda_2(r)$, $\nu_2(r)+\rho_3(r)-1>\lambda_2(r)$ for some $\lambda_2$ in L. Also, there is at least one $Nq-nbd$ $\nu_3$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that, for every $r$ in X, $\nu_3(r)+\eta_1(r)-1\le\lambda_3(r)$ , $\nu_3(r)+\eta_2(r)-1\le\lambda_3(r)$, $\nu_3(r)+\eta_3(r)-1>\lambda_3(r)$ for some $\lambda_3$ in L . Let $\nu=\nu_1\wedge\nu_2\wedge\nu_3$, so $\nu$ is also $Nq-nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ and $\nu_1(r)+(\mu_1\vee\rho_1)(r)-1\le(\lambda_1\vee\lambda_2\vee\lambda_3)(r)$, $\nu_2(r)+(\mu_2\vee\rho_2)(r)-1\le(\lambda_1\vee\lambda_2\vee\lambda_3)(r)$, $\nu_3(r)+(\mu_3\vee\rho_3)(r)-1>(\lambda_1\vee\lambda_2\vee\lambda_3)(r)$, for every $r$ in X. Therefore, by finite additively of neutrosophic fuzzy ideal as $\lambda_1\vee\lambda_2\vee\lambda_3$ in L, $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}\notin(\mu\vee\rho)^*$. Hence $P(\mu\cup\rho)^*(L,\tau_i)\subseteq P\mu^*(L,\tau_i)\cup\rho^*(L,\tau_i)$. Clearly, both $\mu$ and $\rho\subseteq\mu\cup\rho$ which implies $NP\mu^*(L,\tau_i)\cup\rho^*(L,\tau_i)\subseteq NP\big(\mu=<\mu_1,\mu_2,\mu_3>\cup\rho=<\rho_1,\rho_2,\rho_3>\big)^*(L,\tau_i)$ and this the proof .

vi- Clear.

**4. Basic Structure of Generated Neutrosophic Fuzzy Bitopology.**

Let $(X,\tau_i)$, $i\epsilon\{1,2\}$ be a $nfbts$ with neutrosophic fuzzy ideal $L$ on $X$. Let us define $\tau_i-Npcl^*\big(\mu=<\mu_1,\mu_2,\mu_3>\big)=\mu=<\mu_1,\mu_2,\mu_3>\cup NP\mu^*(L,\tau_i)$, $i\epsilon\{1,2\}$ for any neutrosophic fuzzy set $\mu=<\mu_1,\mu_2,\mu_3>$ in $1_N$ . Clearly $\tau_i-Npcl^*(\mu=<\mu_1,\mu_2,\mu_3>)$ represent a neutrosophic fuzzy closure operator. Let ${\tau^*}_i(L)$ be the neutrosophic fuzzy bitopology generated by $\tau_i-Npcl^*\big(\mu=<\mu_1,\mu_2,\mu_3>\big)$, i.e. ${\tau^*}_i(L)=\{\mu=<\mu_1,\mu_2,\mu_3>:\tau_i-Npcl^*(\mu^c)=\mu^c\}$. Now, let $L=\{0_N\}\Longrightarrow\tau_i-Ncl^*\big(\mu=<\mu_1,\mu_2,\mu_3>\big)=\mu\cup NP\mu^*(L,\tau_i)=\mu\cup\tau_i-Ncl(\mu)=\tau_i-Ncl(\mu)$ $i\epsilon\{1,2\}$, for every $\mu=<\mu_1,\mu_2,\mu_3>$ in $1_N$, so ${\tau^*}_i(\{0_N\})=\tau_i$, $i\epsilon\{1,2\}$. Again let $L=\{1_N\}\Longrightarrow\tau_i-Ncl^*\big(\mu=<\mu_1,\mu_2,\mu_3>\big)=\mu\cup P\mu^*(L,\tau_i)=\mu\cup\{0_N\}=\mu$, so ${\tau^*}_i(1_N)$, $i\epsilon\{1,2\}$ is neutrosophic fuzzy discrete bitopology on X. We can conclude by Theorem 3.1 (ii), ${\tau^*}_i(\{0_N\})\subseteq{\tau^*}_i(L)\subseteq{\tau^*}_i(1_N)$, i.e. $\tau_i\subseteq{\tau^*}_i$ , $L\subseteq J\Longrightarrow{\tau^*}_i(L)\subseteq{\tau^*}_i(J)$. Let $\mu=<\mu_1,\mu_2,\mu_3>$ be a $Nq-nbd$ of a neutrosophic fuzzy point $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in ${\tau^*}_i$ – neutrosophic fuzzy bitopology . Therefore , there exist $\rho=<\rho_1,\rho_2,\rho_3>$ in ${\tau^*}_i$, $i\epsilon\{1,2\}$ such that b, $\varepsilon_1+\rho_1(x)>1$, $\varepsilon_2+\rho_2(x)>1$, $\varepsilon_3+\rho_3(x)<1$ or $\varepsilon_1+\rho_1(x)>1$, $\varepsilon_2+\rho_2(x)<1$, $\varepsilon_3+\rho_3(x)<1$ and $\rho=<\rho_1,\rho_2,\rho_3>\subseteq\mu=<\mu_1,\mu_2,\mu_3>$ . Now , $\mu=<\mu_1,\mu_2,\mu_3>$ in ${\tau^*}_i\Leftrightarrow\mu^c$ is ${\tau^*}_i$ -closed $\Leftrightarrow\tau_i-Ncl^*(\mu)=\mu^c\Leftrightarrow NP(\mu^c)^*\subseteq\mu^c\Leftrightarrow\mu\subseteq(NP(\mu^c)^*)^c$. Therefore

$\varepsilon_1 + \mu_1(x) > 1 \Longrightarrow \varepsilon_1 + \{(\mu_1{}^c)^*\}(x) > 1 \Rightarrow \varepsilon_1 + 1 - NP(\mu_1{}^c)^*(x) > 1, \varepsilon_1 > (\mu_1{}^c)^*(x) \Rightarrow x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \notin \left(\mu = < \mu_1, \mu_2, \mu_3 >^c\right)^*$ , $\varepsilon_2 + \mu_2(x) > 1 \Longrightarrow \varepsilon_2 + \{(\mu_2{}^c)^*\}(x) > 1 \Rightarrow \varepsilon_2 + 1 - NP(\mu_2{}^c)^*(x) > 1, \varepsilon_2 > (\mu_2{}^c)^*(x) \Rightarrow x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \notin \left(\mu =< \mu_1, \mu_2, \mu_3 >^c\right)^*$ , $\varepsilon_3 + \mu_3(x) < 1 \Longrightarrow \varepsilon_3 + \{(\mu_3{}^c)^*\}(x) < 1 \Rightarrow \varepsilon_3 + 1 - NP(\mu_3{}^c)^*(x) < 1, \varepsilon_3 \leq (\mu_3{}^c)^*(x) \Rightarrow x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \notin \left(\mu =< \mu_1, \mu_2, \mu_3 >^c\right)^*$ . This implies there exists at least one $Nq - nbd$ $\nu_1$, of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in \tau_i$ such that for every $r$ in $X, \nu_1(r) + \mu_1{}^c(r) - 1 \leq \lambda_1(x)$, $\nu_1(r) + \mu_2{}^c(r) - 1 \leq \lambda_1(x)$, $\nu_1(r) + \mu_3{}^c(r) - 1 > \lambda_1(x)$ for some $\lambda_1$ in L. i.e. $\nu_1(r) - \lambda_1(r) \leq \lambda_1(x)$ for every r in X, there exists at least one $Nq - nbd$ $\nu_2$, of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in \tau_i$ such that for every r in $X, \nu_2(r) + \mu_1{}^c(r) - 1 \leq \lambda_1(x)$ , $\nu_2(r) + \mu_2{}^c(r) - 1 \leq \lambda_1(x)$ , $\nu_2(r) + \mu_3{}^c(r) - 1 > \lambda_1(x)$ for some $\lambda_1$ in L . i.e. $\nu_2(r) - \lambda_1(r) \leq \lambda_1(x)$ for every r in X, there exists at least one $Nq - nbd$ $\nu_3$, of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}$ (in $\tau_i$) such that for every r in $X, \nu_3(r) + \mu_1{}^c(r) - 1 \leq \lambda_1(x)$, $\nu_3(r) + \mu_2{}^c(r) - 1 \leq \lambda_1(x)$, $\nu_3(r) + \mu_3{}^c(r) - 1 > \lambda_1(x)$ for some $\lambda_1$ in L. i.e. $\nu_3(r) - \lambda_1(r) \leq \lambda_1(x)$ for every r in X, . Therefore, as $\nu_1$ $Nq - nbd$ of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in \tau_i$, $\nu_2$ $Nq - nbd$ of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in \tau_i$, $\nu_2$ $Nq - nbd$ of $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>} \in \tau_i$, there is a $\nu =< \nu_1, \nu_2, \nu_3 >$ in $\tau_i$ such that $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}\, Nq\ \nu =< \nu_1, \nu_2, \nu_3 > \subseteq \nu_1$ , $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}\, Nq\ \nu =< \nu_1, \nu_2, \nu_3 > \subseteq \nu_2$ , $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}\, Nq\ \nu =< \nu_1, \nu_2, \nu_3 > \subseteq \nu_3$ and by heredity property of neutrosophic fuzzy ideal we have $\lambda$ in L for which $x_{<\varepsilon_1,\varepsilon_2,\varepsilon_3>}\, Nq \left(\nu =< \nu_1, \nu_2, \nu_3 > - \lambda\right) \subseteq \mu$, where $\left(\nu =< \nu_1, \nu_2, \nu_3 > - \lambda\right)(r) = \max\{\nu(r) - \lambda(r), 0\}$ for every r in X. Hence , for $\mu =< \mu_1, \mu_2, \mu_3 >$ in $\tau^*{}_i$, we have a $\nu =< \nu_1, \nu_2, \nu_3 >$ $\tau_i$ and $\lambda$ in L such that ,$(\nu =< \nu_1, \nu_2, \nu_3 > - \lambda) \subseteq \mu$. Let us denote $\beta(L, \tau_i) = \{\nu - \lambda: \nu \text{ in } \tau_i, \lambda \text{ in } L\}$. Then we have the following Theorem.

**Theorem 4.1:** $\beta(L, \tau_i)$ from a basis for the generated neutrosophic fuzzy bitopology $\tau^*{}_i(L)$ of the nfbts $(X, \tau_i), i\epsilon\{1,2\}$ with neutrosophic fuzzy ideal L on X, the class $\beta(L, \tau_i) = \{\{\mu - \lambda\}: \mu \text{ in } \tau_i, \lambda \text{ in } L, i\epsilon\{1,2\}\}$ may be the base for the neutrosophic fuzzy bitopology $\tau^*{}_i$.

**Proof:** Straightforward

**Theorem 4.2.** If $L_1$ and $L_2$ are two neutrosophic fuzzy ideals on nfbts $(X, \tau_i), i\epsilon\{1,2\}, \mu$ in $1_N$,then,

i- $NP\mu^*(L_1, \tau_i) \geq NP\mu^*(L_2, \tau_i)$ for every neutrosophic fuzzy set $\mu$ and $L_1 \leq L_2$.

ii- $\tau^*{}_i(L_1) \leq \tau^*{}_i(L_2)$ and $L_1 \leq L_2$.

iii- $NP\mu^*(L_1 \cap L_2, \tau_i) = NP\mu^*(L_1, \tau_i) \cup NP\mu^*(L_2, \tau_i)$.

iv- $NP\mu^*(L_1 \vee L_2, \tau_i) = NP\mu^*(L_1, \tau^*{}_i(L_2)) \cap NP\mu^*(L_2, \tau^*{}_i(L_1))$.

**Proof.** i and ii are clear.

iii- Let $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_1, \tau_i) \cup x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_2, \tau_i)$. So $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ is not contained in both $NP\mu^*(L_1,\tau_i)$ and $NP\mu^*(L_2,\tau_i)$. Now $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_1,\tau_i)$ implies there is at least one $Nq-nbd\, \nu_1$ in $\tau_i$, of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ such that for every $r$ in $X, \nu_1(r)+\mu_1(r)-1 \le \lambda_1(r)$, $\nu_1(r)+\mu_2(r)-1 \le \lambda_1(r)$, $\nu_1(r)+\mu_3(r)-1 > \lambda_1(r)$ for some $\lambda_1$ in L. Again $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_2,\tau_i)$ and similarly, there is at least one $Nq-nbd\, \nu_2$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that, for every $r$ in $X, \nu_2(r)+\mu_1(r)-1 \le \lambda_2(x)$, $\nu_2(r)+\mu_2(r)-1 \le \lambda_2(x), \nu_2(r)+\mu_3(r)-1 > \lambda_2(x))$ for some $\lambda_2$ in L , similarly, there is at least one $Nq-nbd\, \nu_3$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that, for every $r$ in $X, \nu_3(r)+\mu_1(r)-1 \le \lambda_3(x)$, $\nu_3(r)+\mu_2(r)-1 \le \lambda_3(x), \nu_3(r)+\mu_3(r)-1 > \lambda_3(x))$ for some $\lambda_3$ in L. Therefore, we have $\nu = \nu_1 \cap \nu_2 \cap \nu_3$, so $(\nu =< \nu_1, \nu_2, \nu_3 >$ may be also $Nq-nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ and $\nu_1(r)+\mu_1(r)-1 \le \lambda_1 \cap \lambda_2 \cap \lambda_3(r)$, $\nu_2(r)+\mu_2(r)-1 \le \lambda_1 \cap \lambda_2 \cap \lambda_3(r)$, $\nu_3(r)+\mu_3(r)-1 > \lambda_1 \cap \lambda_2 \cap \lambda_3(r)$, for every $r$ in X. Since $\nu =< \nu_1, \nu_2, \nu_3 >$ may be also $Nq-nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ and $\lambda_1 \cap \lambda_2 \cap \lambda_3$ in $\nu$, therefore $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_1 \cap L_2, \tau_i)$, so that $NP\mu^*(L_1 \cap L_2, \tau_i) \subseteq NP\mu^*(L_1, \tau_i) \cup NP\mu^*(L_2, \tau_i)$. Also $(L_1 \cap L_2)$ included in both $L_1$ and $L_2$ , so by Theorem 3.1. (ii), reverse inclusion is obvious, which completes the proof of (iii).

iv) Let $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin, NP\mu^*(L_1 \vee L_2, \tau_i)$ implies there is at least one $Nq-nbd\, \nu_1$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that for every $r$ in $X, \nu_1(r)+\mu_1(r)-1 \le \lambda_1(r)$, $\nu_1(r)+\mu_2(r)-1 \le \lambda_1(r)$, $\nu_1(r)+\mu_3(r)-1 > \lambda_1(r)$ for some $\lambda_1$ in $L_1 \vee L_2$, there is at least one $Nq-nbd\, \nu_2$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that for every $r$ in $X, \nu_2(r)+\mu_1(r)-1 \le \lambda_2(r)$, $\nu_2(r)+\mu_2(r)-1 \le \lambda_2(r)$ , $\nu_2(r)+\mu_3(r)-1 > \lambda_2(r)$ for some $\lambda_2$ in $L_1 \vee L_2$, there is at least one $Nq-nbd\, \nu_3$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau_i$ such that for every $r$ in $X, \nu_3(r)+\mu_1(r)-1 \le \lambda_3(r)$, $\nu_3(r)+\mu_2(r)-1 \le \lambda_3(r)$ , $\nu_3(r)+\mu_3(r)-1 > \lambda_3(r)$ for some $\lambda_3$ in $L_1 \vee L_2$ . Therefore, by heredity of the neutrosophic fuzzy ideals and considering the structure of neutrosophic fuzzy $\tau_i$-open sets generated neutrosophic fuzzy bitopology, we can find $\nu_1, \nu_2, \nu_3$ the $Nq-nbd$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau^*{}_i(L_1)$ or $\tau^*{}_i(L_2)$ respectively, such that, for every $r$ in $X, \nu_1(r)+\mu(r)-1 \le \lambda_1(r)$ or $\nu_2(r)+\mu(r)-1 \le \lambda_2(r)$ or $\nu_3(r)+\mu(r)-1 > \lambda_3(r)$ for some $\lambda_2$ in $L_2$ or $\lambda_1$ in $L_1$ or $\lambda_2$ in $L_2$ or $\lambda_3$ in $L_1$ for every $r$ in X. This implies $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_1, \tau^*{}_i(L_2))$ or $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_2, \tau^*{}_i(L_1))$ . Thus we have $NP\mu^*(L_1, \tau^*{}_i(L_2)) \cap NP\mu^*(L_2, \tau^*{}_i(L_1)) \subseteq NP\mu^*(L_1 \vee$

$L_2, \tau_i)$. Conversely, let $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin NP\mu^*(L_1, \tau^*{}_i(L_2))$. This implies there may be least one on $Nq-nbd$ $\nu =< \nu_1, \nu_2, \nu_3 >$ of $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>}$ in $\tau^*{}_i$ such that for every r in X, $\nu(r) + \mu(r) - 1 \leq \lambda_1 \cup \lambda_2 \cup \lambda_2(r)$, for some $\lambda_1$ in $L_1$ and for some $\lambda_2$ in $L_2$, $\lambda_3$ in $L_1$.i.e., $x_{<\mathcal{E}_1,\mathcal{E}_2,\mathcal{E}_3>} \notin, NP\mu^*(L_1 \vee L_2, \tau_i)$. Thus,

$NP\mu^*(L_1 \vee L_2, \tau_i) \subseteq NP\mu^*(L_1, \tau^*{}_i(L_2))$ and $NP\mu^*(L_2, \tau^*{}_i(L_1))$. Then

$NP\mu^*(L_1 \vee L_2, \tau_i) \subseteq NP\mu^*(L_1, \tau^*{}_i(L_2)) \cap NP\mu^*(L_2, \tau^*{}_i(L_1))$ and this completes the proof.

An important result follows from the above theorem that $\tau^*{}_i(L)$ and $\tau^{**}{}_i(L)$ are Equal for any neutrosophic fuzzy ideal on X.

**Corollary 4.1:** Let $(X,\tau_i), i\epsilon\{1,2\}$ be a nfbts with neutrosophic fuzzy ideal L. Then $\tau^*{}_i(L) = \tau^{**}{}_i(L)$

Proof. By taking $L_1 = L_2 = L$ in the above Theorem, we have the required result .

Corollary 3.2: If $L_1$ and $L_2$ are two neutrosophic fuzzy ideals on nfbt $(X, \tau_i)$ then,

i- $\tau^*{}_i(L_1 \vee L_2, \tau_i) = [\tau^{**}{}_i(L_2, \tau_i)](L_1) = [\tau^{**}{}_i(L_1, \tau_i)](L_2)$,

ii- $\tau^*{}_i(L_1 \vee L_2, \tau_i) = [\tau^*{}_i(L_1, \tau_i)] \vee [\tau^*{}_i(L_2, \tau_i)]$,

iii-$\tau^*{}_i(L_1 \cap L_2, \tau_i) = [\tau^*{}_i(L_1, \tau_i)] \cap [\tau^*{}_i(L_2, \tau_i)]$ .

**5. Some Applications in Neutrosophic Fuzzy Ideal Function.**

**Application 5.1.** In this example we illustrate the neutrosophic degrees, it produces three types of chips that are represented $X = \{ x_1 < 1,1,1 > \}$ , it represents the total production of the plant, where $A = \{ x_1 < 0.6,0.3,0.4 > \}$ represents the neutrosophic component of the first type production, $B = \{ x_1 < 0.3,0.5,0.7 > \}$ represents the neutrosophic component of the second type production, $C = \{ x_1 < 0.1,0.7,0.9 > \}$ represents the neutrosophic component of the third type production. We defined the $N\tau_{T_1}$ is a neutrosophic bitopological space of the total production $N\tau_{T_i} = \{ 0_N, X_N A, B , C \}, i \in \{1,2\},$ , NL is a neutrosophic ideal space of the total production $FNL = \{ 0_N, A, B , C \}$, $A^* = B^* = C^* = \{ < 0,0,0 > \}$. Let $D = \{ x_1 < 0.6,0.1,0.9 > \} \notin N\tau_{T_i}, i \in \{1,2\}$, then $D^* = \{ < 0.6,0.3,0.9 > \}$, FNInt(D)=A, we, compute the complement of a neutrosophic bitopological space $co(N\tau_{T_i}) = \{ X_N, 0_N, co(A), co(B), co(C) \}, i \in \{1,2\}$, $co(A) =< 0.4,0.7,0.6 >$, $co(B) =< 0.7,0.5,0.3 >$, $co(C) =< 0.4,0.7,0.6 >$, $co(D) =< 0.4,0.9,0.1 >$, NCL(D) = co(C). In the above Example, we conclude and add a new production with the new type D such that $D^*$ as generalized of the production neutrosophic ideal subspace D, the following Table 5.1. represent the new Matrix for the type for projections.

| N-type | NINT | * | NCL |
|---|---|---|---|
| A | $< 0.6,0.3,0.4 >$ | $< 0,0,0 >$ | $< 0.4,0.7,0.6 >$ |
| B | $< 0.3,0.5,0.7 >$ | $< 0,0,0 >$ | $< 0.7,0.5,0.3 >$ |
| C | $< 0.1,0.7,0.9 >$ | $< 0,0,0 >$ | $< 0.9,0.3,0.1 >$ |
| Proposed D new type | $< 0.6,0.3,0.4 >$ | $< 0.6,0.3,0.9 >$ | $< 0.9,0.3,0.1 >$ |

Table 5.1. Neutrosophic Matrix for Projections.

Note That: $\text{Nint}\ (D) \leq D \leq D^{*} \leq \text{Ncl}(D)$

**Application 5.2.** The following example illustrates a construction of the neutrosophic topological space for an aircraft with two engines and we study the degrees of wear on the two engines by building a neutrosophic topological space to support and make the right decision, we defined universal set $X = \{x_1 < 1,1,1 >\}$ , degrees of damage in the first engine $A = \{x_1 < 0.01,0.05,0.99 >\}$, degrees of damage in the second engine $B = \{x_1 < 0.1,0.7,0.9 >\}$, Degrees of damage in the two engines together $A \cap B = \{x_1 < 0.001,0.007,0.999 >\}$, degrees of damage in the first A or second B engine $A \cup B = \{x_1 < 0.1,0.7,0.9 >\}$, neutrosophic topological space to degrees damages $NT_{T_i} = \{0_N, X_N, A, B, A \cup B, A \cap B\}, i \in \{1,2\}$, we defined neutrosophic topological space to degrees the right competence $co(NT_{T_i}) = \{X_N, 0_N, co(A), co(B), co(A \cup B), co(A \cap B)\}, i \in \{1,2\}$, we introduce $co(A) = \{x_1 < 0.99,0.05,0.01 >\}$, $co(B) = \{x_1 < 0.9,0.3,0.1 >\}$, $co(A \cap B) = \{x_1 < 0.999,0.993,0.001 >\}$, $co(A \cup B) = \{x_1 < 0.9,0.3,0.1 >\}$, we defined neutrosophic bitopological ideal space to degrees the right competence $NL = \{0_N, co(A), co(B),, co(A \cap B)\}$, and

$(co(A))^{*} = \{x_1 < 0.99,0.993,0.01 >\}$, $(co(B))^{*} = \{x_1 < 0.9,0.993,0.1 >\}$, $(co(A \cap B))^{*} = \{x_1 < 0.99,0.993,0.01 >\}$.

From the above information, we found that the efficiency of the second engine type2 is correct and it is less than the certainty for the correct first engine type1. The degree of diffraction for the two motors is equal, the degree of uncertainty of the second plane's proper motion is greater than the degree of uncertainty of the first correct aircraft movement.

Also, we found that the degree of certainty of the efficiency of the two flying engines together is very high, we find that the degree of uncertainty of the efficiency of the two engines together is very small. From the above, we conclude that the degree of efficiency of the aircraft while operating the two engines together is of a high degree of efficiency.

**6. Conclusion**

There is no doubt that the neutrosophic fuzzy topology and bitopological spaces were unfathomable aspects, except the activity of some brilliant authors in publishing dozens of papers related to the structural of neutrosophic fuzzy bitopological spaces, neutrosophic fuzzy ideals, neutrosophic fuzzy local function, neutrosophic fuzzy pairwise local function. In this paper the authors suggested new theorems that give some general properties of the above mentioned concepts. Finally, some applied problems in neutrosophic fuzzy ideals function have been introduced.

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