Neutrosophic Separation Axioms

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Abstract:

Neutrosophic set, developed by Smarandache, is characterized by a truth membership function, an indeterminacy function and a falsity membership function. Neutrosophic sets have been employed to model uncertainty in several areas of application such as decision making, pattern recognition, image segmentation, etc. Neutrosophic separation axioms are interesting concepts via neutrosophic topology. In this paper, we introduce the notion of neutrosophic T_i-spaces (i = 0, 1, 2, 3, 4) via neutrosophic topological spaces, and investigate their different properties. By defining neutrosophic T_i-spaces (i = 0, 1, 2, 3, 4), we prove some interesting results on neutrosophic separation axioms via neutrosophic topological spaces.

Keywords: Neutrosophic Set; Neutrosophic Topological Spaces; Neutrosophic Separation Axioms.

1. Introduction:

Based on neutrosophy [1], Neutrosophic Set (NS) was grounded by Smarandache [1], which is the generalization of Fuzzy Set (FS) [2] and intuitionistic FS [3]. Later on, Salama and Alblowi [4] presented the notion of Neutrosophic Topological Space (NTS). Arokiarani et al. [5] introduced the concept of Neutrosophic Point (NP) in NTSs. AL-Nafee et al. [6] studied some separation axioms on neutrosophic crisp topological spaces. Das and Pramanik [7] presented the generalized neutrosophic b-open sets in NTS. Das and Pramanik [8] developed the neutrosophic Φ-open sets and neutrosophic Φ-continuous functions in NTSs. Maji [9] grounded the idea of neutrosophic soft sets. Bera and Mahapatra [10] introduced the neutrosophic soft topological space. Das and Pramanik [11] presented the neutrosophic simply soft open set in Neutrosophic Soft Topological Space (NSTS). Gunnuz Aras et al. [12] presented the separation axioms on neutrosophic soft topological spaces. Mehmood et al. [13] worked on generalized neutrosophic separation axioms in NSTS in this article the worked on Neutrosophic soft p-separation structures are the most imperative and fascinating notions in neutrosophic soft topology. Acikgoz and Esenbel [14] studied on separation axioms in NTS by defining neutrosophic quasi-coincidence and neutrosophic Ri-spaces, i= 0, 1 and established some basic results. Khattak et al. [15] presented soft b-separation axioms in NSTS. Suresh and Palaniammal [16] worked on "NS(WG) separation axioms in NTS

Research Gap: No investigation on neutrosophic separation axioms [neutrosophic T_i-spaces, I = 0, 1, 2, 3, 4] on NTSs has been reported in the neutrosophic literature.

Motivation: To fill the research gap, we present the notion of neutrosophic separation axioms [neutrosophic T_i-spaces, I = 0, 1, 2, 3, 4] on NTSs.

The rest of the article has been split into following sections:

In section 2, we recall the basic definitions on NSs and NTSs. In section 3, we present the notion of neutrosophic separation axioms [neutrosophic T_i-spaces, i = 0, 1, 2, 3, 4] on NTSs, and examine several relationships between them. Section 4 presents the concluding remarks. In this section, we also state some future scope of research in this direction.

Throughout this article, we use the acronym for the clarity of the presentation (see Table 1).

Table 1. List of Short terms

String of words	acronym/ *abbreviation*
Neutrosophic Set	NS
Neutrosophic Topology	NT
Neutrosophic Topological Space	NTS
Neutrosophic Soft Topological Space	NSTS
Neutrosophic Open Set	NOS
Neutrosophic Closed Set	NCS
Neutrosophic Point	NP
Neutrosophic T_0-Space	N-T_0-S
Neutrosophic T_1-Space	N-T_1-S
Neutrosophic T_2-Space	N-T_2-S

2. Some Relevant Definitions:

In this section, we recall some basic definitions and results on NSs and NTSs.

Definition 2.1. An NS [1] R over a non-empty fixed set X is defined as follows:

$$R = \{(r, T_R(r), I_R(r), F_R(r)): r\in X\},$$

where T, I, F : $X\rightarrow]^-0, 1^+[$ are the truth, indeterminacy and false membership functions respectively.

Definition 2.2. The null NS (0_N) [1] and absolute NS (1_N) over X are defined as follows:

(i) $0_N = \{(r, 0, 1, 1) : r\in X\}$;

(ii) $1_N = \{(r, 1, 0, 0) : r\in X\}$.

Definition 2.3. Let $H = \{(r, T_H(r), I_H(r), F_H(r)) : r\in X\}$ and $K = \{(r, T_K(r), I_K(r), F_K(r)) : r\in X\}$ be two NSs over a fixed set X. Then, the following results [1] hold:

(i) $H^c = \{(r, 1-T_H(r), 1-I_H(r), 1-F_H(r)) : r\in X\}$;

(ii) $H \subseteq K$ if and only if $T_H(r)\le T_K(r)$, $I_H(r)\ge I_K(r)$, $F_H(r)\ge F_K(r)$, for all $r\in X$.

(iii) $H \cup K = \{(r, T_H(r)\vee T_K(r), I_H(r)\wedge I_K(r), F_H(r)\wedge F_K(r)) : r\in X\}$;

(iv) $H \cap K = \{(r, T_H(r)\wedge T_K(r), I_H(r)\vee I_K(r), F_H(r)\vee F_K(r)) : r\in X\}$.

Definition 2.3. A non-empty collection τ of NSs over a fixed set X is called a neutrosophic topology (NT) [4] on X if the following three axioms hold:
(i) 0_N and 1_N are the members of τ;
(ii) $R_1, R_2 \in \tau \Rightarrow R_1 \cap R_2 \in \tau$;
(iii) $\cup \{R_i : i \in \Delta\} \in \tau$, for every $\{R_i : i \in \Delta\} \subseteq \tau$.

If τ is an NT on X, then the structure (X, τ) is called a neutrosophic topological space (NTS) [4]. Every member of τ is said to be a neutrosophic open set (NOS). If $R \in \tau$, then R^c is called a neutrosophic closed set (NCS).

Definition 2.4. Suppose that p, q, r be real standard and non-standard subsets of]-0, 1+[. An NS $z_{p.q.r}$ is called a Neutrosophic Point (NP) [5] over a fixed set X defined by

$$z_{p.q.r}(y) = \begin{cases} (p,q,r), & \text{if } z=y \\ (0,1,1), & \text{if } z\neq y) \end{cases}$$

where p, q, r ($\in]^-0, 1^+[$) are the truth, indeterminacy and falsity membership values of z. Undoubtedly, every NS is the union of its NPs.

Example 2.1. Suppose that X = {r_1, r_2} be a fixed set. Clearly, ${r_1}_{0.2,0.3,0.7}$ and ${r_2}_{0.6,0.5,0.5}$ are two NPs over X. Then, the neutrosophic set R = {(r_1, 0.2, 0.3, 0.7), (r_2, 0.6, 0.5, 0.5)} is the union of neutrosophic points ${r_1}_{0.2,0.3,0.7}$ and ${r_2}_{0.6,0.5,0.5}$.

Definition 2.5. An NP $z_{p.q.r}$ is contained in a neutrosophic set R (i.e., $z_{p.q.r} \in R$) [5] if and only if $p \leq T_R(z)$, $q \geq I_R(z)$, $r \geq F_R(z)$.

Definition 2.6. A one to one and onto function $\xi : (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a neutrosophic continuous mapping [5] if $\xi^{-1}(K)$ is an NOS in X, whenever K is an NOS in Y.

Definition 2.7. A function $\xi : (X, \tau_1) \rightarrow (Y, \tau_2)$ is called a neutrosophic open mapping [5] if ξ(K) is an NOS in Y, whenever K is an NOS in X.

3. Neutrosophic T_i–Spaces:

In this section, we present the notion of neutrosophic separation axioms via NTSs, and investigate different relationships among them.

Definition 3.1. An NTS (X, τ) is called a neutrosophic T_0-space (N-T_0-S) if for any pair of NPs $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ ($x \neq y$) in X, there exists an NOS R such that $x_{\alpha,\beta,\gamma} \in R$, $y_{\theta,\lambda,\mu} \notin R$ or $x_{\alpha,\beta,\gamma} \notin R$, $y_{\theta,\lambda,\mu} \in R$.

Example 3.1. Suppose that X = {x, y} & τ ={0_N, 1_N, {<x, 0.5, 0.4, 0.7>, <y, 0.3, 0.4, 0.3>}, { <x, 0.5, 0.4, 0.7>}}. Clearly, (X, τ) is a neutrosophic T_0-space.

Theorem 3.1. Suppose that $\xi : (X, \tau_1) \rightarrow (Y, \tau_2)$ is both one-one and neutrosophic continuous function from an NTS (X, τ_1) to another NTS (Y, τ_2). If (Y, τ_2) be an N-T_0-S, then (X, τ_1) is also an N-T_0-S.

Proof. Assume that (Y, τ_2) is an N-T_0-S. Also, let $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ ($x \neq y$) be any two NPs in (X, τ_1). Since, $\xi : (X, \tau_1) \rightarrow (Y, \tau_2)$ is a one-one function, so $\xi(x_{\alpha,\beta,\gamma})$, $\xi(y_{\theta,\lambda,\mu})$ are also distinct NPs in (Y, τ_2). Since, (Y, τ_2) is an N-T_0-S, so there exists an NOS R in Y such that $\xi(x_{\alpha,\beta,\gamma}) \in R$, $\xi(y_{\theta,\lambda,\mu}) \notin R$ or $\xi(x_{\alpha,\beta,\gamma}) \notin R$, $\xi(y_{\theta,\lambda,\mu}) \in R$. Therefore, $x_{\alpha,\beta,\gamma} \in \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \notin \xi^{-1}(R)$ or $x_{\alpha,\beta,\gamma} \notin \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \in \xi^{-1}(R)$. Since, ξ is a neutrosophic continuous function, so $\xi^{-1}(R)$ is an NOS in (X, τ_1). Therefore, for any pair of distinct NPs $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ in (X, τ_1), there exists an NOS $\xi^{-1}(R)$ such that $x_{\alpha,\beta,\gamma} \in \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \notin \xi^{-1}(R)$ or $x_{\alpha,\beta,\gamma} \notin \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \in \xi^{-1}(R)$. Hence, (X, τ_1) is an N-T_0-S.

Definition 3.2. An NTS (X, τ) is called a neutrosophic T_1-space (N-T_1-S) if for any pair of NPs $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ ($x \neq y$) in X, there exist two NOSs R and S such that $x_{\alpha,\beta,\gamma} \in R$, $x_{\alpha,\beta,\gamma} \notin S$ and $y_{\theta,\lambda,\mu} \notin R$, $y_{\theta,\lambda,\mu} \in S$.

Obviously, every neutrosophic T_1-space is also a neutrosophic T_0-space.

Example 3.2. Suppose that X = {x, y}. Let τ = {0_N, 1_N, {<x, 0.5, 0.5, 0.1>, <y, 0.7, 0.2, 0.3>}, { <x, 0.5, 0.5, 0.1>}{<y, 0.7, 0.2, 0.3>} be an NT on X. Clearly, (X, τ) is a neutrosophic T_1-space.

Theorem 3.2. Assume that $\xi : (X, \tau_1)\rightarrow(Y, \tau_2)$ be both one-one and neutrosophic continuous function from an NTS (X, τ_1) to another NTS (Y, τ_2). If (Y, τ_2) be an N-T_1-S, then (X, τ_1) is also an N-T_1-S.

Proof. Let (Y, τ_2) be an N-T_1-S. Also, let $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ ($x \neq y$) be any two NPs in X. Since, $\xi : (X, \tau_1)\rightarrow(Y, \tau_2)$ is a one-one function, so $\xi(x_{\alpha,\beta,\gamma})$, $\xi(y_{\theta,\lambda,\mu})$ are also distinct NPs in Y. Since, (Y, τ_2) is an N-T_1-S, so there exist two NOSs R, S in Y such that $\xi(x_{\alpha,\beta,\gamma}) \in R$, $\xi(x_{\alpha,\beta,\gamma}) \notin S$ or $\xi(y_{\theta,\lambda,\mu}) \notin R$, $\xi(y_{\theta,\lambda,\mu}) \in S$. Therefore, $x_{\alpha,\beta,\gamma} \in \xi^{-1}(R)$, $x_{\alpha,\beta,\gamma} \notin \xi^{-1}(S)$ or $y_{\theta,\lambda,\mu} \notin \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \in \xi^{-1}(S)$. Since, ξ is a neutrosophic continuous function, both $\xi^{-1}(R)$, $\xi^{-1}(R)$ are NOSs in X. Therefore, for any pair of distinct NPs $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ in X, there exist two NOSs $\xi^{-1}(R)$, $\xi^{-1}(S)$ such that $x_{\alpha,\beta,\gamma} \in \xi^{-1}(R)$, $x_{\alpha,\beta,\gamma} \notin \xi^{-1}(S)$ or $y_{\theta,\lambda,\mu} \notin \xi^{-1}(R)$, $y_{\theta,\lambda,\mu} \in \xi^{-1}(S)$. Hence, (X, τ_1) is an N-T_1-S.

Theorem 3.3. If an NTS (X, τ) is an N-T_1-S, then every NP in X is an NCS.

Proof. Suppose that (X, τ) is an N-T_1-S. Assume that $x_{\alpha,\beta,\gamma}$ is an arbitrary NP in X. Now, we can take an NP $y_{\theta,\lambda,\mu} \subseteq x^c_{\alpha,\beta,\gamma}$ ($y \neq x$) in X. Since, (X, τ) is an N-T_1-S, so there exist two NOSs R and S such that $x^c_{\alpha,\beta,\gamma} \in R$, $x^c_{\alpha,\beta,\gamma} \notin S$ and $y_{\theta,\lambda,\mu} \notin R$, $y_{\theta,\lambda,\mu} \in S$. Therefore, $x^c_{\alpha,\beta,\gamma} = \cup_{y_{\theta,\lambda,\mu} \subseteq x^c_{\alpha,\beta,\gamma}}$ {R, S : $x^c_{\alpha,\beta,\gamma} \in R$, $x^c_{\alpha,\beta,\gamma} \notin S$ and $y_{\theta,\lambda,\mu} \notin R$, $y_{\theta,\lambda,\mu} \in S$}. Since, $\cup_{y_{\theta,\lambda,\mu} \subseteq x^c_{\alpha,\beta,\gamma}}$ {R, S : $x^c_{\alpha,\beta,\gamma} \in R$, $x^c_{\alpha,\beta,\gamma} \notin S$ and $y_{\theta,\lambda,\mu} \notin R$, $y_{\theta,\lambda,\mu} \in S$} is an NOS in X, so $x^c_{\alpha,\beta,\gamma}$ is an NOS in X. Hence, $x_{\alpha,\beta,\gamma}$ is an NCS in X.

Remark 3.1. Assume that (X, τ) is an NTS. Then, X is an N-T_1-S if and only if $x_{\alpha,\beta,\gamma} = \cap\{N_{cl}(R) : x_{\alpha,\beta,\gamma} \in N_{cl}(R)\}$.

Definition 3.3. An NTS (X, τ) is said to be a neutrosophic T_2-space (N-T_2-S) or neutrosophic Hausdorff space if for any pair of NPs $x_{\alpha,\beta,\gamma}$, $y_{\theta,\lambda,\mu}$ ($x \neq y$) in X, there exist two NOSs R and S such that $x_{\alpha,\beta,\gamma} \in R$, $x_{\alpha,\beta,\gamma} \notin S$ and $y_{\theta,\lambda,\mu} \notin R$, $y_{\theta,\lambda,\mu} \in S$ with $R \subseteq S^c$.

Obviously, every N-T_2-S is an N-T_1-S.

Example 3.3. Let X = {x, y, z} be a fixed set. Let τ = {0_N, 1_N, {<x, 0.5, 0.4, 0.7>, <y, 0.3, 0.4, 0.3>}, {<x, 0.5, 0.4, 0.7>, <z, 0.3, 0.5, 0.8>}, { <y, 0.3, 0.4, 0.3>, <z, 0.3, 0.5, 0.8>}, { <x, 0.5, 0.4, 0.7>} {<y, 0.3, 0.4, 0.3>}, { <z, 0.3,0.5,0.8>}} be an NT on X. Clearly, (X, τ) is an N-T_2-S.

Remark 3.2. In an N-T_2-S (X, τ), every NOS is an NCS.

Theorem 3.4. Assume that $\xi : (X, \tau_1)\rightarrow(Y, \tau_2)$ be both one-one and neutrosophic continuous function from an NTS (X, τ_1) to another NTS (Y, τ_2). If (Y, τ_2) is an N-T_2-S, then (X, τ_1) is also an N-T_2-S.

Proof. Since ξ is a neutrosophic continuous function, so inverse image of an NOS in (Y, τ_2) is also an NOS in (X, τ_1). Also, it is known that, the complement of NOS is NCS in an NTS. Here, since (Y, τ_2) is an N-T_2-S, so every NOS in (Y, τ_2) is also an NCS in (Y, τ_2).

Now, ξ is a neutrosophic continuous function

$\Rightarrow \xi(X) = Y$ is an NOS in τ_2.

$\Rightarrow \xi^{-1}(Y) = X$ is an NOS in τ_1.

Therefore, (Y, τ_2) is an N-T_2-S $\Rightarrow$ ($\xi^{-1}(Y)$, τ_1) is an N-T_2-S. Hence, (X, τ_1) is an N-T_2-S.

Theorem 3.5. Assume that (X, τ) is an N-T_1-S with the condition that complement of each NOS is also an NOS, then (X, τ) is an N-T_2-S.

Proof. Assume that (X, τ) is an N-T_1-S with the condition that complement of each NOS is also an NOS. That is for N, an NOS in (X, τ), $N^c = N$ (1)

Suppose that N is an NOS in (X, τ). Therefore, N^c is an NCS in (X, τ). Again, by equation (1), N^c is an NOS in (X, τ). So, $N^c = N$. Again $(N^c)^c = N^c = N$. Therefore, every NCS in (X, τ) is both NOS and NCS in (X, τ). Hence, by the Remark 3.2, (X, τ) is an N-T_2-S.

Definition 3.4. Assume that (X, τ) is an NTS. Then, X is called a neutrosophic regular-space if for any NP $x_{\alpha,\beta,\gamma}$ in X, and NCS Q with $x_{\alpha,\beta,\gamma} \in Q^c$, there exist two NOSs R and S such that $x_{\alpha,\beta,\gamma} \in R$, $Q \subseteq S$ and $R \subseteq S^c$.

Example 3.4. A zero-dimensional space (every finite open cover of the NT space has a refinement that is a finite open cover such that any NP point in the space is contained in exactly one NOS of this refinement.) with respect to the small inductive dimension has a base consisting of cl-open (NCS and NOS) sets. Every such space is neutrosophic regular-space.

Definition 3.5. An NTS (X, τ) is said to be a neutrosophic T_3-space (N-T_3-S) if it is an N-T_1-S and a neutrosophic regular space.

Obviously, every N-T_3-S is an N-T_2-S.

Example 3.5. The neutrosophic discrete topological space (X, τ) is a neutrosophic regular-space as well as N-T_1-S. Therefore, (X, τ) is an N-T_3-S.

Theorem 3.6. For any NTS (X, τ), the following results are equivalent:

(i) X is a neutrosophic regular-space.

(ii) For any NP $x_{\alpha,\beta,\gamma}$ and any NOS R containing $x_{\alpha,\beta,\gamma}$, there exists an NOS S such that $x_{\alpha,\beta,\gamma} \in S \subseteq N_{cl}(S) \subseteq R$.

Proof. (i)⇒(ii)

Suppose that (X, τ) iss a neutrosophic regular-space. So, for any NP $x_{\alpha,\beta,\gamma}$ in X, and an NCS Q with $x_{\alpha,\beta,\gamma} \in Q^c$, there exist two NOSs S and P such that $x_{\alpha,\beta,\gamma} \in S$, $Q \subseteq P$ and $S \subseteq P^c$.

Again, since R^c is an NCS so there exists an NCS H (say) such that $S \subseteq H$ and so $N_{cl}(S) \subseteq H$.

Again, for an NCS H there exists an NOS R such that $H \subseteq R$. Therefore, $x_{\alpha,\beta,\gamma} \in S \subseteq N_{cl}(S) \subseteq H \subseteq R$. This implies that, $x_{\alpha,\beta,\gamma} \in S \subseteq N_{cl}(S) \subseteq R$. Hence proved.

(ii)⇒(i)

The result is obvious for the neutrosophic regular space.

Definition 3.6. An NTS (X, τ) is said to be a neutrosophic normal-space if for any pair of NCSs G and H with $G \subseteq H^c$ in X, there exist two NOSs R and S in X such that $G \subseteq R$, $H \subseteq S$ and $R \subseteq S^c$.

Example 3.6. Let X = {x, y} be a fixed set. Let τ = {0_N, 1_N, {<x, 0.6, 0.4, 0.1>, <y, 0.7, 0.4, 0.3>}, {<x, 0.6, 0.4, 0.1>}, {<y, 0.7, 0.4, 0.3>}}. Consider the closed set = {0_N, 1_N, {<x, 0.4, 0.6, 0.9>, <y, 0.3, 0.6, 0.7>}, {<x, 0.4, 0.6, 0.9>}}. Clearly, (X, τ) is a neutrosophic normal-space.

Definition 3.7. An NTS (X, τ) is said to be a neutrosophic T_4-space (N-T_4-S) if it is both N-T_1-S and neutrosophic-normal-space.

Obviously, every N-T_4-S is also an N-T_1-S.

Example 3.7. To show the real-life example of separation axioms via NTS, we consider three department, namely, Mathematics = x, Physics = y, Chemistry = z of Tripura University. Based on different activities and departmental work, NAAC provides a degree of members as a neutrosophic set as given below:

{<x, 0.6, 0.3, 0.1>, <y, 0.7, 0.1, 0.3>, <z, 0.5, 0.3, 0.4>}. To analyze the comparison of different developmental work as well as future decision-making, we may consider different neutrosophic topological properties. Here, X = {x, y, z}, and consider the following NSs

A_1 = {<x, 0.6, 0.3, 0.1>}

A_2 = {<y, 0.7, 0.1, 0.3>}

A_3 = <z, 0.5, 0.3, 0.4>}.

A_4 = {<z, 0.5, 0.3, 0.4>, <y, 0.7, 0.1, 0.3>}

A_5= {<x, 0.6, 0.3, 0.1>, <z, 0.5, 0.3, 0.4>}

A_6= {<x, 0.6, 0.3, 0.1>, <y, 0.7, 0.1, 0.3>}

We consider, τ = {0_N, 1_N, A_1, A_2, A_3, A_4, A_5, A_6}. Clearly, we can say that (X, τ) is a neutrosophic topological space, and it is an N-T_2-S.

Theorem 3.7. For any NTS (X, τ), the following results are equivalent:

(i) X is a neutrosophic normal-space.

(ii) For every NCS K and NOS U with $K \subseteq$ U, there exists an NOS V such that $K \subseteq V \subseteq N_{cl}(V) \subseteq U$.

Proof. (i)⇒(ii)

Assume that X is a normal-space neutrosophic. As a result, according to the concept of neutrosophic normal-space, there exist two NOSs U and V in X for any two NCSs K and H with K⊆ H^c in X, such that K⊆ U, H ⊆V, and U ⊆V^c.

For every NCS K and H with K⊆ H^c, we have K⊆ H^c ⇒K∩H=∅ (for any NCS K and H with K ⊆H^c).

Consider the NOS U that contains K and V that contains H, i.e., K⊆U and H⊆V. There are two NOS P and Q that are UP and VQ, where P and Q may or may not be disjoint NOS.

In terms of the first part, we have K⊆U and $\overline{U}$ ⊆P⇒ K⊆U ⊆ $\overline{U}$ ⊆P.

Assuming that U=V and P=U are both NOS, we obtain the following result.

(ii)⇒(i)

There exists an NOS V such that $K \subseteq V \subseteq N_{cl}(V) \subseteq U$ for any NCS K and NOS U with $K \subseteq$ U.

Now, it is necessary to demonstrate that X is a neutrosophic normal-space.

There are two NOSs U and V for any two NCSs K and H with K⊆H^c, such that K ⊆ U, H ⊆ V as K∩H=∅.

As a result, any two NCSs K and H with K⊆ H^c in X have two NOSs U and V, and we have K⊆ U, H ⊆V, and U ⊆V^c. As a result, X is a normal-space neutrosophic.

4. Conclusions:

In this study, we introduce the notion of neutrosophic T_i-spaces (i = 0, 1, 2, 3, 4) via NTS, and study their different properties. By defining neutrosophic T_i-spaces (i = 0, 1, 2, 3, 4), we prove some interesting results on neutrosophic separation axioms via NTSs. Further, we hope that, many new investigations can be done in the future based on the developed notions of neutrosophic separation axioms via NTSs. The notion of neutrosophic T_i-spaces (i = 0, 1, 2, 3, 4) can also be used for introducing the pairwise separation axioms under the neutrosophic bi-topological space. We further hope that the proposed theories can be explored in pentapartitioned neutrosophic set [17] environment.

Conflict of interest: The authors declare that they have no conflict of interest.

Authors' Contribution: All the authors have equally contributed for the preparation of this article.

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