



# Neutrosophic Multi Fuzzy Ideals of $\gamma$ Near Ring

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**Abstract:** The theory of neutrosophic multi fuzzy ideals of  $\gamma$  near ring is dispensed in this work and various algebraic properties such as intersection, union of neutrosophic multi fuzzy ideals of  $\gamma$  near ring are examined.

**Keywords:** Neutrosophic fuzzy set,  $\gamma$  near ring, Neutrosophic multi fuzzy set, neutrosophic multi fuzzy ideal of  $\gamma$  near ring.

#### 1. Introduction

In 1965, Zadeh[25] proposed the notion of fuzzy set. Later A. Rosenfeld[16] developed fuzzy groups. The numerous authors like Bh. Satyanarayana[3,4,5] proposed the concept of fuzzy Y near ring. The authors S. Ragamai, Y. Bhargavi, T. Eswarlal[19] developed theory of fuzzy and L fuzzy ideals of Y near ring. Later the properties of Y near ring in multi fuzzy sets were extended by K. Hemabala and Srinivasa kumar[13]. Florentin Smarandache[7,8] established as a new field of philosophy which is a neutrosophic theory, in 1995. The main base of neutrosophic logic is neutrosophy that includes indeterminacy. It is an augmentation of fuzzy set and intuitionstic fuzzy set. In neutrosophic logic each proposition is estimated by three components T,I,F. The neutrosophic fuzzy set. A. Solairaju and S. Thiruveni[2] verified the algebraic properties of fuzzy neutrosophic set in near rings. In fuzzy neutrosophic set, the three components T,I,F can take single values between 0 and 1. There is some ambiquity irrespective of the distance to the element is. The neutrosophic fuzzy set theory on its own is not sufficient to study real world problems. F. Smarandache[9] developed notion of neutrosophic multi sets, an extension of neutrosophic multi fuzzy set in groups and verified the group properties. We carry the neutrosophic multi fuzzy notion in Y near ring and hence some properties of algebra are verified.

#### 2. Preliminaries:

Basic definitions of fuzzy set, multi fuzzy set, neutrosophic set and neutrosophic multi set,  $\gamma$  near ring are presenting in this section. Fuzzy set can take a single value between [0,1].

# 2.1 Definition:

Let  $\mathcal{H}$  be a non empty set and  $\mathcal{T}$  be a fuzzy set over  $\mathcal{H}$  is defined by[25]

 $\mathcal{T} = \{\mathcal{T}(\mathfrak{X}) \mid x \in \mathcal{H}\}$  where  $\mathcal{T}: \mathcal{H} \to [0,1]$ .

#### 2.2 Definition:

Let  $\mathcal{H}$  be a non empty set and  $\mathcal{S}$  be a multi fuzzy set over  $\mathcal{H}$  is defined as[20,21]

$$\mathcal{S} = \{\langle \mathfrak{X}, \mathcal{S}_1(\mathfrak{X}), \mathcal{S}_2(\mathfrak{X}), \mathcal{S}_3(\mathfrak{X}), \dots, \mathcal{S}_s(\mathfrak{X}) \rangle : \mathfrak{X} \in \mathcal{H} \}$$
 where  $\mathcal{S}_m : \mathcal{H} \to [0,1]$  for all  $m \in \{1, 2, \dots, s\}$  and  $\mathfrak{X} \in \mathcal{H}$ 

#### 2.3 Definition:

Let  $\mathcal H$  be a non empty set then neutrosophic fuzzy set  $\mathcal J$ [7] in  $\mathcal H$  is defined as

 $\mathcal{J} = \{\mathfrak{X}, t_{\mathcal{J}}(\mathfrak{X}), i_{\mathcal{J}}(\mathfrak{X}), \mathfrak{f}_{\mathcal{J}}(\mathfrak{X}) > \mathfrak{X} \in \mathcal{H} \text{ and } t_{\mathcal{J}}(\mathfrak{X}), i_{\mathcal{J}}(\mathfrak{X}), \mathfrak{f}_{\mathcal{J}}(\mathfrak{X}) \in [0,1] \}$ 

Where  $t_{\mathcal{J}}(\mathfrak{X})$  is the truth membership function,  $i_{\mathcal{J}}(\mathfrak{X})$  is the indeterminancy membership function and  $f_{\mathcal{J}}(\mathfrak{X})$  falsity membership function and  $0 \leq t_{\mathcal{J}}(\mathfrak{X}) + i_{\mathcal{J}}(\mathfrak{X}) \leq 1$ .

# 2.4 Definition:

Let  ${\mathcal H}$  be a non empty set. A neutrosophic multi fuzzy set  ${\mathcal L}$  on  ${\mathcal H}$  can be defined as follows

$$\mathcal{L}_{=} \{ < \mathfrak{X}, (t_{\mathcal{L}}^{1}(\mathfrak{X}), t_{\mathcal{L}}^{2}(\mathfrak{X}), ...t_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X})), (i_{\mathcal{L}}^{1}(\mathfrak{X}), i_{\mathcal{L}}^{2}(\mathfrak{X}), ...., i_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X})), (\mathfrak{f}_{\mathcal{L}}^{1}(\mathfrak{X}), \mathfrak{f}_{\mathcal{L}}^{2}(\mathfrak{X}), ...., \mathfrak{f}_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X})) > \mathfrak{X} \in \mathcal{H} \}$$

$$Where \ t_{\mathcal{L}}^{1}(\mathfrak{X}), t_{\mathcal{L}}^{2}(\mathfrak{X}), ...t_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X}): \mathcal{H} \rightarrow [0, 1]$$

$$i_{\mathcal{L}}^{1}(\mathfrak{X}), i_{\mathcal{L}}^{2}(\mathfrak{X}), ...., i_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X}): \mathcal{H} \rightarrow [0, 1]$$

$$\mathfrak{f}_{\mathcal{L}}^{1}(\mathfrak{X}), \mathfrak{f}_{\mathcal{L}}^{2}(\mathfrak{X}), ...., \mathfrak{f}_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X}): \mathcal{H} \rightarrow [0, 1]$$

$$o \leq \operatorname{sunt}^{m}(\mathfrak{X}) + \operatorname{sund}^{m}(\mathfrak{X}) \leq 1 \quad \text{for mettree}$$

 $0 \leq \sup t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}) + \sup i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}) + \sup t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}) \leq 1$  for m=1 to s

 $(t_{\mathcal{L}}^{1}(\mathfrak{X}), t_{\mathcal{L}}^{2}(\mathfrak{X}), ...t_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X})), (i_{\mathcal{L}}^{1}(\mathfrak{X}), i_{\mathcal{L}}^{2}(\mathfrak{X}), ..., i_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X})), (f_{\mathcal{L}}^{1}(\mathfrak{X}), f_{\mathcal{L}}^{2}(\mathfrak{X}), ..., f_{\mathcal{L}}^{\mathfrak{s}}(\mathfrak{X}))$  are the sequences of truth membership values, indeterminacy membership values and falsity membership values. In addition s is called the dimension of neutrosophic multi fuzzy set  $\mathcal{L}$  denoted by  $d(\mathcal{L})$ . The sequence of truth membership values are arranged in decreasing order, but the corresponding indeterminacy membership and falsity membership values may not be in any order.

#### 2.5 Definition:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy sets where  $\mathcal{L} = \{(t_{\mathcal{L}}^{1}(\mathfrak{X}), t_{\mathcal{L}}^{2}(\mathfrak{X}), ..., t_{\mathcal{L}}^{s}(\mathfrak{X})), (i_{\mathcal{L}}^{1}(\mathfrak{X}), i_{\mathcal{L}}^{2}(\mathfrak{X}), ..., i_{\mathcal{L}}^{s}(\mathfrak{X})), (f_{\mathcal{L}}^{1}(\mathfrak{X}), f_{\mathcal{L}}^{2}(\mathfrak{X}), ..., f_{\mathcal{L}}^{s}(\mathfrak{X}))\}$  and  $\mathcal{R} = \{(t_{\mathcal{R}}^{1}(\mathfrak{X}), t_{\mathcal{R}}^{2}(\mathfrak{X}), ..., t_{\mathcal{R}}^{s}(\mathfrak{X})), (i_{\mathcal{R}}^{1}(\mathfrak{X}), i_{\mathcal{R}}^{2}(\mathfrak{X}), ..., i_{\mathcal{R}}^{s}(\mathfrak{X})), (i_{\mathcal{R}}^{1}(\mathfrak{X}), i_{\mathcal{R}}^{2}(\mathfrak{X}), ..., i_{\mathcal{R}}^{s}(\mathfrak{X})), (i_{\mathcal{R}}^{1}(\mathfrak{X}), f_{\mathcal{R}}^{2}(\mathfrak{X}), ..., f_{\mathcal{R}}^{s}(\mathfrak{X}))\}$  then we have the following relations and operations

1. 
$$\mathcal{L} \subseteq \mathcal{R}$$
 iff  $t_{\mathcal{L}}^{m}(\mathfrak{X}) \leq t_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $i_{\mathcal{L}}^{m}(\mathfrak{X}) \geq i_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $f_{\mathcal{L}}^{m}(\mathfrak{X}) \geq f_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $\mathfrak{X} \in \mathcal{H}$  and m=1 to s.

2. 
$$\mathcal{L} = \mathcal{R}$$
 iff  $t_{\mathcal{L}}^{m}(\mathfrak{X}) = t_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $i_{\mathcal{L}}^{m}(\mathfrak{X}) = i_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $f_{\mathcal{L}}^{m}(\mathfrak{X}) = f_{\mathcal{R}}^{m}(\mathfrak{X})$ ,  $x \in \mathcal{H}$  and m=1 to s.

- 3.  $\mathcal{L} \cup \mathcal{R} = \{\mathfrak{X}, \max(\mathfrak{t}_{\mathcal{L}}^{\mathfrak{m}}(\mathfrak{X}), \mathfrak{t}_{\mathcal{R}}^{\mathfrak{m}}(\mathfrak{X})), \min(\mathfrak{i}_{\mathcal{L}}^{\mathfrak{m}}(\mathfrak{X}), \mathfrak{i}_{\mathcal{R}}^{\mathfrak{m}}(\mathfrak{X})), \min(\mathfrak{f}_{\mathcal{L}}^{\mathfrak{m}}(\mathfrak{X}), \mathfrak{f}_{\mathcal{R}}^{\mathfrak{m}}(\mathfrak{X}))\}, \mathfrak{X} \in \mathcal{H} \text{ and } \mathfrak{m} = 1 \text{ to s}$
- 4.  $\mathcal{L} \cap \mathcal{R} = \{\mathfrak{X}, \min(\mathfrak{t}_{\mathcal{L}}^{m}(\mathfrak{X}) \leq \mathfrak{t}_{\mathcal{R}}^{m}(\mathfrak{X})), \max(\mathfrak{i}_{\mathcal{L}}^{m}(\mathfrak{X}), \mathfrak{i}_{\mathcal{R}}^{m}(\mathfrak{X})), \max(\mathfrak{f}_{\mathcal{L}}^{m}(\mathfrak{X}), \mathfrak{f}_{\mathcal{R}}^{m}(\mathfrak{X}))\}, \mathfrak{X} \in \mathcal{H} \text{ and }$

m=1 to s

#### 2.6 Definition:

A non empty set  $\mathcal{H}$  with the binary operations '+'(addition) and '.'(multiplication) is called a near ring[3] if the following conditions hold:

- 1.  $(\mathcal{H}, +)$  is a group
- 2.  $(\mathcal{H}, .)$  is a semigroup
- 3.  $(\mathbf{e}_1 + \mathbf{e}_2) \cdot \mathbf{e}_3 = \mathbf{e}_1 \cdot \mathbf{e}_3 + \mathbf{e}_2 \cdot \mathbf{e}_3$  for all  $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \in \mathcal{H}$

To be precise, it is called right near ring .Since it satisfies the right distributive law. But the word near ring is intended to mean right near ring. We use **gh** instead of **g**. **h** 

- A  $\gamma$  near ring is a triple ( $\xi$ ,+,  $\gamma$ ) where
- 1.  $(\xi, +)$  is a group
- 2.  $\gamma$  is a non empty set of binary operations on  $\xi$  such that  $\tau \in \gamma$ ,  $(\xi, +, \tau)$  is a near ring.
- 3.  $\mathbf{e}_1 \tau (\mathbf{e}_2 \sigma \mathbf{e}_3) = (\mathbf{e}_1 \tau \mathbf{e}_2) \sigma \mathbf{e}_3$  for all  $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \in \xi_{and} \tau, \sigma \in \gamma$ .

## 4. Neutrosophic multi fuzzy set of $\gamma$ near ring

In this section, we introduce the definition of neutrosophic multi fuzzy sets of  $\gamma$  near ring. We proved that union of two neutrosophic multi fuzzy ideals  $\mathcal{L}and\mathcal{R}$  is neutrosophic multi fuzzy ideal whenever  $\mathcal{L} \subseteq \mathcal{R}$ . We also prove that the intersection of two neutrosophic multi fuzzy ideals  $\mathcal{L}and\mathcal{R}$  is also a neutrosophic multi fuzzy ideal.

# 3.1 **Definition:**

A neutrosophic multi fuzzy set  $\mathcal{L} = \{(t_{\mathcal{L}}^{1}(\mathfrak{X}), t_{\mathcal{L}}^{2}(\mathfrak{X}), \dots, t_{\mathcal{L}}^{s}(\mathfrak{X})), (i_{\mathcal{L}}^{1}(\mathfrak{X}), i_{\mathcal{L}}^{2}(\mathfrak{X}), \dots, i_{\mathcal{L}}^{s}(\mathfrak{X})), (f_{\mathcal{L}}^{1}(\mathfrak{X}), f_{\mathcal{L}}^{2}(\mathfrak{X}), \dots, f_{\mathcal{L}}^{s}(\mathfrak{X}))\}$  in a  $\gamma$  near ring  $\xi$  is called neutrosophic multi fuzzy sub  $\gamma$  near ring of  $\xi$  if

i)  $t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}-\mathfrak{z}) \geq \min(t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}), t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{z})),$ 

$$i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}-\mathfrak{Z}) \leq \max(i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}), i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{Z})),$$

$$f_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{Z}) \leq \max(f_{\mathcal{L}}^{m}(\mathfrak{X}), f_{\mathcal{L}}^{m}(\mathfrak{Z})), m=1 \text{ to s.}$$

ii) 
$$t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}\tau\mathfrak{z}) \geq \min(t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}))$$

$$t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{z}), i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}\tau\mathfrak{z}) \leq \max(i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}), i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{z})),$$

$$f_{\ell}^{m}(\mathfrak{X}\mathfrak{T}\mathfrak{Z}) \leq \max(f_{\ell}^{m}(\mathfrak{X}), f_{\ell}^{m}(\mathfrak{Z})), m=1 \text{ to s.}$$

#### 3.2 Definition:

Let  $\xi$  be a  $\gamma$  near ring. A neutrosophic multi fuzzy set  $\mathcal{L}$  in a  $\gamma$  near ring  $\xi$  is called neutrosophic multi fuzzy ideal left(resp. right) of  $\xi$  if for all  $\mathfrak{X}, \mathfrak{z}, \theta_1, \theta_2 \in \xi, \tau \in \gamma$ 

i)  $t_{\mathcal{L}}^{m}(\mathfrak{X} - \mathfrak{z}) \geq \min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{z})), m=1 \text{ to s}$ 

$$i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}-\mathfrak{z}) \leq \max(i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}), i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{z})), \, \mathrm{m=1 \, to \, s}$$

$$f_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{Z}) \leq \max(f_{\mathcal{L}}^{m}(\mathfrak{X}), f_{\mathcal{L}}^{m}(\mathfrak{Z})), m=1 \text{ to s.}$$

ii)  $t_{\mathcal{L}}^{\mathbf{m}}(\mathfrak{z} + \mathfrak{x} - \mathfrak{z}) \ge t_{\mathcal{L}}^{\mathbf{m}}(\mathfrak{x}), \mathbf{m} = 1 \text{ to s}$ 

$$i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{z}+\mathfrak{x}-\mathfrak{z}) \leq i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}), \, \mathrm{m}=1 \, \mathrm{to} \, \mathrm{s}$$

$$f_{\mathcal{L}}^{\mathbf{m}}(\mathfrak{z}+\mathfrak{x}-\mathfrak{z}) \leq f_{\mathcal{L}}^{\mathbf{m}}(\mathfrak{x}), \mathbf{m}=1 \text{ to s}$$

iii)  $t_{\mathcal{L}}^{\mathbf{m}}(\theta_{1}\tau(\mathfrak{X}+\theta_{2})-\theta_{1}\tau\theta_{2}) \geq t_{\mathcal{L}}^{\mathbf{m}}(\mathfrak{X}), \mathbf{m}=1 \text{ to s}$ 

$$i_{\mathcal{L}}^{\mathrm{m}}(\theta_{1}\tau(\mathfrak{X}+\theta_{2})-\theta_{1}\tau\theta_{2}) \leq i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}), \mathrm{m=1 to s}$$

$$f_{\mathcal{L}}^{m}(\theta_{1}\tau(\mathfrak{X}+\theta_{2})-\theta_{1}\tau\theta_{2}) \leq f_{\mathcal{L}}^{m}(\mathfrak{X}), m=1 \text{ to s}$$

[resp. right

$$t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}\tau\theta_{1}) \geq t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{x}), \mathrm{m}=1 \mathrm{ to s}$$

$$i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{t}\tau\theta_{1}) \leq i_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{t}), \mathrm{m}=1 \mathrm{ to s}$$

$$f_{\mathcal{L}}^{m}(\mathfrak{X} \tau \theta_{1}) \leq f_{\mathcal{L}}^{m}(\mathfrak{X}), = 1 \text{ to s}$$

 $\mathcal{L}$  is called a neutrosophic multi fuzzy ideal of  $\boldsymbol{\xi}$  if  $\mathcal{L}$  both left and right neutrosophic multi fuzzy ideal of  $\boldsymbol{\xi}$ .

# Example:

Let  $\xi$  be the set of the 2x2 matrices over the set of integers and  $I_{2x2} \in \gamma$ , Then  $\xi$  is a  $\gamma$  near ring, Define a neutrosophic multi fuzzy subset  $\mathcal{L}$  of  $\xi$  as follows

$$\xi(x) = \begin{cases} \{(1,1,1), (0,0,0), (0,0,0)\} & \text{if } x \in \begin{pmatrix} p & q \\ 0 & 0 \end{pmatrix} \\ \{(0.6,0.7,0.7), (0.1,0.2,0.1), (0.3,0.1,0.2)\} & \text{otherwise} \end{cases}$$

Then clearly  $\mathcal{L}$  is a neutrosophic multi fuzzy ideal of  $\gamma$  near ring  $\xi$ .

#### 3.1 **Theorem:**

Let  $\mathcal{L}$ and  $\mathcal{R}$  neutrosophic multi fuzzy left ideal of  $\boldsymbol{\xi}$ . If  $\mathcal{L} \subset \mathcal{R}$  then  $\mathcal{L} \cup \mathcal{R}$  is a neutrosophic multi fuzzy left ideal of  $\boldsymbol{\xi}$ .

# **Proof:**

Let  $\mathcal{L}and\mathcal{R}$  neutrosophic multi fuzzy left ideal of  $\boldsymbol{\xi}$ .

Let  $\mathfrak{X}$ ,  $\mathfrak{z}, \theta_1, \theta_2 \in \xi, \tau \in \gamma$ 

i) 
$$t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z}) = \max\{t_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z})\}$$
  

$$\geq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \max(t_{\mathcal{L}}^{m}(\mathfrak{Z}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{Z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{Z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X}), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X}), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\begin{aligned} \text{ii)} \qquad t_{\mathcal{L}\cup\mathcal{R}}^{m}(3+\mathfrak{x}-3) &= \max\{t_{\mathcal{L}}^{m}(3+\mathfrak{x}-3), t_{\mathcal{R}}^{m}(3+\mathfrak{x}-3)\} \\ &\geq \max\{t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})\} \\ &\geq t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})\} \\ &\leq t_{\mathcal{L}\cup\mathcal{R}}^{m}(3+\mathfrak{x}-3) = \min\{t_{\mathcal{L}}^{m}(3+\mathfrak{x}-3), t_{\mathcal{R}}^{m}(3+\mathfrak{x}-3)\} \\ &\leq \min\{t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})\} \\ &\leq t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}) = \min\{t_{\mathcal{L}}^{m}(3+\mathfrak{x}-3), t_{\mathcal{R}}^{m}(3+\mathfrak{x}-3)\} \\ &\leq \min\{t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})\} \\ &\leq \min\{t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})\} \\ &\leq t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}) \end{aligned}$$

$$\begin{aligned} \text{iii)} \quad \boldsymbol{t}_{\mathcal{L}\cup\mathcal{R}}^{m} \Big( (\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}) \\ &= \max\{\boldsymbol{t}_{\mathcal{L}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}), \boldsymbol{t}_{\mathcal{R}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2})\} \\ &\geq \max\{\boldsymbol{t}_{\mathcal{L}}^{m}(\boldsymbol{x}), \boldsymbol{t}_{\mathcal{R}}^{m}(\boldsymbol{x})\} \\ &\geq \boldsymbol{t}_{\mathcal{L}\cup\mathcal{R}}^{m} (\boldsymbol{x}) \\ \boldsymbol{i}_{\mathcal{L}\cup\mathcal{R}}^{m} \Big( (\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}) \\ &= \min\{\boldsymbol{i}_{\mathcal{L}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}), \boldsymbol{i}_{\mathcal{R}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2})\} \\ &\leq \min\{\boldsymbol{i}_{\mathcal{L}}^{m}(\boldsymbol{x}), \boldsymbol{i}_{\mathcal{R}}^{m}(\boldsymbol{x})\} \\ &\leq \boldsymbol{i}_{\mathcal{L}\cup\mathcal{R}}^{m} (\boldsymbol{x}) \\ \boldsymbol{f}_{\mathcal{L}\cup\mathcal{R}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}) \\ &= \min\{\boldsymbol{f}_{\mathcal{L}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2}), \boldsymbol{f}_{\mathcal{R}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_{2})\} \\ &= \min\{\boldsymbol{f}_{\mathcal{L}}^{m} ((\theta_{1}\tau(\boldsymbol{x}+\theta_{2})-\theta_{1}\tau\theta_$$

 $\leq \min \{ f_{\mathcal{L}}^{m}(\mathfrak{X}), f_{\mathcal{R}}^{m}(\mathfrak{X}) \}$ 

 $\leq f_{\mathcal{L} \cup \mathcal{R}}^{m}(x)$ 

:  $\mathcal{L} \cup \mathcal{R}$  is a neutrosophic multi fuzzy left ideal of  $\xi$ .

# 3.2 Theorem:

Let  $\mathcal{L}$ and $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $\xi$ . If  $\mathcal{L} \subset \mathcal{R}$  then  $\mathcal{L} \cup \mathcal{R}$  is a neutrosophic multi fuzzy right ideal of  $\xi$ .

# **Proof:**

Let  $\mathcal{L}$ and  $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $\xi$ .

Let 
$$\mathfrak{x}, \mathfrak{z} \theta_{1}, \theta_{2} \in \xi, \tau \in \gamma$$
  
i)  $t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}-\mathfrak{z}) = \max\{t_{\mathcal{L}}^{m}(\mathfrak{x}-\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{x}-\mathfrak{z})\}$   
 $\geq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})), \max(t_{\mathcal{L}}^{m}(\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\geq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x})), \max(t_{\mathcal{L}}^{m}(\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\geq \min\{t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{z}), \max(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\leq \min\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{x}), \min(t_{\mathcal{R}}^{m}(\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\leq \max(t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{L}\cup\mathcal{R}}^{m}(\mathfrak{z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$   
 $\leq \max\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{x}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{x}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}\}$ 

$$\leq \max \left( f_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x}), f_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{s}) \right)$$
ii)  $t_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) = \max\{t_{\mathcal{L}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}), t_{\mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) \}$ 

$$\geq \max\{t_{\mathcal{L}}^{m}(\tilde{x}), t_{\mathcal{R}}^{m}(\tilde{x}) \}$$

$$\geq t_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

$$i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) = \min\{i_{\mathcal{L}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}), i_{\mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) \}$$

$$\leq \min\{i_{\mathcal{L}}^{m}(\tilde{x}), i_{\mathcal{R}}^{m}(\tilde{x}) \}$$

$$\leq i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

$$f_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) = \min\{f_{\mathcal{L}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}), f_{\mathcal{R}}^{m}(\tilde{s} + \tilde{x} - \tilde{s}) \}$$

$$\leq i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

$$i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x} + \tilde{x} - \tilde{s}) = \min\{f_{\mathcal{L}}^{m}(\tilde{x}, i_{\mathcal{R}}^{m}(\tilde{x}) \}$$

$$\leq f_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

$$i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x} + \tilde{x} - \tilde{s}) = \max\{t_{\mathcal{L}}^{m}(\tilde{x}, i_{\mathcal{R}}^{m}(\tilde{x}) \}$$

$$\geq max\{t_{\mathcal{L}}^{m}(\tilde{x}), t_{\mathcal{R}}^{m}(\tilde{x}) \}$$

$$\geq t_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

$$i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x} + \tilde{x} - \tilde{s}) = \max\{t_{\mathcal{L}}^{m}(\tilde{x} + \tilde{x} - \tilde{s}), f_{\mathcal{R}}^{m}(\tilde{x} + \tilde{x} - \tilde{s}) \}$$

$$\leq i_{\mathcal{L} \cup \mathcal{R}}^{m}(\tilde{x})$$

# 3.3 Theorem:

Let  $\mathcal{L}$ and  $\mathcal{R}$  neutrosophic multi fuzzy ideal of  $\xi$ . If  $\mathcal{L} \subset \mathcal{R}$  then  $\mathcal{L} \cup \mathcal{R}$  is a neutrosophic multi fuzzy ideal of  $\boldsymbol{\xi}$ .

**Proof:** It is clear.

### 3.4 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy left ideal of  $\xi$  then  $\mathcal{L} \cap \mathcal{R}$  is a neutrosophic multi fuzzy left ideal of ξ.

#### **Proof:**

Let  $\mathcal{L}$ and  $\mathcal{R}$  neutrosophic multi fuzzy left ideal of  $\xi$ .

Let 
$$\mathfrak{X}, \mathfrak{Z}, \theta_1, \theta_2 \in \xi, \tau \in \gamma$$

i) 
$$t_{\mathcal{L}\cap\mathcal{R}}^m(\mathfrak{X}-\mathfrak{z})=\min\{t_{\mathcal{L}}^m(\mathfrak{X}-\mathfrak{z}), t_{\mathcal{R}}^m(\mathfrak{X}-\mathfrak{z})\}$$

 $\geq \min\{\{\min(t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{X}), t_{\mathcal{L}}^{\mathrm{m}}(\mathfrak{Z})), \min(t_{\mathcal{R}}^{\mathrm{m}}(\mathfrak{X}), t_{\mathcal{R}}^{\mathrm{m}}(\mathfrak{Z}))\}\}$ 

 $\geq \min\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \min(t_{\mathcal{L}}^{m}(\mathfrak{Z}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$ 

 $\geq \min(t^m_{\mathcal{L}\cap\mathcal{R}}(\mathfrak{x})), t^m_{\mathcal{L}\cap\mathcal{R}}(\mathfrak{z}))$ 

 $i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z})=\max\left\{i_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{z}), i_{\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z})\right\}$ 

$$\leq \max \{ \max (i_{\mathcal{L}}^{m}(\mathfrak{X}), i_{\mathcal{L}}^{m}(\mathfrak{Z})), \max (i_{\mathcal{R}}^{m}(\mathfrak{X}), i_{\mathcal{R}}^{m}(\mathfrak{Z})) \} \}$$

 $\leq \max(i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}), i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{Z}))$ 

 $f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z}) = \max \{f_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{z}), f_{\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z})\}$  $\leq \max \{ \{ \max \left( \mathscr{f}_{\mathcal{L}}^{m}(\mathfrak{X}), \mathscr{f}_{\mathcal{L}}^{m}(\mathfrak{Z}) \right), \max \left( \mathscr{f}_{\mathcal{R}}^{m}(\mathfrak{X}), \mathscr{f}_{\mathcal{R}}^{m}(\mathfrak{Z}) \right) \} \}$  $\leq \max \left\{ \left\{ \max \left( \pounds_{\mathcal{L}}^{m}(\mathfrak{X}), \pounds_{\mathcal{R}}^{m}(\mathfrak{X}) \right), \max \left( \pounds_{\mathcal{L}}^{m}(\mathfrak{Z}), \pounds_{\mathcal{R}}^{m}(\mathfrak{Z}) \right) \right\} \right\}$ 

$$\leq \max\left( f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}), f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{Z}) \right)$$

$$\begin{split} \text{ii)} \quad \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s} + \mathbf{x} - \hat{s}) &= \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s} + \mathbf{x} - \hat{s}), \quad \mathcal{C}_{\mathcal{R}}^{m}(\hat{s} + \mathbf{x} - \hat{s})\} \\ &\geq \min\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s})\} \\ &\geq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\geq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})\} \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s} + \hat{s} - \hat{s}), \quad \mathcal{C}_{\mathcal{R}}^{m}(\hat{s} + \hat{s} - \hat{s})\} \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq max(\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}) \\ &\leq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\geq \mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\geq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\geq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\geq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\geq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}) \\ &\leq \min\{\mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s})) \\ &\leq \max\{\mathcal{C}_{\mathcal{L}\Pi,\mathbb{R}}^{m}(\hat{s}, \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{R}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{L}}^{m}(\hat{s}), \mathcal{C}_{\mathcal{L}}^{$$

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 $\leq f_{\mathcal{L}\cap\mathcal{R}}^{m}(x)$ 

:  $\mathcal{L}\cap \mathcal{R}$  is a neutrosophic multi fuzzy left ideal of  $\xi$ .

#### 4.5 Theorem:

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $\mathcal{E}$  then  $\mathcal{L} \cap \mathcal{R}$  is a neutrosophic multi fuzzy right ideal of  $\boldsymbol{\xi}$ .

# **Proof:**

Let  $\mathcal{L}$  and  $\mathcal{R}$  neutrosophic multi fuzzy right ideal of  $\boldsymbol{\xi}$ .

Let  $\mathfrak{X}$ ,  $\mathfrak{z}, \theta_1, \theta_2 \in \xi, \tau \in \gamma$ 

i) 
$$t_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z}) = \min\{t_{\mathcal{L}}^{m}(\mathfrak{X}-\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{X}-\mathfrak{z})\}$$

$$\geq \min\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{z})), \min(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\geq \min\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \min(t_{\mathcal{L}}^{m}(\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\geq \min\{\{\min(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \min(t_{\mathcal{L}}^{m}(\mathfrak{z}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\geq \min\{t_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{z}), \min(\mathfrak{L}_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}$$

$$\leq \max\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{z}))\}\}$$

$$\leq \max\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})), \max(t_{\mathcal{L}}^{m}(\mathfrak{Z}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

$$\leq \max\{\{\max(t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{L}}^{m}(\mathfrak{Z})), \max(t_{\mathcal{R}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{Z}))\}\}$$

 $\leq \max\left( f_{\mathcal{L} \cap \mathcal{R}}^{m}(\mathfrak{X}), f_{\mathcal{L} \cap \mathcal{R}}^{m}(\mathfrak{Z}) \right)$ 

ii) 
$$t_{\mathcal{L}\cap\mathcal{R}}^m(\mathfrak{z}+\mathfrak{x}-\mathfrak{z}) = \min\{t_{\mathcal{L}}^m(\mathfrak{z}+\mathfrak{x}-\mathfrak{z}), t_{\mathcal{R}}^m(\mathfrak{z}+\mathfrak{x}-\mathfrak{z})\}$$

$$\geq \min\{t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})\}$$

$$\geq t_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X})$$

$$i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z}) = \max\{t_{\mathcal{L}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z}), t_{\mathcal{R}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z})\}$$

$$\leq \max\{t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})\}$$

$$\leq i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X})$$

$$f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z}) = \max\{f_{\mathcal{L}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z}), f_{\mathcal{R}}^{m}(\mathfrak{Z} + \mathfrak{X} - \mathfrak{Z})\}$$

$$\leq \max\{f_{\mathcal{L}}^{m}(\mathfrak{X}), f_{\mathcal{R}}^{m}(\mathfrak{X})\}$$

$$\leq f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X})$$
iii) 
$$t_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X} + \mathfrak{r}\theta_{1}) = \min\{t_{\mathcal{L}}^{m}(\mathfrak{X} + \mathfrak{r}\theta_{1}), t_{\mathcal{R}}^{m}(\mathfrak{X} + \mathfrak{r}\theta_{1})\}$$

$$\geq \min\{t_{\mathcal{L}}^{m}(\mathfrak{X}), t_{\mathcal{R}}^{m}(\mathfrak{X})\}$$
$$\geq t_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X})$$

$$\begin{split} i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X} \tau \theta_{1}) &= \max \left\{ i_{\mathcal{L}}^{m}(\mathfrak{X} \tau \theta_{1}) , \ i_{\mathcal{R}}^{m}(\mathfrak{X} \tau \theta_{1}) \right\} \\ &\leq \max \left\{ i_{\mathcal{L}}^{m}(\mathfrak{X}), i_{\mathcal{R}}^{m}(\mathfrak{X}) \right\} \\ &\leq i_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X}) \\ f_{\mathcal{L}\cap\mathcal{R}}^{m}(\mathfrak{X} \tau \theta_{1}) &= \max \left\{ f_{\mathcal{L}}^{m}(\mathfrak{X} \tau \theta_{1}) , \ f_{\mathcal{R}}^{m}(\mathfrak{X} \tau \theta_{1}) \right\} \\ &\leq \max \left\{ f_{\mathcal{L}}^{m}(\mathfrak{X}), f_{\mathcal{R}}^{m}(\mathfrak{X}) \right\} \end{split}$$

$$\leq f_{\mathcal{L}\cap \mathcal{R}}^{m}(x)$$

:  $\mathcal{L} \cap \mathcal{R}$  is a neutrosophic multi fuzzy right ideal of  $\xi$ .

4.6 Theorem:

# Let $\mathcal{L}$ and $\mathcal{R}$ neutrosophic multi fuzzy ideal of $\xi$ then $\mathcal{L} \cap \mathcal{R}$ is a neutrosophic multi fuzzy ideal of $\xi$ .

**Proof:** It is clear.

#### 5. Conclusion:

To conclude, the notion of neutrosophic multi fuzzy gamma near-ring, neutrosophic multi fuzzy ideals of gamma near-rings have been discussed. The proof for the theorem that states Union and Intersection of two neutrosophic multi fuzzy ideals of gamma near-ring is also a Neutrosophic multi fuzzy ideal of gamma near-ring has been provided.

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