On Neutrosophic Multiplication Module

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Abstract: In this article, we investigate some new results of Neutrosophic multiplication module E (shortly Ne(E)). We additionally introduce some light points about some concepts in which have relationship with Neutrosophic multiplication module. We prove that if E is Neutrosophic Artinian multiplication module and Neutrosophic Jacobson radical of E is a Neutrosophic small submodule of Ne(E), then Ne(E) is a Neutrosophic cyclic module. Finally, we show that if E is a Neutrosophic divisible module over Neutrosophic integral domain, then E is a Neutrosophic multiplication module if and only if E is a Neutrosophic cyclic module.

Keywords: Cyclic module; multiplication module; neutrosophic sets; neutrosophic submodule; neutrosophic multiplication module.

1. Introduction

Multiplication module is one of the important concepts in module theory. Several researchers have studied this module in an abstract way, but in this paper, we will present an indeterminacy to study some properties of this module. In 1999, the neutrosophy introduced by Smarandache [1] as a generalization of intuitionistic fuzzy set. Accordingly, he introduced the concept of neutrosophic logic and neutrosophic set where all notion in neutrosophic logic is approximated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F so that this neutrosophic logic is called an extension of fuzzy logic especially to intuitionistic fuzzy logic. In fact, neutrosophic set is the generalization of fuzzy set [2], classical set [3], intuitionistic fuzzy set [4], while neutrosophic group and neutrosophic ring are the generalizations of fuzzy group and ring classical group. In the same way, by the generalization of the classical module, we get the neutrosophic multiplication module. By using the idea of neutrosophic theory, several researchers have studied neutrosophic algebraic structures by inserting an indeterminate element in the algebraic structure. Modules are so much important in algebraic structures as they are in almost all algebraic structures theory [5, 6]. Modules are thought as old algebra due to its rich structure compared to other notions. A few researchers [7- 10] have studied certain type of modules with favourable results. Hence, we will use neutrosophic groups [11] to study the neutrosophic notions formation. In this paper, we will introduce a new hyper algebraic concept that is neutrosophic multiplication module.

The paper is organized as follows. After the literature review in section 1, the preliminaries are reviewed in section 2. The neutrosophic multiplication module is introduced in section 3 along with several relevant section 3, and conclusion in section 4.

2. Preliminaries

In this section, we recall some definitions to be used in this paper.

Definitional 2.1 [12] Suppose that T is a commutative ring with unity. We say that E is a *T-module* if:

$T \times E \to E\ (r, v) \to rv\ such\ that\ E$ is a commutative group with T and satisfies the following.

1. $(rv_1)v = r(v_1 v)$
2. $(r_1 + r_2)v = r_1 v + r_2 v$
3. $r(v_1 + v_2) = rv_1 + rv_2$
4. $1.v = v = v.1$

Definition 2.2 [12] A subset E_1 is called the submodule of E $(E_1 \leq E)$ if closed with (+) and scalar multiplication, that is

(*) $a + b \in E_1,\ \forall a, b \in E_1$

(*) $r\ a \in E_1,\ \forall r \in T,\ a \in E_1$

Definition 2.3 [13] Let U be a universal set. The *neutrosophic U*, in short *Ne (U)* is defined as

$$H=\{(\xi,\ t_H(\xi),\ i_H(\xi),\ f_H(\xi) : \xi \in U\} \ni t_H,\ i_H,\ f_H : U \to [0, 1].$$

Remark 2.4 t_H denotes the percentage of truth, i_H denotes the percentage of indeterminacy and f_H denotes the percentage of falsity.

Remark 2.5 V^U denotes the set of all neutrosophic subsets of U.

Definition 2.6 [13] Let U be an initial universe and if we take Ne(H_1) and Ne(H_2) be two neutrosophic subsets of U. Then Ne(H_1)⊆Ne(H_2) $H_1 \subseteq H_2$ if and only if

$$t_{Ne}(H_1) \leq t_{Ne}(H_2),\ i_{Ne}(H_1) \leq i_{Ne}(H_2),\ f_{Ne}(H_1) \geq f_{Ne}(H_2).$$

Definition 2.7 [13] Let $(T, +, .)$ be a ring and let *Ne(T)* be a neutrosophic set by T and I. So $Ne(T) = \{T(I), +, .\}$ is a neutrosophic ring.

i.e. the set $< T \cup I > = \{t_1 + t_2 I : t_1, t_2 \in T\}$ is a neutrosophic ring generated by T and I with operation of T such that I represented the percentage of determinacy.

Definition 2.8 [13] If we have $N_1e(T)$ as a neutrosophic ring and if we take $N_2e(T)$ as a subset of $N_1e(T)$, we define $N_1e(T)$ as a neutrosophic subring precisely when

1)$N_2e(T) \neq \varphi$

2)$N_2e(T)$ itself is a neutrosophic ring.

3) $N_2e(T)$ must has a proper subset which is a ring.

We know that if *Ne(T)* is a neutrosophic ring and such that *J* is an ideal of *T*. Hence *Ne(J)* is called the neutrosophic ideal of neutrosophic ring *T* if :

$$j_1 - j_2 \in Ne(J) \ni j_1 \in Ne(j) \; and \; j_2 \in Ne(J).$$

$$rj \,, jr \in Ne\,(J) \ni r \in Ne(T) \; and \; j \in Ne(J).$$

Definition 2.9 [14]. Let (*E*,+,.) be a module over the ring *T*. Then (*E*(*I*),+,.) is called a weak neutrosophic module over the ring *T*, and it is called a strong neutrosophic module if it is a module over the neutrosophic ring *T*(*I*).

Definition 2.10 [15]. Let $P=\{(t_P(\eta), i_P(\eta), f_P(\eta)): \eta \in R\}$ be an Ne(R). Then P is called a neutrosophic ideal of R if it satisfies the following conditions $\forall \; \eta, \; \theta \in R$ *Ne*(*E*) be a neutrosophic of module over *Ne*(*T*). Then any neutrosophic subset *Ne(K)* of *Ne(E)* is called neutrosophic submodule if:

(1) $t_P(\eta - \theta) \geq t_P(\eta) \wedge t_P(\theta)$

(2) $i_P(\eta - \theta) \geq i_P(\eta) \wedge i_P(\theta)$

(3) $f_P(\eta - \theta) \leq f_P(\eta) \vee f_P(\theta)$

(4) $t_P(\eta\theta) \geq t_P(\eta) \vee t_P(\theta)$

(5) $i_P(\eta\theta) \geq i_P(\eta) \vee i_P(\theta)$

(6) $f_P(\eta\theta) \leq f_P(\eta) \wedge f_P(\theta)$

Note that any neutrosophic set *Ne(K)* in *E* is called a neutrosophic submodule if

$K(0) = U : t_k(0) = 1, i_k(0) = 1 \; and \; f_k(0) = 0.$

$K(a + b) \geq k(a) \wedge k(b) \; a, b \in E$:

$t_k(a + b) \geq t_k(a) \wedge t_k(b), i_k(a + b) \geq i_k(b) \; and \; f_k(a + b) \leq f_k(a) \vee f(_k(b).$

$k(ra) \geq k(a) \,, a \in E, r \in T$:

$t_k(ra) \geq t_k(a), i_k(ra) \geq i_k(a)$ and $f(ra) \leq f(a)$.

Remark 2.11 More details on neutrosophic module and neutrosophic submodule are discussed by Ameri [16].

3. Neutrosophic Multiplication Module

In this section, we define the concept of a neutrosophic multiplication module over a neutrosophic ring. We investigate and obtain some results on the relationship between neutrosophic multiplication module and other concepts.

Definition 3.l Let *E* be a neutrosophic *T-module*. Then *E* is called the neutrosophic multiplication module in case for every *Ne(K)* of *Ne(E)*, $\exists$ $Ne(J)$ an neutrosophic ideal of *Ne(T)* such that

$$Ne(K) = Ne(J)\,Ne(E).$$

Here, we consider neutrosophic multiplication module *E* over neutrosophic invariant rings *Ne(T)*.

Definition 3.2 A ring *T* is called the neutrosophic invariants ring if every right (left) neutrosophic ideal is a neutrosophic ideal *Ne(J)*.

Theorem 3.3 Let *E* be a neutrosophic multiplication module *Ne(E)* over neutrosophic ring *T*. If *K* is a neutrosophic submodule of *Ne(E)* such that

$$Ne(K) \cap Ne(E)Ne(J) = Ne(K)Ne(J)$$

and *Ne(J)* is a neutrosophic ideal of *Ne(E)*, then *Ne(K)* is a neutrosophic multiplication module.

Proof:

Let $Ne(H) \leq Ne(K)$. Since Ne(E) is a neutrosophic multiplications module, there exists a

Ne(J) of *Ne(T)* $\ni$ *Ne(H) = Ne(E) Ne(J)*.

We have *Ne(K)* $\cap$ *Ne(E) Ne (J) = Ne(K) Ne(J)*.

Then

$$Ne(H) = Ne(E)Ne(J) \subseteq Ne((K) \cap Ne(E)Ne(J)$$

$$= Ne(K)Ne(J) \subseteq Ne(E)Ne(J)$$

$$= Ne(H)$$

Thus

$$Ne(H) = Ne(K)Ne(J)$$

Hence K is a neutrosophic multiplication module.

Definition 3.4 A T-module E is called neutrosophic cyclic module if $Nec(E) = Ne(E)x\ (I) \ni x(I)$ is a neutrosophic $(x(I) = y + ZI)$.

Theorem 3.5. Let E be a neutrosophic multiplication module over neutrosophic ring T and let J be a neutrosophic maximal ideal of T. Then $\frac{Ne(E)}{Ne(E)Ne(J)}$ is a neutrosophic cyclic module with at most two neutrosophic submodules and *Ne(E)=Ne(E)Ne(J)* or *Ne(E)Ne(T)* is a neutrosophic maximal submodule of *Ne(E)*.

Proof:

We know that $\frac{Ne(E)}{Ne(E)Ne(J)}$ is a neutrosophic multiplication module over simple neutrosophic ring of $\frac{T}{J}\left(Ne\left(\frac{T}{J}\right)\right)$. If $\frac{Ne(E)}{Ne(E)Ne(I)} = 0$, then the $\frac{Ne(E)}{Ne(E)Ne(I)}$ is a cyclic with only one neutrosophic submodule. If $\frac{Ne(E)}{Ne(E)Ne(I)} \neq 0$ then *Ne(E)Ne(J)* is a neutrosophic maximal submodule of neutrosophic module E (*Ne(E)*). Note that $\frac{Ne(E)}{Ne(E)Ne(J)}$ is a neutrosophic cyclic module having only two neutrosophic submodules.

Theorem 3.6 Let T be a neutrosophic ring with commutative neutrosophic multiplication ideals, *Ne(E)* be a neutrosophic multiplication T-module and J be a neutrosophic maximal ideal of *Ne(T)*. If J does not contain neutrosophic annihilator of any neutrosophic cyclic submodule of *Ne(E)*, then *Ne(K) = Ne(K) Ne(J)* for every neutrosophic cyclic submodule of *Ne(E).*

Proof:

Suppose that K be a neutrosophic cyclic submodule of neutrosophic module E, i.e. *Ne(K) ≤ Ne(E)*. We have *Ne(r)(Ne(K))* $\not\subset$ J where J is a neutrosophic maximal ideal, and *T = J + Ne(r)(Ne (K))*.

Thus

Ne(K) = Ne(K) Ne(T)

= Ne(K) (Ne(J) + Ne(r)(Ne(K))

= Ne(K) Ne(J) + Ne(K)Ne(r)(Ne(K))

= Ne(K) (Ne(J).

Corollary 3.7 For a neutrosophic module *E* over a neutrosophic ring, if for every neutrosophic submodule *K* of a neutrosophic module *E* ($Ne(K) \leq Ne(E)$), there exists a set $\{k_i\};\ i \in I$ of neutrosophic ideals of *T* such that $Ne(K) = \sum_{i\in I} Ne(K_i)$ and $Ne(K_c) = Ne(E)Ne(J);\ i \in I,$ then E is a neutrosophic multiplication module.

Proof:

Suppose that *Ne(K)* is a submodule of *Ne(E).* There exists $Ne\{k_i\}$ and $Ne(J_i)$ of

$$Ne(T) \ni Ne(Ki) = \sum Ne(k_i)\ and\ K_i = Ne(E)Ne(J) \forall i \in I.$$

Let $Ne(J) = \sum Ne(J_i)$.

Hence

$$Ne(K) = \sum Ne(K_i) = \sum Ne(E)Ne(J_i) = Ne(t)(\sum Ne(J_i))$$

$$= Ne(E)Ne(J).$$

Thus *E* is a neutrosophic multiplication module.

Recall that a module *E* is called neutrosophic artinian module if *E* satisfy neutrosophic descending chain condition. *E* is neutrosophic divisible module if $Ne(r)Ne(E) = Ne(E)$ for every $0 \neq r \in Ne(T)$.

Theorem 3.8 Let *E* be n neutrosophic artinan multiplication module. Then if *Ne(J(E))* is a small neutrosophic submodule of *Ne(E)*, then *Ne(E)* is a neutrosophic cyclic module.

Proof:

Since $\left(\frac{Ne(E)}{Ne(J(E))}\right)$ is a neutrosophic cyclic module over neutrosophic submodule *K* of neutrosophic module *E* ($Ne(K) \leq Ne(E)$) $\ni$ $Ne(E) = Ne(K) + Ne(J(E))$, so *Ne(J(E))* is a small neutrosophic of *Ne(E)* ($Ne(J(E)) << Ne(E)$). Hence $Ne(E) = Ne(K)$. Then *E* is a neutrosophic cyclic module**.**

Corollary 3.9 For a neutrosophic artinian multiplication module *E*, if *Ne(E)* is a neutrosophic finitely generated module, then *Ne(E)* is a neutrosophic cyclic module.

Proof:

Suppose that E is a neutrosophic finitely generated module. Then $Ne(J(E))$ is a neutrosophic small submodule of $Ne(E)$ ($Ne(J(E)) \leq Ne(E)$). Thus from Theorem 3.8, $Ne(E)$ is a neutrosophic cyclic module.

Note that a module E is called neutrosophic semi-prime submodule if for each $Ne(r) \in Ne(T)$, $Ne(x) \in Ne(E)$, $Ne(s) \in Ne(Z^+)$ with $Ne(r^k) Ne(x) \in K$ implies that $Ne(r) Ne(x) \in Ne(K)$.

Proposition 3.10 Let E be a neutrosophic multiplication module. Then K is a neutrosophic semi-prime submodule of E if and only if $Ne(r) Ne(K) = Ne(K)$.

Proof:

$\Rightarrow$

We know that $Ne(k) \subseteq Ne(r) (Ne(K))$, where K is a neutrosophic submodule of E. Suppose that K is a neutrosophic semi-prime submodule of E and let $Ne(a) \in N(r) (Ne(K))$. Thus, for some $k \in Z^+$; $(Ne(a))^k \subseteq Ne(K)$. Now for some $Ne(a) \in Ne(K)$ and $Ne(K)$ being a neutrosophic semi-prime submodule, we then obtain $Ne(K) = Ne(r) (Ne(K))$.

$\Leftarrow$

Suppose that $Ne(r) / Ne(K) = Ne(k)$ and let $(Ne(a))^n \subseteq Ne(K); n \in Z^+$. Therefore some $Ne(a) \in Ne(K)$. Thus, we get $Ne(K)$ to be a neutrosophic semi-prime submodule of Ne(E).

Corollary 3.11 Let E be a neutrosophic divisible module over neutrosophic integral domain. Then E is a neutrosophic multiplication module if and only if E is a neutrosophic cyclic module.

Proof:

$\Rightarrow$

It is clear that every neutrosophic cyclic module is neutrosophic multiplication module.

$\Leftarrow$

Assume that E is a neutrosophic multiplication module. Let $0 \neq K$ be a neutrosophic submodule of E. So there exists a neutrosophic $Ne(J)$ such that

$$Ne(K) = Ne(J)Ne(E) = Ne(E)$$

Definition 3.12 Let U be an initial universe. If $Ne(H_1)$ and $Ne(H_2)$ are two neutrosophic subsets of U, then $Ne(s) = Ne(H_1) \cap Ne(H_2)$ is also neutrosophic defined as follows.

$t_{Ne(s)}(K) = min(t_{Ne(H_1)}(K), t_{Ne(H_2)}(K))$

$I_{Ne(s)}(K) = min(I_{Ne(H_1)}(K), I_{Ne(H_2)}(K))$

$f_{Ne(s)}(K) = min(f_{Ne(H_1)}(K), f_{Ne(H_2)}(K))$

$\forall k \in U, t(K)_{Ne(H_1)}, I(K)_{Ne(H_1)}, f(K)_{Ne(H_1)} \in [0,1],$

$\forall k \in U, t(K)_{Ne(H_2)}, I(K)_{Ne(H_2)}, f(K)_{Ne(H_2)} \in [0,1][1,0].$

Theorem 3.13 Let E be a neutrosophic multiplication T-module and let K be a neutrosophic prime submodule of E. If $\mathrm{K}_1, \mathrm{K}_2, \dots, \mathrm{K}_\mathrm{n}$ are neutrosophic submodules of E, then the following are equivalent.

(1) $Ne(K_j) \subseteq Ne(K)$, $1 \le j \le K$ $Ne(K_j) \subseteq Ne(K)$.

(2) Neutrosophic of the intersect of $K_j \subseteq Ne(K)$.

(3) $Ne(\pi_{i=1}^{n}(K_r) \subseteq Ne(K)$.

Proof:

(1) $\Rightarrow$(2): Obvious.

(2) $\Rightarrow$(3): We know that from (2), $Ne(\pi_{i=1}^{n}(K_r) \subseteq Ne(\cap(K_i)) \subseteq Ne(k)$

(3) $\Rightarrow$(1): For some J_i , $1 \le i \le n$ such that I_j an ideal of T, we have $Ne(k_i) = Ne(J_i)Ne(E)$. Hence $Ne(k_1,k_2,\dots.k_n) = Ne(J_1J_2\dots\dots J_n\, Ne(E) \subseteq Ne(k)$. Then $Ne\,(J_1J_2\dots..J_n) \subseteq Ne\,(k_iE)$. But $Ne\,(k_iE)$ is a prime neutrosophic ideal of T, i.e. $Ne(P.I) \subseteq Ne\,(k_iE)$ for some $1 \le i \le n$. Thus $Ne(k_i) = Ne(J_i)Ne(E) \subseteq Ne(k)$ for some i , $1 \le i \le n$.

Definition 3.14. Let E be a neutrosophic multiplication T-module. A non-empty neutrosophic subset S^* of $Ne(E)$ is called neutrosophic multiplicatively closed, $Ne(MC)$.

Theorem 3.15. Suppose that E is a neutrosophic T- module. Then the following are equivalent.

(1) K is a proper neutrosophic prime submodule of $Ne(E)$.

(2) $Ne(\frac{E}{k})$ is a $Ne(M.C)$.

Proof:

Suppose that condition (1) is true. Let m_1, $m_2 \in Ne(\frac{E}{k})$. From condition (1), we have k is a neutrosophic prime submodule and $m_1, m_2 \not\subset Ne(k)$. So $m_1, m_2 \cap Ne(\frac{E}{k}) \neq \emptyset$.

Now suppose that condition (2) is true. Let m_1, $m_2 \notin Ne(k)$. Hence $m_1, m_2 \in Ne(\frac{E}{k})$. But $Ne(\frac{E}{k})$ is a $Ne(M.C)$, $m_1, m_2 \cap Ne(\frac{E}{k}) \neq \emptyset$. Thus $m_1, m_2 \not\subset Ne(k)$ (see[10]).

4. Conclusion

Neutrosophic module is one of many important concepts in module theory. In this paper we have defined neutrosophic multiplication *T*-module as an algebraic structure. Some basic properties have been introduced. It has been shown that if a neutrosophic Artinian multiplication module is a neutrosophic cyclic, then it is a neutrosophic finitely generated. The main result is if *E* is a neutrosophic divisible module over neutrosophic integral domain, then *E* is a neutrosophic multiplication module if and only if *E* is a neutrosophic cyclic module. Our future research is to further develop more types of neutrosophic multiplication modules, such as those on Q-fuzzy [17-20], Q-neutrosophic [21-28], soft intuitionistic [29], multiparameterized soft set [30], vague soft set [31-32], neutrosophic bipolar [33], neutrosophic cubic [34] and to be used in neurogenetic algorithms [35], numerical analysis for root convergence [36-41] interval complex neutrosophic [42,43] and some algebraic structures [44-46].

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