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The neutrosophic integrals by partial fraction

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Abstract: The purpose of this article is to study the neutrosophic integrals by partial fraction, where the neutrosophic fraction function is defined, in addition, four cases of the neutrosophic proper rational function were discussed, also, integral of the neutrosophic improper rational functions were introduced.Where detailed examples were given to clarify each case.

Keywords: neutrosophic partial fraction; neutrosophic proper rational function; neutrosophic integrals; neutrosophic improper rational functions.

1. Introduction

As an alternative to the existing logics, Smarandache proposed the Neutrosophic Logic to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction, where the concept of neutrosophy is a new branch of philosophy introduced by Smarandache [3-13]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist, he defined the standard form of neutrosophic complex number, and found root index $n \geq 2$ of a neutrosophic real and complex number [2-4], studying the concept of the Neutrosophic probability [3-5], the Neutrosophic statistics [4][6], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1-8]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [9]. Edalatpanah proposed a new direct algorithm to solve the neutrosophic linear programming where the variables and right-hand side represented with triangular neutrosophic numbers [10]. Chakraborty used pentagonal neutrosophic number in networking problem, and Shortest Path Problem [11-12]. Y.Alhasan studied the concepts of neutrosophic complex numbers, the general exponential form of a neutrosophic complex, the neutrosophic integrals and integration methods, and the neutrosophic integrals by parts [7-14-21-22]. On the other hand, M.Abdel-Basset presented study in the science of neutrosophic about an approach of TOPSIS technique for developing supplier selection with group decision making under type-2 neutrosophic number [15].

Also, neutrosophic sets played an important role in applied science such as health care, industry, and optimization [16-17-18-19].

Integration is important in human life, and one of its most important applications is the calculation of area, size and arc length. In our reality we find things that cannot be precisely defined, and that contain an indeterminacy part.

Paper consists of 4 sections. In 1th section, provides an introduction, in which neutrosophic science review has given. In 2th section, some definitions and theories of The neutrosophic integrals and are discussed. The 3th section frames neutrosophic integrals by partial fraction, in which four cases of the neutrosophic proper rational function were discussed, also, integral of the neutrosophic improper rational functions were introduced. In 4th section, a conclusion to the paper is given.

2. Preliminaries

2.1. Neutrosophic integration by substitution method [24]

Definition2.1.1

Let $f: D_f \subseteq R \to R_f \cup \{I\}$, to evaluate $\int f(x)dx$

Put: $x = g(u) \quad \Rightarrow \quad dx = \acute{g}(u)du$

By substitution, we get:

$$\int f(x)dx = \int f(u)\acute{g}(u)du$$

then we can directly integral it.

Theorme2.1.1:

If $\int f(x,I)dx = \varphi(x,I)$ *then*,

$$\int f\big((a+bI)x + c + dI\big)\,dx = \left(\frac{1}{a} - \frac{b}{a(a+b)}I\right)\varphi\big((a+bI)x + c + dI\big) + C$$

where C is an indeterminate real constant, $a \neq 0\,, a \neq -b$ *and* b, c, d are real numbers, while $I =$ indeterminacy.

Theorme2.1.2:

Let $f: D_f \subseteq R \to R_f \cup \{I\}$ then:

$$\int \frac{\acute{f}(x,I)}{f(x,I)}dx = ln|f(x,I)| + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I =$ indeterminacy).

Theorme2.1.3:

Let $f: D_f \subseteq R \to R_f \cup \{I\}$, then:

$$\int \frac{\acute{f}(x,I)}{\sqrt{f(x,I)}}dx = 2\sqrt{f(x,I)} + C$$

where C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while $I =$ indeterminacy).

Theorme2.1.4:

$f: D_f \subseteq R \to R_f \cup \{I\}$, then:

$$\int [f(x,I)]^n \acute{f}(x)\,dx = \frac{[f(x,I)]^{n+1}}{n+1} + C$$

Where n is any rational number. C is an indeterminate real constant (i.e. constant of the form $a + bI$, where a, b are real numbers, while I = indeterminacy).

2.2. Integrating products of neutrosophic trigonometric function [24]

I. $\int \sin^m(a+bI)x \cos^n(a+bI)x\ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following two cases:

- Case n is odd:
  - Split of $\cos(a+bI)x$
  - Apply $\cos^2(a+bI)x = 1 - \sin^2(a+bI)x$
  - We substitution $u = \sin(a+bI)x$
- Case m is odd:
  - Split of $\sin(a+bI)x$
  - Apply $\sin^2(a+bI)x = 1 - \cos^2(a+bI)x$
  - We substitution $u = \cos(a+bI)x$

II. $\int \tan^m(a+bI)x \sec^n(a+bI)x\ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case n is even:
  - Split of $\sec^2(a+bI)x$
  - Apply $\sec^2(a+bI)x = 1 + \tan^2(a+bI)x$
  - We substitution $u = \tan(a+bI)x$
- Case m is odd:
  - Split of $\sec(a+bI)x \tan(a+bI)x$
  - Apply $\tan^2(a+bI)x = \sec^2(a+bI)x - 1$
  - We substitution $u = \sec(a+bI)x$
- Case m even and n odd:
  - Apply $\tan^2(a+bI)x = \sec^2(a+bI)x - 1$
  - We substitution $u = \sec(a+bI)x$ or $u = \tan(a+bI)x$, depending on the case.

III. $\int \cot^m(a+bI)x \csc^n(a+bI)x\ dx$, where m and n are positive integers.
To find this integral, we can distinguish the following cases:

- Case n is even:
  - Split of $\csc^2(a+bI)x$
  - Apply $\csc^2(a+bI)x = 1 + \cot^2(a+bI)x$
  - We substitution $u = \cot(a+bI)x$
- Case m is odd:
  - Split of $\csc(a+bI)x \cot(a+bI)x$
  - Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
  - We substitution $u = \csc(a+bI)x$

- Case m even and n odd:
  - Apply $\cot^2(a+bI)x = \csc^2(a+bI)x - 1$
  - We substitution $u = \csc(a+bI)x$ or $u = \cot(a+bI)x$, depending on the case.

2.3. Neutrosophic trigonometric identities [24]

1) $sin(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI)x + \sin(a+bI-c-dI)x]$

2) $cos(a+bI)x\sin(c+dI)\,x = \frac{1}{2}\,[\sin(a+bI+c+dI)x - \sin(a+bI-c-dI)x]$

3) $cos(a+bI)x\cos(c+dI)\,x = \frac{1}{2}\,[\cos(a+bI+c+dI)x + \cos(a+bI-c-dI)\,x]$

4) $sin(a+bI)x\sin(c+dI)\,x = \frac{-1}{2}\,[\cos(a+bI+c+dI)x - \cos(a+bI-c-dI)x]$

Where $a \neq c\ (not\ zero)\ and\ b, d$ are real numbers, while I = indeterminacy.

3. The neutrosophic integrals by partial fraction

Definition3.1
A polynomial whose coefficients (at least one of them containing I) are neutrosophic numbers is called neutrosophic real polynomials, and take the form:

$$P(x,I) = (a_0 + b_0 I) + (a_1 + b_1 I)x + (a_2 + b_2 I)x^2 + \cdots + (a_n + b_n I)x^n$$

Where $a_0, b_0, a_1, b_1, a_2, b_2 \dots a_n, b_n$ are real number, I represent indeterminacy and n.is positive integer.

Definition3.2
Neutrosophic fraction function is a function which can be written in the form of:

$$f(x,I) = \frac{P(x,I)}{Q(x,I)}$$

Where $P(x,I)$, $Q(x,I)$ are neutrosophic real polynomials and $Q(x,I) \neq 0$, the numerator or denominator, at least, can be a neutrosophic real polynomials.

Example3.1:

1) $f(x,I) = \dfrac{(3+7I)x^3 + 4Ix - 2}{2Ix + 8 - 5I}$

2) $f(x,I) = \dfrac{(7+2I)x}{(3+7I)x^2 + 4Ix - 2}$

3) $f(x,I) = \dfrac{1}{(3+7I)x^2 + 4Ix - 2}$

Remark3.1

- If degree of $P(x,I)$ is less than degree of $Q(x,I)$, then: $f(x,I) = \frac{P(x,I)}{Q(x,I)}$ is an neutrosophic proper rational function.
- If degree of $P(x,I)$ is greater than degree of $Q(x,I)$, then: $f(x,I) = \frac{P(x,I)}{Q(x,I)}$ is an neutrosophic improper rational function.

3.1 Integral of the neutrosophic proper rational functions

3.1.2 There are four cases of the neutrosophic proper rational function

- state1: When the denominator can be expressed as the product of non-repeated linear factors.

Let $Q(x,I) = \big((a_1+b_1I)x+c_1+d_1I\big)\big((a_2+b_2I)x+c_2+d_2I\big)\ldots\big((a_n+b_nI)x+c_n+d_nI\big)$, then we can write:

$$\frac{P(x,I)}{Q(x,I)}=\frac{A_1}{(a_1+b_1I)x+c_1+d_1I}+\frac{A_2}{(a_2+b_2I)x+c_2+d_2I}+\cdots+\frac{A_n}{(a_n+b_nI)x+c_n+d_nI}$$

Where $A_1, A_2, \ldots, A_n$ are constants whose values are to be determined.

- State2: When the denominator can be expressed as the product of repeated linear factors.

Let $Q(x,I) = \big((a+bI)x+c+dI\big)\big((a+bI)x+c+dI\big)\ldots\big((a+bI)x+c+dI\big)$
$= \big((a+bI)x+c+dI\big)^n$, then we can write:

$$\frac{P(x,I)}{Q(x,I)}=\frac{A_1}{(a+bI)x+c+dI}+\frac{A_2}{\big((a+bI)x+c+dI\big)^2}+\cdots+\frac{A_n}{\big((a+bI)x+c+dI\big)^n}$$

Where $A_1, A_2, \ldots, A_n$ are constants whose values are to be determined.

- State3: When the denominator can be expressed as the product of repeated and non-repeated linear factors.

Let $Q(x,I) = \big((a_1+b_1I)x+c_1+d_1I\big)\big((a_2+b_2I)x+c_2+d_2I\big)\ldots\big((a_n+b_nI)x+c_n+d_nI\big)\big((a+bI)x+c+dI\big)^m$, then we can write:

$$\frac{P(x,I)}{Q(x,I)}=\frac{A_1}{(a_1+b_1I)x+c_1+d_1I}+\frac{A_2}{(a_2+b_2I)x+c_2+d_2I}+\cdots+\frac{A_n}{(a_n+b_nI)x+c_n+d_nI}$$
$$+\frac{B_1}{(a+bI)x+c+dI}+\frac{B_2}{\big((a+bI)x+c+dI\big)^2}+\cdots+\frac{B_m}{\big((a+bI)x+c+dI\big)^m}$$

Where $A_1, A_2, \ldots, A_n,\ B_1, B_2, \ldots, B_m$ are constants whose values are to be determined.

- State4: When the denominator can be expressed as the product of non-repeated quadratic factors which cannot be further factorized to linear factors.

Let $Q(x,I) = \big((a_1+b_1I)x^2+(c_1+d_1I)x+e_1+k_1I\big)\big((a_2+b_2I)x^2+(c_2+d_2I)x+e_2+k_2I\big)\ldots\big((a_n+b_nI)x^2+(c_n+d_nI)x+e_n+k_nI\big)$, then we can write:

$$\frac{P(x,I)}{Q(x,I)}=\frac{A_1x+B_1}{(a_1+b_1I)x^2+(c_1+d_1I)x+e_1+k_1I}+\frac{A_2x+B_2}{(a_2+b_2I)x^2+(c_2+d_2I)x+e_2+k_2I}+\cdots$$
$$+\frac{A_nx+B_n}{(a_n+b_nI)x^2+(c_n+d_nI)x+e_n+k_nI}$$

Where $A_1, A_2, \ldots, A_n,\ B_1, B_2, \ldots, B_n$ are constants whose values are to be determined.

3.1.2 Algorithm for finding Integral of the neutrosophic proper rational functions

To evaluate $\int \frac{P(x,I)}{Q(x,I)}$ we follow the following steps:

1) Reformulate the form of the function in one of the previous four cases according to the form of the denominator as sum of neutrosophic partial fractions.
2) Integrate both sides.

Example3.1.1:

Evaluate:

$$\int \frac{dx}{(x+2-I)(x+4+2I)}$$

Solution:

$$\frac{1}{(x+2-I)(x+4+2I)} = \frac{A}{x+2-I} + \frac{B}{x+4+2I} \quad (*)$$

To find value of A, We multiply both sides $(*)$ by $(x+2-I)$:

$$\frac{1}{x+4+2I} = A + \frac{(x+2-I)B}{x+4+2I} \quad (1)$$

by substitution $x = -2+I$ in (1), we get:

$$A = \frac{1}{-2+I+4+2I} = \frac{1}{2+3I}$$

To find value of B, We multiply both sides $(*)$ by $(x+4+2I)$:

$$\frac{1}{x+2-I} = \frac{(x+4+2I)A}{x+2-I} + B \quad (2)$$

by substitution $x = -4-2I$ in (2), we get:

$$B = \frac{1}{-4-2I+2-I} = \frac{1}{-2-3I}$$

by substitution in (*) we get:

$$\frac{1}{(x+2-I)(x+4+2I)} = \frac{\frac{1}{2+3I}}{x+2-I} + \frac{\frac{-1}{2+3I}}{x+4+2I}$$

$$\Rightarrow \int \frac{dx}{(x+2-I)(x+4+2I)}dx = \int \left(\frac{\frac{1}{2+3I}}{x+2-I} + \frac{\frac{-1}{2+3I}}{x+4+2I} \right) dx$$

$$= \frac{1}{2+3I} ln|x+2-I| - \frac{1}{2+3I} ln|x+4+2I|$$

$$= \frac{1}{2+3I} (ln|x+2-I| - ln|x+4+2I|)$$

$$= \frac{1}{2+3I} ln\left|\frac{x+2-I}{x+4+2I}\right|$$

$$=\left(\frac{1}{2}-\frac{3}{10}I\right)ln\left|\frac{x+2-I}{x+4+2I}\right|+C$$

Example3.1.2:

Evaluate:

$$\int\frac{5+2Idx}{((2+I)x+2-I)((-1+2I)x+4+2I)}$$

Solution:

$$\frac{5+2I}{((2+I)x+2-I)((-1+2I)x+4+2I)}=\frac{A}{(2+I)x+2-I}+\frac{B}{(-1+2I)x+4+2I}\quad(*)$$

To find value of A, We multiply both sides $(*)$ by $((2+I)x+2-I)$:

$$\frac{5+2I}{(-1+2I)x+4+2I}=A+\frac{((2+I)x+2-I)B}{(-1+2I)x+4+2I}\quad(1)$$

by substitution $x=\frac{-2+I}{2+I}=-1+\frac{2}{3}I$ in (1), we get:

$$A=\frac{5+2I}{(-1+2I)(-1+\frac{2}{3}I)+4+2I}=\frac{5+2I}{5+\frac{2}{3}I}=1+\frac{4}{17}I$$

To find value of B, We multiply both sides $(*)$ by $((-1+2I)x+4+2I)$:

$$\frac{5+2I}{(2+I)x+2-I}=\frac{((-1+2I)x+4+2I)A}{(2+I)x+2-I}+B\quad(2)$$

by substitution $x=\frac{-4-2I}{-1+2I}=4-10I$ in (2), we get:

$$B=\frac{5+2I}{(2+I)(4-10I)+2-I}=\frac{5+2I}{10-25I}=\frac{1}{2}-\frac{29}{30}I$$

by substitution in (*) we get:

$$\frac{5+2I}{((2+I)x+2-I)((-1+2I)x+4+2I)}=\frac{1+\frac{4}{17}I}{(2+I)x+2-I}+\frac{\frac{1}{2}-\frac{29}{30}I}{(-1+2I)x+4+2I}$$

$$\Rightarrow\int\frac{5+2Idx}{((2+I)x+2-I)((-1+2I)x+4+2I)}=\int\left(\frac{1+\frac{4}{17}I}{(2+I)x+2-I}+\frac{\frac{1}{2}-\frac{29}{30}I}{(-1+2I)x+4+2I}\right)dx$$

$$=\frac{1+\frac{4}{17}I}{2+I}ln|(2+I)x+2-I|+\frac{\frac{1}{2}-\frac{29}{30}I}{-1+2I}ln|(-1+2I)x+4+2I|$$

$$=\left(\frac{1}{2}-\frac{3}{2}I\right)ln|(2+I)x+2-I|+\left(-\frac{1}{2}+\frac{1}{30}I\right)ln|(-1+2I)x+4+2I|$$

Example3.1.3:

Evaluate:

$$\int \frac{3x-5+4I}{(x+1+I)^2(x+3-2I)}dx$$

Solution:

$$\frac{3x-5+4I}{(x+1+I)^2(x+3-2I)} = \frac{A}{x+1+I} + \frac{B}{(x+1+I)^2} + \frac{D}{x+3-2I} \quad (*)$$

To find value of C, We multiply both sides $(*)$ by $(x+3-2I)$:

$$\frac{3x-5+4I}{(x+1+I)^2} = \frac{(x+3-2I)A}{x+1+I} + \frac{(x+3-2I)B}{(x+1+I)^2} + D \quad (1)$$

by substitution $x=-3+2I$ in (1), we get:

$$D = \frac{-9+6I-5+4I}{(3+2I+1+I)^2} = \frac{-14+10I}{(-2+3I)^2}$$

$$= \frac{-14+10I}{4-12I+9I} \Rightarrow \quad D = \frac{-14+10I}{4-3I}$$

To find value of B, We multiply both sides $(*)$ by $(x+1+I)^2$:

$$\frac{3x-5+4I}{x+3-2I} = \frac{(x+1+I)^2A}{x+1+I} + B + \frac{(x+1+I)^2C}{x+3-2I} \quad (2)$$

by substitution $x=-1-I$ in (2), we get:

$$B = \frac{-3-3I-5+4I}{-1-I+3-2I} = \frac{-8+I}{2-3I}$$

To find value of A, we substitute value of B, D and any value of x so that it does not nullify the denominator in (*), let it be $x=0$, we get:

$$\frac{-5+4I}{(1+I)^2(3-2I)} = \frac{A}{1+I} + \frac{\frac{-8+I}{2-3I}}{(1+I)^2} + \frac{\frac{-14+10I}{4-3I}}{3-2I}$$

$$\frac{-5+4I}{3+I} = \frac{A}{1+I} + \frac{-8+I}{2-6I} + \frac{14+10I}{12-11I}$$

$$\frac{A}{1+I} = \frac{7}{2} - \frac{3}{2}I$$

$$A = (1+I)\left(\frac{7}{2} - \frac{3}{2}I\right) = \frac{7}{2} - \frac{1}{2}I$$

by substitution in (*) we get:

$$\frac{3x-5+4I}{(x+1+I)^2(x+3-2I)} = \frac{\frac{7}{2}-\frac{1}{2}I}{x+1+I} + \frac{\frac{-8+I}{2-3I}}{(x+1+I)^2} + \frac{\frac{-14+10I}{4-3I}}{x+3-2I}$$

$$\Rightarrow \int \frac{3x-5+4I}{(x+1+I)^2(x+3-2I)}dx = \int \left(\frac{\frac{7}{2}-\frac{1}{2}I}{x+1+I} + \frac{\frac{-8+I}{2-3I}}{(x+1+I)^2} + \frac{\frac{-14+10I}{4-3I}}{x+3-2I} \right) dx$$

$$= \left(\frac{7}{2} - \frac{1}{2}I\right) ln|x + 1 + I| + \frac{-8 + I}{2 - 3I} . \frac{-1}{x + 1 + I} + \frac{-14 + 10I}{4 - 3I} ln|x + 3 - 2I|$$

$$= \left(\frac{7}{2} - \frac{1}{2}I\right) ln|x + 1 + I| + (4 - 11I). \frac{-1}{x + 1 + I} - \left(\frac{7}{2} - \frac{1}{2}I\right) ln|x + 3 - 2I|$$

$$= \left(\frac{7}{2} - \frac{1}{2}I\right) ln\left|\frac{x + 1 + I}{x + 3 - 2I}\right| + (4 - 11I). \frac{-1}{x + 1 + I} + C$$

Example3.1.4:

Evaluate:

$$\int \frac{5I}{x^2 - 4 + 3I} dx$$

Solution:
to find the denominator factors

$$x^2 - 4 + 3I = x^2 - \left(\sqrt{4 - 3I}\right)^2$$

Let's find $\sqrt{4 - 3I}$

$$\sqrt{4 - 3I} = \alpha + \beta I$$
$$4 - 3I = \alpha^2 + 2\alpha\beta I + \beta^2 I$$

$$4 - 3I = \alpha^2 + (2\alpha\beta + \beta^2)I$$

then:

$$\begin{cases} \alpha^2 = 4 \\ 2\alpha\beta + \beta^2 = -3 \end{cases}$$

$$\begin{cases} \alpha = \pm 2 \\ \alpha^2 + 2\alpha\beta + 3 = 0 \end{cases}$$

Find β:

- When $\alpha = 2 \quad \Rightarrow \beta^2 + 4\beta + 3 = 0$

$$(\beta + 3)(\beta + 1) = 0 \quad \Rightarrow \beta = -3 , \beta = -1$$

$$(2, -3), (2, -1)$$

- When $\alpha = -2 \quad \Rightarrow \beta^2 - 4\beta + 3 = 0$

$$(\beta - 3)(\beta - 1) = 0 \quad \Rightarrow \beta = 3 , \beta = 1$$

$$(-2,3), (-2,1)$$

Thus, the denominator factors can be written in two cases:
Case1:

$$x^2 - 4 + 3I = (x - 2 + 3I)(x + 2 - 3I)$$

$$\frac{5I}{x^2 - 4 + 3I} = \frac{5I}{(x - 2 + 3I)(x + 2 - 3I)}$$

$$\frac{5I}{(x - 2 + 3I)(x + 2 - 3I)} = \frac{A}{x - 2 + 3I} + \frac{B}{x + 2 - 3I} \quad (*)$$

To find value of A, We multiply both sides $(*)$ by $(x - 2 + 3I)$:

$$\frac{5I}{x+2-3I} = A + \frac{(x-2+3I)B}{x+2-3I} \quad (1)$$

by substitution $x = 2 - 3I$ in (1), we get:

$$A = \frac{5I}{2-3I+2-3I} = \frac{I5}{4-6I}$$

To find value of A, We multiply both sides $(*)$ by $(x+2-3I)$:

$$\frac{5I}{x-2+3I} = \frac{(x+2-3I)A}{x-2+3I} + B \quad (2)$$

by substitution $x = -2 + 3I$ in (2), we get:

$$B = \frac{5I}{-2+3I-2+3I} = \frac{I5}{-4+6I} = \frac{-I5}{4-6I}$$

by substitution in (*) we get:

$$\frac{5I}{(x-2+3I)(x+2-3I)} = \frac{\frac{I5}{4-6I}}{x-2+3I} + \frac{\frac{-I5}{4-6I}}{x+2-3I}$$

$$\Rightarrow \int \frac{5I}{(x-2+3I)(x+2-3I)}dx = \int \left(\frac{\frac{I5}{4-6I}}{x-2+3I} + \frac{\frac{-I5}{4-6I}}{x+2-3I} \right) dx$$

$$= \frac{I5}{4-6I} ln|x-2+3I| - \frac{I5}{4-6I} ln|x+2-3I|$$

$$= \frac{I5}{4-6I} (ln|x-2+3I| - ln|x+2-3I|)$$

$$= \frac{I5}{4-6I} ln \left| \frac{x-2+3I}{x+2-3I} \right|$$

$$= \left(-\frac{5}{2}I\right) ln \left| \frac{x-2+3I}{x+2-3I} \right| + C$$

Case2:

$$x^2 - 4 + 3I = (x-2+I)(x+2-I)$$

$$\frac{5I}{x^2-4+3I} = \frac{5I}{(x-2+I)(x+2-I)}$$

$$\frac{5I}{(x-2+I)(x+2-I)} = \frac{A}{x-2+I} + \frac{B}{x+2-I} \quad (*)'$$

To find value of A, We multiply both sides $(*)$ by $(x-2+I)$:

$$\frac{5I}{x+2-I} = A + \frac{(x-2+I)B}{x+2-I} \qquad (3)$$

by substitution $x = 2 - I$ in (3), we get:

$$A = \frac{5I}{2-I+2-I} = \frac{I5}{4-2I}$$

To find value of A, We multiply both sides $(*)$ by $(x+2-I)$:

$$\frac{5I}{x-2+I} = \frac{(x+2-I)A}{x-2+I} + B \qquad (4)$$

by substitution $x = -2 + I$ in (4), we get:

$$B = \frac{5I}{-2+I-2+I} = \frac{I5}{-4+2I} = \frac{-I5}{4-2I}$$

by substitution in $(*)'$, we get:

$$\frac{5I}{(x-2+I)(x+2-I)} = \frac{\frac{I5}{4-2I}}{x-2+I} + \frac{\frac{-I5}{4-2I}}{x+2-I}$$

$$\Rightarrow \int \frac{5I}{(x-2+I)(x+2-I)} dx = \int \left(\frac{\frac{I5}{4-2I}}{x-2+I} + \frac{\frac{-I5}{4-2I}}{x+2-I} \right) dx$$

$$= \frac{I5}{4-2I} ln|x-2+I| - \frac{I5}{4-2I} ln|x+2-I|$$

$$= \frac{I5}{4-2I} (ln|x-2+I| - ln|x+2-I|)$$

$$= \frac{I5}{4-2I} ln\left|\frac{x-2+I}{x+2-I}\right|$$

$$= \left(\frac{5}{2}I\right) ln\left|\frac{x-2+I}{x+2-I}\right| + C$$

Hence:

$$\int \frac{5I}{x^2-4+3I} dx = \begin{cases} \left(\frac{1}{4} - \frac{5}{2}I\right) ln\left|\frac{x-2+3I}{x+2-3I}\right| + C \\ \left(\frac{5}{4} + \frac{5}{2}I\right) ln\left|\frac{x-2+I}{x+2-I}\right| + C \end{cases}$$

Example3.1.5:

Evaluate:

$$\int \frac{1+2I}{x^2+(4-I)x+2I} dx$$

Solution:

to find the denominator factors, we write it as an equation:

$$x^2 + (4 - I)x + 2I = 0$$
$$\Delta = (4 - I)^2 - 8I = 16 - 15I$$

$$x = \frac{-(4 - I) \pm \sqrt{16 - 15I}}{2} \qquad (**)$$

$$\sqrt{16 - 15I} = \alpha + \beta I$$

$$16 - 15I = \alpha^2 + 2\alpha\beta I + \beta^2 I$$

$$16 - 15I = \alpha^2 + (2\alpha\beta + \beta^2)I$$

then:

$$\begin{cases} \alpha^2 = 16 \\ 2\alpha\beta + \beta^2 = -15 \end{cases}$$

$$\begin{cases} \alpha = \pm 4 \\ \beta^2 + 2\alpha\beta + 15 = 0 \end{cases}$$

Find β:

- When $\alpha = 4 \quad \Longrightarrow \beta^2 + 8\beta + 15 = 0$

$$(\beta + 3)(\beta + 5) \quad \Rightarrow \beta = -3\,, \beta = -5$$

$$(4, -3), (4, -5)$$

- When $\alpha = -4 \quad \Longrightarrow \beta^2 - 8\beta + 15 = 0$

$$(\beta - 3)(\beta - 5) \quad \Rightarrow \beta = 3\,, \beta = 5$$

$$(-4,3), (-4,5)$$

Then:

$$(\alpha, \beta) = (4, -3), (4, -5), (-4,3), (-4,5)$$

$$\sqrt{16 - 15I} = 4 - 3I \; or \; 4 - 5I \; or - 4 + 3I \; or - 4 + 5I$$

We can note $4 - 3I$ and $-4 + 3I$ give the same values for x. Similarly, $4 - 5I$ and $-4 + 5I$.

So, we can now to find x in (**):

$$\begin{cases} x_1 = \dfrac{-(4 - I) + 4 - 3I}{2} = -I \\ x_2 = \dfrac{-(4 - I) - 4 + 3I}{2} = -4 + 2I \end{cases}$$

$$\begin{cases} x_3 = \dfrac{-(4 - I) + 4 - 5I}{2} = -2I \\ x_4 = \dfrac{-(4 - I) - 4 + 5I}{2} = -4 + 3I \end{cases}$$

Thus, the denominator factors can be written in two cases:

Case1:

$$x^2 + (4 - I)x + 2I = (x + I)(x + 4 - 2I)$$

$$\frac{1 + 2I}{x^2 + (4 - I)x + 2I} = \frac{1 + 2I}{(x + I)(x + 4 - 2I)}$$

$$\frac{1+2I}{(x+I)(x+4-2I)}=\frac{A}{x+I}+\frac{B}{x+4-2I} \quad (*)$$

To find value of A, We multiply both sides $(*)$ by $(x+I)$:

$$\frac{1+2I}{x+4-2I}=A+\frac{(x+I)B}{x+4-2I} \quad (1)$$

by substitution $x=-I$ in (1), we get:

$$A=\frac{1+2I}{-I+4-2I}=\frac{1+2I}{4-3I}$$

To find value of A, We multiply both sides $(*)$ by $(x+4-2I)$:

$$\frac{1+2I}{x+I}=\frac{(x+4-2I)A}{x+I}+B \quad (2)$$

by substitution $x=-4+2I$ in (2), we get:

$$B=\frac{1+2I}{-4+2I+I}=\frac{1+2I}{-4+3I}$$

by substitution in (*) we get:

$$\frac{1+2I}{(x+I)(x+4-2I)}=\frac{\frac{1+2I}{4-3I}}{x+I}+\frac{\frac{1+2I}{-4+3I}}{x+4-2I}$$

$$\Rightarrow \int \frac{1+2I}{(x+I)(x+4-2I)}dx=\int\left(\frac{\frac{1+2I}{4-3I}}{x+I}+\frac{\frac{1+2I}{-4+3I}}{x+4-2I}\right)dx$$

$$=\frac{1+2I}{4-3I}ln|x+I|-\frac{1+2I}{4-3I}ln|x+4-2I|$$

$$=\frac{1+2I}{4-3I}(ln|x+I|-ln|x+4-2I|)$$

$$=\frac{1+2I}{4-3I}ln\left|\frac{x+I}{x+4-2I}\right|$$

$$=\left(\frac{1}{4}-\frac{11}{4}I\right)ln\left|\frac{x+I}{x+4-2I}\right|+C$$

Case2:

$$x^2+(4-I)x+2I=(x+2I)(x+4-3I)$$

$$\frac{1+2I}{x^2+(4-I)x+2I}=\frac{1+2I}{(x+2I)(x+4-3I)}$$

$$\frac{1+2I}{(x+2I)(x+4-3I)}=\frac{A}{x+2I}+\frac{B}{x+4-3I} \quad (*)$$

To find value of A, We multiply both sides $(*)$ by $(x+2I)$:

$$\frac{1+2I}{x+4-3I}=A+\frac{(x+2I)B}{x+4-3I} \quad (1)$$

by substitution $x=-2I$ in (1), we get:

$$A=\frac{1+2I}{-2I+4-3I}=\frac{1+2I}{4-5I}$$

To find value of A, We multiply both sides $(*)$ by $(x+4-3I)$:

$$\frac{1+2I}{x+2I}=\frac{(x+4-3I)A}{x+2I}+B \quad (2)$$

by substitution $x=-4+3I$ in (2), we get:

$$B=\frac{1+2I}{-4+3I+2I}=\frac{1+2I}{-4+5I}$$

by substitution in (*) we get:

$$\frac{1+2I}{(x+2I)(x+4-3I)}=\frac{\frac{1+2I}{4-3I}}{x+2I}+\frac{\frac{1+2I}{-4+3I}}{x+4-3I}$$

$$\Rightarrow \int \frac{1+2I}{(x+2I)(x+4-3I)}dx=\int\left(\frac{\frac{1+2I}{4-5I}}{x+2I}+\frac{\frac{1+2I}{-4+5I}}{x+4-3I}\right)dx$$

$$=\frac{1+2I}{4-5I}ln|x+2I|-\frac{1+2I}{4-5I}ln|x+4-3I|$$

$$=\frac{1+2I}{4-5I}(ln|x+2I|-ln|x+4-3I|)$$

$$=\frac{1+2I}{4-5I}ln\left|\frac{x+2I}{x+4-3I}\right|$$

$$=\left(\frac{1}{4}-\frac{13}{4}I\right)ln\left|\frac{x+2I}{x+4-3I}\right|+C$$

Hence:

$$\int \frac{1+2I}{x^2+(4-I)x+2I}dx=\begin{cases}\left(\frac{1}{4}-\frac{11}{4}I\right)ln\left|\frac{x+I}{x+4-2I}\right|+C\\ \left(\frac{1}{4}-\frac{13}{4}I\right)ln\left|\frac{x+2I}{x+4-3I}\right|+C\end{cases}$$

Example3.1.6:

Evaluate:

$$\int \frac{3I}{(x-2+3I)(x^2+1+I)}dx$$

Solution:

We note that (x^2+1+I) cannot be analyzing, because:

$$x^2+1+I = x^2-\left(\sqrt{-1-I}\right)^2$$

Let's find $\sqrt{-1-I}$

$$\sqrt{-1-I} = \alpha+\beta I$$

$$-1-I = \alpha^2+2\alpha\beta I+\beta^2 I$$

$$-1-I = \alpha^2+(2\alpha\beta+\beta^2)I$$

then:

$$\alpha^2 = -1 \text{ (impossible in real number)}$$

So:

$$\frac{3I}{(x-2+3I)(x^2+1+I)} = \frac{A}{x-2+3I}+\frac{Bx+D}{x^2+1+I} \quad (*)$$

To find value of A, We multiply both sides $(*)$ by $(x-2+3I)$:

$$\frac{5I}{x-2+3I} = A+\frac{(x-2+3I)(Bx+C)}{x-2+3I} \quad (1)$$

by substitution $x=2-3I$ in (1), we get:

$$A = \frac{5I}{2-3I+2-3I} = \frac{5I}{4-6I}$$

To find value of B, We multiply both sides $(*)$ by x:

$$\frac{3Ix}{(x-2+3I)(x^2+1+I)} = \frac{Ax}{x-2+3I}+\frac{Bx^2+D}{x^2+1+I} \quad (2)$$

By take limit both sides in (2), when $x\longrightarrow\infty$, we get:

$$0 = A+B \implies B=-A=\frac{-5I}{4-6I}$$

To find value of D, we substitute value of A, B and let be $x=0$, in (*), we get:

$$\frac{3I}{(0-2+3I)(0^2+1+I)} = \frac{\frac{5I}{4-6I}}{0-2+3I}+\frac{\frac{-5I}{4-6I}(0)+D}{0^2+1+I} \implies D=3I$$

$$\frac{3I}{-2+4I} = \frac{5I}{-4+6I}+\frac{D}{1+I}$$

$$\frac{D}{1+I} = \frac{-7}{8}+4I \implies D=\frac{-7}{8}+\frac{57}{8}I$$

by substitution in (*) we get:

$$\frac{3I}{(x-2+3I)(x^2+1+I)}=\frac{\frac{5I}{4-6I}}{x-2+3I}+\frac{\frac{-5I}{4-6I}x+\frac{-7}{8}+\frac{57}{8}I}{x^2+1+I}$$

$$\Rightarrow\int\frac{3I}{(x-2+3I)(x^2+1+I)}dx=\int\left(\frac{\frac{5I}{4-6I}}{x-2+3I}+\frac{\frac{-5I}{4-6I}x+\frac{-7}{8}+\frac{57}{8}I}{x^2+1+I}\right)dx$$

$$=\int\frac{\frac{5I}{4-6I}}{x-2+3I}dx+\int\frac{\frac{-5I}{4-6I}x}{x^2+1+I}dx+\int\frac{\frac{-7}{8}+\frac{57}{8}I}{x^2+1+I}dx$$

$$=\frac{I5}{4-6I}ln|x-2+3I|-\frac{I5}{8-12I}ln|x^2+1+I|+\int\frac{\frac{-7}{8}+\frac{57}{8}I}{x^2+1+I}dx \quad (*)'$$

Let's now find:

$$\int\frac{\frac{-7}{8}+\frac{57}{8}I}{x^2+1+I}dx$$

$$x^2+9+7I=x^2+\left(\sqrt{1+I}\right)^2$$

Let's find $\sqrt{1+I}$

$$\sqrt{1+I}=\alpha+\beta I$$

$$1+I=\alpha^2+2\alpha\beta I+\beta^2 I$$

$$1+I=\alpha^2+(2\alpha\beta+\beta^2)I$$

then:

$$\begin{cases}\alpha^2=1\\2\alpha\beta+\beta^2=1\end{cases}$$

$$\begin{cases}\alpha=\pm 1\\\beta^2+2\alpha\beta-1=0\end{cases}$$

Find β:

- When $\alpha=1 \quad \Rightarrow \beta^2+2\beta-1=0$

$$\Rightarrow \beta=-1+\sqrt{2},\beta=-1-\sqrt{2}$$

$$\left(1,-1+\sqrt{2}\right),\left(1,-1-\sqrt{2}\right)$$

- When $\alpha=-1 \quad \Rightarrow \beta^2-2\beta-1=0$

$$\Rightarrow \beta=1+\sqrt{2},\beta=1-\sqrt{2}$$

$$\left(1,1+\sqrt{2}\right),\left(1,1-\sqrt{2}\right)$$

$$(\alpha,\beta)=\left(1,-1+\sqrt{2}\right),\left(1,-1-\sqrt{2}\right),\left(1,1+\sqrt{2}\right),\left(1,1-\sqrt{2}\right)$$

$$\sqrt{1+I}=1+\left(-1+\sqrt{2}\right)I \ or \ 1+\left(-1-\sqrt{2}\right)I \ or -1+\left(1+\sqrt{2}\right)I \ or \ 1+(1-\sqrt{2})I$$

Thus, the denominator factors can be written in two cases:
Case1:

$$x^2+9+7I=x^2+\left(1+\left(-1+\sqrt{2}\right)I\right)^2$$

$$\int \frac{4+I}{x^2+9+7I}dx = \int \frac{4+I}{x^2+\left(1+\left(-1+\sqrt{2}\right)I\right)^2}dx$$

$$= \left(\frac{4+I}{1+\left(-1+\sqrt{2}\right)I}\right)tan^{-1}\left(\frac{x}{1+\left(-1+\sqrt{2}\right)I}\right)+C$$

$$= \left(4+\left(\frac{5-4\sqrt{2}}{\sqrt{2}}\right)I\right)tan^{-1}\left(\left(1+\left(\frac{-1+\sqrt{2}}{\sqrt{2}}\right)I\right)x\right)+C$$

$$= \left(4-\left(\frac{5\sqrt{2}}{2}-4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{-\sqrt{2}}{2}+1\right)I\right)x\right)+C$$

Case2:

$$x^2-4+3I = x^2+\left(1+\left(-1-\sqrt{2}\right)I\right)^2$$

$$\int \frac{4+I}{x^2+9+7I}dx = \int \frac{4+I}{x^2+\left(1+\left(-1-\sqrt{2}\right)I\right)^2}dx$$

$$= \left(\frac{4+I}{1+\left(-1-\sqrt{2}\right)I}\right)tan^{-1}\left(\frac{x}{1+\left(-1-\sqrt{2}\right)I}\right)+C$$

$$= \left(4-\left(\frac{5+4\sqrt{2}}{\sqrt{2}}\right)I\right)tan^{-1}\left(\left(1+\left(\frac{1+\sqrt{2}}{\sqrt{2}}\right)I\right)x\right)+C$$

$$= \left(4-\left(\frac{5\sqrt{2}}{2}+4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{\sqrt{2}}{2}+1\right)I\right)x\right)+C$$

Hence:

$$\int \frac{4+I}{x^2+9+7I}dx = \begin{cases} \left(4-\left(\frac{5\sqrt{2}}{2}-4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{-\sqrt{2}}{2}+1\right)I\right)x\right)+C \\ \left(4-\left(\frac{5\sqrt{2}}{2}+4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{\sqrt{2}}{2}+1\right)I\right)x\right)+C \end{cases}$$

by substitution in $(*)'$, we get:

$$\int \frac{4+I}{x^2+9+7I}dx =$$

$$\begin{cases} \frac{I5}{4-6I}ln|x-2+3I| - \frac{I5}{8-12I}ln|x^2+1+I| + \left(4-\left(\frac{5\sqrt{2}}{2}-4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{-\sqrt{2}}{2}+1\right)I\right)x\right)+C \\ \frac{I5}{4-6I}ln|x-2+3I| - \frac{I5}{8-12I}ln|x^2+1+I| + \left(4-\left(\frac{5\sqrt{2}}{2}+4\right)I\right)tan^{-1}\left(\left(1+\left(\frac{\sqrt{2}}{2}+1\right)I\right)x\right)+C \end{cases}$$

Result3.1:
When decomposing the neutrosophic function into factors, it can give us more than one analysis, and thus we get more than one result in the case the integral of the neutrosophic rational functions, as example3.1.4 and example3.1.5

3.2 Integral of the neutrosophic improper rational functions

If the degree of the numerator is greater than the degree of the denominator, then we use long division method or using synthetic division method to facilitate the integration process.

Example3.2.1:

Evaluate:

$$\int \frac{x^3 + (3+2I)x^2 + (-5+I)x + 8 - 4I}{x - 1 - 2I} dx$$

Solution:
By using synthetic division method, we get:

$1+2I$	1	$3+2I$	$-5+I$	$8-4I$
		$1+2I$	$4+20I$	$-1+84I$
	1	$4+4I$	$-1+21I$	$-1+80I$

Then:

$$\frac{x^3 + (3+2I)x^2 + (-5+I)x + 8 - 4I}{x - 1 - 2I} = x^2 + (4+4I)x + (-1+21I) + \frac{-1+80I}{x-1-2I}$$

$$\Rightarrow \int \frac{x^3 + (3+2I)x^2 + (-5+I)x + 8 - 4I}{x - 1 - 2I} dx = \int \left(x^2 + (4+4I)x - 1 + 21I + \frac{-1+80I}{x-1-2I}\right) dx$$

$$= \frac{x^3}{3} + (2+2I)x^2 + (-1+21I)x + (-1+80I)ln|x-1-2I| + C$$

Example3.2.2:

Evaluate:

$$\int \frac{(1+I)x^2 + (2-3I)x + 4 - 5I}{x - 2 - 7I} dx$$

Solution:
By using synthetic division method, we get:

$2+7I$	$1+I$	$2-3I$	$4-5I$
		$2+11I$	$8-44I$
	$1+I$	$4-8I$	$12-49I$

Then:

$$\frac{(1+I)x^2+(2-3I)x+4-5I}{x-2-7I}=(1+I)x+(4-8I)+\frac{12-49I}{x-2-7I}$$

$$\Rightarrow \int \frac{(1+I)x^2+(2-3I)x+4-5I}{x-2-7I}dx=\int\left((1+I)x+(4-8I)+\frac{12-49I}{x-2-7I}\right)dx$$

$$=\left(\frac{1}{2}+\frac{1}{2}I\right)x^2+(4-8I)x+(12-49I)ln|x-2-7I|+C$$

4. Conclusions

This paper is an extension of the papers I presented in the field of neutrosophic integrals. Integrals are important in our life, as they facilitate many mathematical operations in our reality, and this is what led us to study the neutrosophic integrals by partial fraction, and I concluded that more than one result can be obtained in the case of integration of the neutrosophic fraction function. In addition, this paper is considered an introduction to the applications in neutrosophic integrals.

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