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Single Valued Pentapartitioned Neutrosophic Graphs

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Abstract
Background:
The notion of single valued pentapartitioned neutrosophic set is the extension of single valued neutrosophic set and quadripartitioned single valued neutrosophic set. The single valued pentapartitioned neutrosophic set is a powerful mathematical tool that comprehensively deals with indeterminacy by splitting it into three independent components, namely, unknown, contradiction, ignorance. We apply the concept of single valued pentapartitioned neutrosophic set to graph theory.
Findings:
We develop the notions of Single-Valued Pentapartitioned Neutrosophic graph (SVPN-graph) as an extension of single valued neutrosophic graph theory. Besides, we introduce the notions of degree, size and order of an SVPN-graph. Further, we furnish a few suitable examples on SVPN-graph. Single valued pentapartitioned neutrosophic set.
Limitations:
Pentapartitioned neutrosophic graph is proposed in this model which is based on pentapartitioned neutrosophic sets. A few studies on pentapartitioned neutrosophic sets are reported in the literature.
Future directions:
In future, the single valued pentapartitioned neutrosophic graph can be extended to regular and irregular single valued pentapartitioned neutrosophic graph, single-valued pentapartitioned neutrosophic intersection graphs, single-valued pentapartitioned neutrosophic hypergraphs, and so on. The single-valued pentapartitioned neutrosophic graph can be employed in modeling the computer networks, expert systems, image processing, social network , and telecommunication.

Keywords: Neutrosophic Set; Pentapartitioned NS; Neutrosophic Graph; SVPN-Graph.

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1. Introduction:

Graph theory [1] is generally used as a tool to deal with the combinatorial problems in number theory, geometry, topology, algebra, etc. Euler presented the concept of graph theory [2] in 1736. When there exists uncertainty in the description of a graph, traditional graph theory fails to deal with the problem. To deal with such situation, Rosenfeld [3] developed the Fuzzy Graph (FG) by considering fuzzy relation [4] on Fuzzy Set (FS) [5]. Sunitha and Mathew [6] presented a survey of fuzzy graph in 2013. Shannon and Atanassov [7] developed intuitionistic FG based on Intuitionistic FS (IFS) [8]. Intuitionistic FGs have been further studied in [9-15].

To deal with inconsistency and indeterminacy, Smarandache [16] developed the Neutrosophic Set (NS) in 1998. The Neutrosophic Graphs (NGs) using the NSs were developed by several authors[17-19]. Akram [20] presented the Single Valued Neutrosophic (SVN) planar graph. NGs have been further studied in [21-24]. Broumi et al. [25] presented interval NGs, which have been further studied in [26-27]. NGs have been further studied in different hybrid environment such as neutrosophic soft graph [28], bipolar SVN graphs [29], rough neutrosophic diagraph [30], neutrosophic soft rough graph [31], etc. Recent trends in graph theory have been depicted in [32] in different environments.

Recently, Mallick and Pramanik [33] defined Pentapartitioned Neutrosophic Set (PNS) using the n-valued logic [34]. PNS is a powerful mathematical tool, which is capable of dealing with uncertainty and indeterminacy comprehensively as indeterminacy is divided into three independent components, namely, unknown, contradiction, and ignorance.

In this study, we procure the Single Valued Pentapartitioned Neutrosophic (SVPN) graph and establish some basic its properties.

Research Gap: No investigation on SVPN-graph has been reported in the literature.

Motivation: To fill the research gap, we present the concept of SVPN-graph.

The rest of the article has been organized into four sections:

In section 2, we recall some relevant definitions on PNS those are relevant to the main results of this article. In section 3, we procure the notion of SVPN-graph, and investigate some properties of different types of degree, size and order of an SVPN-graph. Section 4 presents results and discussion section. Section 5 concludes the paper with stating the future scope of research.

2. Some Relevant Definitions:

In this section, we present some existing definitions those are relevant to the main results of this article.

Definition 2.1.[33] Suppose that Ω be a fixed set. Then, a Single Valued Pentapartitioned Neutrosophic Set (SVPN-set) P over Ω is defined by:

$$P=\{(\kappa, T_P(\kappa), C_P(\kappa), R_P(\kappa), U_P(\kappa), F_P(\kappa)) : \kappa\in\Omega\}.$$

Here, T_P, C_P, R_P, U_P and F_P are the truth, contradiction, ignorance, unknown and falsity membership functions respectively from Ω to [0, 1]. So, $0 \leq T_P(\kappa) + C_P(\kappa) + R_P(\kappa) + U_P(\kappa) + F_P(\kappa) \leq 5$, for all $\kappa\in\Omega$.

Definition 2.2.[33] Suppose that $X=\{(\kappa, T_X(\kappa), C_X(\kappa), R_X(\kappa), U_X(\kappa), F_X(\kappa)) : \kappa\in\Omega\}$ and $Y=\{(\kappa, T_Y(\kappa), C_Y(\kappa), R_Y(\kappa), U_Y(\kappa), F_Y(\kappa)) : \kappa\in\Omega\}$ be two SVPN-sets over Ω. Then, an SVPN-set X is said to be a subset of a SVPN-set Y (i.e., $X\subseteq Y$) if and only if $T_X(\kappa) \leq T_Y(\kappa)$, $C_X(\kappa) \leq C_Y(\kappa)$, $R_X(\kappa) \geq R_Y(\kappa)$, $U_X(\kappa) \geq U_Y(\kappa)$, $F_X(\kappa) \geq F_Y(\kappa)$, $\forall\kappa\in\Omega$.

Definition 2.3.[33] Suppose that $X=\{(\kappa, T_X(\kappa), C_X(\kappa), R_X(\kappa), U_X(\kappa), F_X(\kappa)) : \kappa\in\Omega\}$ and $Y=\{(\kappa, T_Y(\kappa), C_Y(\kappa), R_Y(\kappa), U_Y(\kappa), F_Y(\kappa)) : \kappa\in\Omega\}$ be two SVPN-sets over Ω. Then, union of X and Y is defined by $X\cup Y=\{(\kappa, \max\{T_X(\kappa), T_Y(\kappa)\}, \max\{C_X(\kappa), C_Y(\kappa)\}, \min\{R_X(\kappa), R_X(\kappa)\}, \min\{U_X(\kappa), U_X(\kappa)\}, \min\{F_X(\kappa), F_X(\kappa)\}) : \kappa\in\Omega\}$.

Definition 2.4.[33] Suppose that $X=\{(\kappa, T_X(\kappa), C_X(\kappa), R_X(\kappa), U_X(\kappa), F_X(\kappa)) : \kappa\in\Omega\}$ and $Y=\{(\kappa, T_Y(\kappa), C_Y(\kappa), R_Y(\kappa), U_Y(\kappa), F_Y(\kappa)) : \kappa\in\Omega\}$ be any two SVPN-sets over Ω. Then, the complement of X is defined by $X^c=\{(\kappa, F_X(\kappa), U_X(\kappa), 1-R_X(\kappa), C_X(\kappa), T_X(\kappa)) : \kappa\in\Omega\}$.

Definition 2.5.[33] Suppose that $X=\{(\kappa, T_X(\kappa), C_X(\kappa), R_X(\kappa), U_X(\kappa), F_X(\kappa)) : \kappa\in\Omega\}$ and $Y=\{(\kappa, T_Y(\kappa), C_Y(\kappa), R_Y(\kappa), U_Y(\kappa), F_Y(\kappa)) : \kappa\in\Omega\}$ be two SVPN-sets over Ω. Then, intersection of X and Y is defined by $X\cap Y=\{(\kappa, \min\{T_X(\kappa), T_Y(\kappa)\}, \min\{C_X(\kappa), C_Y(\kappa)\}, \max\{R_X(\kappa), R_X(\kappa)\}, \max\{U_X(\kappa), U_X(\kappa)\}, \max\{F_X(\kappa), F_X(\kappa)\}) : \kappa\in\Omega\}$.

Definition 2.6.[18] Suppose that V be a fixed set of n vertex. Assume that E be the set of edges between the vertices. Then, $\hat{G}=(P, Q)$ is called a single valued neutrosophic graph (in short SVN-graph), where (i) $T_P, I_P, F_P : V\rightarrow[0, 1]$ denotes the truth, indeterminacy and false membership functions of a vertex $\kappa_i\in V$ respectively, such that $0 \le T_P(k_i) + I_P(k_i) + F_P(k_i) \le 3$ ($\forall k_i\in V$, i=1, 2, …., n).
(ii) $T_Q, I_Q, F_Q : E\subseteq V\times V\rightarrow[0, 1]$ defined by $T_Q(k_i, k_j) \le \min\{T_P(k_i), T_P(k_j)\}$, $I_Q(k_i, k_j) \ge \max\{I_P(k_i), I_P(k_j)\}$, $F_Q(k_i, k_j) \ge \max\{F_P(k_i), F_P(k_j)\}$ denotes the truth, indeterminac y and false membership functions of the edge $(k_i, k_j)\in E$ respectively, such that $0 \le T_Q(k_i, k_j) + I_Q(k_i, k_j) + F_Q(k_i, k_j) \le 3$ ($\forall(k_i, k_j)\in E$, i=1, 2, …., n).

Here, P is said to be the SVN vertex set of V and Q is said to be the SVN edge set of E, respectively.

3. Single-Valued Pentapartitioned Neutrosophic-Graph

Here, we introduce the notions of degree, size, and order of SVPN-graph and present few illustrative examples.

Definition 3.1. Suppose that $V=\{k_i: i=1, 2, …, n\}$ be a fixed set of vertices and $E=\{(k_i, k_j): i, j=1, 2, …, n\}$ be the set of edges between the vertices of V. An SVPN-graph of $\hat{G}^*=(V, E)$ is defined by $\hat{G}=(P, Q)$, where (i) $T_P: V\rightarrow[0, 1]$, $C_P: V\rightarrow[0, 1]$, $R_P: V\rightarrow[0, 1]$, $U_P: V\rightarrow[0, 1]$ and $F_P: V\rightarrow[0, 1]$ denotes the truth, contradiction, ignorance, unknown and false membership functions of the vertices $k_i\in V$ respectively, such that $0 \le T_P(k_i) + C_P(k_i) + R_P(k_i) + U_P(k_i) + F_P(k_i) \le 5$, $\forall k_i \in V$ (i=1, 2, …, n);
(ii) $T_Q : E\subseteq V\times V\rightarrow[0, 1]$, $C_Q : E\subseteq V\times V\rightarrow[0, 1]$, $R_Q : E\subseteq V\times V\rightarrow[0, 1]$, $U_Q : E\subseteq V\times V\rightarrow[0, 1]$ and $F_Q : E\subseteq V\times V\rightarrow[0, 1]$ defined by $T_Q(k_i, k_j) \le \min\{T_P(k_i), T_P(k_j)\}$, $C_Q(k_i, k_j) \le \min\{C_P(k_i), C_P(k_j)\}$, $R_Q(k_i, k_j) \ge \max\{R_P(k_i), R_P(k_j)\}$, $U_Q(k_i, k_j) \ge \max\{U_P(k_i), U_P(k_j)\}$, and $F_Q(k_i, k_j) \ge \max\{F_P(k_i), F_P(k_j)\}$, indicates the truth, contradiction, ignorance, unknown and false-membership functions from $E\subseteq V\times V$ to [0, 1], respectively, such that $0 \le T_P(k_i) + C_P(k_i) + R_P(k_i) + U_P(k_i) + F_P(k_i) \le 5$, $\forall(k_i, k_j)\in E$ (i, j = 1, 2, …., n).

Here, P is the SVN vertex set of V and Q is the SVN edge set of E respectively. Therefore, $\hat{G}=(P, Q)$ is an SVPN-graph of $\hat{G}^*=(V, E)$ if and only if $T_Q(k_i, k_j) \le \min\{T_P(k_i), T_P(k_j)\}$; $C_Q(k_i, k_j) \le \min\{C_P(k_i), C_P(k_j)\}$; $R_Q(k_i, k_j) \ge \max\{R_P(k_i), R_P(k_j)\}$; $U_Q(k_i, k_j) \ge \max\{U_P(k_i), U_P(k_j)\}$; and $F_Q(k_i, k_j) \ge \max\{F_P(k_i), F_P(k_j)\}$.

Clearly, both P and Q are the SVPN-set over V and E respectively.

Example 3.1. Assume that $\hat{G}=(V, E)$ is a graph, where $V=\{k_1, k_2, k_3, k_4\}$ and $E=\{(k_1, k_2), (k_2, k_3), (k_3, k_4), (k_4, k_1)\}$. Suppose that P is an SVPN vertex set of V and Q is an SVPN edge set of E defined by the Table 1 and Table 2.:

Table 1. Tabular representation of Example 3.1

	k_1	k_2	k_3	k_4	k_5
T_P	0.4	0.3	0.4	0.5	0.2
C_P	0.5	0.5	0.5	0.5	0.5
R_P	0.3	0.4	0.5	0.3	0.4
U_P	0.4	0.3	0.6	0.6	0.3
F_P	0.4	0.5	0.5	0.4	0.5

Table 2. Tabular representation of Example 3.1

	(k_1, k_2)	(k_2, k_3)	(k_3, k_4)	(k_4, k_5)	(k_5, k_1)
T_P	0.3	0.2	0.2	0.1	0.2
C_P	0.4	0.4	0.3	0.5	0.2
R_P	0.5	0.7	0.8	0.6	0.5
U_P	0.4	0.8	0.9	0.7	0.5
F_P	0.6	0.6	0.9	0.8	0.8

The graph of Example 3.1 is presented in Figure 1.

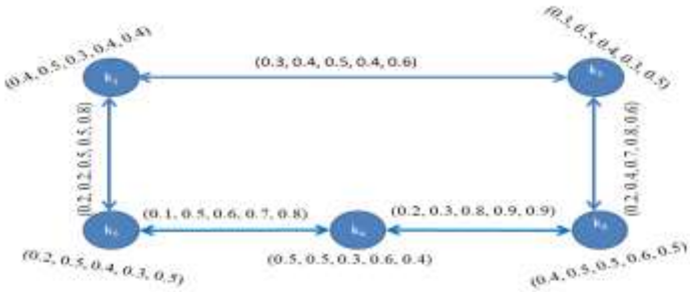

Figure 1: SPVN graph for Example 3.1

Therefore, $\hat{G}$= (P, Q) is an SVPN-graph of $\hat{G}^*$=(V, E).

Remark 3.1. Assume that $\hat{G}$=(P, Q) is an SVPN-graph. Then, the edge (k_i, k_j) is said to be incident at k_i and k_j.

Definition 3.2. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph. Then,

(i) $(k_i, T_P(k_i), C_P(k_i), R_P(k_i), U_P(k_i), F_P(k_i))$ is called a Single Valued Pentapartitioned Neutrosophic (SVPN) vertex (in short SVPN-vertex).

(ii) $((k_i, k_j), T_Q((k_i, k_j)), C_Q((k_i, k_j)), R_Q((k_i, k_j)), U_Q((k_i, k_j)), F_Q((k_i, k_j)))$ is called an SVPN edge (in short SVPN-edge).

Definition 3.3. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph. Then, H=(P′, Q′) is called an SVPN sub-graph (SVPN-sub-graph) of $\hat{G}$=(P, Q) if H=(P′, Q′) is also an SVPN-graph such that:

(i) $P'\subseteq P$ i.e. $T'_{Pi} \leq T_{Pi}$, $C'_{Pi} \leq C_{Pi}$, $R'_{Pi} \geq R_{Pi}$, $U'_{Pi} \geq U_{Pi}$, and $F'_{Pi} \geq F_{Pi}$, $\forall k_i \in V$;

(ii) $Q'\subseteq Q$ i.e. $T'_{Qi} \leq T_{Qi}$, $C'_{Qi} \leq C_{Qi}$, $R'_{Qi} \geq R_{Qi}$, $U'_{Qi} \geq U_{Qi}$, and $F'_{Qi} \geq F_{Qi}$, $\forall (k_i, k_j) \in E$.

Example 3.2. Assume that $\hat{G}$=(P, Q) be an SVPN-graph as shown in Example 3.1. Then, H=(P′, Q′), where $V'=\{k_1, k_2, k_5\}$, $E'=\{(k_1, k_2), (k_1, k_5)\}$ defined by the Table 3 and Table 4:

Table 3. Tabular representation of Example 3.2

	k_1	k_2	k_5
$T_{P'}$	0.3	0.2	0.2
$C_{P'}$	0.3	0.4	0.2
$R_{P'}$	0.5	0.6	0.5
$U_{P'}$	0.6	0.4	0.4
$F_{P'}$	0.6	0.6	0.8

Table 4. Tabular representation of Example 3.2

	(k_1, k_2)	(k_1, k_5)
$T_{P'}$	0.1	0.1
$C_{P'}$	0.3	0.2
$R_{P'}$	0.8	0.6
$U_{P'}$	0.6	0.8
$F_{P'}$	0.8	0.9

Then, the graph H=(P′, Q′) is represented in Figure 2.

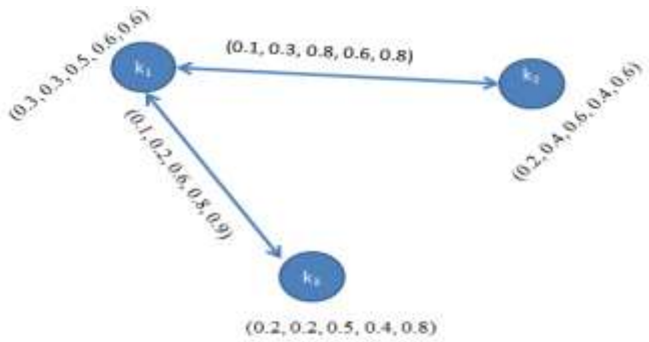

Figure 2: Graph of Example 3.2

Here, H=(P′, Q′) is an SVPN-sub-graph of $\hat{G}$=(P, Q).

Definition 3.4. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph of $\hat{G}^*$=(V, E). Then, the complement of $\hat{G}$=(P, Q) is an SVPN-graph $\overline{\hat{G}}$ of $\hat{G}^*$=(V, E), where

(ii) $\overline{T}_P(k_i)= T_P(k_i)$, $\overline{C}_P(k_i)=C_P(k_i)$, $\overline{R}_P(k_i)=R_P(k_i)$, $\overline{U}_P(k_i)=U_P(k_i)$, $\overline{F}_P(k_i)=F_P(k_i)$, $\forall k_i \in V$;

(iii) $\overline{T}_Q(k_i, k_j) = \min\{T_P(k_i), T_P(k_j)\} - T_Q(k_i, k_j)$, $\overline{C}_Q(k_i, k_j) = \min\{C_P(k_i), C_P(k_j)\} - C_Q(k_i, k_j)$, $\overline{R}_Q(k_i, k_j) = \max\{R_P(k_i), R_P(k_j)\} - R_Q(k_i, k_j)$, $\overline{U}_Q(k_i, k_j) = \max\{U_P(k_i), U_P(k_j)\} - U_Q(k_i, k_j)$ and $\overline{F}_Q(k_i, k_j) = \max\{F_P(k_i), F_P(k_j)\} - F_Q(k_i, k_j)$, $\forall (k_i, k_j) \in E$.

Definition 3.5. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph. Then, the vertices k_i and k_j are called adjacent in $\hat{G}$=(P, Q) if and only if $T_Q(k_i, k_j) = \min\{T_P(k_i), T_P(k_j)\}$, $C_Q(k_i, k_j) = \min\{C_P(k_i), C_P(k_j)\}$, $R_Q(k_i, k_j) = \max\{R_P(k_i), R_P(k_j)\}$, $U_Q(k_i, k_j) = \max\{U_P(k_i), U_P(k_j)\}$ and $F_Q(k_i, k_j) = \max\{F_P(k_i), F_P(k_j)\}$.

Example 3.3. Assume that $\hat{G}$=(P, Q) be an SVPN-graph, which is defined in Table 5 and Table 6.

Table 5. Tabular representation of Example 3.3

	k_1	k_2	k_3
T_P	0.3	0.2	0.3
C_P	0.3	0.8	0.4
R_P	0.5	0.6	0.6
U_P	0.6	0.5	0.7
F_P	0.6	0.5	0.8

Table 6. Tabular representation of Example 3.3

	(k_1, k_2)	(k_2, k_3)	(k_3, k_1)
T_P	0.2	0.1	0.3
C_P	0.3	0.4	0.3
R_P	0.6	0.8	0.6
U_P	0.6	0.7	0.7
F_P	0.6	0.9	0.8

The representation of the graph of Example 3 is shown in Figure-3.

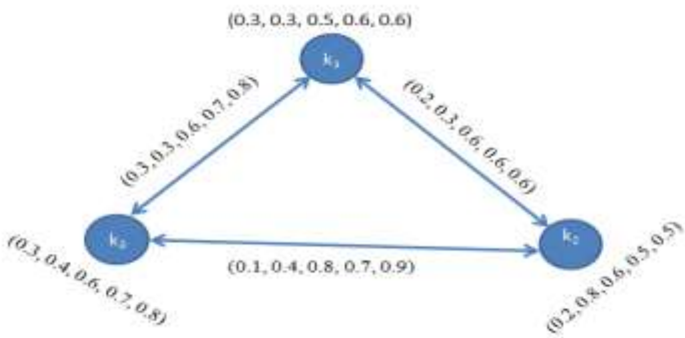

Figure 3: Graph of Example 3.3

Here, the vertices k_1 and k_2 are adjacent in the SVPN-graph $\hat{G}$=(P, Q). Similarly, the vertices k_3 and k_1 are adjacent in the SVPN-graph $\hat{G}$=(P, Q). But, the vertices k_2 and k_3 are not adjacent in the SVPN-graph $\hat{G}$=(P, Q).

Definition 3.6. In an SVPN-graph $\hat{G}$=(P, Q), a vertex $k_j \in V$ is called an isolated vertex if there exists no edge incident at k_j.

Example 3.4. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph, which is defined in Table 7 and Table 8.

Table 7. Tabular representation of Example 3.4

	k_1	k_2	k_3	k_4
T_P	0.3	0.2	0.5	0.3
C_P	0.3	0.8	0.6	0.4
R_P	0.5	0.6	1.0	0.6
U_P	0.6	0.5	0.8	0.7
F_P	0.6	0.5	0.8	0.8

Table 8. Tabular representation of Example 3.4

	(k_1, k_2)	(k_2, k_4)	(k_4, k_1)
T_P	0.3	0.1	0.3
C_P	0.3	0.4	0.3
R_P	0.6	0.7	0.6
U_P	0.7	0.8	0.7
F_P	0.8	0.8	0.8

The graph of Example 3.4 is represented in Figure 4.

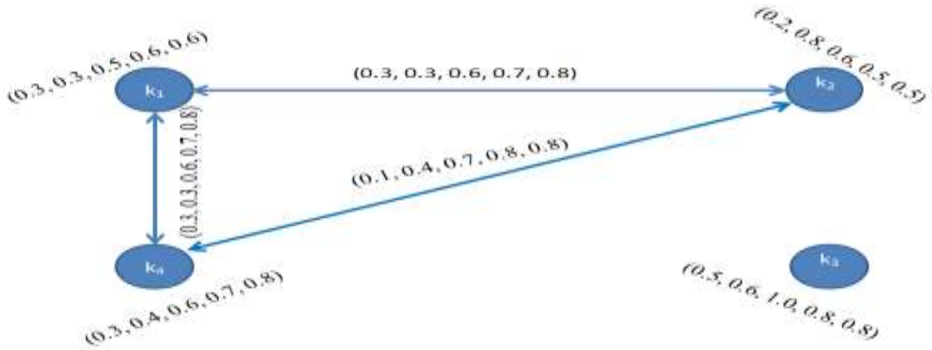

Figure 4: Graph of Example 3.4

In the above SVPN-graph $\hat{G}$=(P, Q), the vertex k_3 is an isolated vertex.

Definition 3.7. Suppose that $\hat{G}$=(P, Q) is an SVPN-graph. Assume that k_0 and k_n be two vertices in $\hat{G}$=(P, Q). Then, an SVPN path P(k_0, k_n) in an SVPN-graph $\hat{G}$=(P, Q) is a sequence of distinct vertices k_0, k_1, k_2, k_3, …., k_n such that $T_Q(k_{i-1}, k_i) > 0$, $C_Q(k_{i-1}, k_i) > 0$, $R_Q(k_{i-1}, k_i) > 0$, $U_Q(k_{i-1}, k_i) > 0$ and $F_Q(k_{i-1}, k_i) > 0$, where $0 \le i \le n$. Here, n (≥ 1) is called the length of the path P(k_0, k_n). The consecutive pairs (k_{i-1}, k_i) ($0 \le i \le n$) are called the edges of the path P(k_0, k_n). The path P(k_0, k_n) is called a cycle if $k_0 = k_n$, where $n \ge 3$.

Definition 3.8. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph. Then, $\hat{G}$=(P, Q) is said to be an SVPN Connected graph (in short SVPN-C-graph) if there exists at least one SVPN-path between two vertices.

Remark 3.2. If an SVPN-graph Ĝ=(P, Q) is not an SVPN-C-graph, then it is called an SVPN Dis-Connected graph (in short SVPN-DC-graph).

Definition 3.9. Assume that Ĝ=(P, Q) be an SVPN-graph. Then, a vertex having exactly one edge incident on it is called a pendent vertex. If a vertex is not a pendent vertex, then it is called a non-pendent vertex.

Remark 3.3. (i) If an edge is incident with a pendent vertex, then the edge is said to be a pendent edge. Otherwise, it is called a non-pendent edge.

(ii) If a vertex is adjacent to a pendent vertex, then the vertex is said to be a support of that pendent edge.

Example 3.5. Let Ĝ=(P, Q) be an SVPN-graph, which is defined by Table 9 and Table 10.

Table 9. Tabular representation of Example 3.5

	k_1	k_2	k_3	k_4
T_P	0.3	0.2	0.5	0.3
C_P	0.3	0.8	0.6	0.4
R_P	0.5	0.6	1.0	0.6
U_P	0.6	0.5	0.8	0.7
F_P	0.6	0.5	0.8	0.8

Table 10. Tabular representation of Example 3.5

	(k_1, k_2)	(k_2, k_3)	(k_3, k_4)
T_P	0.1	0.2	0.3
C_P	0.2	0.5	0.4
R_P	0.7	1.0	1.0
U_P	0.7	0.9	0.8
F_P	0.7	0.8	0.8

The representation of the graph for Example 3.5 is presented in Figure 5.

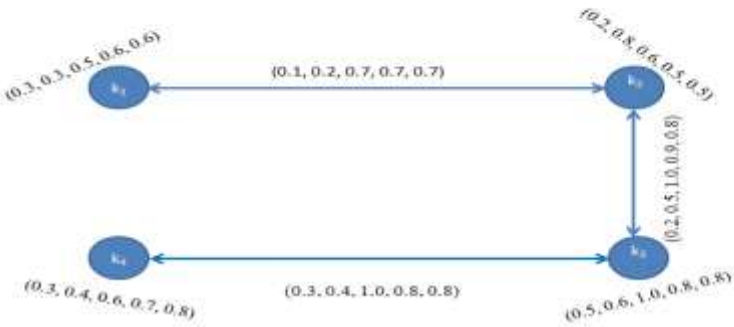

Figure 5: Graph for Example 3.5

In the above SVPN-graph Ĝ=(P, Q), the vertices k_1 and k_4 are the pendent vertices. But the vertices k_2 and k_3 are the non-pendent vertices. Similarly, the edges (k_1, k_2) and (k_3, k_4) are the pendent edges. But the edge (k_2, k_3) is a non-pendent edge. The vertex k_3 is support of the pendent edge (k_3, k_4). But k_2 is not the support of the pendent edge (k_1, k_2).

Definition 3.10. A SVPN-graph Ĝ=(P, Q) of $\hat{G}^*$=(V, E) is said to be a complete SVPN-graph if

$T_Q(k_i, k_j)= \min\{T_P(k_i), T_P(k_j)\}$;

$C_Q(k_i, k_j)= \min\{C_P(k_i), C_P(k_j)\}$;

$R_Q(k_i, k_j)= \max\{R_P(k_i), R_P(k_j)\}$;

$U_Q(k_i, k_j)= \max\{U_P(k_i), U_P(k_j)\}$;

and $F_Q(k_i, k_j)= \max\{F_P(k_i), F_P(k_j)\}$, $\forall k_i, k_j \in V$.

Example 3.6. Assume that $\hat{G}^*$=(V, E) is a graph, where V = {k_1, k_2, k_3} and E = {(k_1, k_2), (k_2, k_3), (k_3, k_1)}. Suppose that $\hat{G}$=(P, Q) is an SVPN-graph defined by Table 11 and Table 12.

Table 11. Tabular representation of Example 3.6

	k_1	k_2	k_3
T_P	0.4	0.3	0.4
C_P	0.5	0.5	0.5
R_P	0.3	0.4	0.5
U_P	0.4	0.3	0.6
F_P	0.4	0.5	0.5

Table 12. Tabular representation of Example 3.6

	(k_1, k_2)	(k_2, k_3)	(k_3, k_1)
T_P	0.3	0.3	0.4
C_P	0.5	0.5	0.5
R_P	0.4	0.5	0.5
U_P	0.4	0.6	0.6
F_P	0.5	0.5	0.5

The representation of the graph for Example 3.6 is presented in Figure 6.

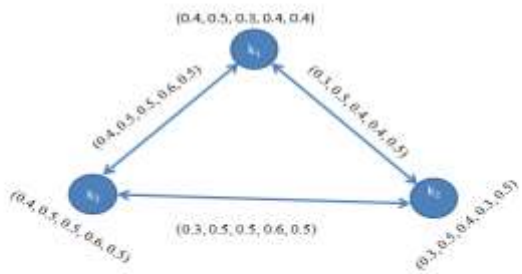

Figure 6: Graph of Example 3.6.

Here, the above SVPN-graph is a complete SVPN-graph.

Definition 3.11. An SVPN-graph $\hat{G}$=(P, Q) of $\hat{G}^*$=(V, E) is called a bipartite SVPN-graph if the graph $\hat{G}^*$=(V, E) is a bipartite graph.

Example 3.7. Assume that $\hat{G}^*$=(V, E) be a graph, where V= {k_1, k_2, k_3, k_4, k_5, k_6} and E={(k_1, k_2), (k_2, k_3), (k_3, k_1)}. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph defined by Table 13 and Table 14.

Table 13. Tabular representation of Example 3.7

	k_1	k_2	k_3	k_4	k_5	k_6
T_P	0.4	0.3	0.4	0.6	0.9	0.8
C_P	0.5	0.5	0.5	0.3	0.8	0.4
R_P	0.3	0.4	0.5	0.5	0.5	0.3
U_P	0.4	0.3	0.6	0.8	0.7	0.6
F_P	0.4	0.5	0.5	0.4	0.8	0.5

Table 14. Tabular representation of Example 3.7

	(k_1, k_2)	(k_1, k_3)	(k_1, k_6)	(k_3, k_5)	(k_2, k_4)	(k_2, k_6)	(k_3, k_6)	(k_4, k_6)	(k_5, k_6)
T_P	0.3	0.3	0.4	0.4	0.3	0.3	0.4	0.6	0.8
C_P	0.5	0.5	0.4	0.5	0.3	0.4	0.4	0.3	0.4
R_P	0.4	0.5	0.3	0.5	0.5	0.4	0.5	0.5	0.5
U_P	0.4	0.6	0.6	0.6	0.8	0.6	0.6	0.8	0.7
F_P	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.8

The representation of the graph of Example 3.7 is presented in Figure 7.

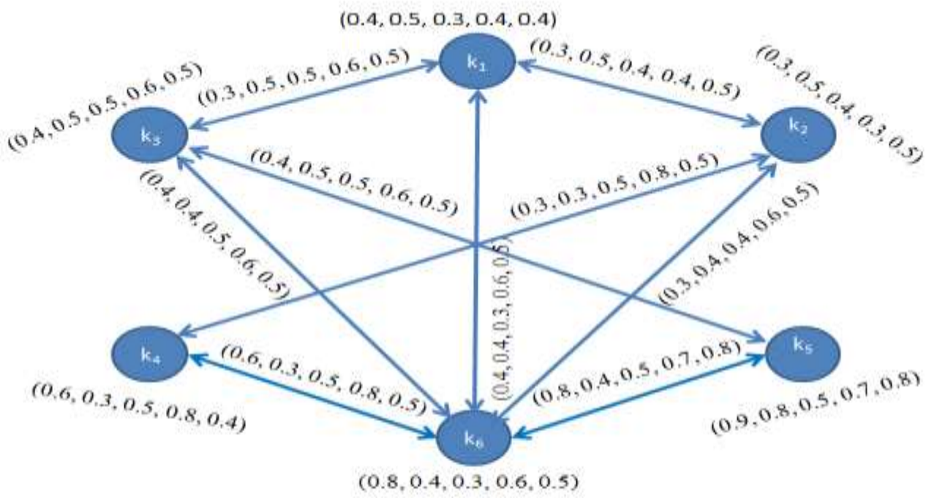

Figure 7: Graph of Example 3.7

Here, the crisp graph $\hat{G}^*$=(V, E) is a bipartite graph and $\hat{G}$=(P, Q) is a SVPN-graph of $\hat{G}^*$=(V, E). Hence $\hat{G}$=(P, Q) is a bipartite SVPN-graph.

Definition 3.12. Suppose that $\hat{G}$=(P, Q) be an SVPN-graph. Then, the degree of the vertex k is defined by d(k)=(d_T(k), d_C(k), d_R(k), d_U(k), d_F(k)),

where, d_T(k) = degree of the truth-membership vertex = sum of the truth-membership of all edges those are incident on the vertex k = $\sum_{u\neq k} T_Q(u, k)$;

d_C(k) = degree of the contradiction-membership vertex = sum of the contradiction-membership of all edges those are incident on the vertex k = $\sum_{u\neq k} C_Q(u, k)$;

d_R(k) = degree of the ignorance-membership vertex = sum of the ignorance-membership of all edges those are incident on the vertex k = $\sum_{u\neq k} R_Q(u, k)$;

d_U(k) = degree of the unknown-membership vertex = sum of the unknown-membership of all edges those are incident on the vertex k = $\sum_{u\neq k} U_Q(u, k)$;

d_F(k) = degree of the falsity-membership vertex = sum of the false-membership of all edges those are incident on the vertex k = $\sum_{u\neq k} F_Q(u, k)$.

Example 3.8. Assume that $\hat{G}$=(P, Q) be an SVPN-graph of $\hat{G}^*$=(V, E) defined by Table 15, Table 16.

Table 15. Tabular representation of example 3.8

	k_1	k_2	k_3	k_4
T_P	0.3	0.2	0.5	0.3
C_P	0.3	0.8	0.6	0.4
R_P	0.5	0.6	1.0	0.6
U_P	0.6	0.5	0.8	0.7
F_P	0.6	0.5	0.8	0.8

Table 16. Tabular representation of example 3.8

	(k_1, k_2)	(k_2, k_3)	(k_3, k_4)	(k_4, k_1)	(k_2, k_3)	(k_2, k_4)
T_P	0.1	0.2	0.3	0.2	0.1	0.1
C_P	0.2	0.5	0.4	0.3	0.4	0.3
R_P	0.7	1.0	1.0	0.8	1.0	0.7
U_P	0.7	0.9	0.8	0.8	0.9	0.9
F_P	0.7	0.8	0.8	0.9	0.8	0.9

The representation of the graph of example 3.8 is shown in Figure 8.

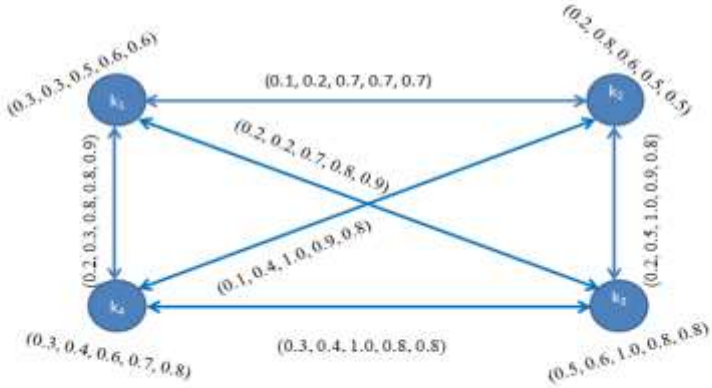

Figure 8: Graph of Example 3.8

Then, d(k_1) = (0.3, 0.5, 1.5, 1.5, 1.6), d(k_2) = (0.5, 1.4, 3.4, 3.4, 3.2), d(k_3)= (0.6, 1.3, 3.0, 2.6, 2.4), and d(k_4)= (0.6, 1.0, 2.5, 2.5, 2.6).

Definition 3.13. Suppose that $\hat{G}$=(P, Q) is an SVPN-graph of $\hat{G}^*$=(V, E). Then, $\hat{G}$=(P, Q) is called a constant SVPN-graph if degree of each vertex is same i.e., d(k) = (y_1, y_2, y_3, y_4, y_5), $\forall k \in V$.

Example 3.9. Assume that $\hat{G}$=(P, Q) be an SVPN-graph, which is defined by Table 17 and Table 18.

Table 17. Tabular representation of example 3.9

	k_1	k_2	k_3	k_4
T_P	0.4	0.2	0.4	0.3
C_P	0.3	0.4	0.6	0.5
R_P	0.6	0.6	0.7	0.6
U_P	0.7	0.6	0.7	0.7
F_P	0.7	0.4	0.8	0.7

Table 18. Tabular representation of example 3.9

	(k_1, k_2)	(k_2, k_3)	(k_3, k_4)	(k_4, k_1)
T_P	0.2	0.1	0.2	0.1
C_P	0.2	0.3	0.2	0.3
R_P	0.7	0.9	0.7	0.9
U_P	0.8	0.8	0.8	0.8
F_P	0.9	0.9	0.9	0.9

The representation of the graph for Example 3.9 is shown in Figure 9.

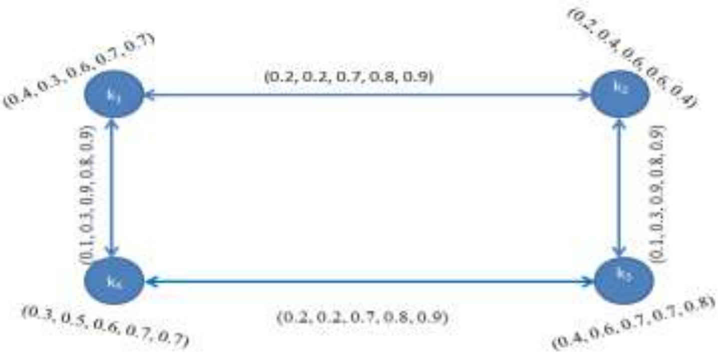

Figure 9: Graph of Example 3.9

In the above SVPN-graph $\hat{G}$= (P, Q), the degree of the vertices k_1, k_2, k_3, and k_4 are d(k_1) = (0.3, 0.5, 1.6, 1.6, 1.8), d(k_2) = (0.3, 0.5, 1.6, 1.6, 1.8), d(k_3) = (0.3, 0.5, 1.6, 1.6, 1.8) and d(k_4) = (0.3, 0.5, 1.6, 1.6, 1.8). Hence, $\hat{G}$= (P, Q) is a constant SVPN-graph.

Definition 3.14. Assume that $\hat{G}$= (P, Q) be a SVPN-graph. Then, the order of $\hat{G}$= (P, Q), denoted by O($\hat{G}$) is defined by O($\hat{G}$)= ($O_T(\hat{G})$, $O_C(\hat{G})$, $O_R(\hat{G})$, $O_U(\hat{G})$, $O_F(\hat{G})$), where

$O_T(\hat{G}) = \sum_{k\in V} T_P$ denotes the T-order of $\hat{G}$= (P, Q);

$O_C(\hat{G}) = \sum_{k\in V} C_P$ denotes the C-order of $\hat{G}$= (P, Q)

$O_R(\hat{G}) = \sum_{k\in V} R_P$ denotes the R-order of $\hat{G}$= (P, Q);

$O_U(\hat{G}) = \sum_{k\in V} U_P$ denotes the U-order of $\hat{G}$= (P, Q);

$O_F(\hat{G}) = \sum_{k\in V} F_P$ denotes the F-order of $\hat{G}$= (P, Q).

Example 3.10. Assume that $\hat{G}$= (P, Q) is an SVPN-graph of $\hat{G}^*$= (V, E) as shown in Example 3.6. Then, order of the SVPN-graph $\hat{G}$= (P, Q) is O($\hat{G}$)= (1.3, 2.1, 2.7, 2.6, 2.7).

Definition 3.15. Suppose that $\hat{G}$= (P, Q) is an SVPN-graph. Then, the size of $\hat{G}$= (P, Q), denoted by S($\hat{G}$) is defined by S($\hat{G}$)= ($S_T(\hat{G})$, $S_C(\hat{G})$, $S_R(\hat{G})$, $S_U(\hat{G})$, $S_F(\hat{G})$), where

$S_T(\hat{G}) = \sum_{u\neq k} T_Q(u, k)$ denotes the T-size of $\hat{G}$= (P, Q);

$S_C(\hat{G}) = \sum_{u\neq k} C_Q(u, k)$ denotes the C-size of $\hat{G}$= (P, Q);

$S_R(\hat{G}) = \sum_{u\neq k} R_Q(u, k)$ denotes the R-size of $\hat{G}$= (P, Q);

$S_U(\hat{G}) = \sum_{u\neq k} U_Q(u, k)$ denotes the U-size of $\hat{G}$= (P, Q);

$S_F(\hat{G}) = \sum_{u\neq k} F_Q(u, k)$ denotes the F-size of $\hat{G}$= (P, Q).

Example 3.11. Assume that $\hat{G}$= (P, Q) is an SVPN-graph of $\hat{G}^*$= (V, E) as shown in Example 3.6. Then, size of the SVPN-graph $\hat{G}$= (P, Q) is S($\hat{G}$) = (1.0, 2.1, 5.2, 5, 4.9).

4. Result and discussion

Graph theory is utilized to deal with many real- problems in operations research. In real-life situation, however, indeterminacy and uncertainty may exist in almost every graph theoretic problem. SVPN-graph is a useful graph theory to model uncertainty and indeterminacy in convincing way based on pentapartitioned neutrosophic set which is an extension of neutrosophic set. So, there is a possibility that SVPN-graph will be more successful in dealing with graph theoretic problems having indeterminacy in the form of three independent components, namely, unknown, contradiction, and ignorance.

5. Conclusions

In this article, we have presented the notions of SVPN-graph. Also, we have defined the degree, order, size of a SVPN-graph and investigated some properties of them. By defining degree, order, size of SVPN-graphs, we formulate some results on SVPN-graphs. Further, we give few examples to justify the definitions and results. We hope that the approach presented in this paper will open up new avenues of research on SVPN-graph for its application in real life problems in the current neutrosophic area.

In future study, the single valued pentapartitioned neutrosophic graph can be extended to regular and irregular single valued pentapartitioned neutrosophic graph. The proposed single valued pentapartitioned graph can be extended to single-valued pentapartitioned neutrosophic intersection graphs, single-valued pentapartitioned neutrosophic hypergraphs, and so on. The single-valued pentapartitioned neutrosophic graph can be employed to model the computer networks, expert systems, image processing, social network , and telecommunication.

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