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Neutrosophic Hypersoft Expert Set: Theory and Applications

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Abstract. Soft set-like models deal with single argument approximate functions while hypersoft set, an extension of the soft set, deals with multi-argument approximate functions. The soft set cannot handle situations when attributes are required to be further divided into disjoint attribute-valued sets. To overcome this situation, a hypersoft set has been developed. In different fields like decision making and medical diagnosis, many researchers developed models based on the soft set for the solution of many problems. But these models deal with only one expert who creates many problems for the users, primarily in designing questionnaires. To remove this discrepancy, we present a neutrosophic hypersoft expert set. This model not only solves the problem of dealing with one expert but also solves the problem of different parametric-valued sets parallel to different characteristics. In this study, we first introduce the concept of neutrosophic hypersoft expert sets, which is a amalgam of both structures i.e., neutrosophic set and hypersoft expert sets. Certain essential basic characteristics (i.e., subset, equal set, agree, disagree set, null set, whole relative set, and whole absolute set), aggregation operations (i.e., complement, restricted union, extended intersection, AND and OR), and results (i.e., idempotent, absorption, domination, identity, commutative, associative and distributive law) are discussed with examples. Some hybrid structures of the neutrosophic hypersoft expert set are developed with illustrated examples. In the end, a decision-making application is presented for the validity of the proposed theory.

Keywords: Soft Set; Soft Expert Set; Neutrosophic set; Hypersoft Set; Neutrosophic Hypersoft Expert Set.

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1. Introduction

In some real-life issues in professional and information systems where we have a situation to deal with the truth-membership along with the falsity-membership for a correct description of an object in an uncertain and an ambiguous environment. Smarandache [1–3] characterized neutrosophic set as a generalization of classical sets, fuzzy set, intuitionistic fuzzy set. Membership functions are use to define fuzzy sets [4], while membership and non-membership functions both are used for intuitionistic fuzzy sets [5] and are used for solving problems having the data of imprecise, indeterminacy and inconsistent. Neutrosophic set (NS) has wide applications in different fields like decision making, medical diagnosis, data bases, control theory and topology etc. Wang et al [6] introduced single valued neutrosophic set (SVNS) and presented its set operations and different properties. The use NS and its hybridized structures in various fields has been continuing quickly [7]- [32].

Molodtsov [34] constructed soft set by taking the advantage of parameterization tool. Rahman et al. [35, 36] conceptualized m-convexity (m-concavity) and (m, n)-convexity ((m, n)-concavity) on soft sets with some properties. Maji et al. [37] made development by introducing fuzzy soft set to solve parametrization problems with uncertainty. Many researchers [38]- [44] advanced this theory and used in many fields. Rahman et al. [45] conceptualized (m-n)-convexity(concavity)on fuzzy soft set with applications in first and second senses. Alkhazaleh et al. [46, 47] made extensions in soft set by introducing soft and fuzzy soft expert sets. They used these structures for applications in decision-making problems(DMPs). Ihsan et al. [48,49] conceptualized convexity on soft expert set and fuzzy soft expert set with certain properties. Broumi et al. [50] conceptualized intuitionistic fuzzy soft expert set and made its use in DMPs. Mehmet et al. [33] defined neutrosophic soft expert sets and applied it in DMPs.

In 2018, Smarandache [51] extended soft set to hypersoft set and used in daily life problems. In 2020, Saeed et al. [52] advanced this theory and explained its structures. In 2020, Rahman et al. [53], [54] worked on hypersoft set and introduced its some new structures like complex fuzzy hypersoft set. They also gave the concept of convexity (concavity) on it and proved its some basic properties. Ihsan et al. [58, 59] introduced the structures of hypersoft expert set and fuzzy hypersoft expert set with applications in DMPs. Kamaci and Saqlain [60] worked on n-ary fuzzy hypersoft expert set and applied in real life problem. Kamaci [61] gave hybrid structures of hypersoft set and rough set and applied in DMPs. He [62] introduced the structure of simplified neutrosophic multiplicative refined sets and their correlation coefficients with Application in medical pattern recognition. The neutrosophic soft set like structures have been investigated and applied in different fields like game theory and DMPs in [63–65].

Having motivation from [33]- [50], new notions of neutrosophic hypersoft expert set are developed and some hybrids of neutrosophic hypersoft expert set are established.

The remaining portion of the paper is constructed as: Section 2 describes the basic definitions of soft set, fuzzy set, intuitionistic fuzzy set, neutrosophic set, fuzzy soft expert set, hypersoft set and relevant definitions used in the proposed work. Section 3, presents notions of fuzzy hypersoft expert set, neutrosophic hypersoft expert set with properties. Section 4, describes the set theoretic operations of NHSES. Section 5, presents the basic properties and laws of

NHSES. Section 6 shows the hybrids of NHSES. Section 7, presents an application in decision making and section 8 contains the conclusions of the paper.

2. Preliminaries

In this portion, some elementary definitions are presented from the literature.
Suppose W be a set of experts and $\mathbf{O}$ be a set of opinions, $\mathbf{T} = \mathbf{F} \times W \times \mathbf{O}$. Taking $S \subseteq \mathbf{T}$ and Δ as a set of universe with $P(\Delta)$ is the power set of universe, while parameters set is $\mathbf{F}$.

Definition 2.1. [40] A set "F_z" is called a *fuzzy set* written as $F_z = \{(\hat{r}, B(\hat{r}))|\hat{r} \in \Delta\}$ with $B : \Delta \to \mathbb{I}$ and $B(\hat{r})$ represents the membership value of $\hat{r} \in F_z$.

Definition 2.2. [41] A set " $\mathcal{J}$" is called an *intuitionistic fuzzy set* written as $\mathcal{J} = \{(\check{a}, < Z_{\mathcal{J}}(\check{a}), X_{\mathcal{J}}(\check{a}) >), |(\check{a}) \in \Delta\}$ with $Z_{\mathcal{J}} : \mathbb{I} \to \Delta$, $X_{\mathcal{J}} : \mathbb{I} \to \Delta$ and $Z_{\mathcal{J}}(\check{a})$, $X_{\mathcal{J}}(\check{a})$ represent the truth, falsity membership functions of $\check{a} \in \Delta$ satisfying the inequality $0 \leq Z_{\mathcal{J}}(\check{a}) + X_{\mathcal{J}}(\check{a}) \leq 1$.

Definition 2.3. [39] A neutrosophic set $\mathbf{N}$ in Δ is defined by
$N = \{< y, (T_N(y), I_N(y), F_N(y)) >: y \in \mathbf{F}, T_N, I_N, F_N \in]^{-}0, 1^{+}[\}$
where T_N, I_N, F_N are truth, indeterminacy, and falsity membership functions and are real standard or nonstandard subsets of $]^{-}0, 1^{+}[$. Their sum does not have any restriction, that is, $0^{-} \leq T_N(y), I_N(y), F_N(y) \leq 3^{+}$. Here $]^{-}0, 1^{+}[$ is named the nonstandard subset, which is the extension of real standard subsets $[0, 1]$ where the nonstandard number $1^{+} = 1 + \epsilon$, 1 is named the standard part, and ϵ is named the nonstandard part. $^{-}0 = 0 - \epsilon$, 0 is the standard part and ϵ is named the nonstandard part, where ϵ is closed to positive real number zero.

Definition 2.4. [34] A pair $(\Psi_M, \mathbf{F})$ is named as *soft set* and Ψ_M is characterized by a mapping

$$\Psi_M : \mathbf{F} \to P(\Delta)$$

where $P(\Delta)$ is the power set of universe of discourse.

Definition 2.5. [37] Let $C \subseteq \mathbf{F}$. A fuzzy soft set is a pair (R, C) and R is characterized as

$$R : C \to I^{\Delta}$$

where I^{Δ} represents the collection of all fuzzy subsets of Δ.

Definition 2.6. [46] A soft expert set is a pair (Φ_H, S) with Φ_H is characterized by a mapping

$$\Phi_H : S \to P(\Delta)$$

where $S \subseteq \mathbf{F} \times W \times \mathbf{O}$.

Definition 2.7. [47] A fuzzy soft expert set is a pair (Ψ_F, S) with Ψ_F is characterized by a mapping

$$\Psi_F : S \to I^{\Delta}$$

where $S \subseteq \mathbf{F} \times W \times \mathbf{O}$ and I^{Δ} represents the collection of all fuzzy subsets of Δ.

Definition 2.8. [6] Let $\mathcal{K}_{\eth}$,$\mathcal{L}_{\eth}$ and $\mathcal{M}_{\eth}$ represent truth, indeterminacy and falsity membership functions, then $\eth$ represents a single valued neutrosophic set such that $0 \leq \mathcal{K}_{\eth}(\beta) + \mathcal{L}_{\eth}(\beta) + \mathcal{M}_{\eth}(\beta) \leq 3$. While $\mathcal{K}_{\eth}$, $\mathcal{L}_{\eth}$, $\mathcal{M}_{\eth} \in [0, 1]$ for all β in Δ.

Definition 2.9. [51]
Let $\ltimes_1, \ltimes_2, \ltimes_3, \ldots.., \ltimes_\epsilon$, with $\epsilon \geq 1$, be ϵ different characters having parrallel characteristics values are the sets $\overline{\barwedge}_1, \overline{\barwedge}_2, \overline{\barwedge}_3, \ldots.., \overline{\barwedge}_\epsilon$, with $\overline{\barwedge}_p \cap \overline{\barwedge}_q = \emptyset$, for $p \neq q$, and $p, q \in \{1, 2, 3, \ldots, \epsilon\}$. Then hypersoft expert set is a pair (Υ, L) with Υ is characterized by a mapping

$$\Upsilon : L \to P(\Delta)$$

where $L = \overline{\barwedge}_1 \times \overline{\barwedge}_2 \times \overline{\barwedge}_3 \times \ldots.. \times \overline{\barwedge}_\epsilon$.

3. Neutrosophic Hypersoft Expert Set (NHSES)

In this portion, neutrosophic hypersoft expert set has been developed with the help of existing concept of neutrosophic soft expert set and some basic properties are presented.

Definition 3.1. [59] Fuzzy Hypersoft Expert Set (FHSES)
A pair $(\pounds, \mathcal{F})$ represents a FHSES with $\pounds$ is characterized by a mapping

$$\xi : \mathcal{F} \to I^{\Delta}$$

- I^{Δ} is being used a collection of all fuzzy subsets of Δ
- $\mathcal{F} \subseteq \mathcal{H} = \text{Æ} \times \text{Œ} \times \text{Å}$
- $\text{Æ} = \text{Æ}_1 \times \text{Æ}_2 \times \text{Æ}_3 \times \ldots. \times \text{Æ}_p$ where Æ_i are different characteristics-valued sets parallel to different characteristics $\text{æ}_i, i = 1, 2, 3, \ldots, p$
- Œ represents an expert set
- Å represents a conclusion set.

Definition 3.2. Neutrosophic Hypersoft Expert Set (NHSES)
A *neutrosophic hypersoft expert set* represents a pair $(\hbar, \mathbb{G})$ if

$$\hbar : \mathbb{G} \to NF^{\Delta}$$

with NF^{Δ} is being used as collection of all neutrosophic subsets of Δ and $\text{Æ} \subseteq \mathcal{H} = \text{Æ} \times \text{Œ} \times \text{Å}$.

Example 3.3. Assume that a worldwide organization expects to continue the assessment of specific experts about its sure items. Let $\Delta = \{w_1, w_2, w_3, w_4\}$ be a set of products and $\AE_1 = \{p_{11}, p_{12}\}$, $\AE_2 = \{p_{21}, p_{22}\}$, $\AE_3 = \{p_{31}, p_{32}\}$, be different characteristics sets for different characteristics p_1= simple to use, p_2= nature, p_3= modest. Now $\AE = \AE_1 \times \AE_2 \times \AE_3$

$$\AE = \left\{ \begin{array}{c} \upsilon_1 = (p_{11}, p_{21}, p_{31}), \upsilon_2 = (p_{11}, p_{21}, p_{32}), \upsilon_3 = (p_{11}, p_{22}, p_{31}), \upsilon_4 = (p_{11}, p_{22}, p_{32}), \\ \upsilon_5 = (p_{12}, p_{21}, p_{31}), \upsilon_6 = (p_{12}, p_{21}, p_{32}), \upsilon_7 = (p_{12}, p_{22}, p_{31}), \upsilon_8 = (p_{12}, p_{22}, p_{32}) \end{array} \right\}$$

Now $\mathcal{H} = \AE \times \OE \times \AA$.

$$\mathcal{H} = \left\{ \begin{array}{c} (\upsilon_1, c, 0), (\upsilon_1, c, 1), (\upsilon_1, d, 0), (\upsilon_1, d, 1), (\upsilon_1, e, 0), (\upsilon_1, e, 1), (\upsilon_2, c, 0), (\upsilon_2, c, 1), \\ (\upsilon_2, d, 0), (\upsilon_2, d, 1), (\upsilon_2, e, 0), (\upsilon_2, e, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_3, d, 0), (\upsilon_3, d, 1), \\ (\upsilon_3, e, 0), (\upsilon_3, e, 1), (\upsilon_4, c, 0), (\upsilon_4, c, 1), (\upsilon_4, d, 0), (\upsilon_4, d, 1), (\upsilon_4, e, 0), (\upsilon_4, e, 1), \\ (\upsilon_5, c, 0), (\upsilon_5, c, 1), (\upsilon_5, d, 0), (\upsilon_5, d, 1), (\upsilon_5, e, 0), (\upsilon_5, e, 1), (\upsilon_6, c, 0), (\upsilon_6, c, 1), \\ (\upsilon_6, d, 0), (\upsilon_6, d, 1), (\upsilon_6, e, 0), (\upsilon_6, e, 1), (\upsilon_7, c, 0), (\upsilon_7, c, 1), (\upsilon_7, d, 0), (\upsilon_7, d, 1), \\ (\upsilon_7, e, 0), (\upsilon_7, e, 1), (\upsilon_8, c, 0), (\upsilon_8, c, 1), (\upsilon_8, d, 0), (\upsilon_8, d, 1), (\upsilon_8, e, 0), (\upsilon_8, e, 1) \end{array} \right\}.$$

let

$$\mathbb{G} = \left\{ \begin{array}{c} (\upsilon_1, c, 0), (\upsilon_1, c, 1), (\upsilon_1, d, 0), (\upsilon_1, d, 1), (\upsilon_1, e, 0), (\upsilon_1, e, 1), \\ (\upsilon_2, c, 0), (\upsilon_2, c, 1), (\upsilon_2, d, 0), (\upsilon_2, d, 1), (\upsilon_2, e, 0), (\upsilon_2, e, 1), \\ (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_3, d, 0), (\upsilon_3, d, 1), (\upsilon_3, e, 0), (\upsilon_3, e, 1), \end{array} \right\}$$

be a subset of $\mathcal{H}$ and $\OE = \{c, d, e, \}$ be a set of specialists.

Following check relates the varieties of three specialists:

$$\hbar_1 = \hbar(\upsilon_1, c, 1) = \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.5>}, \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.1,0.3,0.6>} \right\},$$

$$\hbar_2 = \hbar(\upsilon_1, d, 1) = \left\{ \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.8,0.1,0.5>}, \frac{w_3}{<0.4,0.5,0.6>}, \frac{w_4}{<0.2,0.5,0.3>} \right\},$$

$$\hbar_3 = \hbar(\upsilon_1, e, 1) = \left\{ \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.3,0.6>}, \frac{w_3}{<0.6,0.3,0.7>}, \frac{w_4}{<0.3,0.5,0.6>} \right\},$$

$$\hbar_4 = \hbar(\upsilon_2, c, 1) = \left\{ \frac{w_1}{<0.9,0.1,0.3>}, \frac{w_2}{<0.4,0.5,0.4>}, \frac{w_3}{<0.7,0.2,0.6>}, \frac{w_4}{<0.3,0.4,0.8>} \right\},$$

$$\hbar_5 = \hbar(\upsilon_2, d, 1) = \left\{ \frac{w_1}{<0.4,0.5,0.6>}, \frac{w_2}{<0.8,0.1,0.7>}, \frac{w_3}{<0.3,0.6,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \right\},$$

$$\hbar_6 = \hbar(\upsilon_2, e, 1) = \left\{ \frac{w_1}{<0.5,0.4,0.7>}, \frac{w_2}{<0.3,0.6,0.4>}, \frac{w_3}{<0.6,0.2,0.5>}, \frac{w_4}{<0.8,0.1,0.6>} \right\},$$

$$\hbar_7 = \hbar(\upsilon_3, c, 1) = \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.9,0.1,0.4>}, \frac{w_3}{<0.4,0.5,0.7>}, \frac{w_4}{<0.5,0.4,0.8>} \right\},$$

$$\hbar_8 = \hbar(\upsilon_3, d, 1) = \left\{ \frac{w_1}{<0.4,0.3,0.2>}, \frac{w_2}{<0.6,0.3,0.1>}, \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.9,0.1,0.4>} \right\},$$

$$\hbar_9 = \hbar(\upsilon_3, e, 1) = \left\{ \frac{w_1}{<0.7,0.2,0.6>}, \frac{w_2}{<0.3,0.5,0.7>}, \frac{w_3}{<0.5,0.4,0.5>}, \frac{w_4}{<0.2,0.7,0.8>} \right\},$$

$$\hbar_{10} = \hbar(\upsilon_1, c, 0) = \left\{ \frac{w_1}{<0.3,0.2,0.1>}, \frac{w_2}{<0.2,0.4,0.5>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.1,0.8,0.3>} \right\},$$

$$\hbar_{11} = \hbar(\upsilon_1, d, 0) = \left\{ \frac{w_1}{<0.1,0.8,0.4>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.7,0.5>} \right\},$$

$$\hbar_{12} = \hbar(\upsilon_1, e, 0) = \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.1,0.8,0.6>}, \frac{w_3}{<0.3,0.5,0.7>}, \frac{w_4}{<0.5,0.4,0.6>} \right\},$$

$$\hbar_{13} = \hbar(\upsilon_2, c, 0) = \left\{ \frac{w_1}{<0.8,0.1,0.6>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.5,0.4,0.8>}, \frac{w_4}{<0.7,0.2,0.9>} \right\},$$

$$\hbar_{14} = \hbar(\upsilon_2, d, 0) = \left\{ \frac{w_1}{<0.7,0.2,0.5>}, \frac{w_2}{<0.2,0.6,0.4>}, \frac{w_3}{<0.9,0.1,0.6>}, \frac{w_4}{<0.4,0.5,0.7>} \right\},$$

$$\hbar_{15} = \hbar(\upsilon_2, e, 0) = \left\{ \frac{w_1}{<0.6,0.2,0.5>}, \frac{w_2}{<0.7,0.2,0.4>}, \frac{w_3}{<0.3,0.5,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \right\},$$

$$\hbar_{16} = \hbar(\upsilon_3, c, 0) = \left\{ \frac{w_1}{<0.1,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.7>}, \frac{w_3}{<0.7,0.2,0.9>}, \frac{w_4}{<0.8,0.2,0.4>} \right\},$$

$$\hbar_{17} = \hbar(\upsilon_3, d, 0) = \left\{ \frac{w_1}{<0.2,0.7,0.4>}, \frac{w_2}{<0.9,0.1,0.6>}, \frac{w_3}{<0.8,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.7>} \right\},$$

$$\hbar_{18} = \hbar(\upsilon_3, e, 0) = \left\{ \frac{w_1}{<0.5,0.4,0.2>}, \frac{w_2}{<0.3,0.6,0.1>}, \frac{w_3}{<0.6,0.3,0.2>}, \frac{w_4}{<0.1,0.8,0.3>} \right\}.$$

The NHSES can be described as $(\hbar, \mathbb{G}) =$

$$\left\{ \begin{array}{ll}
\left((\upsilon_1, c, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.5>}, \\ \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.1,0.3,0.6>} \end{array} \right\} \right), & \left((\upsilon_1, d, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.8,0.1,0.5>}, \\ \frac{w_3}{<0.4,0.5,0.6>}, \frac{w_4}{<0.2,0.5,0.3>} \end{array} \right\} \right), \\
\left((\upsilon_1, e, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.3,0.6>}, \\ \frac{w_3}{<0.6,0.3,0.7>}, \frac{w_4}{<0.3,0.5,0.6>} \end{array} \right\} \right), & \left((\upsilon_2, c, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.9,0.1,0.3>}, \frac{w_2}{<0.4,0.5,0.4>}, \\ \frac{w_3}{<0.7,0.2,0.6>}, \frac{w_4}{<0.3,0.4,0.8>} \end{array} \right\} \right), \\
\left((\upsilon_2, d, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.4,0.5,0.6>}, \frac{w_2}{<0.8,0.1,0.7>}, \\ \frac{w_3}{<0.3,0.6,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \end{array} \right\} \right), & \left((\upsilon_2, e, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.5,0.4,0.7>}, \frac{w_2}{<0.3,0.6,0.4>}, \\ \frac{w_3}{<0.6,0.2,0.5>}, \frac{w_4}{<0.8,0.1,0.6>} \end{array} \right\} \right), \\
\left((\upsilon_3, c, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.9,0.1,0.4>}, \\ \frac{w_3}{<0.4,0.5,0.7>}, \frac{w_4}{<0.5,0.4,0.8>} \end{array} \right\} \right), & \left((\upsilon_3, d, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.4,0.3,0.2>}, \frac{w_2}{<0.6,0.3,0.1>}, \\ \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.9,0.1,0.4>} \end{array} \right\} \right), \\
\left((\upsilon_3, e, 1), \left\{ \begin{array}{c} \frac{w_1}{<0.7,0.2,0.6>}, \frac{w_2}{<0.3,0.5,0.7>}, \\ \frac{w_3}{<0.5,0.4,0.5>}, \frac{w_4}{<0.2,0.7,0.8>} \end{array} \right\} \right), & \left((\upsilon_1, c, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.3,0.2,0.1>}, \frac{w_2}{<0.2,0.4,0.5>}, \\ \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.1,0.8,0.3>} \end{array} \right\} \right), \\
\left((\upsilon_1, d, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.1,0.8,0.4>}, \frac{w_2}{<0.9,0.1,0.2>}, \\ \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.7,0.5>} \end{array} \right\} \right), & \left((\upsilon_1, e, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.1,0.8,0.6>}, \\ \frac{w_3}{<0.3,0.5,0.7>}, \frac{w_4}{<0.5,0.4,0.6>} \end{array} \right\} \right), \\
\left((\upsilon_2, c, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.8,0.1,0.6>}, \frac{w_2}{<0.3,0.6,0.7>}, \\ \frac{w_3}{<0.5,0.4,0.8>}, \frac{w_4}{<0.7,0.2,0.9>} \end{array} \right\} \right), & \left((\upsilon_2, d, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.7,0.2,0.5>}, \frac{w_2}{<0.2,0.6,0.4>}, \\ \frac{w_3}{<0.9,0.1,0.6>}, \frac{w_4}{<0.4,0.5,0.7>} \end{array} \right\} \right), \\
\left((\upsilon_2, e, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.6,0.2,0.5>}, \frac{w_2}{<0.7,0.2,0.4>}, \\ \frac{w_3}{<0.3,0.5,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \end{array} \right\} \right), & \left((\upsilon_3, c, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.1,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.7>}, \\ \frac{w_3}{<0.7,0.2,0.9>}, \frac{w_4}{<0.8,0.2,0.4>} \end{array} \right\} \right), \\
\left((\upsilon_3, d, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.2,0.7,0.4>}, \frac{w_2}{<0.9,0.1,0.6>}, \\ \frac{w_3}{<0.8,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.7>} \end{array} \right\} \right), & \left((\upsilon_3, e, 0), \left\{ \begin{array}{c} \frac{w_1}{<0.5,0.4,0.2>}, \frac{w_2}{<0.3,0.6,0.1>}, \\ \frac{w_3}{<0.6,0.3,0.2>}, \frac{w_4}{<0.1,0.8,0.3>} \end{array} \right\} \right),
\end{array} \right\}.$$

Definition 3.4. Neutrosophic Hypersoft Expert Subset

A NHSES $(\hbar_1, \mathbb{G})$ is said to be NHSE subset of $(\hbar_2, \mathbb{P})$, if

(i) $\mathbb{G} \subseteq \mathbb{P}$, (ii) $\forall\ \gamma \in \mathbb{G}, \hbar_1(\gamma) \subseteq \hbar_2(\gamma)$ and denoted by $(\hbar_1, \mathbb{G}) \subseteq (\hbar_2, \mathbb{P})$.

Example 3.5. Considering Example 3.3, with two NHSESs

$$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1) \Big\}$$

$$\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_1, d, 0), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1), (\upsilon_1, e, 1) \Big\}.$$

It is clear that $\mathbb{G}_1 \subset \mathbb{G}_2$. Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ be defined as following

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l}
\left((\upsilon_1, c, 1), \left\{\frac{w_1}{<0.1,0.6,0.7>}, \frac{w_2}{<0.6,0.5,0.8>}, \frac{w_3}{<0.4,0.6,0.9>}, \frac{w_4}{<0.1,0.8,0.6>}\right\}\right), \\
\left((\upsilon_1, d, 1), \left\{\frac{w_1}{<0.3,0.4,0.5>}, \frac{w_2}{<0.6,0.4,0.6>}, \frac{w_3}{<0.2,0.5,0.7>}, \frac{w_4}{<0.1,0.5,0.6>}\right\}\right), \\
\left((\upsilon_3, d, 1), \left\{\frac{w_1}{<0.2,0.6,0.4>}, \frac{w_2}{<0.5,0.4,0.7>}, \frac{w_3}{<0.6,0.5,0.8>}, \frac{w_4}{<0.8,0.6,0.4>}\right\}\right), \\
\left((\upsilon_3, e, 1), \left\{\frac{w_1}{<0.6,0.4,0.3>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.5,0.3>}, \frac{w_4}{<0.1,0.7,0.4>}\right\}\right), \\
\left((\upsilon_1, e, 0), \left\{\frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.1,0.7,0.4>}, \frac{w_3}{<0.2,0.7,0.6>}, \frac{w_4}{<0.1,0.6,0.7>}\right\}\right), \\
\left((\upsilon_3, c, 0), \left\{\frac{w_1}{<0.1,0.8,0.6>}, \frac{w_2}{<0.3,0.6,0.5>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.7,0.2,0.6>}\right\}\right), \\
\left((\upsilon_3, d, 0), \left\{\frac{w_1}{<0.1,0.7,0.4>}, \frac{w_2}{<0.6,0.3,0.6>}, \frac{w_3}{<0.7,0.2,0.5>}, \frac{w_4}{<0.2,0.7,0.4>}\right\}\right)
\end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l}
\left((\upsilon_1, c, 1), \left\{\frac{w_1}{<0.2,0.3,0.6>}, \frac{w_2}{<0.7,0.4,0.7>}, \frac{w_3}{<0.5,0.4,0.8>}, \frac{w_4}{<0.2,0.4,0.5>}\right\}\right), \\
\left((\upsilon_1, d, 1), \left\{\frac{w_1}{<0.4,0.3,0.4>}, \frac{w_2}{<0.8,0.3,0.5>}, \frac{w_3}{<0.4,0.3,0.6>}, \frac{w_4}{<0.2,0.6,0.5>}\right\}\right), \\
\left((\upsilon_3, c, 1), \left\{\frac{w_1}{<0.1,0.3,0.4>}, \frac{w_2}{<0.9,0.1,0.3>}, \frac{w_3}{<0.4,0.5,0.4>}, \frac{w_4}{<0.5,0.3,0.4>}\right\}\right), \\
\left((\upsilon_3, d, 1), \left\{\frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.6,0.3,0.6>}, \frac{w_3}{<0.7,0.4,0.5>}, \frac{w_4}{<0.9,0.5,0.2}\right\}\right), \\
\left((\upsilon_1, e, 1), \left\{\frac{w_1}{<0.7,0.2,0.4>}, \frac{w_2}{<0.5,0.2,0.6>}, \frac{w_3}{<0.6,0.2,0.7>}, \frac{w_4}{<0.3,0.5,0.8}\right\}\right), \\
\left((\upsilon_3, e, 1), \left\{\frac{w_1}{<0.7,0.3,0.1>}, \frac{w_2}{<0.3,0.5,0.4>}, \frac{w_3}{<0.5,0.4,0.2>}, \frac{w_4}{<0.2,0.6,0.3>}\right\}\right), \\
\left((\upsilon_1, e, 0), \left\{\frac{w_1}{<0.2,0.5,0.1>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.4>}, \frac{w_4}{<0.5,0.3,0.5>}\right\}\right), \\
\left((\upsilon_1, d, 0), \left\{\frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.9,0.1,0.6>}, \frac{w_3}{<0.6,0.3,0.8>}, \frac{w_4}{<0.2,0.6,0.8>}\right\}\right), \\
\left((\upsilon_3, c, 0), \left\{\frac{w_1}{<0.2,0.7,0.4>}, \frac{w_2}{<0.4,0.5,0.3>}, \frac{w_3}{<0.7,0.2,0.1>}, \frac{w_4}{<0.8,0.1,0.5>}\right\}\right), \\
\left((\upsilon_3, d, 0), \left\{\frac{w_1}{<0.2,0.5,0.1>}, \frac{w_2}{<0.7,0.2,0.3>}, \frac{w_3}{<0.8,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.2>}\right\}\right)
\end{array} \right\}$$

which shows that $(\hbar_1, \mathbb{G}_1) \subseteq (\hbar_2, \mathbb{G}_2)$.

Definition 3.6. Two NHSESs $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ are said to be equal if $(\hbar_1, \mathbb{G}_1)$ is a NHSE subset of $(\hbar_2, \mathbb{G}_2)$ and $(\hbar_2, \mathbb{G}_2)$ is a neutrosophic hypersoft expert subset of $(\hbar_1, \mathbb{G}_1)$.

Definition 3.7. The complement of a NHSES is characterized by as
$(\hbar, \mathbb{G})^c = \tilde{c}(\hbar(\varsigma))\ \forall\ \varsigma \in \Delta$ while $\tilde{c}$ is a neutrosophic complement.

Example 3.8. Finding complement of NHSES find in 3.3, we have

$$(\hbar,\mathbb{G})^{c}=\left\{\begin{array}{l}
\left((\upsilon_1,c,1),\left\{\frac{w_1}{<0.4,0.5,0.2>},\frac{w_2}{<0.5,0.8,0.7>},\frac{w_3}{<0.6,0.6,0.5>},\frac{w_4}{<0.6,0.7,0.1>}\right\}\right),\\
\left((\upsilon_1,d,1),\left\{\frac{w_1}{<0.3,0.8,0.6>},\frac{w_2}{<0.5,0.9,0.8>},\frac{w_3}{<0.6,0.5,0.4>},\frac{w_4}{<0.3,0.5,0.2>}\right\}\right),\\
\left((\upsilon_1,e,1),\left\{\frac{w_1}{<0.3,0.8,0.7>},\frac{w_2}{<0.6,0.7,0.5>},\frac{w_3}{<0.7,0.7,0.6>},\frac{w_4}{<0.6,0.5,0.3>}\right\}\right),\\
\left((\upsilon_2,c,1),\left\{\frac{w_1}{<0.3,0.9,0.9>},\frac{w_2}{<0.4,0.5,0.4>},\frac{w_3}{<0.6,0.8,0.7>},\frac{w_4}{<0.8,0.6,0.3>}\right\}\right),\\
\left((\upsilon_2,d,1),\left\{\frac{w_1}{<0.6,0.5,0.4>},\frac{w_2}{<0.7,0.9,0.8>},\frac{w_3}{<0.5,0.4,0.3>},\frac{w_4}{<0.7,0.4,0.2>}\right\}\right),\\
\left((\upsilon_2,e,1),\left\{\frac{w_1}{<0.7,0.6,0.5>},\frac{w_2}{<0.4,0.4,0.3>},\frac{w_3}{<0.5,0.8,0.6>},\frac{w_4}{<0.6,0.9,0.8>}\right\}\right),\\
\left((\upsilon_3,c,1),\left\{\frac{w_1}{<0.5,0.3,0.2>},\frac{w_2}{<0.4,0.9,0.9>},\frac{w_3}{<0.7,0.5,0.4>},\frac{w_4}{<0.8,0.6,0.5>}\right\}\right),\\
\left((\upsilon_3,d,1),\left\{\frac{w_1}{<0.2,0.7,0.4>},\frac{w_2}{<0.1,0.7,0.6>},\frac{w_3}{<0.3,0.8,0.7>},\frac{w_4}{<0.4,0.9,0.9>}\right\}\right),\\
\left((\upsilon_3,e,1),\left\{\frac{w_1}{<0.6,0.8,0.7>},\frac{w_2}{<0.7,0.5,0.3>},\frac{w_3}{<0.5,0.6,0.5>},\frac{w_4}{<0.8,0.3,0.2>}\right\}\right),\\
\left((\upsilon_1,c,0),\left\{\frac{w_1}{<0.1,0.8,0.3>},\frac{w_2}{<0.5,0.6,0.2>},\frac{w_3}{<0.8,0.5,0.4>},\frac{w_4}{<0.3,0.2,0.1>}\right\}\right),\\
\left((\upsilon_1,d,0),\left\{\frac{w_1}{<0.4,0.2,0.1>},\frac{w_2}{<0.2,0.9,0.9>},\frac{w_3}{<0.4,0.7,0.6>},\frac{w_4}{<0.5,0.3,0.2>}\right\}\right),\\
\left((\upsilon_1,e,0),\left\{\frac{w_1}{<0.5,0.3,0.2>},\frac{w_2}{<0.6,0.2,0.1>},\frac{w_3}{<0.7,0.5,0.3>},\frac{w_4}{<0.6,0.6,0.5>}\right\}\right),\\
\left((\upsilon_2,c,0),\left\{\frac{w_1}{<0.6,0.9,0.8>},\frac{w_2}{<0.7,0.4,0.3>},\frac{w_3}{<0.8,0.6,0.5>},\frac{w_4}{<0.9,0.8,0.7>}\right\}\right),\\
\left((\upsilon_2,d,0),\left\{\frac{w_1}{<0.5,0.8,0.7>},\frac{w_2}{<0.4,0.4,0.2>},\frac{w_3}{<0.6,0.9,0.9>},\frac{w_4}{<0.7,0.5,0.4>}\right\}\right),\\
\left((\upsilon_2,e,0),\left\{\frac{w_1}{<0.5,0.8,0.6>},\frac{w_2}{<0.4,0.8,0.7>},\frac{w_3}{<0.4,0.5,0.3>},\frac{w_4}{<0.6,0.3,0.2>}\right\}\right),\\
\left((\upsilon_3,c,0),\left\{\frac{w_1}{<0.5,0.3,0.1>},\frac{w_2}{<0.7,0.5,0.3>},\frac{w_3}{<0.9,0.8,0.8>},\frac{w_4}{<0.4,0.8,0.8>}\right\}\right),\\
\left((\upsilon_3,d,0),\left\{\frac{w_1}{<0.4,0.3,0.2>},\frac{w_2}{<0.6,0.9,0.9>},\frac{w_3}{<0.4,0.8,0.8>},\frac{w_4}{<0.7,0.5,0.3>}\right\}\right),\\
\left((\upsilon_3,e,0),\left\{\frac{w_1}{<0.2,0.6,0.5>},\frac{w_2}{<0.1,0.4,0.3>},\frac{w_3}{<0.2,0.7,0.6>},\frac{w_4}{<0.3,0.2,0.1>}\right\}\right)
\end{array}\right\}.$$

Definition 3.9. An agree-NHSES is described by as $(\hbar,Æ)_{ag}=\{\hbar_{ag}(\varsigma):\varsigma\in Æ\times Œ\times\{1\}\}$.

Example 3.10. Finding agree-NHSES calculated in 3.3, we get

$$(\hbar,\mathbb{G})=\left\{\begin{array}{l}
\left((\upsilon_1,c,1),\left\{\frac{w_1}{<0.2,0.5,0.4>},\frac{w_2}{<0.7,0.2,0.5>},\frac{w_3}{<0.5,0.4,0.6>},\frac{w_4}{<0.1,0.3,0.6>}\right\}\right),\\
\left((\upsilon_1,d,1),\left\{\frac{w_1}{<0.4,0.2,0.3>},\frac{w_2}{<0.8,0.1,0.5>},\frac{w_3}{<0.4,0.5,0.6>},\frac{w_4}{<0.2,0.5,0.3>}\right\}\right),\\
\left((\upsilon_1,e,1),\left\{\frac{w_1}{<0.7,0.2,0.3>},\frac{w_2}{<0.5,0.3,0.6>},\frac{w_3}{<0.6,0.3,0.7>},\frac{w_4}{<0.3,0.5,0.6>}\right\}\right),\\
\left((\upsilon_2,c,1),\left\{\frac{w_1}{<0.9,0.1,0.3>},\frac{w_2}{<0.4,0.5,0.4>},\frac{w_3}{<0.7,0.2,0.6>},\frac{w_4}{<0.3,0.4,0.8>}\right\}\right),\\
\left((\upsilon_2,d,1),\left\{\frac{w_1}{<0.4,0.5,0.6>},\frac{w_2}{<0.8,0.1,0.7>},\frac{w_3}{<0.3,0.6,0.5>},\frac{w_4}{<0.2,0.6,0.7>}\right\}\right),\\
\left((\upsilon_2,e,1),\left\{\frac{w_1}{<0.5,0.4,0.7>},\frac{w_2}{<0.3,0.6,0.4>},\frac{w_3}{<0.6,0.2,0.5>},\frac{w_4}{<0.8,0.1,0.6>}\right\}\right),\\
\left((\upsilon_3,c,1),\left\{\frac{w_1}{<0.2,0.7,0.5>},\frac{w_2}{<0.9,0.1,0.4>},\frac{w_3}{<0.4,0.5,0.7>},\frac{w_4}{<0.5,0.4,0.8>}\right\}\right),\\
\left((\upsilon_3,d,1),\left\{\frac{w_1}{<0.4,0.3,0.2>},\frac{w_2}{<0.6,0.3,0.1>},\frac{w_3}{<0.7,0.2,0.3>},\frac{w_4}{<0.9,0.1,0.4>}\right\}\right),\\
\left((\upsilon_3,e,1),\left\{\frac{w_1}{<0.7,0.2,0.6>},\frac{w_2}{<0.3,0.5,0.7>},\frac{w_3}{<0.5,0.4,0.5>},\frac{w_4}{<0.2,0.7,0.8>}\right\}\right)
\end{array}\right\}.$$

Definition 3.11. A disagree-NHSES is described by as
$(\hbar,Æ)_{dag}=\{\hbar_{dag}(\varsigma):\varsigma\in Æ\times Œ\times\{0\}\}$.

Example 3.12. Getting disagree-NHSES calculated in 3.3,

$$(\hbar,\mathbb{G})=\left\{\begin{array}{l}\left((\upsilon_1,c,0),\left\{\frac{w_1}{<0.3,0.2,0.1>},\frac{w_2}{<0.2,0.4,0.5>},\frac{w_3}{<0.4,0.5,0.8>},\frac{w_4}{<0.1,0.8,0.3>}\right\}\right),\\ \left((\upsilon_1,d,0),\left\{\frac{w_1}{<0.1,0.8,0.4>},\frac{w_2}{<0.9,0.1,0.2>},\frac{w_3}{<0.6,0.3,0.4>},\frac{w_4}{<0.2,0.7,0.5>}\right\}\right),\\ \left((\upsilon_1,e,0),\left\{\frac{w_1}{<0.2,0.7,0.5>},\frac{w_2}{<0.1,0.8,0.6>},\frac{w_3}{<0.3,0.5,0.7>},\frac{w_4}{<0.5,0.4,0.6>}\right\}\right),\\ \left((\upsilon_2,c,0),\left\{\frac{w_1}{<0.8,0.1,0.6>},\frac{w_2}{<0.3,0.6,0.7>},\frac{w_3}{<0.5,0.4,0.8>},\frac{w_4}{<0.7,0.2,0.9>}\right\}\right),\\ \left((\upsilon_2,d,0),\left\{\frac{w_1}{<0.7,0.2,0.5>},\frac{w_2}{<0.2,0.6,0.4>},\frac{w_3}{<0.9,0.1,0.6>},\frac{w_4}{<0.4,0.5,0.7>}\right\}\right),\\ \left((\upsilon_2,e,0),\left\{\frac{w_1}{<0.6,0.2,0.5>},\frac{w_2}{<0.7,0.2,0.4>},\frac{w_3}{<0.3,0.5,0.4>},\frac{w_4}{<0.2,0.7,0.6>}\right\}\right),\\ \left((\upsilon_3,c,0),\left\{\frac{w_1}{<0.1,0.7,0.5>},\frac{w_2}{<0.4,0.5,0.7>},\frac{w_3}{<0.7,0.2,0.9>},\frac{w_4}{<0.8,0.2,0.4>}\right\}\right),\\ \left((\upsilon_3,d,0),\left\{\frac{w_1}{<0.2,0.7,0.4>},\frac{w_2}{<0.9,0.1,0.6>},\frac{w_3}{<0.8,0.2,0.4>},\frac{w_4}{<0.3,0.5,0.7>}\right\}\right),\\ \left((\upsilon_3,e,0),\left\{\frac{w_1}{<0.5,0.4,0.2>},\frac{w_2}{<0.3,0.6,0.1>},\frac{w_3}{<0.6,0.3,0.2>},\frac{w_4}{<0.1,0.8,0.3>}\right\}\right),\end{array}\right\}.$$

Definition 3.13. A NHSES $(\hbar_1,\mathbb{G}_1)$ is called a relative null NHSES w.r.t $\mathbb{G}_1\subset\mathbb{G}$, denoted by $(\hbar_1,\mathbb{G}_1)$, if $\hbar_1(g)=\emptyset$, $\forall$ g $\in\mathbb{G}_1$.

Example 3.14. Considering Example 3.3, we
$(\hbar_1,\mathbb{G}_1)=\{((w_1,c,1),\emptyset),((w_2,d,1),\emptyset),((w_3,e,1),\emptyset)\}$.

Definition 3.15. A NHSES $(\hbar_2,\mathbb{G}_2)$ is called a relative whole NHSES w.r.t $\mathbb{G}_2\subset\mathbb{G}$, denoted by $(\hbar_2,\mathbb{G}_2)_\Delta$, if $\hbar_1(g)=\Delta$, $\forall$ g $\in\mathbb{G}_2$.

Example 3.16. Considering Example 3.3, we have
$(\hbar_2,\mathbb{G}_2)_\Delta=\{((w_1,c,1),\Delta),((w_2,d,1),\Delta),((w_3,e,1),\Delta)\}$ where $\mathbb{G}_2\subseteq\mathbb{G}$.

Definition 3.17. A NHSES $(\hbar,\mathbb{G})$ is called absolute whole NHSES denoted by $(\hbar,\mathbb{G})_\Delta$, if $\hbar(g)=\Delta$, $\forall$ g $\in\mathbb{G}$.

Example 3.18. Considering Example 3.3, we have

$$(\Psi,\mathbb{S})_\Delta=\left\{\begin{array}{l}((w_1,c,1),\Delta),((w_1,d,1),\Delta),((w_1,e,1),\Delta),((w_3,c,1),\Delta),\\ ((w_3,d,1),\Delta),((w_3,e,1),\Delta),((w_5,c,1),\Delta),((w_5,d,1),\Delta),\\ ((w_5,e,1),\Delta),((w_1,c,0),\Delta),((w_1,d,0),\Delta),((w_1,e,0),\Delta),\\ ((w_3,c,0),\Delta),((w_3,d,0),\Delta),((w_3,e,0),\Delta),((w_5,c,0),\Delta),\\ ((w_5,d,0),\Delta),((w_5,e,0),\Delta)\end{array}\right\}.$$

Proposition 3.19. *Suppose $(\hbar_1,\mathbb{G}_1)_\Delta$, $(\hbar_2,\mathbb{G}_2)_\Delta$, $(\hbar_3,\mathbb{G}_3)_\Delta$, be three NHSES-sets over Δ, then*

- $(\hbar_1,\mathbb{G}_1)\subset(\hbar_2,\mathbb{G}_2)_\Delta$,
- $(\hbar_1,\mathbb{G}_1)_\hbar\subset(\hbar_1,\mathbb{G}_1)$,
- $(\hbar_1,\mathbb{G}_1)\subset(\hbar_1,\mathbb{G}_1)$,
- *If* $(\hbar_1,\mathbb{G}_1)\subset(\hbar_2,\mathbb{G}_2)$, *and* $(\hbar_2,\mathbb{G}_2)\subset(\hbar_3,\mathbb{G}_3)$, *then* $(\hbar_1,\mathbb{G}_1)\subset(\hbar_3,\mathbb{S}_3)$.

- *If* $(\hbar_1, \mathbb{G}_1) = (\hbar_2, \mathbb{G}_2)$, *and* $(\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_3)$, *then* $(\hbar_1, \mathbb{G}_1) = (\hbar_3, \mathbb{G}_3)$.

Proposition 3.20. *If* $(\hbar, \mathbb{G})$ *is a NHSES over* Δ, *then*

(1) $((\hbar, \mathbb{G})^c)^c = (\hbar, \mathbb{G})$
(2) $(\hbar, \mathbb{G})^c_{ag} = (\hbar, \mathbb{G})_{dag}$
(3) $(\hbar, \mathbb{G})^c_{dag} = (\hbar, \mathbb{G})_{ag}$.

4. Set Theoretic Operations of NHSES

In this portion, some set theoretic operations are presented with detailed examples.

Definition 4.1. The union of $(\hbar_1, \mathbb{G})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{G} \cup \mathbb{R}$, defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{G} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{G} \\ \cup(\hbar_1(\varsigma), \hbar_2(\varsigma)) & ; \varsigma \in \mathbb{G} \cap \mathbb{R}. \end{cases}$$

where $\cup(\hbar_1(\varsigma), \hbar_2(\varsigma)) = \{< u, \max\{\upsilon_1(\varsigma), \upsilon_2(\varsigma)\}, 1/2\{\nu_1(\varsigma) + \nu_2(\varsigma)\}, \min\{\omega_1(\varsigma), \omega_2(\varsigma)\} >: u \in \Delta\}$.

Example 4.2. Considering Example 3.3, we see

$$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1) \Big\}$$

$$\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_1, e, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1), (\upsilon_1, d, 0) \Big\}.$$

Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ are two NHSESs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.3,0.2>}, \frac{w_3}{<0.4,0.5,0.1>}, \frac{w_4}{<0.1,0.8,0.5>} \right\} \right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.5>}, \frac{w_2}{<0.6,0.2,0.3>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.5,0.3>} \right\} \right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.6,0.7>}, \frac{w_2}{<0.5,0.2,0.3>}, \frac{w_3}{<0.6,0.3,0.5>}, \frac{w_4}{<0.8,0.1,0.9>} \right\} \right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.2,0.4>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.1,0.5,0.4>} \right\} \right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.3,0.5>}, \frac{w_2}{<0.1,0.7,0.6>}, \frac{w_3}{<0.2,0.7,0.4>}, \frac{w_4}{<0.4,0.6,0.8>} \right\} \right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\} \right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.7,0.3>}, \frac{w_2}{<0.8,0.1,0.2>}, \frac{w_3}{<0.7,0.2,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \right\} \right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l}
\left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.3,0.4>}, \frac{w_2}{<0.7,0.4,0.5>}, \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.2,0.4,0.7>} \right\} \right), \\
\left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.3,0.8>}, \frac{w_2}{<0.8,0.3,0.5>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \right\} \right), \\
\left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.3,0.6>}, \frac{w_2}{<0.9,0.1,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\} \right), \\
\left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.7,0.4,0.5>}, \frac{w_4}{<0.9,0.5,0.7} \right\} \right), \\
\left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.2,0.4>}, \frac{w_3}{<0.6,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.6} \right\} \right), \\
\left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.3,0.7>}, \frac{w_2}{<0.3,0.5,0.6>}, \frac{w_3}{<0.5,0.4,0.3>}, \frac{w_4}{<0.2,0.6,0.4>} \right\} \right), \\
\left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.6>}, \frac{w_4}{<0.5,0.3,0.7>} \right\} \right), \\
\left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.6,0.3>} \right\} \right), \\
\left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.6>}, \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.8,0.1,0.4>} \right\} \right), \\
\left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.3>}, \frac{w_3}{<0.8,0.2,0.6>}, \frac{w_4}{<0.3,0.5,0.7>} \right\} \right)
\end{array} \right\}.$$

Then $(\hbar_1, \mathbb{G}_1) \cup (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_3)$

$$(\hbar_3, \mathbb{G}_3) = \left\{ \begin{array}{l}
\left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.45,0.4>}, \frac{w_2}{<0.7,0.35,0.2>}, \frac{w_3}{<0.5,0.45,0.1>}, \frac{w_4}{<0.2,0.6,0.5>} \right\} \right), \\
\left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.35,0.5>}, \frac{w_2}{<0.8,0.25,0.3>}, \frac{w_3}{<0.4,0.4,0.5>}, \frac{w_4}{<0.2,0.55,0.3>} \right\} \right), \\
\left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.30,0.6>}, \frac{w_2}{<0.9,0.10,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\} \right), \\
\left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.40,0.3>}, \frac{w_2}{<0.6,0.25,0.3>}, \frac{w_3}{<0.7,0.35,0.5>}, \frac{w_4}{<0.9,0.30,0.7} \right\} \right), \\
\left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.20,0.3>}, \frac{w_2}{<0.5,0.20,0.4>}, \frac{w_3}{<0.6,0.20,0.4>}, \frac{w_4}{<0.3,0.50,0.6} \right\} \right), \\
\left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.20,0.4>}, \frac{w_2}{<0.3,0.50,0.6>}, \frac{w_3}{<0.5,0.3,0.3>}, \frac{w_4}{<0.2,0.5,0.4>} \right\} \right), \\
\left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.40,0.4>}, \frac{w_2}{<0.2,0.65,0.3>}, \frac{w_3}{<0.3,0.6,0.4>}, \frac{w_4}{<0.5,0.45,0.7>} \right\} \right), \\
\left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.60,0.3>}, \frac{w_2}{<0.9,0.10,0.2>}, \frac{w_3}{<0.6,0.30,0.4>}, \frac{w_4}{<0.2,0.60,0.3>} \right\} \right), \\
\left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.65,0.5>}, \frac{w_2}{<0.4,0.55,0.6>}, \frac{w_3}{<0.7,0.15,0.2>}, \frac{w_4}{<0.8,0.15,0.3>} \right\} \right), \\
\left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.60,0.3>}, \frac{w_2}{<0.8,0.15,0.2>}, \frac{w_3}{<0.8,0.20,0.4>}, \frac{w_4}{<0.3,0.60,0.6>} \right\} \right)
\end{array} \right\}.$$

Definition 4.3. Restricted Union of two NHSESs $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{G}_1 \cap \mathbb{G}_2$, defined as $\hbar_3(\varsigma) = \hbar_1(\varsigma) \cup_{\mathbb{R}} \hbar_2(\varsigma)$ for $\varsigma \in \mathbb{G}_1 \cap \mathbb{G}_2$.

Example 4.4. Considering Example 3.3, we see
$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1) \Big\}$, $\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_1, d, 0), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1), (\upsilon_1, e, 1) \Big\}$.

Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ are two NHSESs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.3,0.2>}, \frac{w_3}{<0.4,0.5,0.1>}, \frac{w_4}{<0.1,0.8,0.5>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.5>}, \frac{w_2}{<0.6,0.2,0.3>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.5,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.6,0.7>}, \frac{w_2}{<0.5,0.2,0.3>}, \frac{w_3}{<0.6,0.3,0.5>}, \frac{w_4}{<0.8,0.1,0.9>} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.2,0.4>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.1,0.5,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.3,0.5>}, \frac{w_2}{<0.1,0.7,0.6>}, \frac{w_3}{<0.2,0.7,0.4>}, \frac{w_4}{<0.4,0.6,0.8>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.7,0.3>}, \frac{w_2}{<0.8,0.1,0.2>}, \frac{w_3}{<0.7,0.2,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.3,0.4>}, \frac{w_2}{<0.7,0.4,0.5>}, \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.2,0.4,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.3,0.8>}, \frac{w_2}{<0.8,0.3,0.5>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.3,0.6>}, \frac{w_2}{<0.9,0.1,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.7,0.4,0.5>}, \frac{w_4}{<0.9,0.5,0.7} \right\}\right), \\ \left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.2,0.4>}, \frac{w_3}{<0.6,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.6} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.3,0.7>}, \frac{w_2}{<0.3,0.5,0.6>}, \frac{w_3}{<0.5,0.4,0.3>}, \frac{w_4}{<0.2,0.6,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.6>}, \frac{w_4}{<0.5,0.3,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.6,0.3>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.6>}, \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.8,0.1,0.4>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.3>}, \frac{w_3}{<0.8,0.2,0.6>}, \frac{w_4}{<0.3,0.5,0.7>} \right\}\right) \end{array} \right\}.$$

Then $(\hbar_1, \mathbb{G}_1) \cup_{\mathbb{R}} (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{L})$

$$(\hbar_3, \mathbb{L}) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.45,0.4>}, \frac{w_2}{<0.7,0.35,0.2>}, \frac{w_3}{<0.5,0.45,0.1>}, \frac{w_4}{<0.2,0.6,0.5>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.35,0.5>}, \frac{w_2}{<0.8,0.25,0.3>}, \frac{w_3}{<0.4,0.4,0.5>}, \frac{w_4}{<0.2,0.55,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.40,0.3>}, \frac{w_2}{<0.6,0.25,0.3>}, \frac{w_3}{<0.7,0.35,0.5>}, \frac{w_4}{<0.9,0.30,0.7} \right\}\right), \\ \left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.20,0.3>}, \frac{w_2}{<0.5,0.20,0.4>}, \frac{w_3}{<0.6,0.20,0.4>}, \frac{w_4}{<0.3,0.50,0.6} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.20,0.4>}, \frac{w_2}{<0.3,0.50,0.6>}, \frac{w_3}{<0.5,0.3,0.3>}, \frac{w_4}{<0.2,0.5,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.60,0.4>}, \frac{w_2}{<0.2,0.10,0.3>}, \frac{w_3}{<0.3,0.30,0.4>}, \frac{w_4}{<0.5,0.60,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.65,0.5>}, \frac{w_2}{<0.4,0.55,0.6>}, \frac{w_3}{<0.7,0.15,0.2>}, \frac{w_4}{<0.8,0.15,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.60,0.3>}, \frac{w_2}{<0.8,0.15,0.2>}, \frac{w_3}{<0.8,0.20,0.4>}, \frac{w_4}{<0.3,0.60,0.6>} \right\}\right) \end{array} \right\}.$$

Proposition 4.5. *If* $(\hbar_1, \mathbb{G}_1)$,$(\hbar_2, \mathbb{G}_2)$ *and* $(\hbar_3, \mathbb{G}_3)$ *are three NHSESs, then*

(1) $(\hbar_1, \mathbb{G}_1) \cup (\hbar_2, \mathbb{G}_2) = (\hbar_2, \mathbb{G}_2) \cup (\hbar_1, \mathbb{G}_1)$

(2) $((\hbar_1, \mathbb{G}_1) \cup (\hbar_2, \mathbb{G}_2)) \cup (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2) \cup (\hbar_3, N_3))$

(3) $(\hbar, \mathbb{G}) \cup \Phi = (\hbar, \mathbb{G})$.

Definition 4.6. The intersection of two NHSESs $(\hbar_1, \mathbb{G})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{G} \cap \mathbb{R}$, defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{G} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{G} \\ \cap(\hbar_1(\varsigma), \hbar_2(\varsigma)) & ; \varsigma \in \mathbb{G} \cap \mathbb{R} \end{cases}$$

where $\cap(\hbar_1(\varsigma), \hbar_2(\varsigma)) = \{< u,\ \min\{\upsilon_1(\varsigma), \upsilon_2(\varsigma)\},\ 1/2\{\nu_1(\varsigma) + \nu_2(\varsigma)\},\ \max\{\omega_1(\varsigma), \omega_2(\varsigma)\} >: u \in \Delta\}$.

Example 4.7. Reconsidering Example 3.3, we have

$$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1) \Big\}$$

$$\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_1, d, 0), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1), , (\upsilon_1, e, 1) \Big\}$$

Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ are two NHSESs over Δ such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.3,0.2>}, \frac{w_3}{<0.4,0.5,0.1>}, \frac{w_4}{<0.1,0.8,0.5>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.5>}, \frac{w_2}{<0.6,0.2,0.3>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.5,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.6,0.7>}, \frac{w_2}{<0.5,0.2,0.3>}, \frac{w_3}{<0.6,0.3,0.5>}, \frac{w_4}{<0.8,0.1,0.9>} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.2,0.4>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.1,0.5,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.3,0.5>}, \frac{w_2}{<0.1,0.7,0.6>}, \frac{w_3}{<0.2,0.7,0.4>}, \frac{w_4}{<0.4,0.6,0.8>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.7,0.3>}, \frac{w_2}{<0.8,0.1,0.2>}, \frac{w_3}{<0.7,0.2,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.3,0.4>}, \frac{w_2}{<0.7,0.4,0.5>}, \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.2,0.4,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.3,0.8>}, \frac{w_2}{<0.8,0.3,0.5>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.3,0.6>}, \frac{w_2}{<0.9,0.1,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.7,0.4,0.5>}, \frac{w_4}{<0.9,0.5,0.7} \right\}\right), \\ \left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.2,0.4>}, \frac{w_3}{<0.6,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.6} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.3,0.7>}, \frac{w_2}{<0.3,0.5,0.6>}, \frac{w_3}{<0.5,0.4,0.3>}, \frac{w_4}{<0.2,0.6,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.6>}, \frac{w_4}{<0.5,0.3,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.6,0.3>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.6>}, \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.8,0.1,0.4>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.3>}, \frac{w_3}{<0.8,0.2,0.6>}, \frac{w_4}{<0.3,0.5,0.7>} \right\}\right) \end{array} \right\}$$

Then $(\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_3)$

$$(\hbar_3, \mathbb{G}_3) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.45,0.4>}, \frac{w_2}{<0.6,0.35,0.5>}, \frac{w_3}{<0.4,0.45,0.6>}, \frac{w_4}{<0.1,0.6,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.35,0.8>}, \frac{w_2}{<0.6,0.25,0.5>}, \frac{w_3}{<0.2,0.4,0.6>}, \frac{w_4}{<0.1,0.55,0.7>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.4,0.7>}, \frac{w_2}{<0.5,0.25,0.5>}, \frac{w_3}{<0.6,0.35,0.5>}, \frac{w_4}{<0.8,0.30,0.7>} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.25,0.7>}, \frac{w_2}{<0.2,0.60,0.6>}, \frac{w_3}{<0.4,0.35,0.5>}, \frac{w_4}{<0.1,0.55,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.40,0.5>}, \frac{w_2}{<0.1,0.65,0.6>}, \frac{w_3}{<0.2,0.60,0.6>}, \frac{w_4}{<0.4,0.45,0.8>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.65,0.9>}, \frac{w_2}{<0.3,0.55,0.7>}, \frac{w_3}{<0.6,0.15,0.3>}, \frac{w_4}{<0.7,0.15,0.4>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.60,0.4>}, \frac{w_2}{<0.8,0.15,0.3>}, \frac{w_3}{<0.7,0.20,0.6>}, \frac{w_4}{<0.2,0.60,0.7>} \right\}\right), \end{array} \right\}.$$

Definition 4.8. Extended intersection of two NHSESs $(\hbar_1, \mathbb{S})$ and $(\hbar_2, \mathbb{R})$ over Δ is $(\hbar_3, \mathbb{L})$ with $\mathbb{L} = \mathbb{S} \cup \mathbb{R}$, defined as

$$\hbar_3(\varsigma) = \begin{cases} \hbar_1(\varsigma) & ; \varsigma \in \mathbb{S} - \mathbb{R} \\ \hbar_2(\varsigma) & ; \varsigma \in \mathbb{R} - \mathbb{S} \\ \hbar_1(\varsigma) \cap \hbar_2(\varsigma) & ; \varsigma \in \mathbb{S} \cap \mathbb{R}. \end{cases}$$

Example 4.9. Considering Example 3.3, we have

$$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1) \Big\}$$

$$\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 1), (\upsilon_3, c, 0), (\upsilon_3, c, 1), (\upsilon_1, d, 1), (\upsilon_3, d, 1), (\upsilon_1, d, 0), (\upsilon_3, d, 0), (\upsilon_1, e, 0), (\upsilon_3, e, 1), (\upsilon_1, e, 1) \Big\}.$$

Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ are two NHSESs over Δ such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.3,0.2>}, \frac{w_3}{<0.4,0.5,0.1>}, \frac{w_4}{<0.1,0.8,0.5>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.5>}, \frac{w_2}{<0.6,0.2,0.3>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.5,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.6,0.7>}, \frac{w_2}{<0.5,0.2,0.3>}, \frac{w_3}{<0.6,0.3,0.5>}, \frac{w_4}{<0.8,0.1,0.9>} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.2,0.4>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.1,0.5,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.3,0.5>}, \frac{w_2}{<0.1,0.7,0.6>}, \frac{w_3}{<0.2,0.7,0.4>}, \frac{w_4}{<0.4,0.6,0.8>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.7,0.3>}, \frac{w_2}{<0.8,0.1,0.2>}, \frac{w_3}{<0.7,0.2,0.4>}, \frac{w_4}{<0.2,0.7,0.6>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{c} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.2,0.3,0.4>}, \frac{w_2}{<0.7,0.4,0.5>}, \frac{w_3}{<0.5,0.4,0.6>}, \frac{w_4}{<0.2,0.4,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.4,0.3,0.8>}, \frac{w_2}{<0.8,0.3,0.5>}, \frac{w_3}{<0.4,0.3,0.5>}, \frac{w_4}{<0.2,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.3,0.6>}, \frac{w_2}{<0.9,0.1,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.4,0.2,0.3>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.7,0.4,0.5>}, \frac{w_4}{<0.9,0.5,0.7} \right\}\right), \\ \left((\upsilon_1, e, 1), \left\{ \frac{w_1}{<0.7,0.2,0.3>}, \frac{w_2}{<0.5,0.2,0.4>}, \frac{w_3}{<0.6,0.2,0.4>}, \frac{w_4}{<0.3,0.5,0.6} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.7,0.3,0.7>}, \frac{w_2}{<0.3,0.5,0.6>}, \frac{w_3}{<0.5,0.4,0.3>}, \frac{w_4}{<0.2,0.6,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.6>}, \frac{w_4}{<0.5,0.3,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.6,0.3>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.2,0.7,0.5>}, \frac{w_2}{<0.4,0.5,0.6>}, \frac{w_3}{<0.7,0.2,0.3>}, \frac{w_4}{<0.8,0.1,0.4>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.7,0.2,0.3>}, \frac{w_3}{<0.8,0.2,0.6>}, \frac{w_4}{<0.3,0.5,0.7>} \right\}\right) \end{array} \right\}.$$

Then $(\hbar_1, \mathbb{G}_1) \cap_E (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{L})$

$$(\hbar_3, \mathbb{L}) = \left\{ \begin{array}{c} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.4,0.5>}, \frac{w_3}{<0.4,0.5,0.6>}, \frac{w_4}{<0.1,0.8,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.8>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, d, 1), \left\{ \frac{w_1}{<0.2,0.6,0.7>}, \frac{w_2}{<0.5,0.4,0.5>}, \frac{w_3}{<0.6,0.4,0.5>}, \frac{w_4}{<0.8,0.1,0.7>} \right\}\right), \\ \left((\upsilon_3, e, 1), \left\{ \frac{w_1}{<0.6,0.3,0.7>}, \frac{w_2}{<0.2,0.7,0.6>}, \frac{w_3}{<0.4,0.4,0.5>}, \frac{w_4}{<0.1,0.6,0.4>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.1,0.5,0.5>}, \frac{w_2}{<0.1,0.6,0.6>}, \frac{w_3}{<0.2,0.7,0.6>}, \frac{w_4}{<0.4,0.6,0.8>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.7,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.2,0.3>}, \frac{w_4}{<0.7,0.2,0.4>} \right\}\right), \\ \left((\upsilon_3, d, 0), \left\{ \frac{w_1}{<0.1,0.7,0.4>}, \frac{w_2}{<0.8,0.2,0.3>}, \frac{w_3}{<0.7,0.2,0.6>}, \frac{w_4}{<0.2,0.7,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 0), \left\{ \frac{w_1}{<0.1,0.6,0.3>}, \frac{w_2}{<0.9,0.1,0.2>}, \frac{w_3}{<0.6,0.3,0.4>}, \frac{w_4}{<0.2,0.6,0.3>} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{<0.2,0.5,0.4>}, \frac{w_2}{<0.2,0.6,0.3>}, \frac{w_3}{<0.3,0.5,0.6>}, \frac{w_4}{<0.5,0.3,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.3,0.6>}, \frac{w_2}{<0.9,0.1,0.7>}, \frac{w_3}{<0.4,0.5,0.8>}, \frac{w_4}{<0.5,0.3,0.5>} \right\}\right), \end{array} \right\}.$$

Proposition 4.10. *If* $(\hbar_1, \mathbb{G}_1)$,$(\hbar_2, \mathbb{G}_2)$ *and* $(\hbar_3, \mathbb{G}_3)$ *are three NHSESs then*

(1) $(\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2) = (\hbar_2, \mathbb{G}_2) \cap (\hbar_1, \mathbb{G}_1)$

(2) $((\hbar_1, \mathbb{G}_1) \cap (\hbar_2, \mathbb{G}_2)) \cap (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2) \cap (\hbar_3, \mathbb{G}_3))$

(3) $(\hbar, \mathbb{G}) \cap \phi = \phi$.

Proposition 4.11. *If* $(\hbar_1, \mathbb{G}_1)$,$(\hbar_2, \mathbb{G}_2)$ *and* $(\hbar_3, \mathbb{G}_3)$ *are three NHSESs, then*

(1) $(\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2) \cap (\hbar_3, \mathbb{G}_3)) =$
$((\hbar_1, \mathbb{G}_1) \cup ((\hbar_2, \mathbb{G}_2)) \cap ((\hbar_1, \mathbb{G}_1) \cup (\hbar_3, \mathbb{G}_3))$

(2) $(\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2) \cup (\hbar_3, \mathbb{G}_3)) = ((\hbar_1, \mathbb{G}_1) \cap ((\hbar_2, \mathbb{G}_2)) \cup ((\hbar_1, \mathbb{G}_1) \cap (\hbar_3, \mathbb{G}_3))$.

Definition 4.12. If $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ are two NHSESs over Δ then $(\hbar_1, \mathbb{G}_1)$ AND $(\hbar_2, \mathbb{G}_2)$ denoted by $(\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2)$ is defined by
$(\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2)$, while $\hbar_3(\varsigma, \gamma) = \hbar_1(\varsigma) \cap \hbar_2(\gamma), \forall (\varsigma, \gamma) \in \mathbb{G}_1 \times \mathbb{G}_2$.

Example 4.13. Considering Example 3.3, we have
$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_1, d, 1), (\upsilon_3, c, 0) \Big\}$, $\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 0), (\upsilon_3, c, 1) \Big\}$.
Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ are two NHSESs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.4,0.5>}, \frac{w_3}{<0.4,0.5,0.6>}, \frac{w_4}{<0.1,0.8,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.8>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\}\right), \end{array} \right\}.$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 0), \left\{ \frac{w_1}{<0.2,0.1,0.3>}, \frac{w_2}{<0.7,0.2,0.4>}, \frac{w_3}{<0.5,0.2,0.5>}, \frac{w_4}{<0.2,0.3,0.6>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.5,0.6>}, \frac{w_2}{<0.4,0.2,0.5>}, \frac{w_3}{<0.7,0.1,0.2>}, \frac{w_4}{<0.8,0.1,0.4>} \right\}\right), \end{array} \right\}.$$

Then $(\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2)$,

$$(\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2) = \left\{ \begin{array}{l} \left(((\upsilon_1, c, 1), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.1,0.35,0.4>}, \frac{w_2}{<0.6,0.30,0.5>}, \frac{w_3}{<0.4,0.35,0.6>}, \frac{w_4}{<0.1,0.55,0.7>} \right\}\right), \\ \left(((\upsilon_1, d, 1), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.2,0.25,0.8>}, \frac{w_2}{<0.6,0.25,0.5>}, \frac{w_3}{<0.2,0.35,0.6>}, \frac{w_4}{<0.1,0.45,0.7>} \right\}\right), \\ \left(((\upsilon_1, d, 1), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.1,0.45,0.8>}, \frac{w_2}{<0.4,0.25,0.5>}, \frac{w_3}{<0.2,0.30,0.6>}, \frac{w_4}{<0.1,0.35,0.7>} \right\}\right), \\ \left(((\upsilon_1, c, 1), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.1,0.55,0.6>}, \frac{w_2}{<0.4,0.30,0.5>}, \frac{w_3}{<0.4,0.30,0.6>}, \frac{w_4}{<0.1,0.45,0.7>} \right\}\right), \\ \left(((\upsilon_3, c, 0), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.1,0.35,0.9>}, \frac{w_2}{<0.3,0.40,0.7>}, \frac{w_3}{<0.5,0.15,0.5>}, \frac{w_4}{<0.2,0.25,0.6>} \right\}\right), \\ \left(((\upsilon_3, c, 0), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.1,0.55,0.9>}, \frac{w_2}{<0.3,0.40,0.7>}, \frac{w_3}{<0.6,0.10,0.2>}, \frac{w_4}{<0.7,0.15,0.4>} \right\}\right), \end{array} \right\}.$$

Definition 4.14. If $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ are two NHSESs over Δ, then $(\hbar_1, \mathbb{G}_1)$ OR $(\hbar_2, \mathbb{G}_2)$ denoted by $(\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2)$ is defined by $(\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2)$, while $\hbar_3(\delta, \gamma) = \hbar_1(\delta) \cup \hbar_2(\gamma), \forall (\delta, \gamma) \in \mathbb{G}_1 \times \mathbb{G}_2$.

Example 4.15. Considering Example 3.3, we see
$\mathbb{G}_1 = \Big\{ (\upsilon_1, c, 1), (\upsilon_1, d, 1), (\upsilon_3, c, 0) \Big\}$, $\mathbb{G}_2 = \Big\{ (\upsilon_1, c, 0), (\upsilon_3, c, 1) \Big\}$.
Suppose $(\hbar_1, \mathbb{G}_1)$ and $(\hbar_2, \mathbb{G}_2)$ over Δ are two NHSESs such that

$$(\hbar_1, \mathbb{G}_1) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{<0.1,0.6,0.4>}, \frac{w_2}{<0.6,0.4,0.5>}, \frac{w_3}{<0.4,0.5,0.6>}, \frac{w_4}{<0.1,0.8,0.7>} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{<0.3,0.4,0.8>}, \frac{w_2}{<0.6,0.3,0.5>}, \frac{w_3}{<0.2,0.5,0.6>}, \frac{w_4}{<0.1,0.6,0.7>} \right\}\right), \\ \left((\upsilon_3, c, 0), \left\{ \frac{w_1}{<0.1,0.6,0.9>}, \frac{w_2}{<0.3,0.6,0.7>}, \frac{w_3}{<0.6,0.1,0.2>}, \frac{w_4}{<0.7,0.2,0.3>} \right\}\right) \end{array} \right\}$$

$$(\hbar_2, \mathbb{G}_2) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 0), \left\{ \frac{w_1}{<0.2,0.1,0.3>}, \frac{w_2}{<0.7,0.2,0.4>}, \frac{w_3}{<0.5,0.2,0.5>}, \frac{w_4}{<0.2,0.3,0.6>} \right\}\right), \\ \left((\upsilon_3, c, 1), \left\{ \frac{w_1}{<0.1,0.5,0.6>}, \frac{w_2}{<0.4,0.2,0.5>}, \frac{w_3}{<0.7,0.1,0.2>}, \frac{w_4}{<0.8,0.1,0.4>} \right\}\right), \end{array} \right\}.$$

Then $(\hbar_3, \mathbb{G}_3) \vee (\hbar_2, \mathbb{G}_2) = (\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2)$,

$$(\hbar_3, \mathbb{G}_1 \times \mathbb{G}_2) = \left\{ \begin{array}{l} \left(((\upsilon_1, c, 1), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.2,0.35,0.3>}, \frac{w_2}{0<0.7,0.30,0.4>}, \frac{w_3}{<0.5,0.35,0.5>}, \frac{w_4}{<0.2,0.55,0.6>} \right\} \right), \\ \left(((\upsilon_1, d, 1), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.3,0.25,0.3>}, \frac{w_2}{<0.7,0.25,0.4>}, \frac{w_3}{<0.5,0.35,0.5>}, \frac{w_4}{<0.2,0.45,0.6>} \right\} \right), \\ \left(((\upsilon_1, d, 1), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.3,0.45,0.6>}, \frac{w_2}{<0.6,0.25,0.5>}, \frac{w_3}{<0.7,0.30,0.2>}, \frac{w_4}{<0.8,0.35,0.4>} \right\} \right), \\ \left(((\upsilon_1, c, 1), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.1,0.55,0.4>}, \frac{w_2}{<0.6,0.30,0.5>}, \frac{w_3}{<0.7,0.30,0.2>}, \frac{w_4}{<0.8,0.45,0.4>} \right\} \right), \\ \left(((\upsilon_3, c, 0), (\upsilon_1, c, 0)), \left\{ \frac{w_1}{<0.2,0.35,0.3>}, \frac{w_2}{<0.7,0.40,0.4>}, \frac{w_3}{<0.6,0.15,0.2>}, \frac{w_4}{<0.7,0.25,0.3>} \right\} \right), \\ \left(((\upsilon_3, c, 0), (\upsilon_3, c, 1)), \left\{ \frac{w_1}{<0.1,0.55,0.6>}, \frac{w_2}{<0.4,0.40,0.5>}, \frac{w_3}{<0.7,0.10,0.2>}, \frac{w_4}{<0.8,0.15,0.4>} \right\} \right), \end{array} \right\}.$$

Proposition 4.16. *If* $(\hbar_1, \mathbb{G}_1)$,$(\hbar_2, \mathbb{G}_2)$ *and* $(\hbar_3, \mathbb{G}_3)$ *are three NHSESs over* Δ, *then*

(1) $((\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2))^c = ((\hbar_1, \mathbb{G}_1))^c \vee ((\hbar_2, \mathbb{G}_2))^c$
(2) $((\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2))^c = ((\hbar_1, \mathbb{G}_1))^c \wedge ((\hbar_2, \mathbb{G}_2))^c$.

Proposition 4.17. *If* $(\hbar_1, \mathbb{G}_1)$,$(\hbar_2, \mathbb{G}_2)$ *and* $(\hbar_3, \mathbb{G}_3)$ *are three NHSESs over* Δ, *then*

(1) $((\hbar_1, \mathbb{G}_1) \wedge (\hbar_2, \mathbb{G}_2)) \wedge (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2) \wedge (\hbar_3, \mathbb{G}_3))$
(2) $((\hbar_1, \mathbb{G}_1) \vee (\hbar_2, \mathbb{G}_2)) \vee (\hbar_3, \mathbb{G}_3) = (\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2) \vee (\hbar_3, \mathbb{G}_3))$
(3) $(\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2) \wedge (\hbar_3, \mathbb{G}_3) = ((\hbar_1, \mathbb{G}_1) \vee ((\hbar_2, \mathbb{G}_2)) \wedge ((\hbar_1, \mathbb{G}_1) \vee (\hbar_3, \mathbb{G}_3))$
(4) $(\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2) \vee (\hbar_3, \mathbb{G}_3)) = ((\hbar_1, \mathbb{G}_1) \wedge ((\hbar_2, \mathbb{G}_2)) \vee ((\hbar_1, \mathbb{G}_1) \wedge (\hbar_3, \mathbb{G}_3))$.

5. Basic Properties and Laws of Neutrosophic Hypersoft Expert Set Operations

In this important part of the paper, certain important characteristics and laws are explained for NHSES.

Here $(\hbar, \mathbb{G}), (\hbar, \mathbb{G}_1), (\hbar, \mathbb{G}_2), (\hbar, \mathbb{G}_3)$ and $(\hbar_1, \mathbb{G})$ are NHSESs over Δ

- Idempotent Laws
  (a) $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})$
  (b) $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})$
- Identity Laws
  (a) $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})_\Phi$
  (b) $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})_\Delta$.
- Domination Laws
  (a) $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})_\Delta$
  (b) $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})_\Phi$.
- Characteristic of Exclusion
  $(\hbar, \mathbb{G}) \cup (\hbar, \mathbb{G})^c = (\hbar, \mathbb{G})_\Delta = (\hbar, \mathbb{G}) \cup_R (\hbar, \mathbb{G})^c$.
- Characteristic of Contradiction
  $(\hbar, \mathbb{G}) \cap (\hbar, \mathbb{G})^c = (\hbar, \mathbb{G})_\Phi = (\hbar, \mathbb{G}) \cap_\varepsilon (\hbar, \mathbb{G})^c$.

- Absorption Laws
  (a) $(\hbar, \mathbb{G}_1) \cup ((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_1)) = (\hbar, \mathbb{G}_1)$
  (b) $(\hbar, \mathbb{G}_1) \cap ((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_1)) = (\hbar, G_1)$
  (c) $(\hbar, \mathbb{G}_1) \cup_R ((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_1)) = (\hbar, \mathbb{G}_1)$
  (d) $(\hbar, \mathbb{G}_1) \cap_\varepsilon ((\hbar, \mathbb{G}_1) \cup_R (\hbar, \mathbb{G}_1)) = (\hbar, \mathbb{G}_1)$.

- Absorption Laws
  (a) $((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_2)) = ((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_2))$
  (b) $((\hbar, \mathbb{G}_1) \cup_R (\hbar, \mathbb{G}_2)) = ((\hbar, \mathbb{G}_1) \cup_R (\hbar, \mathbb{G}_2))$
  (c) $((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_2)) = ((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_2))$
  (d) $((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_2)) = ((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_2))$.

- Associative Laws
  (a) $(\hbar, \mathbb{G}_1) \cup ((\hbar, \mathbb{G}_2) \cup (\hbar_1, G_3)) = ((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_2)) \cup (\hbar_1, \mathbb{G}_3)$
  (b) $(\hbar, \mathbb{G}_1) \cup_R ((\hbar, \mathbb{G}_2) \cup_R (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cup_R (\hbar, G_2)) \cup_R (\hbar_1, \mathbb{G}_3)$
  (c) $(\hbar, \mathbb{G}_1) \cap ((\hbar, \mathbb{G}_2) \cap (\hbar_1, G_3)) = ((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_2)) \cap (\hbar_1, \mathbb{G}_3)$
  (d) $(\hbar, \mathbb{G}_1) \cap_\varepsilon ((\hbar, \mathbb{G}_2) \cap_\varepsilon (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_2)) \cap_\varepsilon (\hbar_1, \mathbb{G}_3)$
  (e) $(\hbar, \mathbb{G}_1) \bigvee((\hbar, \mathbb{G}_2) \bigvee(\hbar_1, G_3)) = ((\hbar, \mathbb{G}_1) \bigvee(\hbar, \mathbb{G}_2)) \bigvee(\hbar_1, \mathbb{G}_3)$
  (e) $(\hbar, \mathbb{G}_1) \bigwedge((\hbar, \mathbb{G}_2) \bigwedge(\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \bigwedge(\hbar, \mathbb{G}_2)) \bigwedge(\hbar_1, \mathbb{G}_3)$.

- De Morgan's Laws
  (a) $((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_2))^c = (\hbar, \mathbb{G}_1)^c \cap_\varepsilon (\hbar, \mathbb{G}_2)^c$
  (b) $((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_2))^c = (\hbar, \mathbb{G}_1)^c \cup (\hbar, \mathbb{G}_2)^c$
  (c) $((\hbar, \mathbb{G}_1) \bigvee(\hbar, \mathbb{G}_2))^c = (\hbar, \mathbb{G}_1)^c \bigwedge(\hbar, \mathbb{G}_2)^c$
  (d) $((\hbar, \mathbb{G}_1) \bigwedge(\hbar, \mathbb{G}_2))^c = (\hbar, \mathbb{G}_1)^c \bigvee(\hbar, \mathbb{G}_2)^c$.

- Distributive Laws
  (a) $(\hbar, \mathbb{G}_1) \cup ((\hbar, \mathbb{G}_2) \cap (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cup (\hbar, \mathbb{G}_2)) \cap ((\hbar, \mathbb{G}_1) \cup (\hbar_1, \mathbb{G}_3))$
  (b) $(\hbar, \mathbb{G}_1) \cap ((\hbar, \mathbb{G}_2) \cup (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_2)) \cup ((\hbar, \mathbb{G}_1) \cap (\hbar_1, \mathbb{G}_3))$
  (c) $(\hbar, \mathbb{G}_1) \cup_R ((\hbar, \mathbb{G}_2) \cap_\varepsilon (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cup_R (\hbar, \mathbb{G}_2)) \cap_\varepsilon ((\hbar, \mathbb{G}_1) \cup_R (\hbar_1, \mathbb{G}_3))$
  (d) $(\hbar, \mathbb{G}_1) \cap_\varepsilon ((\hbar, \mathbb{G}_2) \cup_R (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar, \mathbb{G}_2)) \cup_R ((\hbar, \mathbb{G}_1) \cap_\varepsilon (\hbar_1, \mathbb{G}_3))$
  (c) $(\hbar, \mathbb{G}_1) \cup_R ((\hbar, \mathbb{G}_2) \cap (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cup_R (\hbar, G_2)) \cap ((\hbar, \mathbb{G}_1) \cup_R (\hbar_1, \mathbb{G}_3))$
  (e) $(\hbar, \mathbb{G}_1) \cap ((\hbar, \mathbb{G}_2) \cup_R (\hbar_1, \mathbb{G}_3)) = ((\hbar, \mathbb{G}_1) \cap (\hbar, \mathbb{G}_2)) \cup_R ((\hbar, \mathbb{G}_1) \cap (\hbar_1, \mathbb{G}_3))$.

6. Hybrids of Neutrosophic Hypersoft Expert Set

In this study, some hybridized structures of NHSES are presented. Suppose $\mathbf{Y}$ denotes the set of expert and $\mathbf{O}$ be a set of opinions, $T = \mathbf{F} \times \mathbf{Y} \times \mathbf{O}$. Taking $A \subseteq T$ and $\mathbf{\Delta}$ denotes the universe, while $\mathbf{F}$ used for parameters.

Definition 6.1. A bipolar neutrosophic hypersoft expert set is a pair $(\mathbb{B}, A)$ and is characterized by a mapping

$$\mathbb{B} : A \to P(\Delta)$$

where

$$(\mathbb{B}, A) = \{\langle x, \upsilon^{+}_{B(e)}(x), \nu^{+}_{B(e)}(x), \omega^{+}_{B(e)}(x), \upsilon^{-}_{B(e)}(x), \nu^{-}_{B(e)}(x), \omega^{-}_{B(e)}(x)\rangle : \forall e \in A, x \in \Delta\}$$

, where $\upsilon^{+}_{B(e)}, \nu^{+}_{B(e)}, \omega^{+}_{B(e)} : \Delta \to [0,1]$, $\upsilon^{-}_{B(e)}, \nu^{-}_{B(e)}, \omega^{-}_{B(e)} : \Delta \to [0,1]$.

Example 6.2. Considering Example 3.3 with $\Delta = \{w_1, w_2\}$, we have bipolar neutrosophic hypersoft expert set as

$$(\mathbb{B}, \mathrm{A}) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{\frac{w_1}{<0.2,0.5,0.4,-0.1,-0.2,-0.3>}, \frac{w_2}{<0.1,0.3,0.6,-0.2,-0.3,-0.2>}\right\}\right), \\ \left((\upsilon_1, d, 1), \left\{\frac{w_1}{<0.4,0.2,0.3,-0.1,-0.1,-0.2>}, \frac{w_2}{<0.2,0.5,0.3,-0.1,-0.2,-0.5>}\right\}\right), \\ \left((\upsilon_1, e, 1), \left\{\frac{w_1}{<0.7,0.2,0.3,-0.3,-0.1,-0.2>}, \frac{w_2}{<0.3,0.5,0.6,-0.2,-0.3,-0.4>}\right\}\right), \\ \left((\upsilon_2, c, 1), \left\{\frac{w_1}{<0.9,0.1,0.3,-0.3,-0.2,-0.1>}, \frac{w_2}{<0.3,0.4,0.8,-0.1,-0.7,-0.4>}\right\}\right), \\ \left((\upsilon_2, d, 1), \left\{\frac{w_1}{<0.4,0.5,0.6,-0.2,-0.3,-0.4>}, \frac{w_2}{<0.2,0.6,0.7,-0.1,-0.3,-0.4>}\right\}\right), \\ \left((\upsilon_2, e, 1), \left\{\frac{w_1}{<0.5,0.4,0.7,-0.1,-0.2,-0.3>}, \frac{w_2}{<0.8,0.1,0.6,-0.2,-0.2,-0.3>}\right\}\right), \\ \left((\upsilon_1, c, 0), \left\{\frac{w_1}{<0.3,0.2,0.1,-0.1,-0.2,-0.3>}, \frac{w_2}{<0.1,0.8,0.3,-0.1,-0.7,-0.2>}\right\}\right), \\ \left((\upsilon_1, d, 0), \left\{\frac{w_1}{<0.1,0.8,0.4,-0.1,-0.2,-0.3>}, \frac{w_2}{<0.2,0.7,0.5,-0.1,-0.3,-0.4>}\right\}\right), \\ \left((\upsilon_1, e, 0), \left\{\frac{w_1}{<0.2,0.7,0.5,-0.1,-0.3,-0.4>}, \frac{w_2}{<0.5,0.4,0.6,-0.1,-0.3,-0.4>}\right\}\right), \end{array} \right\}.$$

Definition 6.3. A complex neutrosophic hypersoft expert set $(\mathcal{C}, A)$ is characterized by a mapping

$$\mathcal{C} : A \to \mathcal{C}N^{\Delta}$$

where $\mathcal{C}N^{\Delta}$ denotes the collection of all complex neutrosophic subsets of Δ and
$(\mathbb{C}, A) = \{\langle x, \upsilon_{C(e)}(x), \nu_{C(e)}(x), \omega_{C(e)}(x)\rangle : \forall e \in A, x \in \Delta\}$, where

$$\upsilon_{C(e)}(x) = aC(e)(x).e^{jC(e)(x)}, \nu_{C(e)}(x) = bC(e)(x).e^{jC(e)(x)}, \omega_{C(e)}(x) = cC(e)(x).e^{jC(e)(x)}$$

for all $u \in \Delta$ while $\upsilon_{C(e)}, \nu_{C(e)}, \omega_{C(e)}$ are complex-valued truth, indeterminacy and falsity membership functions and these values lie within the unit circle in the complex plane and both the amplitude terms $aC(e)(x), bC(e)(x), cC(e)(x)$ and the phase terms $\upsilon C(e)(x), \nu C(e)(x), \omega C(e)(x)$ are real valued such that $0 \leq aC(e)(x)+bC(e)(x)+cC(e)(x) \leq 3$ while $aC(e)(x), bC(e)(x), cC(e)(x) \in [0,1]$.

Example 6.4. Considering Example 3.3, we have complex neutrosophic hypersoft expert as

$$(\mathbb{C}, \mathrm{A}) = \left\{ \begin{array}{l} \left((\upsilon_1, c, 1), \left\{ \frac{w_1}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.9e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_1, d, 1), \left\{ \frac{w_1}{\langle 0.2e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_1, e, 1), \left\{ \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.6e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_2, c, 1), \left\{ \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_2, d, 1), \left\{ \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.6e^{j2\pi(0.3)}, 0.8e^{j2\pi(0.3)}, 0.1e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_2, e, 1), \left\{ \frac{w_1}{\langle 0.4e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.4e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_1, c, 0), \left\{ \frac{w_1}{\langle 0.3e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.1e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_1, d, 0), \left\{ \frac{w_1}{\langle 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.1e^{j2\pi(0.3)}, 0.6e^{j2\pi(0.3)}, 0.2e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_1, e, 0), \left\{ \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.5e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.4e^{j2\pi(0.3)} \rangle} \right\}\right), \\ \left((\upsilon_2, c, 0), \left\{ \frac{w_1}{\langle 0.6e^{j2\pi(0.3)}, 0.7e^{j2\pi(0.3)}, 0.5e^{j2\pi(0.3)} \rangle}, \frac{w_2}{\langle 0.7e^{j2\pi(0.3)}, 0.9e^{j2\pi(0.3)}, 0.3e^{j2\pi(0.3)} \rangle} \right\}\right) \end{array} \right\}.$$

Definition 6.5. A pair (F, H) is called a fuzzy parameterized complex neutrosophic hypersoft expert set(FP-CNHSES) over Δ, where F is a mapping given by

$$F : H \to CN^{\Delta}$$

where CN^{Δ} is the collection of all complex neutrosophic subsets of Δ.
It can also be written as $(F,\ H) \ = \ \left\{ \left(t, \left\{ \frac{w}{F(t)(x)} : x \in \Delta \right\}\right) : t \in H \right\}$
where $H \subseteq \mathcal{G} \times \mathcal{D} \times \mathbb{C} = \ \left\{ \left(\frac{\alpha}{\Im(\alpha)} : \beta,\ \gamma \in \Delta \right) : \alpha \in \mathcal{G},\ \beta \in \mathcal{D}, \gamma \in \mathcal{C} \right\}$ with $\Im$ is a corresponding membership function of fuzzy set and

$$(F, H) = \langle x, \upsilon_{C(e)}(x), \nu_{C(e)}(x), \omega_{C(e)}(x) \rangle : \forall e \in H, x \in \Delta,$$

where

$$\upsilon_{C(e)}(x) = aC(e)(x).e^{jC(e)(x)}, \nu_{C(e)}(x) = bC(e)(x).e^{jC(e)(x)}, \omega_{C(e)}(x) = cC(e)(x).e^{jC(e)(x)}$$

for all $x \in \Delta$ while $\upsilon_{C(e)}, \nu_{C(e)}, \omega_{C(e)}$ are complex-valued truth, indeterminacy and falsity membership functions for or the FP-CNHSES and these values lie within the unit circle in the complex plane and both the amplitude terms $aC(e)(x), bC(e)(x), cC(e)(x)$ and the phase terms $vC(e)(x), \nu C(e)(x), \omega C(e)(x)$ are real valued such that $0 \leq aC(e)(x) + bC(e)(x) + cC(e)(x) \leq 3$.

Example 6.6. Considering Example 3.3 with $\Im = \left\{\frac{\upsilon_1}{0.2}, \frac{\upsilon_2}{0.3}, \frac{\upsilon_3}{0.5}\right\}$ as a fuzzy subset of FZ(E). We can define FP-CNHSES as

$$(\mathrm{F},\mathrm{H}) = \left\{\begin{array}{l}
\left((\frac{\upsilon_1}{0.2}, c, 1), \left\{\frac{w_1}{\langle 0.1e^{2\pi(0.2)},\, 0.6e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.9e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.8e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_2}{0.3}, d, 1), \left\{\frac{w_1}{\langle 0.2e^{2\pi(0.2)},\, 0.6e^{2\pi(0.2)},\, 0.3e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.2)},\, 0.8e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_3}{0.5}, e, 1), \left\{\frac{w_1}{\langle 0.6e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.8e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.6e^{2\pi(0.2)},\, 0.9e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_1}{0.2}, c, 1), \left\{\frac{w_1}{\langle 0.4e^{2\pi(0.2)},\, 0.5e^{2\pi(0.2)},\, 0.4e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.3)},\, 0.8e^{2\pi(0.2)},\, 0.5e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_2}{0.3}, d, 1), \left\{\frac{w_1}{\langle 0.4e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.4e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.6e^{2\pi(0.2)},\, 0.8e^{2\pi(0.2)},\, 0.1e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_3}{0.5}, e, 1), \left\{\frac{w_1}{\langle 0.4e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.4e^{2\pi(0.2)},\, 0.6e^{2\pi(0.2)},\, 0.6e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_1}{0.2}, c, 0), \left\{\frac{w_1}{\langle 0.3e^{2\pi(0.3)},\, 0.3e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.3)},\, 0.6e^{2\pi(0.2)},\, 0.1e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_2}{0.3}, d, 0), \left\{\frac{w_1}{\langle 0.7e^{2\pi(0.2)},\, 0.4e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.1e^{2\pi(0.2)},\, 0.6e^{2\pi(0.2)},\, 0.2e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_3}{0.5}, e, 0), \left\{\frac{w_1}{\langle 0.6e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.4e^{22\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.5e^{2\pi(0.2)},\, 0.9e^{2\pi(0.2)},\, 0.4e^{2\pi(0.2)}\rangle}\right\}\right),\\
\left((\frac{\upsilon_1}{0.2}, c, 0), \left\{\frac{w_1}{\langle 0.6e^{2\pi(0.2)},\, 0.7e^{2\pi(0.2)},\, 0.5e^{2\pi(0.2)}\rangle}, \frac{w_2}{\langle 0.7e^{2\pi(0.3)},\, 0.9e^{2\pi(0.2)},\, 0.3e^{2\pi(0.2)}\rangle}\right\}\right)
\end{array}\right\}.$$

Definition 6.7. A pair (V, A) is called a neutrosophic vague hypersoft expert set is a pair (V, A), with V representing a mapping $V : A \to NV^{\Delta}$, and NV^{Δ} is being used for the power neutrosophic vague set of Δ. Let mapping V is defined by as $V(t) = V(t)(x),\ \ \mathrm{x} \in \Delta$. For each $t_i \in A$, $V(t_i) = V(t_i)(x)$, where $V(t_i)$ represents the truth, indeterminacy and falsity membership functions of Δ in $V(t_i)$. Hence $V(t_i)$ can be written as
$V(t_i) = \ \left\{\frac{x_i}{V(t_i)x_i}\right\}$, for $i = 1, 2, 3, ...$
where $V(t_i)(x_i) = [\upsilon - \omega(t_i)(x_i), \upsilon + \omega(t_i)(x_i)], [I - \omega(t_i)(x_i), I + \omega(t_i)(x_i)], [\omega - \omega(t_i)(x_i), \omega + \omega(t_i)(x_i)]$ and $\upsilon + \omega(t_i)(x_i) = 1 - \omega(t_i)(x_i), \omega + \omega(t_i)(x_i) = 1 - \upsilon - \omega(t_i)(x_i)$ with $[\upsilon\omega(t_i)(x_i), \upsilon + \omega(t_i)(x_i)], [I - \omega(t_i)(x_i), I + \omega(t_i)(x_i)]$ representing the truth, indeterminacy and falsity-membership functions of each of the elements $x_i \in \Delta$, respectively.

Example 6.8. Considering Example 3.3 with $\Delta = \{w_1, w_2\}$, we have neutrosophic vague hypersoft expert set as

$$(\mathbb{B},\mathrm{A}) = \left\{\begin{array}{l}
\left((\upsilon_1, c, 1), \left\{\frac{w_1}{<[0.2,0.5],[0.4,0.1],[0.2,0.3]>}, \frac{w_2}{<[0.1,0.3],[0.6,0.2],[0.3,0.2]>}\right\}\right),\\
\left((\upsilon_1, d, 1), \left\{\frac{w_1}{<[0.4,0.2],[0.3,0.1],[0.1,0.2]>}, \frac{w_2}{<[0.2,0.5],[0.3,0.1],[0.2,0.5]>}\right\}\right),\\
\left((\upsilon_1, e, 1), \left\{\frac{w_1}{<[0.7,0.2],[0.3,0.8],[0.1,0.6]>}, \frac{w_2}{<[0.3,0.5],[0.6,0.2],[0.3,0.4]>}\right\}\right),\\
\left((\upsilon_2, c, 1), \left\{\frac{w_1}{<[0.9,0.1],[0.3,0.3],[0.2,0.1]>}, \frac{w_2}{<[0.3,0.4],[0.8,0.1],[0.7,0.4]>}\right\}\right),\\
\left((\upsilon_2, d, 1), \left\{\frac{w_1}{<[0.4,0.5],[0.6,0.2],[0.3,0.4]>}, \frac{w_2}{<[0.2,0.6],[0.7,0.1],[0.3,0.4]>}\right\}\right),\\
\left((\upsilon_2, e, 1), \left\{\frac{w_1}{<[0.5,0.4],[0.7,0.1],[0.2,0.3]>}, \frac{w_2}{<[0.8,0.1],[0.6,0.2],[0.2,0.3]>}\right\}\right),\\
\left((\upsilon_1, c, 0), \left\{\frac{w_1}{<[0.3,0.2],[0.1,0.1],[0.2,0.3]>}, \frac{w_2}{<[0.1,0.8],[0.3,0.1],[0.7,0.2]>}\right\}\right),\\
\left((\upsilon_1, d, 0), \left\{\frac{w_1}{<[0.1,0.8],[0.4,0.1],[0.2,0.3]>}, \frac{w_2}{<[0.2,0.7],[0.5,0.1],[0.3,0.4]>}\right\}\right),\\
\left((\upsilon_1, e, 0), \left\{\frac{w_1}{<[0.2,0.7],[0.5,0.1],[0.3,0.4]>}, \frac{w_2}{<[0.5,0.4],[0.6,0.1],[0.3,0.4]>}\right\}\right),
\end{array}\right\}.$$

7. An Application to Neutrosophic Hypersoft Expert Set

An application of NHSES theory related to the decision-making problem is presented while using an algorithmic technique..

Statement of the problem

Mr Jay needs to buy a mask from a business opportunity for his own wellbeing. He takes help from his a few companions (Henry, John and Watson) who have skill in mask buying.

Proposed Algorithm For Selection Of Mask

The accompanying calculation is embraced for this choice (purchase).

(1) Construct NHSES $(\hbar, \mathbb{G})$,
(2) Determine an Agree and Disagree-NHSES,
(3) Compute $d_i = \sum_i t_{ij}$ for Agree-NHSES,
(4) Determine $q_i = \sum_i t_{ij}$ for Disagree-NHSES,
(5) Determine $g_j = d_j - q_j$ for Agree and Disaree-NHSES,
(6) Compute n, for which $p_n =$ max p_j for best solution of the product.

Step-1

Let eight categories of mask which are being used for the universe of discourse $\Omega = \{b_1, b_2, b_3, b_4, b_5, b_6, b_7, b_8\}$ and X $= \{\rho_1 = Henry, \rho_2 = John, \rho_3 = Watson\}$ be a set of experts. The prescribed attributes for the attribute-valued sets are :

$O_1 = Brand = \{o_1 = new, o_2 = old\}$

$O_2 = Price = \{o_3 = l00dollar, o_4 = 50dollar\}$

$O_3 = Colour = \{o_5 = black, o_6 = blue\}$

$O_4 = Quality = \{o_7 = good, o_8 = better\}$

$O_5 = Shape = \{o_9 = circular, o_{10} = square\}$

and then $O = O_1 \times O_2 \times O_3 \times O_4 \times O_5$

$$O = \left\{ \begin{array}{l} (o_1, o_3, o_5, o_7, o_9), (o_1, o_3, o_5, o_7, o_{10}), (o_1, o_3, o_5, o_8, o_9), (o_1, o_3, o_5, o_8, o_{10}), (o_1, o_3, o_6, o_7, o_9), \\ (o_1, o_3, o_6, o_7, o_{10}), (o_1, o_3, o_6, o_8, o_9), (o_1, o_3, o_6, o_8, o_{10}), (o_1, o_4, o_5, o_7, o_9), (o_1, o_4, o_5, o_7, o_{10}), \\ (o_1, o_4, o_5, o_8, o_9), (o_1, o_4, o_5, o_8, o_{10}), (o_1, o_4, o_6, o_7, o_9), (o_1, o_4, o_6, o_7, o_{10}), (o_1, o_4, o_6, o_8, o_9), \\ (o_1, o_4, o_6, o_8, o_{10}), (o_2, o_3, o_5, o_7, o_9), (o_2, o_3, o_5, o_7, o_{10}), (o_2, o_3, o_5, o_8, o_9), (o_2, o_3, o_5, o_8, o_{10}), \\ (o_2, o_3, o_6, o_7, o_9), (o_2, o_3, o_6, o_7, o_{10}), (o_2, o_3, o_6, o_8, o_9), (o_2, o_3, o_6, o_8, o_{10}), (o_2, o_4, o_5, o_7, o_9), \\ (o_2, o_4, o_5, o_7, o_{10}), (o_2, o_4, o_5, o_8, o_9), (o_2, o_4, o_5, o_8, o_{10}), (o_2, o_4, o_6, o_7, o_9), (o_2, o_4, o_6, o_7, o_{10}), \\ (o_2, o_4, o_6, o_8, o_9), (o_2, o_4, o_6, o_8, o_{10}) \end{array} \right\}$$

Now take $Q \subseteq O$ as

$Q = \{q_1 = (o_1, o_3, o_5, o_7, o_9), q_2 = (o_1, o_3, o_6, o_7, o_{10}), q_3 = (o_1, o_4, o_6, o_8, o_9), q_4 =$

$(o_2, o_3, o_6, o_8, o_9), q_5 = (o_2, o_4, o_6, o_7, o_{10})\}$

$$(\hbar, \mathbb{G}) = \left\{ \begin{array}{l} ((q_1, \rho_1, 1) = \{o_2, o_3, o_4, o_5, o_6, o_8\}), ((q_1, \rho_2, 1) = \{o_1, o_2, o_3, o_7\}), ((q_2, \rho_1, 1) = \{o_5, o_8\}), \\ ((q_2, \rho_2, 1) = \{o_1, o_2, o_3, o_4, o_5, o_6, o_8\}), ((q_3, \rho_1, 1) = \{o_4, o_7\}), ((q_3, \rho_2, 1) = \{o_1, o_2, o_4, o_5, o_8\}), \\ ((q_3, \rho_3, 1) = \{o_1, o_5, o_7, o_8\}), \\ ((q_4, \rho_1, 1) = \{o_1, o_7, o_8\}), ((q_4, \rho_2, 1) = \{o_1, o_4, o_8\}), ((q_4, \rho_3, 1) = \{o_1, o_6, o_7, o_8\}), \\ ((q_5, \rho_1, 1) = \{o_3, o_7, o_8\}), ((q_5, \rho_2, 1) = \{o_1, o_2, o_3, o_4, o_5, o_8\}), ((q_5, \rho_3, 0) = \{o_1, o_3, o_6\}) \\ ((q_5, \rho_3, 1) = \{o_2, o_3, o_5, o_7, o_8\}), ((q_1, \rho_1, 0) = \{o_3, o_5, o_6\}), ((q_1, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_1, \rho_3, 0) = \{o_3, o_4\}), ((q_2, \rho_1, 0) = \{o_1, o_2, o_4, o_5, o_6, o_7\}), ((q_2, \rho_2, 0) = \{o_2, o_7\}), \\ ((q_2, \rho_3, 0) = \{o_2, o_3, o_4, o_5, o_6\}), ((q_3, \rho_1, 0) = \{o_1, o_2, o_6, o_8\}), ((q_3, \rho_2, 0) = \{o_3, o_4, o_6, o_7\}), \\ ((q_3, \rho_3, 0) = \{o_2, o_3, o_4, o_5, o_7\}), ((q_5, \rho_1, 0) = \{o_4, o_6, o_7\}), ((q_4, \rho_1, 0) = \{o_2, o_3, o_3, o_4, o_5, o_7\}), \\ ((q_4, \rho_3, 0) = \{o_2, o_3, o_4, o_5\}), ((q_5, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_1, \rho_3, 1) = \{o_1, o_3, o_4, o_6, o_7, o_8\}), ((q_4, \rho_2, 0) = \{o_2, o_3, o_6, o_7\}), \\ ((q_2, \rho_3, 1) = \{o_1, o_2, o_4, o_7, o_8\}), \end{array} \right\}$$

is a NHSES.

Step-2

The Agree and Disagree-NHSES are represented by Table 1 and Table 2 respectively, also when $o_i \in F_1(\beta)$ then $o_{ij} = \checkmark = 1$ diversely $o_{ij} = \times = 0$, and if

$$o_i \in F_0(\beta)$$

then $o_{ij} = \checkmark = 1$ diversely $o_{ij} = \times = 0$ while o_{ij} are being used as members of Tables 1 and 2.

Step-(3-5)

presents The $d_i = \sum_i o_{ij}$ for Agree-NHSES, $q_i = \sum_i o_{ij}$ for Disagree-NHSES are presented in Table 3 and $g_j = d_j - q_j$ have been shown and to choose product $p_n = \max p_j$ for solution.

Step-6-Decision

Since g_8 is maximum in above Table 3, so category b_8 is preferred to be selected for purchase.

8. Conclusions

In this paper,

- The fundamentals of neutrosophic hypersoft expert set are established and some necessary properties like subset, equal set, agree and disagree set, relative whole and relative null set, absolute whole set are explained with detailed examples.
- Some theoretic operations like union, restricted union, intersection, extended intersection, complement, AND and OR are generalized.
- Some basic laws such as idempotent, absorption, domination, identity, associative and distributive are discussed with examples.

Table 1. Agree-NHSES

B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
(p_1, ρ_1)	×	✓	✓	✓	✓	✓	×	×
(p_2, ρ_1)	×	✓	×	×	×	×	✓	×
(p_3, ρ_1)	×	×	×	✓	×	×	✓	×
(p_4, ρ_1)	✓	×	×	×	×	×	✓	✓
(p_5, ρ_1)	✓	✓	✓	✓	×	×	×	✓
(p_1, ρ_2)	✓	×	✓	×	×	×	✓	×
(p_2, ρ_2)	✓	✓	✓	×	✓	✓	×	✓
(p_3, ρ_2)	×	✓	✓	×	×	×	✓	×
(p_4, ρ_2)	✓	×	×	×	×	×	×	✓
(p_5, ρ_2)	×	×	×	×	✓	×	×	×
(p_1, ρ_3)	✓	×	✓	✓	×	✓	✓	×
(p_2, ρ_3)	✓	✓	×	✓	✓	×	✓	×
(p_3, ρ_3)	✓	×	✓	×	✓	×	✓	×
(p_4, ρ_3)	✓	✓	✓	✓	×	✓	✓	×
(p_5, ρ_3)	×	✓	✓	✓	✓	×	✓	×
$d_j = \sum_i n_{ij}$	$d_1 = 09$	$d_2 = 08$	$d_3 = 9$	$d_4 = 7$	$d_5 = 06$	$d_6 = 4$	$d_7 = 10$	$d_8 = 11$

Table 2. Disagree-NHSES

B	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8
(p_1, ρ_1)	×	×	✓	×	×	✓	×	×
(p_2, ρ_1)	✓	✓	×	✓	×	✓	✓	×
(p_3, ρ_1)	✓	✓	×	×	×	✓	×	✓
(p_4, ρ_1)	×	✓	✓	✓	✓	✓	×	×
(p_5, ρ_1)	×	×	×	✓	×	✓	✓	×
(p_1, ρ_2)	×	✓	✓	×	×	✓	✓	×
(p_2, ρ_2)	×	✓	×	×	×	×	✓	×
(p_3, ρ_2)	✓	✓	×	×	×	✓	×	✓
(p_4, ρ_2)	×	✓	✓	×	×	✓	✓	×
(p_5, ρ_2)	×	✓	✓	×	×	✓	✓	×
(p_1, ρ_3)	×	×	✓	✓	×	×	×	×
(p_2, ρ_3)	×	×	✓	×	✓	✓	×	×
(p_3, ρ_3)	×	✓	✓	✓	✓	✓	×	×
(p_4, ρ_3)	×	✓	✓	✓	✓	×	×	×
(p_5, ρ_3)	×	✓	×	✓	×	✓	×	×
$p_i = \sum_i n_{ij}$	$p_1 = 3$	$p_2 = 11$	$p_3 = 9$	$p_4 = 7$	$p_5 = 4$	$p_6 = 12$	$p_7 = 6$	$p_8 = 2$

- Some hybridized structures of neutrosophic hypersoft expert set are established with illustrative examples.

Table 3. Optimal

$d_i = \sum_i n_{ij}$	$q_i = \sum_i n_{ij}$	$g_j = d_j - q_j$
$d_1 = 09$	$q_1 = 3$	$g_1 = 6$
$d_2 = 8$	$q_2 = 11$	$g_2 = -3$
$d_3 = 9$	$q_3 = 9$	$g_3 = 0$
$d_4 = 7$	$q_4 = 7$	$g_4 = 0$
$d_5 = 06$	$q_5 = 4$	$g_5 = 2$
$d_6 = 4$	$q_6 = 12$	$g_6 = -8$
$d_7 = 10$	$q_7 = 6$	$g_7 = 4$
$d_8 = 11$	$q_8 = 2$	$g_8 = 9$

- An algorithm is developed to explain the procedure of decision making problem.
- An application related to the mask purchasing is described with the help of proposed algorithm.
- Future task may include the extension of the existing work for other neutrosophic hypersoft expert-like hybrids i.e., generalized neutrosophic, generalized interval valued neutrosophic, neutrosophic vague , interval-valued neutrosophic, etc. This new work will give an outstanding extension to existing theories for dealing with truthness, indeterminacy and falsity membership functions.

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