NSS

Neutrosophic Sets and Systems, Vol. 50, 2022

University of New Mexico

Rough Neutrosophic Ideals in a Ring

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Abstract. Aim of this paper is to introduce the notion of rough neutrosophic sets in rings also we discuss the sum and product of rough neutrosophic ideal in a ring. Also we prove the upper and lower approximation of neutrosophic subring is also a neutrosophic subring and some examples are discussed.

Keywords: Rough set; Neutrosophic set; Rough neutrosophic set; Neutrosophic ideal; Rough neutrosophic ideal.)

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1. **Introduction**

Fuzzy set is introduced and described using membership functions by Zadeh in 1965 [14] in 1965. The notion of Rough sets was introduced by Pawlak [7] in his seminal paper of 1982. Crisp set and equivalence relation are the basic elements of Rough set theory. Rough set is based on result of approximating crisp sets known as the lower approximation and the upper approximation of a set introduced by Biswas and Nanda [2] in 1994. Approximation spaces are sets with multiple memberships but, fuzzy sets are with partial memberships. Many scholars Dubios et al [4], Gong et al [5], Leung et al [6], Sun et al [12] has developed many models upon different aspects. Rough sets and fuzzy sets, vague set and Intuitionistic fuzzy sets combine with various notions such as Generalized fuzzy rough sets, Intuitionistic fuzzy rough sets, Rough Intuitionistic fuzzy sets, and Rough vague sets were introduced.
Atanassov (1983) [1] introduced the notion of Intuitionistic fuzzy sets. They are the sets whose elements having degrees of membership and non-membership. Selvan, Senthil Kumar [8–10],

introduced the notion of rough intuitionistic fuzzy ideal(prime ideal) in rings in 2012. The generalizations of the theory of intuitionistic fuzzy sets is the theory of neutrosophic sets. The words neutrosophy and neutrosophic were introduced by Smarandache [11]. Neutrosophic concepts are very much useful in real life problem. For example, If a opinion is asked about a statement one may assign that the possibility that the statement true is 0.6 and the statement false is 0.8 and if it is not sure is 0.2. This idea is very much needful in various problems in real life situation. Neutrosophic sets are characterized by truth membership function , indeterminacy membership function and falsity membership function . Vildan cetkin and Halis Aygun [13] introduced an approach to single valued Neutrosophic ideals over a classical ring and Neutrosophic subring in 2018. Rough neutrosophic set is introduced by Broumi, Smarandache, and Dhar [3].
In this paper, we prove that any neutrosophic subring (ideal) of a ring is an upper and lower rough neutrosophic subring (ideal) of the ring.

2. **Preliminaries**

See [3], [7], [13] for basic concepts which are used in this work.

3. **Operations on Rough Neutrosophic sets in a Ring**

In this section we introduce the notion of RNI in a ring. Some basic properties of these ideals are proved and examples are given. Let C_R denote the congruence relation on R, throughout this section.

Theorem 3.1. *Let C_R and $C_R{}'$ be the two congruence relations on R. If P and Q are any two NS of R, then the following properties are,*

$(a)\underline{C_R}(P) \subseteq P \subseteq \overline{C_R}(P)$
$(b)\underline{C_R}(\underline{C_R}(P)) = \underline{C_R}(P)$
$(c)\overline{C_R}(\overline{C_R}(P)) = \overline{C_R}(P)$
$(d)\overline{C_R}(\underline{C_R}(P)) = \underline{C_R}(P)$
$(e)\underline{C_R}(\overline{C_R}(P)) = \overline{C_R}(P)$
$(f)(\overline{C_R}(P^c))^c = \underline{C_R}(P)$
$(g)(\underline{C_R}(P^c))^c = \overline{C_R}(P)$
$(h)\underline{C_R}(P \cap Q) = \underline{C_R}(P) \cap \underline{C_R}(Q)$
$(i)\overline{C_R}(P \cap Q) \subseteq \overline{C_R}(P) \cap \overline{C_R}(Q)$
$(j)\overline{C_R}(P \cup Q) = \overline{C_R}(P) \cup \overline{C_R}(Q)$
$(k)\underline{C_R}(P \cup Q) \supseteq \underline{C_R}(P) \cup \underline{C_R}(Q)$
$(l)P \subseteq Q \Rightarrow \overline{C_R}(P) \subseteq \overline{C_R}(Q)$
$(m)P \subseteq Q \Rightarrow \underline{C_R}(P) \subseteq \underline{C_R}(Q)$

$(n) C_R \subseteq C_R' \Rightarrow \underline{C_R}(P) \supseteq \underline{C_R}'(P)$
$(o) C_R \subseteq C_R' \Rightarrow \overline{C_R}(P) \subseteq \overline{C_R}'(P)$.

Proof. Proof is obvious. □

Theorem 3.2. *If P and Q are any two NS of R, then $\overline{C_R}(P) + \overline{C_R}(Q) \subseteq \overline{C_R}(P+Q)$.*

Proof. Since P and Q be any two NS of R. Then

$$\overline{C_R}(P) + \overline{C_R}(Q) = \{[\overline{C_R}(P(n_t)) + \overline{C_R}(Q(n_t))], [\overline{C_R}(P(n_i)) + \overline{C_R}(Q(n_i))], [\overline{C_R}(P(n_f)) + \overline{C_R}(Q(n_f))]\}.$$

$$\overline{C_R}(P+Q) = \{[\overline{C_R}(P(n_t) + Q(n_t))], [\overline{C_R}(P(n_i) + Q(n_i))], [\overline{C_R}(P(n_f) + Q(n_f))]\}.$$

we've to prove, $\overline{C_R}(P) + \overline{C_R}(Q) \subseteq \overline{C_R}(P+Q)$.
For this we want to prove, $\forall\ \alpha \in R$
$(\overline{C_R}(P(n_t)) + \overline{C_R}(Q(n_t)))(\alpha) \leq \overline{C_R}(P(n_t) + Q(n_t))(\alpha)$
$(\overline{C_R}(P(n_i)) + \overline{C_R}(Q(n_i)))(\alpha) \geq \overline{C_R}(P(n_i) + Q(n_i))(\alpha)$
$(\overline{C_R}(P(n_f)) + \overline{C_R}(Q(n_f)))(\alpha) \geq \overline{C_R}(P(n_f) + Q(n_f))(\alpha)$
Consider,

$$\begin{aligned}
(\overline{C_R}(P(n_t)) + \overline{C_R}(Q(n_t)))(\alpha) &= \bigvee_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_t))(\beta) \wedge \overline{C_R}(Q(n_t))(\gamma)] \\
&= \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{x\in[\beta]_{C_R}} (P(n_t)(x)) \wedge (\bigvee_{y\in[\gamma]_{C_R}} (Q(n_t)(y))] \\
&= \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{\substack{x\in[\beta]_{C_R} \\ y\in[\gamma]_{C_R}}} (P(n_t)(x) \wedge Q(n_t)(y))] \\
&\leq \bigvee_{\alpha=\beta+\gamma} [(\bigvee_{x+y\in[\beta+\gamma]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y))] \\
&= \bigvee_{x+y\in[\alpha]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{\substack{z\in[\alpha]_{C_R} \\ z=x+y}} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{z\in[\alpha]_{C_R}} \bigvee_{z=x+y} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{z\in[\alpha]_{C_R}} [P(n_t) + Q(n_t)](z) \\
&= \overline{C_R}(P(n_t) + Q(n_t))(z)
\end{aligned}$$

And

$$\begin{aligned}(\overline{C_R}(P(n_i)) + \overline{C_R}(Q(n_i)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_i))(\beta) \vee \overline{C_R}(Q(n_i))(\gamma)] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x\in[\beta]_{C_R}} (P(n_i)(x)) \vee (\bigwedge_{y\in[\gamma]_{C_R}} (Q(n_i)(y))] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{\substack{x\in[\beta]_{C_R}\\ y\in[\gamma]_{C_R}}} (P(n_i)(x) \vee Q(n_i)(y))] \\
&\geq \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x+y\in[\beta+\gamma]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y))] \\
&= \bigwedge_{x+y\in[\alpha]_{C_R}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{\substack{z\in[\alpha]_{C_R}\\ z=x+y}} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z\in[\alpha]_{C_R}} \bigwedge_{z=x+y} (P(n_i)(x) \vee Q(n_i)(y)) \\
&= \bigwedge_{z\in[\alpha]_{C_R}} [P(n_i) + Q(n_i)](z) \\
&= \overline{C_R}(P(n_i) + Q(n_i))(z)\end{aligned}$$

Also,

$$\begin{aligned}(\overline{C_R}(P(n_f)) + \overline{C_R}(Q(n_f)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma} [\overline{C_R}(P(n_f))(\beta) \vee \overline{C_R}(Q(n_f))(\gamma)] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x\in[\beta]_{C_R}} (P(n_f)(x)) \vee (\bigwedge_{y\in[\gamma]_{C_R}} (Q(n_f)(y))] \\
&= \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{\substack{x\in[\beta]_{C_R}\\ y\in[\gamma]_{C_R}}} (P(n_f)(x) \vee Q(n_f)(y))] \\
&\geq \bigwedge_{\alpha=\beta+\gamma} [(\bigwedge_{x+y\in[\beta+\gamma]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y))] \\
&= \bigwedge_{x+y\in[\alpha]_{C_R}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{\substack{z\in[\alpha]_{C_R}\\ z=x+y}} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z\in[\alpha]_{C_R}} \bigwedge_{z=x+y} (P(n_f)(x) \vee Q(n_f)(y)) \\
&= \bigwedge_{z\in[\alpha]_{C_R}} [P(n_f) + Q(n_f)](z) \\
&= \overline{C_R}(P(n_f) + Q(n_f))(z)\end{aligned}$$

Hence Proved.

□

Theorem 3.3. *If P and Q are any two NS of R, then $\underline{C_R}(P) + \underline{C_R}(Q) \subseteq \underline{C_R}(P+Q)$.*

Proof. This proof is similar to Theorem 3.2. □

Theorem 3.4. *Let P and Q are any two NS of R, then $\overline{C_R}(P).\overline{C_R}(Q) \subseteq \overline{C_R}(P.Q)$.*

Proof. Since P and Q be any two NS of R. Then,
$\overline{C_R}(P).\overline{C_R}(Q) = \{[\overline{C_R}(P(n_t)).\overline{C_R}(Q(n_t))], [\overline{C_R}(P(n_i)).\overline{C_R}(Q(n_i))], [\overline{C_R}(P(n_f)).\overline{C_R}(Q(n_f))]\}$
$\overline{C_R}(P.Q) = \{[\overline{C_R}(P(n_t).Q(n_t))], [\overline{C_R}(P(n_i).Q(n_i))], [\overline{C_R}(P(n_f).Q(n_f))]\}$
To prove, $\overline{C_R}(P).\overline{C_R}(Q) \subseteq \overline{C_R}(P.Q)$.
It is enough to prove that, $\forall\ \alpha \in R$
$(\overline{C_R}(P(n_t)).\overline{C_R}(Q(n_t)))(\alpha) \leq \overline{C_R}(P(n_t).Q(n_t))(\alpha)$
$(\overline{C_R}(P(n_i)).\overline{C_R}(Q(n_i)))(\alpha) \geq \overline{C_R}(P(n_i).Q(n_i))(\alpha)$
$(\overline{C_R}(P(n_f)).\overline{C_R}(Q(n_f)))(\alpha) \geq \overline{C_R}(P(n_f).Q(n_f))(\alpha)$
Consider,

$$\begin{aligned}
(\overline{C_R}(P(n_t)).\overline{C_R}(Q(n_t)))(\alpha) &= \bigvee_{\alpha=\beta\gamma} [\overline{C_R}(P(n_t))(\beta) \wedge \overline{C_R}(Q(n_t))(\gamma)] \\
&= \bigvee_{\alpha=\beta\gamma} [(\bigvee_{x\in[\beta]_{C_R}} (P(n_t)(x)) \wedge (\bigvee_{y\in[\gamma]_{C_R}} (Q(n_t)(y))] \\
&= \bigvee_{\alpha=\beta\gamma} [(\bigvee_{\substack{x\in[\beta]_{C_R}\\ y\in[\gamma]_{C_R}}} (P(n_t)(x) \wedge Q(n_t)(y))] \\
&\leq \bigvee_{\alpha=\beta\gamma} [(\bigvee_{xy\in[\beta\gamma]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y))] \\
&= \bigvee_{xy\in[\alpha]_{C_R}} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{\substack{z\in[\alpha]_{C_R}\\ z=xy}} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{z\in[\alpha]_{C_R}} \bigvee_{z=xy} (P(n_t)(x) \wedge Q(n_t)(y)) \\
&= \bigvee_{z\in[\alpha]_{C_R}} [P(n_t).Q(n_t)](z) \\
&= \overline{C_R}(P(n_t).Q(n_t))(z)
\end{aligned}$$

$$\begin{aligned}(\overline{C_R}(P(n_i)).\overline{C_R}(Q(n_i)))(\alpha) &= \bigwedge_{\alpha=\beta\gamma}[\overline{C_R}(P(n_i))(\beta) \vee \overline{C_R}(Q(n_i))(\gamma)] \\ &= \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{x\in[\beta]_{C_R}}(P(n_i)(x)) \vee (\bigwedge_{y\in[\gamma]_{C_R}}(Q(n_i)(y))] \\ &= \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{\substack{x\in[\beta]_{C_R}\\ y\in[\gamma]_{C_R}}}(P(n_i)(x) \vee Q(n_i)(y))] \\ &\geq \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{xy\in[\beta\gamma]_{C_R}}(P(n_i)(x) \vee Q(n_i)(y))] \\ &= \bigwedge_{xy\in[\alpha]_{C_R}}(P(n_i)(x) \vee Q(n_i)(y)) \\ &= \bigwedge_{\substack{z\in[\alpha]_{C_R}\\ z=xy}}(P(n_i)(x) \vee Q(n_i)(y)) \\ &= \bigwedge_{z\in[\alpha]_{C_R}}\bigwedge_{z=xy}(P(n_i)(x) \vee Q(n_i)(y)) \\ &= \bigwedge_{z\in[\alpha]_{C_R}}[P(n_i).Q(n_i)](z) \\ &= \overline{C_R}(P(n_i).Q(n_i))(z)\end{aligned}$$

$$\begin{aligned}(\overline{C_R}(P(n_f)) \cdot \overline{C_R}(Q(n_f)))(\alpha) &= \bigwedge_{\alpha=\beta+\gamma}[\overline{C_R}(P(n_f))(\beta) \vee \overline{C_R}(Q(n_f))(\gamma)] \\ &= \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{x\in[\beta]_{C_R}}(P(n_f)(x)) \vee (\bigwedge_{y\in[\gamma]_{C_R}}(Q(n_f)(y))] \\ &= \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{\substack{x\in[\beta]_{C_R}\\ y\in[\gamma]_{C_R}}}(P(n_f)(x) \vee Q(n_f)(y))] \\ &\geq \bigwedge_{\alpha=\beta\gamma}[(\bigwedge_{xy\in[\beta\gamma]_{C_R}}(P(n_f)(x) \vee Q(n_f)(y))] \\ &= \bigwedge_{xy\in[\alpha]_{C_R}}(P(n_f)(x) \vee Q(n_f)(y)) \\ &= \bigwedge_{\substack{z\in[\alpha]_{C_R}\\ z=xy}}(P(n_f)(x) \vee Q(n_f)(y)) \\ &= \bigwedge_{z\in[\alpha]_{C_R}}\bigwedge_{z=xy}(P(n_f)(x) \vee Q(n_f)(y)) \\ &= \bigwedge_{z\in[\alpha]_{C_R}}[P(n_f).Q(n_f)](z) \\ &= \overline{C_R}(P(n_f).Q(n_f))(z)\end{aligned}$$

Hence proved. □

Theorem 3.5. *Let P and Q are any two NS of R, then*
$\underline{C_R}(P).\underline{C_R}(Q) \subseteq \underline{C_R}(P.Q)$.

Proof. This proof is similar to Theorem 3.4. □

4. Rough Neutrosophic Subring (RNSR) of Ring

Definition 4.1. A NSR is called an $RNSR$ if it is both upper $RNSR$ and lower $RNSR$ of R.

Definition 4.2. A NSR is said to be an lower (upper) $RNSR$ of R if its lower(upper) approximation is also a NSR of R.

Theorem 4.3. *If K be a NSR of R, then K is an upper $RNSR$ of R.*

Proof. Since K is a NSR of R. Now,$\forall\ a, b \in R$

$$\begin{aligned}
\overline{C_R}(K(n_t))(a-b) &= \bigvee_{c\in[a-b]_{C_R}} K(n_t)(c) \\
&= \bigvee_{x-y\in[a]_{C_R}-[b]_{C_R}} K(n_t)(x-y) \\
&\geq \bigvee_{\substack{x\in[a]_{C_R}\\ y\in[b]_{C_R}}} [K(n_t)(x) \wedge K(n_t)(y)] \\
&= [\bigvee_{x\in[a]_{C_R}} K(n_t)(x)] \wedge [\bigvee_{y\in[b]_{C_R}} K(n_t)(y)] \\
&= \overline{C_R}(K(n_t))(x) \wedge \overline{C_R}(K(n_t))(y)
\end{aligned}$$

$$\begin{aligned}
\overline{C_R}(K(n_i))(a-b) &= \bigwedge_{c\in[a-b]_{C_R}} K(n_i)(c) \\
&= \bigwedge_{x-y\in[a]_{C_R}-[b]_{C_R}} K(n_i)(x-y) \\
&\leq \bigwedge_{\substack{x\in[a]_{C_R}\\ y\in[b]_{C_R}}} [P(n_i)(x) \vee K(n_i)(y)] \\
&= [\bigwedge_{x\in[a]_{C_R}} K(n_i)(x)] \vee [\bigwedge_{y\in[b]_{C_R}} K(n_i)(y)] \\
&= \overline{C_R}(K(n_i))(x) \vee \overline{C_R}(K(n_i))(y)
\end{aligned}$$

$$\begin{aligned}
\overline{C_R}(K(n_f))(a-b) &= \bigwedge_{c\in[a-b]_{C_R}} K(n_f)(c) \\
&= \bigwedge_{x-y\in[a]_{C_R}-[b]_{C_R}} K(n_f)(x-y) \\
&\leq \bigwedge_{\substack{x\in[a]_{C_R}\\ y\in[b]_{C_R}}} [K(n_f)(x) \vee K(n_f)(y)] \\
&= [\bigwedge_{x\in[a]_{C_R}} K(n_f)(x)] \vee [\bigwedge_{y\in[b]_{C_R}} K(n_f)(y)] \\
&= \overline{C_R}(K(n_f))(x) \vee \overline{C_R}(K(n_f))(y)
\end{aligned}$$

$\forall a, b \in R$

$$
\begin{aligned}
\overline{C_R}(K(n_t))(ab) &= \bigvee_{c\in[ab]_{C_R}} K(n_t)(c) \\
&= \bigvee_{xy\in[a]_{C_R}[b]_{C_R}} K(n_t)(xy) \\
&\geq \bigvee_{x\in[a]_{C_R}, y\in[b]_{C_R}} [K(n_t)(x) \wedge K(n_t)(y)] \\
&= [\bigvee_{x\in[a]_{C_R}} K(n_t)(x)] \wedge [\bigvee_{y\in[b]_{C_R}} K(n_t)(y)] \\
&= \overline{C_R}(K(n_t))(x) \wedge \overline{C_R}(K(n_t))(y)
\end{aligned}
$$

$$
\begin{aligned}
\overline{C_R}(K(n_i))(ab) &= \bigwedge_{c\in[ab]_{C_R}} K(n_i)(c) \\
&= \bigwedge_{xy\in[a]_{C_R}[b]_{C_R}} K(n_i)(xy) \\
&\leq \bigwedge_{x\in[a]_{C_R}, y\in[b]_{C_R}} [K(n_i)(x) \vee K(n_i)(y)] \\
&= [\bigwedge_{x\in[a]_{C_R}} K(n_i)(x)] \vee [\bigwedge_{y\in[b]_{C_R}} K(n_i)(y)] \\
&= \overline{C_R}(K(n_i))(x) \vee \overline{C_R}(K(n_i))(y)
\end{aligned}
$$

$$
\begin{aligned}
\overline{C_R}(K(n_f))(ab) &= \bigwedge_{c\in[ab]_{C_R}} K(n_f)(c) \\
&= \bigwedge_{xy\in[a]_{C_R}[b]_{C_R}} K(n_f)(xy) \\
&\leq \bigwedge_{x\in[a]_{C_R}, y\in[b]_{C_R}} [K(n_f)(x) \vee K(n_f)(y)] \\
&= [\bigwedge_{x\in[a]_{C_R}} K(n_f)(x)] \vee [\bigwedge_{y\in[b]_{C_R}} K(n_f)(y)] \\
&= \overline{C_R}(K(n_f))(x) \vee \overline{C_R}(K(n_f))(y)
\end{aligned}
$$

Hence, $\overline{C_R}(K)$ is a NSR of R. Thus K is an upper $RNSR$ of R. □

Theorem 4.4. *If K be a NSR of R, then K is a lower $RNSR$ of R.*

Proof. This proof is similar to Theorem 4.3 □

Corollary 4.5. *Let K be the NSR of R. Then K is a rough $RNSR$ of R.*

Proof. By applying Theorem 4.3 and 4.4 we get the result. □

Definition 4.6. A NI is called an RNI if it is both upper RNI and lower RNI of R.

Definition 4.7. A NI is said to be an lower (upper) RNI of R if its lower(upper) approximation is also an NI of R.

Theorem 4.8. *If K be a NI of R, then K is an upper RNI of R.*

Proof. Since K is a NI of R. We've to prove that, $\forall\ a, b \in R$
$\overline{C_R}(K(n_t))(ab) \geq \overline{C_R}(K(n_t))(a) \vee \overline{C_R}(K(n_t))(b)$
$\overline{C_R}(K(n_i))(ab) \leq \overline{C_R}(K(n_i))(a) \wedge \overline{C_R}(K(n_i))(b)$
$\overline{C_R}(K(n_f))(ab) \leq \overline{C_R}(K(n_f))(a) \wedge \overline{C_R}(K(n_f))(b)$
Now,

$$\begin{aligned}
\overline{C_R}(K(n_i))(ab) &= \bigvee_{c\in[ab]_{C_R}} K(n_t)(c) \\
&\geq \bigvee_{c\in[a]_{C_R}[b]_{C_R}} K(n_t)(c) \\
&= \bigvee_{xy\in[a]_{C_R}[b]_{C_R}} K(n_t)(xy) \\
&\geq \bigvee_{x\in[a]_{C_R}, y\in[b]_{C_R}} [K(n_t)(x) \vee K(n_t)(y)] \\
&= [\bigvee_{x\in[a]_{C_R}} K(n_t)(x)] \vee [\bigvee_{y\in[b]_{C_R}} K(n_t)(y)] \\
&= \overline{C_R}(K(n_t))(x) \vee \overline{C_R}(K(n_t))(y)
\end{aligned}$$

$$\begin{aligned}
\overline{C_R}(K(n_i))(ab) &= \bigwedge_{c\in[ab]_{C_R}} K(n_i)(c) \\
&\leq \bigwedge_{c\in[a]_{C_R}[b]_{C_R}} K(n_i)(c) \\
&= \bigwedge_{xy\in[a]_{C_R}[b]_{C_R}} K(n_i)(xy) \\
&\leq \bigwedge_{x\in[a]_{C_R}, y\in[b]_{C_R}} [K(n_i)(x) \wedge K(n_i)(y)] \\
&= [\bigwedge_{x\in[a]_{C_R}} K(n_i)(x)] \wedge [\bigwedge_{y\in[b]_{C_R}} K(n_i)(y)] \\
&= \overline{C_R}(K(n_i))(x) \wedge \overline{C_R}(K(n_i))(y)
\end{aligned}$$

$$\begin{aligned}\overline{C_R}(K(n_f))(ab) &= \bigwedge_{c\in[ab]_{C_R}} K(n_f)(c)\\ &\leq \bigwedge_{c\in[a]_{C_R}[b]_{C_R}} K(n_f)(c)\\ &= \bigwedge_{xy\in[a]_{C_R}[b]_{C_R}} K(n_f)(xy)\\ &\leq \bigwedge_{x\in[a]_{C_R},y\in[b]_{C_R}} [K(n_f)(x)\wedge K(n_f)(y)]\\ &= [\bigwedge_{x\in[a]_{C_R}} K(n_f)(x)]\wedge[\bigwedge_{y\in[b]_{C_R}} K(n_f)(y)]\\ &= \overline{C_R}(K(n_f))(x)\wedge\overline{C_R}(K(n_f))(y)\end{aligned}$$

Hence, $\overline{C_R}(K)$ is a NI of R. Thus K is an upper RNI of R. □

Theorem 4.9. *If K be a NI of R, then K is a lower RNI of R.*

Proof. This proof is similar to Theorem 4.8. □

Corollary 4.10. *If K be the NI of R. then K is a RNI of R.*

Proof. By applying Theorem 4.8 and 4.9 we get the result. □

5. Conclusion

In this paper, we discussed the notion of rough neutrosophic set in a ring and their properties. Also, we proved that any neutrosophic ideal of a ring is an rough neutrosophic ideal of a ring. For further research one can extend this to other algebraic systems.

References

1. Atanassov,K.T. Intuitionsitic fuzzy sets, Fuzzy Sets and Systems, (1986) 20, 8796.
2. Biswas,R.; Nanda,S.; Rough groups and rough subgroups, Bull. PolishAcad. Sci. Math., (1994) 42, 251-254.
3. Broumi; Smarandache; and Dhar. "Rough neutrosophic set", Italian journal of pure and applied mathematics-N. (2014) 32, 493-502.
4. Dubios,D.; Prade,H; "Rough fuzzy sets and fuzzy rough sets" , Int. J.Gen. Syst., (1990) 17, 191-208.
5. Gong, Z.; Sun, B.; Chen,D. Rough set theory for the interval-valuedfuzzy information systems, Inf. Sci., (2008) 178, 1968-1985.
6. Ju-Sheng Mi; Yee Leung; Hui-Yin Zhao; Tao Feng. "Generalized fuzzy rough sets determined by a triangular norm". Information Sciences. (2008) 178, 3203 - 3213.
7. Pawlak,Z. "Rough Sets", Int. J. Comput. Inform. Sci., (1982) 11, 341 - 356.
8. Selvan,V.; Senthil Kumar,G. Rough ideals in semirings, Int. Jr. of Mathematics Sciences and Applications, (2012) 2(2), 557-564.
9. Selvan,V.; Senthil Kumar,G., Lower and Upper approximations of fuzzy ideals in a semiring, International Journal of scienctic and Engineering Research, (2012)3(1).

10. Senthil Kumar,G.; Selvan,V. "Rough Intutionistic fuzzy ideals in a ring", International journal of scientific and engineering research (2012) 3(8).
11. Smarandache,F. "A Unifying Field in Logics. Neutrosophy: NeutrosophicProbability, Set and Logic". Rehoboth: American Research Press, 1999.
12. Sun,B.; Gong,Z.; Chen,D. "Fuzzy rough set theory for the interval-valued fuzzy information systems", Inf. Sci., (2008) 178, 2794-2815.
13. Vildan cetkin and Halis Aygun, "An approch to neutrosophic ideals, Universal journal of mathematics and applications", (2018) 1(2), 132-136.
14. Zadeh, L.A. "Fuzzy sets" Archived 2015-08-13 at the Wayback Machine. Information and Control (1965) 8(3), 338353.
15. Zhang, Z. "On interval type-2 rough fuzzy sets", Knowledge-Based Syst., (2012) 35, 1-13.
16. Wei-Zhi Wu; Ju-ShengMi; Wen-XiuZhang; Generalized Fuzzy Rough Sets,Inf. Sci., (2003) 151, 263-282.

Received: Feb 5, 2022. Accepted: Jun 9, 2022