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On Characterizations of $(\varpi, \varepsilon, \varsigma)$-Single Valued Neutrosophic Hyperrings and Hyperideals

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Abstract. This study included concepts for $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hyperring ($(\varpi, \varepsilon, \varsigma)$-$SVNHR$) and $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hyperideal ($(\varpi, \varepsilon, \varsigma)$-$SVNHI$). $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hyperrings ($(\varpi, \varepsilon, \varsigma)$-$SVNHRs$) and $(\varpi, \varepsilon, \varsigma)$-single valued hyperideals ($(\varpi, \varepsilon, \varsigma)$-$SVNHIs$) are examined and validated in terms of algebraic features and of structural characteristics.

Keywords: Hyperring, Hyperideal, Single valued neutrosophic set, Single valued neutrosophic hyperring, Single valued neutrosophic hyperideal, $(\varpi, \varepsilon, \varsigma)$-Single valued neutrosophic hyperring and $(\varpi, \varepsilon, \varsigma)$-Single valued neutrosophic hyperideal.

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1. **Introduction**

The theory of hyperstructure came into existence in 1934 when Marty [1] defined hypergroups as being generalized. To propose an overlying homomorphism, Corsini [2] developed the concept of hypering and general forms of hypering. The H_v-ring, the H_v-subring, and the H_v-ideals of the H_v-ring, all these are modifying the thoughts introduced by Corsini [2], have been invented by Vougiouklis [3, 4]. Generally, [5] offers a variety of rates in $[0, 1]$ stated by a single real number. In order to relieve ambiguities, a fuzzy set model was created by the Turksen [6], which was utilized to assess membership of the fuzzy set framework. An enhancement of fuzzy sets is intuitionistic sets, suggested in 1986 by Atanassov [7]. This approach was analogous to the interval-valued fuzzy sets described in [8]. Intuitionistic fuzzy sets can execute flawed data and not inexhaustible information, frequently in real-life [8]. Rosenfeld [9] launched the fuzzy algebra work, extending it to several fuzzy models such as intuitionistic fuzzy sets, fuzzy soft sets, and imprecise soft sets. Some artworks related to soft, fuzzy rings and ideal vague soft groups, vague soft rings, and soft ideals are also found in [10–13]. In 1998, to attain these goals, Smarandache suggested the neutrosophic paradigm in [14]. In [15–20], numerous new neutrosophic theoretical fads were launched.

Wang et al. [8] pioneered the theory of a single-valued neutrosophic set ($SVNS$), whereas Smaranache plithogenic was presented into [21] as a refinement of neutrosophic structure. Hyperstructure theory is often used in numerous mathematical ideas. Algebraic hyperstructures have a wide range of applications, including fuzzy sets, design and data, artificial intelligence, lattices, automation, and combinatorics, and etc. As a result of fuzzy algebra research, fuzzy hyperalgebraic theory was produced. Liu [22] created the idea of fuzzy ideals of a ring. A lot of hyperstructure work has been done over the last two decades, such as fuzzy hyperalgebras [23], fuzzy hyperrings [24], fuzzy topological F-polygroups [25], Bipolar-valued fuzzy soft hyper BCK ideals [26], fuzzy hypergroup degree [27], fuzzy hypergraphs [28], hyper-spectral image analysis [29], fundamental relations on fuzzy hypermodules [30], and so on.

There are works available of hyperstructures related to hyperrings in these manuscripts: fuzzy hyperings [31], Γ-hyperrings [32], soft hyperrings [33], topological hyperrings [34], and topological structures of lower and upper rough subsets in a hyperring [35], etc. In [36], Davvaz initiated the generalization of fuzzy hyperideal. Bharathi and Vimala subsequently established the notions of fuzzy l-ideal in [37], and the fuzzy l-ideal was then expanded in [38]. In [39–41], Selvachandran et al. introduced the hypergroup and hyperring theory for imprecise soft sets, and some other important works on fuzzy sets are studied in [42–45].

In this paper, we focus on the theories of $(\varpi, \varepsilon, \varsigma)$-$SVNHR$s and $(\varpi, \varepsilon, \varsigma)$-$SVNHIs$ in order to contribute to the advancement of the neutrosophic theory of hyperalgebraic.

2. Preliminaries

Let Ξ be a set of points where $\hat{n}$ refers to a generic element of Ξ.

Definition 2.1. [8] A $SVNS$ Υ neutrosophic set that is characterized by a truth membership function $\tau_\Upsilon(\hat{n})$, an indeterminacy-membership function $\iota_\Upsilon(\hat{n})$, and a falsity-membership function $\digamma_\Upsilon(\hat{n})$, where $\tau_\Upsilon(\hat{n}), \iota_\Upsilon(\hat{n}), \digamma_\Upsilon(\hat{n}) \in [0, 1]$. This set Υ can thus be written as:

$$\Upsilon = \{\langle \hat{n}, \tau_\Upsilon(\hat{n}), \iota_\Upsilon(\hat{n}), \digamma_\Upsilon(\hat{n})\rangle : \hat{n} \in \Xi)\}.$$

The sum of $\tau_\Upsilon(\hat{n})$, $\iota_\Upsilon(\hat{n})$ and $\digamma_\Upsilon(\hat{n})$ must fulfill the clause $0 \leq \tau_\Upsilon(\hat{n}) + \iota_\Upsilon(\hat{n}) + \digamma_\Upsilon(\hat{n}) \leq 3$. For a $SVNS$ Υ in Ξ, the triplet $(\tau_\Upsilon(\hat{n}), \iota_\Upsilon(\hat{n}), \digamma_\Upsilon(\hat{n}))$ is referred to as a single valued neutrosophic number (SVNN). Let $\hat{n} = (\tau_{\hat{n}}, \iota_{\hat{n}}, \digamma_{\hat{n}})$ stand for a SVNN.

Definition 2.2. [8] Assume Υ and Γ are two $SVNSs$ in a universe Ξ.

(1) Υ is contained in Γ, if $\tau_\Upsilon(\hat{n}) \leq \tau_\Gamma(\hat{n}), \iota_\Upsilon(\hat{n}) \leq \iota_\Gamma(\hat{n})$, and $\digamma_\Upsilon(\hat{n}) \geq \digamma_\Gamma(\hat{n})$, $\forall\ \hat{n} \in \Xi$. This relationship is denoted as $\Upsilon \subseteq \Gamma$.

(2) Υ=Γ if $\Upsilon \subseteq \Gamma$ and $\Gamma \subseteq \Upsilon$.
(3) $\Upsilon^c = (\hat{n}, (F_\Upsilon(\hat{n}), 1 - \iota_\Upsilon(\hat{n}), \tau_\Upsilon(\hat{n})), \forall\ \hat{n} \in \Xi$.
(4) $\Upsilon \cup \Gamma = (\hat{n}, (\bigvee(\tau_\Upsilon, \tau_\Gamma), \bigvee(\iota_\Upsilon, \iota_\Gamma), \bigwedge(F_\Upsilon, F_\Gamma))), \forall\ \hat{n} \in \Xi$.
(5) $\Upsilon \cap \Gamma = (\hat{n}, (\bigwedge(\tau_\Upsilon, \tau_\Gamma), \bigwedge(\iota_\Upsilon, \iota_\Gamma), \bigvee(F_\Upsilon, F_\Gamma))), \forall\ \hat{n} \in \Xi$.

Definition 2.3. [1] A hypergroup $\langle H, \circ \rangle$ is a set H with an associative hyperoperation $(\circ) : H * H \longrightarrow P(H)$ which satisfies $\hat{n} \circ H = H \circ \hat{n} = H$, $\forall\ \hat{n} \in H$ (reproduction axiom).

Definition 2.4. [36] If the following properties satisfy, a hyperstructure $\langle H, \circ \rangle$ is termed a H_v-group:

(1) $\hat{n} \circ (\hat{o} \circ \hat{p}) \cap (\hat{n} \circ \hat{o}) \circ \hat{p} \neq \phi$, $\forall\ \hat{n}, \hat{o}, \hat{p} \in H$, ($H_v$-semigroup).
(2) $\hat{n} \circ H = H \circ \hat{n} = H$, $\forall\ \hat{n} \in H$.

Definition 2.5. [1] A subset W of H is termed as subhypergroup if $\langle W, \circ \rangle$ is a hypergroup.

Definition 2.6. [2] A H_v-ring is a multi-valued system $(R, +, \circ)$ that satisfies the following axioms:

(1) $(R, +)$ must a H_v-group,
(2) $(R, \circ)$ must a H_v-semigroup,
(3) The hyperoperation "$\circ$" is weak distributive over the hyperoperation "$+$", that is for each $\hat{n}, \hat{o}, \hat{p} \in R$ the clauses $\hat{n} \circ (\hat{o} + \hat{p}) \cap ((\hat{n} \circ \hat{o}) + (\hat{n} \circ \hat{p})) \neq \phi$ and $(\hat{n} + \hat{o}) \circ \hat{p} \cap ((\hat{n} \circ \hat{p}) + (\hat{o} \circ \hat{p})) \neq \phi$ must satisfy.

Definition 2.7. [2] A nonempty subset $R^{'}$ of R is a subhyperring of $(R, +, \circ)$ if $(R^{'}, +)$ is a subhypergroup of $(R, +)$ and $\forall\ \hat{n}, \hat{o}, \hat{p} \in R^{'}, \hat{n} \circ \hat{o} \in P^*(R^{'})$, where $P^*(R^{'})$ denotes the set of all non-empty subsets of $R^{'}$.

Definition 2.8. [2] Suppose H_v-ring be R. a nonempty subset I of R is called a left (resp. right) H_v-ideal if the following axioms hold:

(1) $(I, +)$ be a H_v-subgroup of $(R, +)$,
(2) $R \circ I \subseteq I$(resp. $I \circ R \subseteq I$).

If I is both a left and right H_v-ideal of R, then I is called H_v-ideal of R.

3. $(\varpi, \varepsilon, \varsigma)$-Single Valued Neutrosophic Hyperrings

We represent hyperring $(R, +, \circ)$ by R throughout this section.

Definition 3.1. If Υ be a single valued neutrosophic subset of Ξ then $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic Υ subset of Ξ is categorize as,

$$\Upsilon^{(\varpi,\varepsilon,\varsigma)} = \Big\{\langle \hat{n}, \tau^{\varpi}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Upsilon}(\hat{n}), F^{\varsigma}_{\Upsilon}(\hat{n})\rangle | \tau^{\varpi}_{\Upsilon}(\hat{n}) = \bigwedge\{\tau_\Upsilon(\hat{n}), \varpi\}, \iota^{\varepsilon}_{\Upsilon}(\hat{n}) = \bigwedge\{\iota_\Upsilon(\hat{n}), \varepsilon\}, F^{\varsigma}_{\Upsilon}(\hat{n}) = \bigvee\{F_\Upsilon(\hat{n}), \varsigma\}, \hat{n} \in \Xi\Big\},$$

and $0 \leq \tau^{\varpi}_{\Upsilon}(\hat{n})+\iota^{\varepsilon}_{\Upsilon}(\hat{n})+F^{\varsigma}_{\Upsilon}(\hat{n}) \leq 3$, where $\varpi, \varepsilon, \varsigma \in [0,1]$ also $\tau, \iota, F : \Upsilon \to [0,1]$, such that τ^{ϖ}_{Υ}, $\iota^{\varepsilon}_{\Upsilon}$, F^{ς}_{Υ} represents the functions of truth, indeterminacy, and falsity-membership, respectively.

Definition 3.2. Let Υ be a $(\varpi, \varepsilon, \varsigma)$-$SVNS$ over R. Then Υ is called a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R, if,

(1) $\forall\ \hat{k}, \hat{l} \in R$,
$\bigwedge\{\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Upsilon}(\hat{l})\} \leq \inf\{\tau^{\varpi}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\}$,
$\bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l})\} \geq \sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\}$, and
$\bigvee\{F^{\varsigma}_{\Upsilon}(\hat{k}), F^{\varsigma}_{\Upsilon}(\hat{l})\} \geq \sup\{F^{\varsigma}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\}$.

(2) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{l} \in R$ such that $\hat{k} \in \hat{n}+\hat{l}$ and
$\bigwedge\{\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Upsilon}(\hat{k})\} \leq \tau^{\varpi}_{\Upsilon}(\hat{l})$,
$\bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Upsilon}(\hat{k})\} \geq \iota^{\varepsilon}_{\Upsilon}(\hat{l})$, and
$\bigvee\{F^{\varsigma}_{\Upsilon}(\hat{n}), F^{\varsigma}_{\Upsilon}(\hat{k})\} \geq F^{\varsigma}_{\Upsilon}(\hat{l})$

(3) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{m} \in R$ such that $\hat{k} \in \hat{m}+\hat{n}$ and
$\bigwedge\{\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Upsilon}(\hat{k})\} \leq \tau^{\varpi}_{\Upsilon}(\hat{m})$,
$\bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{n}, \iota^{\varepsilon}_{\Upsilon}(\hat{k})\} \geq \iota^{\varepsilon}_{\Upsilon}(\hat{m})$, and
$\bigvee\{F^{\varsigma}_{\Upsilon}(\hat{n}, F^{\varsigma}_{\Upsilon}(\hat{k})\} \geq F^{\varsigma}_{\Upsilon}(\hat{m})$.

(4) $\forall\ \hat{k}, \hat{l} \in R$,
$\bigwedge\{\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Upsilon}(\hat{l}\} \leq \inf\{\tau^{\varpi}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$,
$\bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l})\} \geq \sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$, and
$\bigvee\{F^{\varsigma}_{\Upsilon}(\hat{k}), F^{\varsigma}_{\Upsilon}(\hat{l})\} \geq \sup\{F^{\varsigma}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$.

Example 3.3. The family of t-level sets of $(\varpi, \varepsilon, \varsigma)$-$SVNSs$ over R is a subhyperring of R is resulting below:

$$\Upsilon^{(\varpi,\varepsilon,\varsigma)}_{t} = \{\hat{k} \in R : \tau^{\varpi}_{\Upsilon}(\hat{k}) \geq t,\ \iota^{\varepsilon}_{\Upsilon}(\hat{k}) \leq t,\ F^{\varsigma}_{\Upsilon}(\hat{k}) \leq t\}, \forall\ t \in [0,1].$$

Then Υ over R is a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$.

Theorem 3.4. *Υ is a $(\varpi, \varepsilon, \varsigma)$-$SVNS$ over R. Then Υ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R if and only if Υ is $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic semi hypergroup over $(R, \circ)$ and also a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hypergroup over $(R, +)$.*

Proof. The definition 3.2 readily indicates this proof. □

Proposition 3.5. *If Υ and Γ be two $(\varpi, \varepsilon, \varsigma)$-single-valued neutrosophic subset of ring R then* $(\Upsilon \cap \Gamma)^{(\varpi,\varepsilon,\varsigma)} = \Upsilon^{(\varpi,\varepsilon,\varsigma)} \cap \Gamma^{(\varpi,\varepsilon,\varsigma)}$.

Proof. Assume that Υ and Γ are two $(\varpi,\varepsilon,\varsigma)$-single-valued neutrosophic subset of ring R.

$$(\Upsilon\cap\Gamma)^{(\varpi,\varepsilon,\varsigma)}(\hat{n}) = \Big\{ \min\{\min\{\tau_\Upsilon(\hat{n}),\tau_\Gamma(\hat{n})\},\varpi\}, \min\{\min\{\iota_\Upsilon(\hat{n}),\ \iota_\Gamma(\hat{n})\},\varepsilon\}, \max\{\max\{F_\Upsilon(\hat{n}),F_\Gamma(\hat{n})\},\varsigma\} \Big\}$$

$$= \Big\{ \min\{\min\{\tau_\Upsilon(\hat{n}),\varpi\},\min\{\tau_\Gamma(\hat{n}),\varpi\}\}, \min\{\min\{\iota_\Upsilon(\hat{n}),\varepsilon\},\max\{\iota_\Gamma(\hat{n}),\varepsilon\}\}, \max\{\max\{F_\Upsilon(\hat{n}),\varsigma\},\max\{F_\Gamma(\hat{n}),\varsigma\}\}\Big\}$$

$$= \Big\{ \min(\{\tau^{\varpi}_\Upsilon(\hat{n})\},\{\tau^{\varpi}_\Gamma(\hat{n})\}), \min(\{\iota^{\varepsilon}_\Upsilon(\hat{n})\},\{\iota^{\varepsilon}_\Gamma(\hat{n})\}), \max(\{F^{\varsigma}_\Upsilon(\hat{n})\},\{F^{\varsigma}_\Gamma(\hat{n})\}) \Big\} = \Upsilon^{(\varpi,\varepsilon,\varsigma)}(\hat{n})\cap\Gamma^{(\varpi,\varepsilon,\varsigma)}(\hat{n}),\ \forall\,\hat{n}\in R.$$

□

Theorem 3.6. *Let Υ and Γ be $(\varpi,\varepsilon,\varsigma)$-$SVNHRs$ over R. Then $\Upsilon\cap\Gamma$ is a $(\varpi,\varepsilon,\varsigma)$-$SVNHR$ over R if it is non-null.*

Proof. Let Υ and Γ are $(\varpi,\varepsilon,\varsigma)$-$SVNHR$s over R. By using Definition 3.2, and Proposition 3.5

$$(\Upsilon\cap\Gamma)^{(\varpi,\varepsilon,\varsigma)} = \Upsilon^{(\varpi,\varepsilon,\varsigma)}\cap\Gamma^{(\varpi,\varepsilon,\varsigma)} = \{\langle\hat{k},(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{k}),(\iota^{\varepsilon}_\Upsilon \bigwedge \iota^{\varepsilon}_\Gamma)(\hat{k}),(F^{\varsigma}_\Upsilon \bigvee F^{\varsigma}_\Gamma)(\hat{k})\rangle : \hat{k}\in R\},$$

where

$$\begin{aligned}
(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{k}) &= \bigwedge(\tau^{\varpi}_\Upsilon(\hat{k}),\tau^{\varpi}_\Gamma(\hat{k})),\\
(\iota^{\varepsilon}_\Upsilon \bigwedge \iota^{\varepsilon}_\Gamma)(\hat{k}) &= \bigwedge(\iota^{\varepsilon}_\Upsilon(\hat{k}),\iota^{\varepsilon}_\Gamma(\hat{k})),\\
(F^{\varsigma}_\Upsilon \bigvee F^{\varsigma}_\Gamma)(\hat{k}) &= \bigvee(F^{\varsigma}_\Upsilon(\hat{k}),F^{\varsigma}_\Gamma(\hat{k})).
\end{aligned}$$

Assuming $\forall\,\hat{k},\hat{l}\in R$, we are only proven to include all four clauses for membership terms τ^{ϖ}_Υ, τ^{ϖ}_Γ and indeterminacy terms $\iota^{\varepsilon}_\Upsilon$, $\iota^{\varepsilon}_\Gamma$. Indications for falsity functions of F^{ς}_Υ, F^{ς}_Γ correspondingly derived.

$$\begin{aligned}
(1)\ \bigwedge\{(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{k}),(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{l})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_\Upsilon(\hat{k}),\tau^{\varpi}_\Gamma(\hat{k})),\bigwedge(\tau^{\varpi}_\Upsilon(\hat{l}),\tau^{\varpi}_\Gamma(\hat{l}))\}\\
&\le \bigwedge\{\bigwedge(\tau^{\varpi}_\Upsilon(\hat{k}),\tau^{\varpi}_\Upsilon(\hat{l})),\bigwedge(\tau^{\varpi}_\Gamma(\hat{k}),\tau^{\varpi}_\Gamma(\hat{l}))\}\\
&\le \bigwedge\{\inf\{\tau^{\varpi}_\Upsilon(\hat{m}) : \hat{m}\in\hat{k}+\hat{l}\},\inf\{\tau^{\varpi}_\Gamma(\hat{m}) : \hat{m}\in\hat{k}+\hat{l}\}\}\\
&\le \inf\{\bigwedge(\tau^{\varpi}_\Upsilon(\hat{m}),\tau^{\varpi}_\Gamma(\hat{m})) : \hat{m}\in\hat{k}+\hat{l}\}\\
&= \inf\{(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{m}) : \hat{m}\in\hat{k}+\hat{l}\}.\\
\Rightarrow\ \bigwedge\{(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{k}),(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{l})\} &\le \inf\{(\tau^{\varpi}_\Upsilon \bigwedge \tau^{\varpi}_\Gamma)(\hat{m}) : \hat{m}\in\hat{k}+\hat{l}\}.
\end{aligned}$$

Also

$$\begin{aligned}
\bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})\}(\hat{l})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Gamma}(\hat{k})), \bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{l}), \iota^{\varepsilon}_{\Gamma}(\hat{l}))\} \\
&\geq \bigwedge\{\bigvee(\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l})), \bigvee(\iota^{\varepsilon}_{\Gamma}(\hat{k}), \iota^{\varepsilon}_{\Gamma}(\hat{l}))\} \\
&\geq \bigwedge\{\sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}, \sup\{\iota^{\varepsilon}_{\Gamma}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}\} \\
&\geq \sup\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{m}), \iota^{\varepsilon}_{\Gamma}(\hat{m})) : \hat{m} \in \hat{k} + \hat{l}\} \\
&= \sup\{(\iota^{\varepsilon}_{\Upsilon}(\hat{m}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{m})) : \hat{m} \in \hat{k} + \hat{l}\}. \\
\Rightarrow \bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})\}(\hat{l})\} &\geq \sup\{(\iota^{\varepsilon}_{\Upsilon}(\hat{m}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{m})) : \hat{m} \in \hat{k} + \hat{l}\}.
\end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})(\hat{k}), (F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})\}(\hat{l})\} \geq \sup\{(F^{\varsigma}_{\Upsilon}(\hat{m}) \bigvee F^{\varsigma}_{\Gamma}(\hat{m})) : \hat{m} \in \hat{k} + \hat{l}\}.$$

(2) $\exists \ \forall \ \hat{n}, \hat{k} \in R$ such that $\hat{k} \in \hat{n} + \hat{l}$ then it argues that:

$$\begin{aligned}
\bigwedge\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{n}), (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{k})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Gamma}(\hat{n})), \bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Gamma}(\hat{k}))\} \\
&\leq \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Upsilon}(\hat{k})), \bigwedge(\tau^{\varpi}_{\Gamma}(\hat{n}), \tau^{\varpi}_{\Gamma}(\hat{k}))\} \\
&\leq \{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{l}), \tau^{\varpi}_{\Gamma}(\hat{l}))\} \\
&= (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{l}). \\
\Rightarrow \bigwedge\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{n}), (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{k})\} &\leq \{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{l}) : \hat{k} \in \hat{n} + \hat{l}\}.
\end{aligned}$$

Also, $\exists \ \forall \ \hat{n}, \hat{k} \in R$ such that $\hat{k} \in \hat{n} + \hat{l}$ then it argues that:

$$\begin{aligned}
\bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{n}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Gamma}(\hat{n})), \bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Gamma}(\hat{k}))\} \\
&\geq \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Upsilon}(\hat{k})), \bigwedge(\iota^{\varepsilon}_{\Gamma}(\hat{n}), \iota^{\varepsilon}_{\Gamma}(\hat{k}))\} \\
&\geq \{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{l}), \iota^{\varepsilon}_{\Gamma}(\hat{l}))\} \\
&= \{(\iota^{\varepsilon}_{\Upsilon}(\hat{l}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{l}))\}. \\
\Rightarrow \bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{n}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &\geq \{(\iota^{\varepsilon}_{\Upsilon}(\hat{l}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{l})) : \hat{k} \in \hat{n} + \hat{l}\}
\end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})(\hat{n}), (F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})(\hat{k})\} \geq \{(F^{\varsigma}_{\Upsilon}(\hat{l}) \bigvee F^{\varsigma}_{\Gamma}(\hat{l})) : \hat{k} \in \hat{n} + \hat{l}\}$$

(3) $\forall\ \hat{n}, \hat{k} \in R\ \exists\ \hat{m} \in R$ where $\hat{k} \in \hat{m} + \hat{n}$ can be readily proved that

$$\begin{aligned} \bigwedge\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{n}), (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{k})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Gamma}(\hat{n})), \bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Gamma}(\hat{k}))\} \\ &\leq \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}), \tau^{\varpi}_{\Upsilon}(\hat{k})), \bigwedge(\tau^{\varpi}_{\Gamma}(\hat{n}), \tau^{\varpi}_{\Gamma}(\hat{k}))\} \\ &\leq \{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{m}), \tau^{\varpi}_{\Gamma}(\hat{m}))\} \\ &= (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{m}). \\ \Rightarrow \bigwedge\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{n}), (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{k})\} &\leq \{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{m}) : \hat{k} \in \hat{m} + \hat{n}\}. \end{aligned}$$

Also, $\forall\ \hat{n}, \hat{k} \in R\ \exists\ \hat{m} \in R$ where $\hat{k} \in \hat{m} + \hat{n}$ then it argues that:

$$\begin{aligned} \bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{n}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Gamma}(\hat{n})), \bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Gamma}(\hat{k}))\} \\ &\geq \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}), \iota^{\varepsilon}_{\Upsilon}(\hat{k})), \bigwedge(\iota^{\varepsilon}_{\Gamma}(\hat{n}), \iota^{\varepsilon}_{\Gamma}(\hat{k}))\} \\ &\geq \{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{m}), \iota^{\varepsilon}_{\Gamma}(\hat{m}))\} \\ &= \{(\iota^{\varepsilon}_{\Upsilon}(\hat{m}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{m}))\}. \\ \Rightarrow \bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{n}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &\geq \{(\iota^{\varepsilon}_{\Upsilon}(\hat{m}) \bigwedge \iota^{\varepsilon}_{\Gamma}(\hat{m})) : \hat{k} \in \hat{m} + \hat{n}\} \end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})(\hat{n}), (F^{\varsigma}_{\Upsilon} \bigvee F^{\varsigma}_{\Gamma})(\hat{k})\} \geq \{(F^{\varsigma}_{\Upsilon}(\hat{m}) \bigvee F^{\varsigma}_{\Gamma}(\hat{m})) : \hat{k} \in \hat{m} + \hat{n}\}$$

(4) $\forall\ \hat{k},\ \hat{l} \in R$,

$$\begin{aligned} \bigwedge\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{k}), (\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{l})\} &\leq \inf\{(\tau^{\varpi}_{\Upsilon} \bigwedge \tau^{\varpi}_{\Gamma})(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}, \\ \bigvee\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \tau^{\varepsilon}_{\Gamma})(\hat{k}), (\iota^{\varepsilon}_{\Upsilon} \bigwedge \iota^{\varepsilon}_{\Gamma})(\hat{l})\} &\geq \sup\{(\iota^{\varepsilon}_{\Upsilon} \bigwedge \tau^{\varepsilon}_{\Gamma})\ (\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}, \\ \bigvee\{(F^{\varsigma}_{\Upsilon} \bigwedge F^{\varsigma}_{\Gamma})(\hat{k}), (F^{\varsigma}_{\Upsilon} \bigwedge F^{\varsigma}_{\Gamma})(\hat{l})\} &\geq \sup\{(F^{\varsigma}_{\Upsilon} \bigwedge F^{\varsigma}_{\Gamma})(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}. \end{aligned}$$

Hence, $\Upsilon \cap \Gamma$ is $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R. □

Theorem 3.7. *Let Υ be a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R. Then for every $t \in [0, 1]$, $\Upsilon^{(\varpi,\varepsilon,\varsigma)}_t \neq \phi$ is a subhyperring over R.*

Proof. Let Υ be a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R. $\forall\ t \in [0, 1]$, let $\hat{k}, \hat{l} \in \Upsilon^{(\varpi,\varepsilon,\varsigma)}_t$.
Then $\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Upsilon}(\hat{l}) \geq t,\ \iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l}) \leq t$ and $F^{\varsigma}_{\Upsilon}(\hat{k}), F^{\varsigma}_{\Upsilon}(\hat{l}) \leq t$.
Since Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic subhypergroup of $(R, +)$, we have the following

$$\inf\{\tau^{\varpi}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\} \geq \bigwedge\{\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Upsilon}(\hat{l})\} \geq \bigwedge\{t, t\} = t,$$

$$\sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\} \leq \bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l})\} \leq \bigvee\{t, t\} = t,$$

and

$$\sup\{F^{\varsigma}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\} \leq \bigvee\{F^{\varsigma}_{\Upsilon}(\hat{k}), F^{\varsigma}_{\Upsilon}(\hat{l})\} \leq \bigvee\{t, t\} = t.$$

This implies that $\hat{m} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ and then for every $\hat{m} \in \hat{k}+\hat{l}$, we obtain $\hat{k}+\hat{l} \subseteq \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$.
As such, for every $\hat{m} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$, we obtain $\hat{m}+\Upsilon_t^{(\varpi,\varepsilon,\varsigma)} \subseteq \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$.
Now let $\hat{k},\ \hat{m} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$. Then $\tau_\Upsilon^\varpi(\hat{k}),\ \tau_\Upsilon^\varpi(\hat{m}) \geq t,\ \iota_\Upsilon^\varepsilon(\hat{k}),\ \iota_\Upsilon^\varepsilon(\hat{m}) \leq t$ and $F_\Upsilon^\varsigma(\hat{k}),\ F_\Upsilon^\varsigma(\hat{m}) \leq t$. Υ is a $(\varpi,\varepsilon,\varsigma)$-single valued neutrosophic subhypergroup of $(R,+)$, $\exists\ \hat{l} \in R$ such that $\hat{k} \in \hat{m}+\hat{l}$ and $\tau_\Upsilon^\varpi(\hat{l}) \geq \bigwedge(\tau_\Upsilon^\varpi(\hat{k}),\ \tau_\Upsilon^\varpi(\hat{m})) \geq t,\ \iota_\Upsilon^\varepsilon(\hat{l}) \leq \bigvee(\iota_\Upsilon^\varepsilon(\hat{k}),\ \iota_\Upsilon^\varepsilon(\hat{m})) \leq t,\ F_\Upsilon^\varsigma(\hat{l}) \leq \bigvee(F_\Upsilon^\varsigma(\hat{k}),\ F_\Upsilon^\varsigma(\hat{m})) \leq t$, and this implies that $\hat{l} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$. Therefore, we obtain $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)} \subseteq \hat{m}+\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$.
As such, we obtain $\hat{m}+\Upsilon_t^{(\varpi,\varepsilon,\varsigma)} = \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$. As a result, $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is a subhypergroup of $(R,+)$.
Let $\hat{k},\hat{l} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$, then $\tau_\Upsilon^\varpi(\hat{k}),\ \tau_\Upsilon^\varpi(\hat{l}) \geq t,\ \iota_\Upsilon^\varepsilon(\hat{k}),\ \iota_\Upsilon^\varepsilon(\hat{l}) \leq t$ and $F_\Upsilon^\varsigma(\hat{k}),\ F_\Upsilon^\varsigma(\hat{l}) \leq t$. Since Υ is a $(\varpi,\varepsilon,\varsigma)$-single valued neutrosophic sub-semihypergroup of $(R,\circ)$, then $\forall\ \hat{k},\hat{l} \in R$, we have the following:

$$\inf\{\tau_\Upsilon^\varpi(\hat{m}) : \hat{m} \in \hat{k}\circ\hat{l}\} \geq \bigwedge\{\tau_\Upsilon^\varpi(\hat{k}),\ \tau_\Upsilon^\varpi(\hat{l})\} = t,$$

$$\sup\{\iota_\Upsilon^\varepsilon(\hat{m}) : \hat{m} \in \hat{k}\circ\hat{l}\} \leq \bigvee(\iota_\Upsilon^\varepsilon(\hat{k}),\ \iota_\Upsilon^\varepsilon(\hat{l})) = t,$$

and

$$\sup\{F_\Upsilon^\varsigma(\hat{m}) : \hat{m} \in \hat{k}\circ\hat{l} \leq \bigvee(F_\Upsilon^\varsigma(\hat{k}),\ F_\Upsilon^\varsigma(\hat{l}) = t.$$

This implies that $\hat{m} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ and consequently $\hat{k}\circ\hat{l} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$.
Therefore, for every $\hat{k},\hat{l} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ we obtain $\hat{k}\circ\hat{l} \in P^*(R)$. Hence $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is a subhyperring over R. □

Theorem 3.8. *Let Υ be a $(\varpi,\varepsilon,\varsigma)$-single valued neutrosophic set over R. Then the following statements are equivalent:*

(1) *Υ is a $(\varpi,\varepsilon,\varsigma)$-$SVNHR$ over R.*
(2) *$\forall\ t \in [0,1]$, a non-empty $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is a subhyperring over R.*

Proof. (1)⇒(2) $\forall\ t \in [0,1]$, by Theorem 3.7, $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is subhyperring over R.
(2)⇒(1) Assume that $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is a subhyperring over R. Let $\hat{k},\hat{l} \in \Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ and therefore $\hat{k}+\hat{l} \subseteq \Upsilon_{t_0}^{(\varpi,\varepsilon,\varsigma)}$. Then for every $\hat{m} \in \hat{k}+\hat{l}$ we have $\tau_\Upsilon^\varpi(\hat{m}) \geq t_0,\ \iota_\Upsilon^\varepsilon(\hat{m}) \leq t_0$ and $F_\Upsilon^\varsigma(\hat{m}) \leq t_0$, which implies that:

$$\bigwedge(\tau_\Upsilon^\varpi(\hat{k}),\tau_\Upsilon^\varpi(\hat{l})) \leq \inf\{\tau_\Upsilon^\varpi(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\},$$

$$\bigvee(\iota_\Upsilon^\varepsilon(\hat{k}),\iota_\Upsilon^\varepsilon(\hat{l})) \geq \sup\{\iota_\Upsilon^\varepsilon(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\},$$

and

$$\bigvee(F_\Upsilon^\varsigma(\hat{k}),F_\Upsilon^\varsigma(\hat{l})) \geq \sup\{F_\Upsilon^\varsigma(\hat{m}) : \hat{m} \in \hat{k}+\hat{l}\}.$$

Thus, clause (1) of Definition 3.2 has been fulfilled.
Next, let $\hat{n},\hat{k} \in \Upsilon_{t_1}^{(\varpi,\varepsilon,\varsigma)}$ for every $t_1 \in [0,1]$ which means that $\exists\ \hat{l} \in \Upsilon_{t_1}^{(\varpi,\varepsilon,\varsigma)}$ such that $\hat{k} \in \hat{n}\circ\hat{l}$.

Since $\hat{l} \in \Upsilon_{t_1}^{(\varpi,\varepsilon,\varsigma)}$, we have $\tau_{\Upsilon}^{\varpi}(\hat{l}) \geq t_1,\ \iota_{\Upsilon}^{\varepsilon}(\hat{l}) \leq t_1$ and $F_{\Upsilon}^{\varsigma}(\hat{l}) \leq t_1$, and thus we have

$$\tau_{\Upsilon}^{\varpi}(\hat{l}) \geq t_1 = \bigwedge(\tau_{\Upsilon}^{\varpi}(\hat{k}), \tau_{\Upsilon}^{\varpi}(\hat{m})),$$

$$\iota_{\Upsilon}^{\varepsilon}(\hat{l}) \leq t_1 = \bigvee(\iota_{\Upsilon}^{\varepsilon}(\hat{k}), \iota_{\Upsilon}^{\varepsilon}(\hat{m})),$$

and

$$F_{\Upsilon}^{\varsigma}(\hat{l}) \leq t_1 = \bigvee(F_{\Upsilon}^{\varsigma}(\hat{k}), F_{\Upsilon}^{\varsigma}(\hat{m})).$$

Thus, clause (2) of Definition 3.2 has been fulfilled.

Assurance of (3) of Definition 3.2 can be satisfied in a similar way.

As a result, Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic subhypergroup of $(R, +)$.

Now since $\Upsilon_t^{(\varpi,\varepsilon,\varsigma)}$ is a sub-semihypergroup of the semihypergroup $(R, \circ)$, we got the following. Let $\hat{k}, \hat{l} \in \Upsilon_{t_2}^{(\varpi,\varepsilon,\varsigma)}$ and therefore we have $\hat{k} \circ \hat{l} \in \Upsilon_{t_2}^{(\varpi,\varepsilon,\varsigma)}$. Thus for every $\hat{m} \in \hat{k} \circ \hat{l}$, we obtain $\tau_{\Upsilon}^{\varpi}(\hat{m}) \geq t_2,\ \iota_{\Upsilon}^{\varepsilon}(\hat{m}) \leq t_2$ and $F_{\Upsilon}^{\varsigma}(\hat{m}) \leq t_2$, and therefore it follows that:

$$\bigwedge(\tau_{\Upsilon}^{\varpi}(\hat{k}),\ \tau_{\Upsilon}^{\varpi}(\hat{l})) \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\},$$

$$\bigvee(\iota_{\Upsilon}^{\varepsilon}(\hat{k}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{l})) \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\},$$

and

$$\bigvee(F_{\Upsilon}^{\varsigma}(\hat{k}),\ F_{\Upsilon}^{\varsigma}(\hat{l})) \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}.$$

which reveals that clause (4) of Definition 3.2 is verified.

Hence Υ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHR$ over R. □

4. $(\varpi, \varepsilon, \varsigma)$-Single Valued Neutrosophic Hyperideals

Definition 4.1. Let Υ be a $(\varpi, \varepsilon, \varsigma)$-$SVNS$ over R. Then Υ is $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic left (resp. right) hyperideal over R, if,

(1) $\forall\ \hat{k}, \hat{l} \in R$,
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{k}),\ \tau_{\Upsilon}^{\varpi}(\hat{l})\} \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{k}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{l})\} \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{k}),\ F_{\Upsilon}^{\varsigma}(\hat{l})\} \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$.

(2) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{l} \in R$ such that $\hat{k} \in \hat{n} + \hat{l}$, and
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{n}),\ \tau_{\Upsilon}^{\varpi}(\hat{k})\} \leq \tau_{\Upsilon}^{\varpi}(\hat{l})$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{n}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{k})\} \geq \iota_{\Upsilon}^{\varepsilon}(\hat{l})$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{n}),\ F_{\Upsilon}^{\varsigma}(\hat{k})\} \geq F_{\Upsilon}^{\varsigma}(\hat{l})$.

(3) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{m} \in R$ such that $\hat{k} \in \hat{m} + \hat{n}$, and
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{n}),\ \tau_{\Upsilon}^{\varpi}(\hat{k})\} \leq \tau_{\Upsilon}^{\varpi}(\hat{m})$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{n}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{k})\} \geq \iota_{\Upsilon}^{\varepsilon}(\hat{m})$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{n}),\ F_{\Upsilon}^{\varsigma}(\hat{k})\} \geq F_{\Upsilon}^{\varsigma}(\hat{m})$.

(4) $\forall\ \hat{k}, \hat{l} \in R$,
$\tau_{\Upsilon}^{\varpi}(\hat{l}) \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$ (resp. $\tau_{\Upsilon}^{\varpi}(\hat{k}) \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$),
$\iota_{\Upsilon}^{\varepsilon}(\hat{l}) \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$ (resp. $\iota_{\Upsilon}^{\varepsilon}(\hat{k}) \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$), and
$F_{\Upsilon}^{\varsigma}(\hat{l}) \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$ (resp. $F_{\Upsilon}^{\varsigma}(\hat{k}) \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$).

If Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic left (resp. right) hyperideal of R then Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic subhypergroup of $(R, +)$ by clauses (1), (2) and (3).

Definition 4.2. Let Υ be a $(\varpi, \varepsilon, \varsigma)$-$SVNS$ over R. Then Υ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHI$ over R, if aforementioned clauses are met:

(1) $\forall\ \hat{k}, \hat{l} \in R$,
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{k}),\ \tau_{\Upsilon}^{\varpi}(\hat{l})\} \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{k}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{l})\} \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{k}),\ F_{\Upsilon}^{\varsigma}(\hat{l})\} \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} + \hat{l}\}$.

(2) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{l} \in R$ such that $\hat{k} \in \hat{n} + \hat{l}$, and
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{n}),\ \tau_{\Upsilon}^{\varpi}(\hat{k})\} \leq \tau_{\Upsilon}^{\varpi}(\hat{l})$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{n}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{k})\} \geq \iota_{\Upsilon}^{\varepsilon}(\hat{l})$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{n}),\ F_{\Upsilon}^{\varsigma}(\hat{k})\} \geq F_{\Upsilon}^{\varsigma}(\hat{l})$.

(3) $\forall\ \hat{n}, \hat{k} \in R$, $\exists\ \hat{m} \in R$ such that $\hat{k} \in \hat{m} + \hat{n}$, and
$\bigwedge\{\tau_{\Upsilon}^{\varpi}(\hat{n}),\ \tau_{\Upsilon}^{\varpi}(\hat{k})\} \leq \tau_{\Upsilon}^{\varpi}(\hat{m})$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{n}),\ \iota_{\Upsilon}^{\varepsilon}(\hat{k})\} \geq \iota_{\Upsilon}^{\varepsilon}(\hat{m})$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{n}),\ F_{\Upsilon}^{\varsigma}(\hat{k})\} \geq F_{\Upsilon}^{\varsigma}(\hat{m})$.

(4) $\forall\ \hat{k}, \hat{l} \in R$,
$\bigvee\{\tau_{\Upsilon}^{\varpi}(\hat{k}), \tau_{\Upsilon}^{\varpi}(\hat{l})\} \leq \inf\{\tau_{\Upsilon}^{\varpi}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$,
$\bigvee\{\iota_{\Upsilon}^{\varepsilon}(\hat{k}), \iota_{\Upsilon}^{\varepsilon}(\hat{l})\} \geq \sup\{\iota_{\Upsilon}^{\varepsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$, and
$\bigvee\{F_{\Upsilon}^{\varsigma}(\hat{k}), F_{\Upsilon}^{\varsigma}(\hat{l})\} \geq \sup\{F_{\Upsilon}^{\varsigma}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}$.

Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic subhypergroup of $(R, +)$ by clauses (1), (2) and (3). Clause (4) indicate that Υ is both $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic left hyperideal and $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic right hyperideal.
$\Rightarrow \Upsilon$ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHI$ of R.

Theorem 4.3. *Let Υ be a non-null $(\varpi, \varepsilon, \varsigma)$-$SVNS$ over R. Υ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHI$ over R if and only if Υ is a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic hypergroup over $(R, +)$ and also Υ is both a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic left hyperideal and a $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic right hyperideal of R.*

Proof. With the help of Definitions 4.1 and 4.2, we get the required proof. □

Theorem 4.4. *Let Υ and Γ be two $(\varpi,\varepsilon,\varsigma)$-$SVNHIs$ over R. Then $\Upsilon\cap\Gamma$ is a $(\varpi,\varepsilon,\varsigma)$-$SVNHI$ over R if it is non-null.*

Proof. Let Υ and Γ are $(\varpi,\varepsilon,\varsigma)$-$SVNHIs$ over R. By using Definition 3.2, and Proposition 3.5

$$(\Upsilon\cap\Gamma)^{(\varpi,\varepsilon,\varsigma)}=\Upsilon^{(\varpi,\varepsilon,\varsigma)}\cap\Gamma^{(\varpi,\varepsilon,\varsigma)}=\{\langle\hat{k},(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{k}),(\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})(\hat{k}),(F^{\varsigma}_{\Upsilon}\bigvee F^{\varsigma}_{\Gamma})(\hat{k})\rangle:\hat{k}\in R\},$$

where

$$\begin{aligned}(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{k}) &= \bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}),\tau^{\varpi}_{\Gamma}(\hat{k})),\\ (\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})(\hat{k}) &= \bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}),\iota^{\varepsilon}_{\Gamma}(\hat{k})),\\ (F^{\varsigma}_{\Upsilon}\bigvee F^{\varsigma}_{\Gamma})(\hat{k}) &= \bigvee(F^{\varsigma}_{\Upsilon}(\hat{k}),F^{\varsigma}_{\Gamma}(\hat{k})).\end{aligned}$$

Assuming $\forall\ \hat{k},\hat{l}\in R$, we are only proven to include all four clauses for membership terms τ^{ϖ}_{Υ}, τ^{ϖ}_{Γ} and indeterminacy terms $\iota^{\varepsilon}_{\Upsilon}$, $\iota^{\varepsilon}_{\Gamma}$.

$$\begin{aligned}(1)\ \bigwedge\{(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{k}),(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{l})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}),\tau^{\varpi}_{\Gamma}(\hat{k})),\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{l}),\tau^{\varpi}_{\Gamma}(\hat{l}))\}\\ &\leq \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}),\tau^{\varpi}_{\Upsilon}(\hat{l})),\bigwedge(\tau^{\varpi}_{\Gamma}(\hat{k}),\tau^{\varpi}_{\Gamma}(\hat{l}))\}\\ &\leq \bigwedge\{\inf\{\tau^{\varpi}_{\Upsilon}(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\},\inf\{\tau^{\varpi}_{\Gamma}(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\}\}\\ &\leq \inf\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{m}),\tau^{\varpi}_{\Gamma}(\hat{m})):\hat{m}\in\hat{k}+\hat{l}\}\\ &= \inf\{(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\}.\\ \Rightarrow\ \bigwedge\{(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{k}),(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{l})\} &\leq \inf\{(\tau^{\varpi}_{\Upsilon}\bigwedge\tau^{\varpi}_{\Gamma})(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\}.\end{aligned}$$

Also

$$\begin{aligned}\bigvee\{(\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})(\hat{k}),(\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})\}(\hat{l})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}),\iota^{\varepsilon}_{\Gamma}(\hat{k})),\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{l}),\iota^{\varepsilon}_{\Gamma}(\hat{l}))\}\\ &\geq \bigwedge\{\bigvee(\iota^{\varepsilon}_{\Upsilon}(\hat{k}),\iota^{\varepsilon}_{\Upsilon}(\hat{l})),\bigvee(\iota^{\varepsilon}_{\Gamma}(\hat{k}),\iota^{\varepsilon}_{\Gamma}(\hat{l}))\}\\ &\geq \bigwedge\{\sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\},\sup\{\iota^{\varepsilon}_{\Gamma}(\hat{m}):\hat{m}\in\hat{k}+\hat{l}\}\}\\ &\geq \sup\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{m}),\iota^{\varepsilon}_{\Gamma}(\hat{m})):\hat{m}\in\hat{k}+\hat{l}\}\\ &= \sup\{(\iota^{\varepsilon}_{\Upsilon}(\hat{m})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{m})):\hat{m}\in\hat{k}+\hat{l}\}.\\ \Rightarrow\ \bigvee\{(\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})(\hat{k}),(\iota^{\varepsilon}_{\Upsilon}\bigwedge\iota^{\varepsilon}_{\Gamma})\}(\hat{l})\} &\geq \sup\{(\iota^{\varepsilon}_{\Upsilon}(\hat{m})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{m})):\hat{m}\in\hat{k}+\hat{l}\}.\end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon}\bigvee F^{\varsigma}_{\Gamma})(\hat{k}),(F^{\varsigma}_{\Upsilon}\bigvee F^{\varsigma}_{\Gamma})\}(\hat{l})\}\ \geq\ \sup\{(F^{\varsigma}_{\Upsilon}(\hat{m})\bigvee F^{\varsigma}_{\Gamma}(\hat{m})):\hat{m}\in\hat{k}+\hat{l}\}.$$

(2) $\exists\ \forall\ \hat{n},\hat{k}\in R$ such that $\hat{k}\in\hat{n}+\hat{l}$ then it argues that:

$$\begin{aligned}
\bigwedge\{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{n}),(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{k})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}),\tau^{\varpi}_{\Gamma}(\hat{n})),\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}),\tau^{\varpi}_{\Gamma}(\hat{k}))\}\\
&\leq \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}),\tau^{\varpi}_{\Upsilon}(\hat{k})),\bigwedge(\tau^{\varpi}_{\Gamma}(\hat{n}),\tau^{\varpi}_{\Gamma}(\hat{k}))\}\\
&\leq \{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{l}),\tau^{\varpi}_{\Gamma}(\hat{l}))\}\\
&= (\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{l}).\\
\Rightarrow \bigwedge\{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{n}),(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{k})\} &\leq \{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{l}):\hat{k}\in\hat{n}+\hat{l}\}.
\end{aligned}$$

Also, $\exists\ \forall\ \hat{n},\hat{k}\in R$ such that $\hat{k}\in\hat{n}+\hat{l}$ then it argues that:

$$\begin{aligned}
\bigvee\{(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{n}),(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}),\iota^{\varepsilon}_{\Gamma}(\hat{n})),\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}),\iota^{\varepsilon}_{\Gamma}(\hat{k}))\}\\
&\geq \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}),\iota^{\varepsilon}_{\Upsilon}(\hat{k})),\bigwedge(\iota^{\varepsilon}_{\Gamma}(\hat{n}),\iota^{\varepsilon}_{\Gamma}(\hat{k}))\}\\
&\geq \{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{l}),\iota^{\varepsilon}_{\Gamma}(\hat{l}))\}\\
&= \{(\iota^{\varepsilon}_{\Upsilon}(\hat{l})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{l}))\}.\\
\Rightarrow \bigvee\{(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{n}),(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &\geq \{(\iota^{\varepsilon}_{\Upsilon}(\hat{l})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{l})):\hat{k}\in\hat{n}+\hat{l}\}
\end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon}\ \bigvee\ F^{\varsigma}_{\Gamma})(\hat{n}),(F^{\varsigma}_{\Upsilon}\ \bigvee\ F^{\varsigma}_{\Gamma})(\hat{k})\} \geq \{(F^{\varsigma}_{\Upsilon}(\hat{l})\bigvee F^{\varsigma}_{\Gamma}(\hat{l})):\hat{k}\in\hat{n}+\hat{l}\}$$

(3) $\forall\ \hat{n},\hat{k}\in R\ \exists\ \hat{m}\in R$ where $\hat{k}\in\hat{m}+\hat{n}$ can be readily proved that

$$\begin{aligned}
\bigwedge\{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{n}),(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{k})\} &= \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}),\tau^{\varpi}_{\Gamma}(\hat{n})),\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{k}),\tau^{\varpi}_{\Gamma}(\hat{k}))\}\\
&\leq \bigwedge\{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{n}),\tau^{\varpi}_{\Upsilon}(\hat{k})),\bigwedge(\tau^{\varpi}_{\Gamma}(\hat{n}),\tau^{\varpi}_{\Gamma}(\hat{k}))\}\\
&\leq \{\bigwedge(\tau^{\varpi}_{\Upsilon}(\hat{m}),\tau^{\varpi}_{\Gamma}(\hat{m}))\}\\
&= (\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{m}).\\
\Rightarrow \bigwedge\{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{n}),(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{k})\} &\leq \{(\tau^{\varpi}_{\Upsilon}\ \bigwedge\ \tau^{\varpi}_{\Gamma})(\hat{m}):\hat{k}\in\hat{m}+\hat{n}\}.
\end{aligned}$$

Also, $\forall\ \hat{n},\hat{k}\in R\ \exists\ \hat{m}\in R$ where $\hat{k}\in\hat{m}+\hat{n}$ then it argues that:

$$\begin{aligned}
\bigvee\{(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{n}),(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &= \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}),\iota^{\varepsilon}_{\Gamma}(\hat{n})),\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{k}),\iota^{\varepsilon}_{\Gamma}(\hat{k}))\}\\
&\geq \bigvee\{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{n}),\iota^{\varepsilon}_{\Upsilon}(\hat{k})),\bigwedge(\iota^{\varepsilon}_{\Gamma}(\hat{n}),\iota^{\varepsilon}_{\Gamma}(\hat{k}))\}\\
&\geq \{\bigwedge(\iota^{\varepsilon}_{\Upsilon}(\hat{m}),\iota^{\varepsilon}_{\Gamma}(\hat{m}))\}\\
&= \{(\iota^{\varepsilon}_{\Upsilon}(\hat{m})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{m}))\}.\\
\Rightarrow \bigvee\{(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{n}),(\iota^{\varepsilon}_{\Upsilon}\ \bigwedge\ \iota^{\varepsilon}_{\Gamma})(\hat{k})\} &\geq \{(\iota^{\varepsilon}_{\Upsilon}(\hat{m})\bigwedge\iota^{\varepsilon}_{\Gamma}(\hat{m})):\hat{k}\in\hat{m}+\hat{n}\}
\end{aligned}$$

Similarly,

$$\bigvee\{(F^{\varsigma}_{\Upsilon}\ \bigvee\ F^{\varsigma}_{\Gamma})(\hat{n}),(F^{\varsigma}_{\Upsilon}\ \bigvee\ F^{\varsigma}_{\Gamma})(\hat{k})\} \geq \{(F^{\varsigma}_{\Upsilon}(\hat{m})\bigvee F^{\varsigma}_{\Gamma}(\hat{m})):\hat{k}\in\hat{m}+\hat{n}\}$$

(4) $\forall\ \hat{k},\ \hat{l} \in R$,

$$\begin{aligned} \bigvee\{\tau^{\varpi}_{\Upsilon}(\hat{k}), \tau^{\varpi}_{\Upsilon}(\hat{l})\} &\leq \inf\{\tau^{\varpi}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}, \\ \bigvee\{\iota^{\varepsilon}_{\Upsilon}(\hat{k}), \iota^{\varepsilon}_{\Upsilon}(\hat{l})\} &\geq \sup\{\iota^{\varepsilon}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\},\ and \\ \bigvee\{F^{\varsigma}_{\Upsilon}(\hat{k}), F^{\varsigma}_{\Upsilon}(\hat{l})\} &\geq \sup\{F^{\varsigma}_{\Upsilon}(\hat{m}) : \hat{m} \in \hat{k} \circ \hat{l}\}. \end{aligned}$$

Hence, it is verified that $\Upsilon \cap \Gamma$ is a $(\varpi, \varepsilon, \varsigma)$-$SVNHI$ over R. □

5. Conclusions

This research has introduced the novel concepts of the $(\varpi, \varepsilon, \varsigma)$-single valued neutrosophic theory of hyperrings and hyperideals through the introduction of a few hyperalgebraic structures and the analysis of some basic properties, outcomes, and structural characteristics of these concepts. We plan to meld more hyperalgebraic theory with real-world applications in the future for plithogenic sets for $(\varpi, \varepsilon, \varsigma)$-single-valued neutrosophic sets and $(\varpi, \varepsilon, \varsigma)$-interval-valued neutrosophic sets.

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