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Lie-Algebra of Single-Valued Pentapartitioned Neutrosophic Set

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Abstract:

In this article, we procure the concept of single-valued pentapartitioned neutrosophic Lie (in short SVPN-Lie) algebra under single-valued pentapartitioned neutrosophic set (in short SVPN-set) environment. Besides, we study the notion of SVPN-Lie ideal of SVPN-Lie algebra, and produce several interesting results on SVPN-Lie algebra and SVPN-Lie ideal.

Keywords: *Lie-ideal*; *Lie-algebra*; *Neutrosophic Set*; *SVPN-set*; *SVPN-Lie ideal*.

1. Introduction:

In nineteenth century, Sophus Lie grounded the concept of Lie groups. Sophus Lie also discovered the notion of Lie algebra. Thereafter, Humphreys [30] introduced the concept of representation theory of Lie algebra in 1972. In 2003, Coelho and Nunes [10] proposed an application of Lie algebra to mobile robot control. Till now, the concept of Lie theory has been applied in mathematics, physics, continuum mechanics, cosmology and life sciences. The problems of computer vision can also be solved by using the idea of Lie algebra. In 1965, Zadeh [40] grounded the notion of Fuzzy Set (in short FS) theory. Afterwards, Yehia [38] presented the concept of Fuzzy-Lie ideals and Fuzzy-Lie sub-algebra of Lie algebra in 1996. Later on, Yehia [39] also studied the adjoint representation of Fuzzy-Lie algebra. In 1998, Kim and Lee [31] further studied the Fuzzy-Lie ideals and Fuzzy-Lie sub-algebra. The notion of anti-Fuzzy-Lie ideals of Lie algebra was studied by Akram [1]. Later on, Akram [4] studied the concept of generalized Fuzzy-Lie sub-algebra in 2008. The concept of Fuzzy-Lie ideals of Lie algebra with the interval-valued membership function was studied by Akram [5]. In 1986, Atanassov [8] grounded the idea of Intuitionistic Fuzzy

Set (in short IFS) theory by introducing the idea of non-membership of a mathematical expression. Afterwards, Akram and Shum [7] grounded the concept of Lie algebra on IFSs. The notion of Intuitionistic (S, T)-Fuzzy-Lie ideals was studied by Akram [2]. In 2008, Akram [3] further established several results on Intuitionistic Fuzzy-Lie ideals of Lie algebra.

In 1998, Smarandache [36] grounded the idea of neutrosophic set (in short NS) by introducing the indeterminacy membership function of mathematical expression. Later on, Wang et al. [37] defined single-valued neutrosophic set (in short SVNS) as a generalization of FS and IFS. In 2020, Das et al. [14] proposed a multi-criteria decision making algorithm via SVNS environment. Thereafter, Akram et al. [6] introduced the concept of Lie-Algebra on SVNSs in 2019. Afterwards, Das and Hassan [15] grounded the notion of *d*-ideals on NS. In 2016, Chatterjee et al. [9] presented the idea of single-valued quadripartitioned neutrosophic set (in short SVQN-set) be extending the notion of SVNS. Later on, Mallick and Pramanik [33] grounded the notion of SVPN-set by splitting the indeterminacy membership function into three different membership functions namely contradiction, ignorance and unknown membership functions. Recently, Das et al. [13] studied the concept of *Q*-Ideals on SVPN-sets.

In this article, we procure the idea of SVPN-Lie ideal of SVPN-Lie algebra. Further, we produce several interesting results on SVPN-Lie algebra and SVPN-Lie ideal.

Research gap: No investigation on SVPN-Lie algebra and SVPN-Lie ideal has been reported in the recent literature.

Motivation: To explore the unexplored research, we introduce the notion of SVPN-Lie algebra and SVPN-Lie ideal.

The remaining part of this article has been organized as follows:

In section-2, we recall some basic definitions and results on SVNS, Lie algebra, Lie ideal, SVN-Lie algebra, SVN-Lie ideal and SVPN-set those are useful for the preparation of the main results of this article. Section-3 introduces the idea of SVPN-Lie algebra and SVPN-Lie ideal. In this section, we also formulate several interesting results on them. Section-4 represents the concluding remarks on the work done in this article.

2. Some Relevant Results:

Definition 2.1.[30] Assume that Ω be a field, and L be a vector space on Ω. Consider an operation L×L → L defined by (a, b) → [a, b], for all a, b ∈ L. Then, L is called Lie algebra if the following properties hold:

(*i*) [a, b] is a bilinear,

(*ii*) [a, a] = 0, for all a ∈ L,

(*iii*) [[a, b], c] + [[b, c], a] + [[c, a], b] = 0, for all a, b, c ∈ L.

Definition 2.2.[40] A Fuzzy Set (in short FS) W over a universe of discourse Π is defined as follows:

$$W = \{(\eta, T_W(\eta)) : \eta \in \Pi\},$$

where $T_W(\eta)$ is the truth-membership value of each $\eta \in \Pi$ such that $0 \leq T_W(\eta) \leq 1$.

Definition 2.3.[38] A FS $W = \{(\eta, T_W(\eta)) : \eta \in L\}$ is called a Fuzzy Lie ideal (in short F-L-Ideal) of a Lie algebra L if and only if the following three conditions hold:

(*i*) $T_W(q + r) \geq \min \{ T_W(q), T_W(r)\}$;

(*ii*) $T_W(\alpha q) \geq T_W(q)$;

(*iii*) $T_W([q, r]) \geq T_W(q)$, for all $q, r \in L$, and $\alpha \in \Omega$.

Definition 2.4.[8] An intuitionistic fuzzy set (in short IFS) W over a fixed set Π is defined as follows:

$$W = \{(\eta, T_W(\eta), I_W(\eta)) : \eta \in \Pi\},$$

where T_W, I_W are the membership and non-membership functions from W to [0, 1], and so $0 \leq T_W(\eta) + I_W(\eta) \leq 2$, for all $\eta \in \Pi$.

Definition 2.5.[7] An IFS $W = \{(q, T_W(q), I_W(q)) : q \in L\}$ on Lie algebra L is called an Intuitionistic Fuzzy Lie (in short IF-Lie) algebra if the following condition holds:

(*i*) $T_W(q + r) \geq \min \{T_W(q), T_W(r)\}$ and $I_W(q + r) \leq \max \{I_W(q), I_W(r)\}$;

(*ii*) $T_W(\alpha q) \geq T_W(q)$ and $I_W(\alpha q) \leq I_W(q)$;

(*iii*) $T_W([q, r]) \geq \min \{T_W(q), T_W(r)\}$ and $I_W([q, r]) \leq \max \{I_W(q), I_W(r)\}$, for all $q, r \in L$, and $\alpha \in \Omega$.

Definition 2.6.[37] An Single-Valued Neutrosophic Set (in short SVNS) W over Π is defined as follows:

$$W = \{(\eta, T_W(\eta), I_W(\eta), F_W(\eta)) : \eta \in \Pi\},$$

where T_W, I_W, F_W are truth, indeterminacy and falsity membership mappings from W to [0, 1], and so $0 \leq T_W(\eta) + I_W(\eta) + F_W(\eta) \leq 3$, for all $\eta \in \Pi$.

Definition 2.7.[37] Assume that $Y = \{(c, T_Y(c), I_Y(c), F_Y(c)) : c \in \Pi\}$ be an SVNS over Π. Then, the sets $W(T_Y,\alpha)=\{c \in \Pi : T_Y(c) \geq \alpha\}$, $W(I_Y,\alpha)=\{c \in \Pi : I_Y(c) \leq \alpha\}$, $W(F_Y,\alpha)=\{c \in \Pi : F_Y(c) \leq \alpha\}$ are respectively called T-level α-cut, I-level α-cut, F-level α-cut of Y.

Definition 2.8.[6] An SVNS $W=\{(q, T_W(q), I_W(q), F_W(q)) : q \in L\}$ over a Lie algebra L is called an Single-Valued Neutrosophic Lie (in short SVN-Lie) algebra if the following condition holds:

(*i*) $T_W(q + r) \geq \min \{T_W(q), T_W(r)\}$, $I_W(q + r) \geq \min \{I_W(q), I_W(r)\}$ and $F_W(q + r) \leq \max \{F_W(q), F_W(r)\}$;

(*ii*) $T_W(\alpha q) \geq T_W(q)$, $I_W(\alpha q) \geq I_W(q)$ and $F_W(\alpha q) \leq F_W(q)$;

(*iii*) $T_W([q, r]) \geq \min \{T_W(q), T_W(r)\}$, $I_W([q, r]) \geq \min \{I_W(q), I_W(r)\}$ and $F_W([q, r]) \leq \max \{F_W(q), F_W(r)\}$, for all $q, r \in L$, and $\alpha \in \Omega$.

Example 2.1. Suppose that F = R be the set of all real number. Suppose that $L = R^3 = \{(a, b, c) : a, b, c \in R\}$ be the set of all three-dimensional real vectors. Then, L forms a Lie algebra. We define

$$R^3 \times R^3 \rightarrow R^3$$

$$[a, b] \rightarrow a \times b,$$

where '×' is the usual cross product. Now, we define an SVNS $N = (T_N, I_N, F_N) : R^3 \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ by

$$T_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.1, & a \neq b \neq c \neq 0 \end{cases}$$

$$I_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.1, & a \neq b \neq c \neq 0 \end{cases}$$

$$\text{and } F_N(a, b, c) = \begin{cases} 0.1, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0. \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

Then, N = (T_N, I_N, F_N) is an SVN-Lie algebra of L.

Definition 2.9.[6] Suppose that L be a Lie algebra over a field Ω. An SVNS W={(q, $T_W(q)$, $I_W(q)$, $F_W(q)$):q∈L} on L is called an SVN-Lie ideal if the following conditions hold:

(i) $T_W(r+q) \geq \min \{T_W(r), T_W(q)\}$, $I_W(r+q) \geq \min \{I_W(r), I_W(q)\}$ and $F_W(r+q) \leq \max \{F_W(r), F_W(q)\}$;

(ii) $T_W(\alpha q) \geq T_W(q)$, $I_W(\alpha q) \geq I_W(q)$ and $F_W(\alpha q) \leq F_W(q)$;

(iii) $T_W([r, q]) \geq T_W(r)$, $I_W([r, q]) \geq I_W(r)$ and $F_W([r, q]) \leq F_W(r)$, for all r, q ∈L.

Example 2.2. Suppose that F = R be the set of all real number. Suppose that L = R^3 = {(a, b, c) : a, b, c ∈ R} be the set of all three-dimensional real vectors which forms a Lie algebra. Now, we define an SVNS N = (T_N, I_N, F_N) : $R^3 \rightarrow [0, 1] \times [0, 1] \times [0, 1]$ ('×' is the usual cross product) by

$$T_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$I_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$\text{and } F_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0. \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

Then, N is an SVN-Lie ideal of L.

Remark 2.1. Every SVN-Lie algebra may not be an SVN-Lie ideal. This follows from the following example.

Example 2.3. Let us consider an SVNS N = (T_N, I_N, F_N) over the field L = R as defined in Example 2.1. Then, the SVNS N = {($T_N(x, y, z)$, $I_N(x, y, z)$, $F_N(x, y, z)$) : (x, y, z) ∈ R^3} is an SVN-Lie algebra of L, but it is not an SVN-Lie ideal of L, because

$T_N([(1, 0, 0), (0, 0, 1)]) = T_N(0,-1, 0) = 0.1 \ngeq 0.4$ i.e., $T_N([(1, 0, 0), (0, 0, 1)]) \ngeq T_N(0, 0, 1)$,

$I_N([(1, 0, 0), (0, 0, 1)]) = C_N(0,-1, 0) = 0.1 \ngeq 0.4$ i.e., $C_N([(1, 0, 0), (0, 0, 1)]) \ngeq C_N(0, 0, 1)$,

$F_N([(1, 0, 0), (0, 0, 1)]) = F_N(0,-1, 0) = 0.9 \nleq 0.4$ i.e., $F_N([(1, 0, 0), (0, 0, 1)]) \nleq F_N(0, 0, 1)$.

Remark 2.2.[6] Let W={(q, $T_W(q)$, $I_W(q)$, $F_W(q)$):q∈L} be an SVN-Lie algebra on a Lie algebra L. Then,

(i) $T_W(0) \geq T_W(q)$, $I_W(0) \geq I_W(q)$, $F_W(0) \leq F_W(q)$;

(ii) $T_W(-q) \geq T_W(q)$, $I_W(-q) \geq I_W(q)$, $F_W(-q) \leq F_W(q)$, for all q ∈L.

Definition 2.10.[9] Suppose that Π be a universal set. Then, an Single-Valued Quadripartitioned Neutrosophic Set (in short SVQN-set) W over Π is defined as follows:

$$W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in \Pi\},$$

where $T_W(\eta)$, $C_W(\eta)$, $G_W(\eta)$ and $F_W(\eta)$ ($\in[0, 1]$) are the truth, contradiction, ignorance and false membership values of each $\eta\in\Pi$. So, $0 \leq T_W(\eta) + C_W(\eta) + G_W(\eta) + F_W(\eta) \leq 4$, for all $\eta\in\Pi$.

Definition 2.11.[9] Assume that $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), F_W(\eta)) : \eta \in \Pi\}$ and $E = \{(\eta, T_E(\eta), C_E(\eta), G_E(\eta), F_E(\eta)) : \eta \in \Pi\}$ be two SVQN-sets over a fixed set Π. Then,

(i) $W\subseteq E$ if and only if $T_W(\eta) \leq T_E(\eta)$, $C_W(\eta) \leq C_E(\eta)$, $G_W(\eta) \geq G_E(\eta)$, $F_W(\eta) \geq F_E(\eta)$, $\forall\eta\in\Pi$,

(ii) $W\cup E = \{(\eta, \max\{T_W(\eta), T_E(\eta)\}, \max\{C_W(\eta), C_E(\eta)\}, \min\{G_W(\eta), G_E(\eta)\}, \min\{F_W(\eta), F_E(\eta)\}) : \eta\in\Pi\}$,

(iii) $W\cap E = \{(\eta, \min\{T_W(\eta), T_E(\eta)\}, \min\{C_W(\eta), C_E(\eta)\}, \max\{G_W(\eta), G_E(\eta)\}, \max\{F_W(\eta), F_E(\eta)\}) : \eta\in\Pi\}$,

(iv) $W^c = \{(\eta, F_W(\eta), G_W(\eta), C_W(\eta), T_W(\eta)) : \eta\in\Pi\}$.

Definition 2.12.[33] Suppose that Π be a fixed set. Then, an Single-Valued Pentapartitioned Neutrosophic Set (in short SVPN-set) W over Π is defined by:

$$W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta \in \Pi\},$$

where $T_W(\eta)$, $C_W(\eta)$, $G_W(\eta)$, $U_W(\eta)$ and $F_W(\eta)$ ($\in[0, 1]$) are the truth, contradiction, ignorance, unknown and false membership values of each $\eta\in\Pi$. So, $0 \leq T_W(\eta)+C_W(\eta)+G_W(\eta)+U_W(\eta)+F_W(\eta)\leq 4$, for all $\eta\in\Pi$.

Definition 2.13.[33] Assume that $W=\{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in\Pi\}$ and $E=\{(\eta, T_E(\eta), C_E(\eta), G_E(\eta), U_E(\eta), F_E(\eta)) : \eta\in\Pi\}$ be two SVPN-sets over a fixed set Π. Then,

(i) $W\subseteq E$ if and only if $T_W(\eta)\leq T_E(\eta)$, $C_W(\eta)\leq C_E(\eta)$, $G_W(\eta)\geq G_E(\eta)$, $U_W(\eta)\geq U_E(\eta)$, $F_W(\eta)\geq F_E(\eta)$, $\forall\eta\in\Pi$.

(ii) $W\cup E = \{(\eta, \max\{T_W(\eta), T_E(\eta)\}, \max\{C_W(\eta), C_E(\eta)\}, \min\{G_W(\eta), G_E(\eta)\}, \min\{U_W(\eta), U_E(\eta)\}, \min\{F_W(\eta), F_E(\eta)\}) : \eta\in\Pi\}$.

(iii) $W\cap E = \{(\eta, \min\{T_W(\eta), T_E(\eta)\}, \min\{C_W(\eta), C_E(\eta)\}, \max\{G_W(\eta), G_E(\eta)\}, \max\{U_W(\eta), U_E(\eta)\}, \max\{F_W(\eta), F_E(\eta)\}) : \eta\in\Pi\}$.

(iv) $W^c = \{(\eta, F_W(\eta), U_W(\eta), 1-G_W(\eta), C_W(\eta), T_W(\eta)) : \eta\in\Pi\}$.

3. SVPN-Lie Ideal of SVPN-Lie Algebra:

In this section, we procure the notion of SVPN-Lie ideal of SVPN-Lie algebra. Besides, we study different properties of SVPN-Lie ideal, and formulate several results on it.

Definition 3.1. Let L be a Lie algebra on a field Ω. Then, an SVPN-set $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta \in L\}$ over L is called an SVPN-Lie algebra if the following conditions hold:

(i) $T_W(\eta+\delta) \geq \min\{T_W(\eta), T_W(\delta)\}$, $C_W(\eta+\delta) \geq \min\{C_W(\eta), C_W(\delta)\}$, $G_W(\eta+\delta) \leq \max\{G_W(\eta), G_W(\delta)\}$, $U_W(\eta+\delta) \leq \max\{U_W(\eta), U_W(\delta)\}$ and $F_W(\eta+\delta) \leq \max\{F_W(\eta), F_W(\delta)\}$;

(ii) $T_W(\alpha\eta) \geq T_W(\eta)$, $C_W(\alpha\eta) \geq C_W(\eta)$, $G_W(\alpha\eta) \leq G_W(\eta)$, $U_W(\alpha\eta) \leq U_W(\eta)$ and $F_W(\alpha\eta) \leq F_W(\eta)$;

(iii) $T_W([\eta, \delta]) \geq \min\{T_W(\eta), T_W(\delta)\}$, $C_W([\eta, \delta]) \geq \min\{C_W(\eta), C_W(\delta)\}$, $G_W([\eta, \delta]) \leq \max\{G_W(\eta), G_W(\delta)\}$, $U_W([\eta, \delta]) \leq \max\{U_W(\eta), U_W(\delta)\}$ and $F_W([\eta, \delta]) \leq \max\{F_W(\eta), F_W(\delta)\}$, for all $\eta, \delta \in L$, and $\alpha \in\Omega$.

Example 3.1. Suppose that F = R be the set of all real number. Suppose that L = R^3 = {(a, b, c): a, b, c ∈R} be the set of all three-dimensional real vectors. Then, L forms a Lie algebra. We define

$$R^3 \times R^3 \to R^3$$

$$[a, b] \to a \times b,$$

where '×' is the usual cross product. Now, we define an SVPN-set N = (T_N, C_N, G_N, U_N, F_N) : R^3 → [0, 1] × [0, 1] × [0, 1] by

$$T_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.1, & a \neq b \neq c \neq 0 \end{cases}$$

$$C_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.1, & a \neq b \neq c \neq 0 \end{cases}$$

$$G_N(a, b, c) = \begin{cases} 0.1, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$U_N(a, b, c) = \begin{cases} 0.1, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

and $$F_N(a, b, c) = \begin{cases} 0.1, & a = b = c = 0 \\ 0.4, & a = b = 0, c \neq 0. \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

Then, N = (T_N, C_N, G_N, U_N, F_N) is an SVPN-Lie algebra of L.

Definition 3.2. Let L be a Lie algebra on a field Ω. Then, an SVPN-set W = {(η, $T_W(\eta)$, $C_W(\eta)$, $G_W(\eta)$, $U_W(\eta)$, $F_W(\eta)$) : η∈Π} over L is called an SVPN-Lie ideal if the following condition holds:

(i) $T_W(\eta+\delta) \geq \min\{T_W(\eta), T_W(\delta)\}$, $C_W(\eta+\delta) \geq \min\{C_W(\eta), C_W(\delta)\}$, $G_W(\eta+\delta) \leq \max\{G_W(\eta), G_W(\delta)\}$, $U_W(\eta+\delta) \leq \max\{U_W(\eta), U_W(\delta)\}$ and $F_W(\eta+\delta) \leq \max\{F_W(\eta), F_W(\delta)\}$;

(ii) $T_W(\alpha\eta) \geq T_W(\eta)$, $C_W(\alpha\eta) \geq C_W(\eta)$, $G_W(\alpha\eta) \leq G_W(\eta)$, $U_W(\alpha\eta) \leq U_W(\eta)$ and $F_W(\alpha\eta) \leq F_W(\eta)$;

(iii) $T_W([\eta, \delta]) \geq T_W(\eta)$, $C_W([\eta, \delta]) \geq C_W(\eta)$, $G_W([\eta, \delta]) \leq G_W(\eta)$, $U_W([\eta, \delta]) \leq U_W(\eta)$ and $F_W([\eta, \delta]) \leq F_W(\eta)$, for all η, δ∈L, and α∈Ω.

Example 3.2. Suppose that F = R be the set of all real number. Suppose that L = R^3 = {(a, b, c) : a, b, c ∈ R} be the set of all three-dimensional real vectors which forms a Lie algebra. Now, we define an SVNS N = (T_N, C_N, G_N, U_N, F_N) : R^3 → [0, 1] × [0, 1] × [0, 1] ('×' is the usual cross product) by

$$T_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$C_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$G_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0, \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$U_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0 \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

$$\text{and } F_N(a, b, c) = \begin{cases} 0.9, & a = b = c = 0 \\ 0.9, & a = b = 0, c \neq 0. \\ 0.9, & a \neq b \neq c \neq 0 \end{cases}$$

Then, N = (T_N, C_N, G_N, U_N, F_N) is an SVPN-Lie ideal of L.

Remark 3.1. Every SVPN-Lie algebra may not be an SVPN-Lie ideal. This follows from the following example.

Example 3.3. Let N = (T_N, C_N, G_N, U_N, F_N) be an SVPN-set over the field L = R as defined in Example 3.1. Then, the SVPN-set N = {(T_N(x, y, z), C_N(x, y, z), G_N(x, y, z), U_N(x, y, z), F_N(x, y, z)) : (x, y, z) $\in R^3$} is an SVPN-Lie algebra of L, but it is not an SVPN-Lie ideal of L, because

$T_N([(1, 0, 0), (0, 0, 1)]) = T_N(0,-1, 0) = 0.1 \ngeq 0.4$ i.e., $T_N([(1, 0, 0), (0, 0, 1)]) \ngeq T_N(0, 0, 1)$,

$C_N([(1, 0, 0), (0, 0, 1)]) = C_N(0,-1, 0) = 0.1 \ngeq 0.4$ i.e., $C_N([(1, 0, 0), (0, 0, 1)]) \ngeq C_N(0, 0, 1)$,

$G_N([(1, 0, 0), (0, 0, 1)]) = G_N(0,-1, 0) = 0.9 \nleq 0.4$ i.e., $G_N([(1, 0, 0), (0, 0, 1)]) \nleq G_N(0, 0, 1)$,

$U_N([(1, 0, 0), (0, 0, 1)]) = U_N(0,-1, 0) = 0.9 \nleq 0.4$ i.e., $U_N([(1, 0, 0), (0, 0, 1)]) \nleq U_N(0, 0, 1)$

$F_N([(1, 0, 0), (0, 0, 1)]) = F_N(0,-1, 0) = 0.9 \nleq 0.4$ i.e., $F_N([(1, 0, 0), (0, 0, 1)]) \nleq F_N(0, 0, 1)$.

Theorem 3.1. Suppose that $\{W_i : i \in \Delta\}$ be the family of SVPN-Lie ideals on a Lie-Algebra L. Then, their intersection $\cap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee U_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta \in L\}$ is also an SVPN-Lie ideal of L.

Proof. Suppose that $\{W_i : i \in \Delta\}$ be the family of SVPN-Lie ideals on a Lie-Algebra L. It is known that, $\cap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee U_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta \in L\}$.

Now,

(i) $\wedge T_{N_i}(\eta+\delta) = \min\{T_{N_i}(\eta+\delta) : i \in \Delta\} \geq \min\{\min\{T_{N_i}(\eta), T_{N_i}(\delta)\} : i \in \Delta\} \geq \min\{\wedge T_{N_i}(\eta), \wedge T_{N_i}(\delta)\}$,

$\wedge C_{N_i}(\eta+\delta) = \min\{C_{N_i}(\eta+\delta) : i \in \Delta\} \geq \min\{\min\{C_{N_i}(\eta), C_{N_i}(\delta)\} : i \in \Delta\} \geq \min\{\wedge C_{N_i}(\eta), \wedge C_{N_i}(\delta)\}$,

$\vee G_{N_i}(\eta+\delta) = \max\{G_{N_i}(\eta+\delta) : i \in \Delta\} \leq \max\{\max\{G_{N_i}(\eta), G_{N_i}(\delta)\} : i \in \Delta\} \leq \max\{\vee G_{N_i}(\eta), \vee G_{N_i}(\delta)\}$,

$\vee U_{N_i}(\eta+\delta) = \max\{U_{N_i}(\eta+\delta) : i \in \Delta\} \leq \max\{\max\{U_{N_i}(\eta), U_{N_i}(\delta)\} : i \in \Delta\} \leq \max\{\vee U_{N_i}(\eta), \vee U_{N_i}(\delta)\}$,

$\vee F_{N_i}(\eta+\delta) = \max\{F_{N_i}(\eta+\delta) : i \in \Delta\} \leq \max\{\max\{F_{N_i}(\eta), F_{N_i}(\delta)\} : i \in \Delta\} \leq \max\{\vee F_{N_i}(\eta), \vee F_{N_i}(\delta)\}$.

(ii) $\wedge T_{N_i}(\alpha\eta) = \min\{T_{N_i}(\alpha\eta) : i \in \Delta\} \geq \min\{T_{N_i}(\eta) : i \in \Delta\} \geq \wedge T_{N_i}(\eta)$,

$\wedge C_{N_i}(\alpha\eta) = \min\{C_{N_i}(\alpha\eta) : i \in \Delta\} \geq \min\{C_{N_i}(\eta) : i \in \Delta\} \geq \wedge C_{N_i}(\eta)$,

$\vee G_{N_i}(\alpha\eta) = \max\{G_{N_i}(\alpha\eta) : i \in \Delta\} \leq \max\{G_{N_i}(\eta) : i \in \Delta\} \leq \vee G_{N_i}(\eta)$,

$\vee U_{N_i}(\alpha\eta) = \max\{U_{N_i}(\alpha\eta) : i \in \Delta\} \leq \max\{U_{N_i}(\eta) : i \in \Delta\} \leq \vee U_{N_i}(\eta)$,

$\vee F_{N_i}(\alpha\eta) = \max\{F_{N_i}(\alpha\eta) : i \in \Delta\} \leq \max\{F_{N_i}(\eta) : i \in \Delta\} \leq \vee F_{N_i}(\eta)$.

(iii) $\wedge T_{N_i}([\eta, \delta]) = \min\{T_{N_i}([\eta, \delta]) : i \in \Delta\} \geq \min\{T_{N_i}(\eta) : i \in \Delta\} \geq \wedge T_{N_i}(\eta)$,

$\wedge C_{N_i}([\eta, \delta]) = \min \{C_{N_i}([\eta, \delta]) : i\in\Delta\} \geq \min \{C_{N_i}(\eta) : i\in\Delta\} \geq \wedge C_{N_i}(\eta)$,

$\vee G_{N_i}([\eta, \delta]) = \max \{G_{N_i}([\eta, \delta]) : i\in\Delta\} \leq \max \{G_{N_i}(\eta) : i\in\Delta\} \leq \vee G_{N_i}(\eta)$,

$\vee U_{N_i}([\eta, \delta]) = \max \{U_{N_i}([\eta, \delta]) : i\in\Delta\} \leq \max \{U_{N_i}(\eta) : i\in\Delta\} \leq \vee U_{N_i}(\eta)$,

$\vee F_{N_i}([\eta, \delta]) = \max \{F_{N_i}([\eta, \delta]) : i\in\Delta\} \leq \max \{F_{N_i}(\eta) : i\in\Delta\} \leq \vee F_{N_i}(\eta)$.

Therefore, $\cap W_i = \{(\eta, \wedge T_{N_i}(\eta), \wedge C_{N_i}(\eta), \vee G_{N_i}(\eta), \vee U_{N_i}(\eta), \vee F_{N_i}(\eta)) : \eta\in L\}$ is an SVPN-Lie ideal of L.

Theorem 3.2. Assume that $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ be an SVPN-Lie algebra on a Lie algebra L. Then,

(i) $T_W(0) \geq T_W(\delta)$, $C_W(0) \geq C_W(\delta)$, $G_W(0) \leq G_W(\delta)$, $U_W(0) \leq U_W(\delta)$, $F_W(0) \leq F_W(\delta)$;

(ii) $T_W(-\delta) \geq T_W(\delta)$, $C_W(-\delta) \geq C_W(\delta)$, $G_W(-\delta) \leq G_W(\delta)$, $U_W(-\delta) \leq U_W(\delta)$, $F_W(-\delta) \leq F_W(\delta)$, for all $\delta \in L$.

Proof. The proof is so easy, so omitted.

Lemma 3.1. Every SVPN-Lie ideal is also an SVPN-Lie algebra.

Theorem 3.3. Suppose that $W = \{(\delta, T_W(\delta), C_W(\delta), G_W(\delta), U_W(\delta), F_W(\delta)) : \delta\in L\}$ be an SVPN-Lie ideal of a Lie-Algebra L. Then, the following holds:

(i) $T_W(0) \geq T_W(\delta)$, $C_W(0) \geq C_W(\delta)$, $G_W(0) \leq G_W(\delta)$, $U_W(0) \leq U_W(\delta)$, $F_W(0) \leq F_W(\delta)$;

(ii) $T_W([\delta, \eta]) \geq \max\{T_W(\delta), T_W(\eta)\}$; $C_W([\delta, \eta]) \geq \max\{C_W(\delta), C_W(\eta)\}$; $G_W([\delta, \eta]) \leq \min\{G_W(\delta), G_W(\eta)\}$; $U_W([\delta, \eta]) \leq \min\{U_W(\delta), U_W(\eta)\}$; $F_W([\delta, \eta]) \leq \min\{F_W(\delta), F_W(\eta)\}$;

(iii) $T_W([\delta, \eta]) = T_W(-[\delta, \delta]) = T_W([\eta, \delta])$; $C_W([\delta, \eta]) = C_W(-[\delta, \delta]) = C_W([\eta, \delta])$; $G_W([\delta, \eta]) = G_W(-[\delta, \delta]) = G_W([\eta, \delta])$; $U_W([\delta, \eta]) = U_W(-[\delta, \delta]) = U_W([\eta, \delta])$; $F_W([\delta, \eta]) = F_W(-[\delta, \delta]) = F_W([\eta, \delta])$, for all $\delta, \eta \in L$.

Proof. The proofs are straightforward, so omitted.

Definition 3.3. Assume that $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ be an SVPN-set over a Lie-Algebra L. Suppose that $\alpha, \beta, \gamma, \delta, \lambda \in [0, 1]$. Then, the sets $L(T_W, \alpha) = \{\eta\in L : T_W(\eta)\geq\alpha\}$, $L(C_W, \beta) = \{\eta\in L : C_W(\eta)\geq\beta\}$, $L(G_W, \gamma) = \{\eta\in L : G_W(\eta)\leq\gamma\}$, $L(U_W, \delta) = \{\eta\in L : U_W(\eta)\leq\delta\}$, $L(F_W, \lambda) = \{\eta\in L : F_W(\eta)\leq\lambda\}$ are called T-level α-cut, C-level β-cut, G-level γ-cut, U-level δ-cut and F-level λ-cut of W respectively.

Definition 3.4. Suppose that L be a Lie-Algebra. Assume that $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ be an SVPN-set over L. Suppose that $\alpha, \beta, \gamma, \delta, \lambda \in [0, 1]$. Then, $(\alpha, \beta, \gamma, \delta, \lambda)$-level subset of W is defined by:

$$L(\alpha, \beta, \gamma, \delta, \lambda) = \{\eta\in L : T_W(\eta) \geq \alpha, C_W(\eta) \geq \beta, G_W(\eta) \leq \gamma, U_W(\eta) \leq \delta, F_W(\eta) \leq \delta\}.$$

Remark 3.2. Suppose that L be a Lie-Algebra. If $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ be an SVPN-set over L, then $L(\alpha, \beta, \gamma, \delta, \lambda) = L(T_W, \alpha) \cap L(C_W, \beta) \cap L(G_W, \gamma) \cap L(U_W, \delta) \cap L(F_W, \lambda)$.

Proposition 3.1. Suppose that L be a Lie-Algebra. An SVPN-set $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ is an SVPN-Lie ideal of L if and only if $L(\alpha, \beta, \gamma, \delta, \lambda)$ is a Lie-Ideal of L for every $\alpha, \beta, \gamma, \delta, \lambda \in [0, 1]$.

Proof. The proof is straightforward, so omitted.

Theorem 3.4. Let L be a Lie-Algebra. Assume that $W = \{(\eta, T_W(\eta), C_W(\eta), G_W(\eta), U_W(\eta), F_W(\eta)) : \eta\in L\}$ be an SVPN-Lie ideal of L. Let $\alpha_1, \beta_1, \gamma_1, \delta_1, \lambda_1, \alpha_2, \beta_2, \gamma_2, \delta_2, \lambda_2 \in [0, 1]$. Then, $L(\alpha_1, \beta_1, \gamma_1, \delta_1, \lambda_1) = L(\alpha_2, \beta_2, \gamma_2, \delta_2, \lambda_2)$ if and only if $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$, $\lambda_1 = \lambda_2$.

Proof. Suppose that L be a Lie-Algebra. Let W = {(η, T_W(η), C_W(η), G_W(η), U_W(η), F_W(η)) : η∈L} be an SVPN-Lie ideal of L. Let α_1, β_1, γ_1, δ_1, λ_1, α_2, β_2, γ_2, δ_2, λ_2 ∈ [0, 1] such that L(α_1, β_1, γ_1, δ_1, λ_1) = L(α_2, β_2, γ_2, δ_2, λ_2). Therefore, {η∈L : T_W(η) ≥ α_1, C_W(η) ≥ β_1, G_W(η) ≤ γ_1, U_W(η) ≤ δ_1, F_W(η) ≤ λ_1} = {η∈L : T_W(η) ≥ α_2, C_W(η) ≥ β_2, G_W(η) ≤ γ_2, U_W(η) ≤ δ_2, F_W(η) ≤ λ_2}. This is possible only when $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$, $\lambda_1 = \lambda_2$. Therefore, L(α_1, β_1, γ_1, δ_1, λ_1) = L(α_2, β_2, γ_2, δ_2, λ_2) implies $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$, $\lambda_1 = \lambda_2$. Conversely, let $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$, $\lambda_1 = \lambda_2$.

Now, L(α_1, β_1, γ_1, δ_1, λ_1)

= {η∈L : T_W(η) ≥ α_1, C_W(η) ≥ β_1, G_W(η) ≤ γ_1, U_W(η) ≤ δ_1, F_W(η) ≤ λ_1}

= {η∈L : T_W(η) ≥ α_2, C_W(η) ≥ β_2, G_W(η) ≤ γ_2, U_W(η) ≤ δ_2, F_W(η) ≤ λ_2}

= L(α_2, β_2, γ_2, δ_2, λ_2)

Therefore, $\alpha_1 = \alpha_2$, $\beta_1 = \beta_2$, $\gamma_1 = \gamma_2$, $\delta_1 = \delta_2$, $\lambda_1 = \lambda_2$ implies L(α_1, β_1, γ_1, δ_1, λ_1) = L(α_2, β_2, γ_2, δ_2, λ_2).

Definition 3.5. Assume that L_1 and L_2 be two Lie-Algebras on a common field Ω. Suppose that f be a bijective mapping from L_1 to L_2. If M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L} be an SVPN-set in L_2, then f^{-1}(M) defined by f^{-1}(M) = {(η, f^{-1}(T_M(η)), f^{-1}(C_M(η)), f^{-1}(G_M(η)), f^{-1}(U_M(η)), f^{-1}(F_M(η))) : η∈L} is also an SVPN-set in L_1.

Theorem 3.5. Assume that L_1 and L_2 be two Lie-Algebras on a common field Ω. Suppose that f be an onto homomorphism from L_1 to L_2. If M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L} is an SVPN-Lie ideal of L_2, then f^{-1}(M) = {(η, f^{-1}(T_M(η)), f^{-1}(C_M(η)), f^{-1}(G_M(η)), f^{-1}(U_M(η)), f^{-1}(F_M(η))) : η∈L} is also an SVPN-Lie ideal of L_1.

Proof. The proof is so easy, so omitted.

Proposition 3.2. Suppose that L_1 and L_2 be two Lie-Algebras. Let f be an epimorphism from L_1 to L_2. If M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L} be an SVPN-Lie ideal of L_2, then $f^{-1}(M^c) = (f^{-1}(M))^c$ is also an SVPN-Lie ideal of L_1.

Proof. The proof is straightforward, so omitted.

Theorem 3.6. Suppose that L_1 and L_2 be two Lie-Algebras. Let f be an epimorphism from L_1 to L_2. If M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L} be an SVPN-Lie ideal of L_2, then f^{-1}(M) = {(η, f^{-1}(T_M(η)), f^{-1}(C_M(η)), f^{-1}(G_M(η)), f^{-1}(U_M(η)), f^{-1}(F_M(η))) : η∈L} is also an SVPN-Lie ideal of L_1.

Proof. The proof is directly holds from Definitions 3.2 and Definition 3.5.

Definition 3.6. Let us consider two Lie-Algebras L_1 and L_2. Let f be a mapping from a L_1 to L_2. If W = {(η, T_W(η), C_W(η), G_W(η), U_W(η), F_W(η)) : η∈L} be an SVPN-set in L_1, then the image of W = {(η, T_W(η), C_W(η), G_W(η), U_W(η), F_W(η)) : η∈L} under f denoted by f(W) is an SVPN-set in L_2, defined as follows:

$$f(T_W)(r)=\begin{cases}\max_{u\in f^{-1}(r)} T_W(u), & \text{if } f^{-1}(r)\neq\emptyset \\ 0, & \text{otherwise}\end{cases}, \text{ for each } r\in L_2$$

$$f(C_W)(r)=\begin{cases}\max_{u\in f^{-1}(r)} C_W(u), & \text{if } f^{-1}(r)\neq\emptyset \\ 0, & \text{otherwise}\end{cases}, \text{ for each } r\in L_2$$

$$f(G_W)(r)=\begin{cases}\min_{u\in f^{-1}(r)} G_W(u), & \text{if } f^{-1}(r)\neq\emptyset \\ 1, & \text{otherwise}\end{cases}, \text{ for each } r\in L_2,$$

$f(U_W)(r)=\begin{cases}\min_{u\in f^{-1}(r)} U_W(u), & \text{if } f^{-1}(r)\neq\emptyset\\ 1, & \text{otherwise}\end{cases}$, for each $r\in L_2$,

$f(F_W)(r)=\begin{cases}\min_{u\in f^{-1}(r)} F_W(u), & \text{if } f^{-1}(r)\neq\emptyset\\ 1, & \text{otherwise}\end{cases}$, for each $r\in L_2$.

Theorem 3.7. Let us consider two Lie-Algebras L_1 and L_2. Suppose that f be an epimorphism from L_1 to L_2. If W = {(η, T_W(η), C_W(η), G_W(η), U_W(η), F_W(η)) : η∈L} is an SVPN-Lie ideal in L_1, then the image of W = {(η, T_W(η), C_W(η), G_W(η), U_W(η), F_W(η)) : η∈L} i.e., f(W) is also an SVPN-Lie ideal in L_2.

Proof. The proof is directly holds from Definition 3.2 and Definition 3.6.

Definition 3.7. Let us consider two Lie-Algebras L_1 and L_2. Suppose that f be an onto homomorphism from L_1 to L_2. Let M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L} be an SVPN-set in L_2. Then, we define $L^f = \{(\eta, T^f_M(\eta), C^f_M(\eta), G^f_M(\eta), U^f_M(\eta), F^f_M(\eta)) : \eta\in L_1\}$ in L_1 by $T^f_M(\eta)=T_M(f(\eta))$, $C^f_M(\eta)=C_M(f(\eta))$, $G^f_M(\eta)=G_M(f(\eta))$, $U^f_M(\eta)=U_M(f(\eta))$, $F^f_M(\eta)=F_M(f(\eta))$, for all $\eta\in L_1$. Clearly, L^f is an SVPN-set in L_1.

Theorem 3.8. Suppose that L_1 and L_2 be two Lie-Algebras on a common field Ω. Assume that f be an onto homomorphism from L_1 to L_2. If M = {(η, T_M(η), C_M(η), G_M(η), U_M(η), F_M(η)) : η∈L_2} is an SVPN-Lie ideal of L_2, then $L^f = \{(\eta, T^f_M(\eta), C^f_M(\eta), G^f_M(\eta), U^f_M(\eta), F^f_M(\eta)): \eta\in L_1\}$ is also an SVPN-Lie ideal of L_1.

Proof. Suppose that L_1 and L_2 be two Lie-Algebras on a common field Ω. Assume that η, δ ∈ L_1 and a∈ Ω. Then, we have

(i) $T^f_N(\eta+\delta)$

$=T_N(f(\eta+\delta))$

$=T_N(f(\eta)+f(\delta))$

$\geq\min\{T_N(f(\eta)), T_N(f(\delta))\}$

$=\min\{T^f_N(\eta), T^f_N(\delta)\}$,

$C^f_N(\eta+\delta)$

$=C_N(f(\eta+\delta))$

$=C_N(f(\eta)+f(\delta))$

$\geq\min\{C_N(f(\eta)), C_N(f(\delta))\}$

$=\min\{C^f_N(\eta), C^f_N(\delta)\}$,

$G^f_N(\eta+\delta)$

$=G_N(f(\eta+\delta))$

$=G_N(f(\eta)+f(\delta))$

$\leq\max\{G_N(f(\eta)), G_N(f(\delta))\}$

$=\max\{G^f_N(\eta), G^f_N(\delta)\}$,

$U^f_N(\eta+\delta)$

$=U_N(f(\eta+\delta))$

$=U_N(f(\eta)+f(\delta))$

$\leq \max\{U_N(f(\eta)), U_N(f(\delta))\}$

$=\max\{U_N^f(\eta), U_N^f(\delta)\}$,

$F_N^f(\eta+\delta)$

$=F_N(f(\eta+\delta))$

$=F_N(f(\eta)+f(\delta))$

$\leq \max\{F_N(f(\eta)), F_N(f(\delta))\}$

$=\max\{F_N^f(\eta), F_N^f(\delta)\}$,

(ii) $T_N^f(a\eta) = T_N(f(a\eta)) = T_N(af(\eta)) \geq T_N(f(\eta)) = T_N^f(\eta)$,

$C_N^f(a\eta) = C_N(f(a\eta)) = C_N(af(\eta)) \geq C_N(f(\eta)) = C_N^f(\eta)$,

$G_N^f(a\eta) = G_N(af(\eta)) = G_N(af(\eta)) \leq G_N(f(\eta)) = G_N^f(\eta)$,

$U_N^f(a\eta) = U_N(af(\eta)) = U_N(af(\eta)) \leq U_N(f(\eta)) = U_N^f(\eta)$,

$F_N^f(a\eta) = F_N(af(\eta)) = F_N(af(\eta)) \leq F_N(f(\eta)) = F_N^f(\eta)$.

(iii) $T_N^f([\eta,\delta]) = T_N(f([\eta,\delta])) = T_N(f(\eta), f(\delta)]) \geq T_N(f(\eta)) = T_N^f(\eta)$,

$C_N^f([\eta,\delta]) = C_N(f([\eta,\delta])) = C_N(f(\eta), f(\delta)]) \geq C_N(f(\eta)) = C_N^f(\eta)$,

$G_N^f([\eta,\delta]) = G_N(f([\eta,\delta])) = G_N(f(\eta), f(\delta)]) \leq G_N(f(\eta)) = G_N^f(\eta)$,

$U_N^f([\eta,\delta]) = U_N(f([\eta,\delta])) = U_N(f(\eta), f(\delta)]) \leq U_N(f(\eta)) = U_N^f(\eta)$,

$F_N^f([\eta,\delta]) = F_N(f([\eta,\delta])) = F_N(f(\eta), f(\delta)]) \leq F_N(f(\eta)) = F_N^f(\eta)$.

Therefore, $L^f = \{(\eta, T_M^f(\eta), C_M^f(\eta), G_M^f(\eta), U_M^f(\eta), F_M^f(\eta)) : \eta \in L_1\}$ satisfies all the conditions for being an SVPN-Lie ideal of L_1. Hence, L^f is an SVPN-Lie ideal of L_1.

Theorem 3.9. Assume that L_1 and L_2 be two Lie-Algebras on a common field Ω. Suppose that f be an onto homomorphism from L_1 to L_2. Then, $L^f=\{(w, T_M^f(w), C_M^f(w), G_M^f(w), U_M^f(w), F_M^f(w)): w \in L_1\}$ is an SVPN-Lie ideal of L_1 iff M={(w, $T_M(w)$, $C_M(w)$, $G_M(w)$, $U_M(w)$, $F_M(w)$) : w∈L} is an SVPN-Lie ideal of L_2.

Proof. The sufficiency of this theorem directly follows from the previous theorem.

Now, we just need to prove the necessity part of this theorem. Since, the mapping f is a onto mapping, so for any w, q ∈ L_2 there are w_1, q_1 ∈ L_2 such that $w = f(w_1), q = f(q_1)$. Therefore, $T_N(w) = T_N^f(w_1)$, $T_N(q) = T_N^f(q_1)$, $C_N(w) = C_N^f(w_1)$, $C_N(q) = C_N^f(q_1)$, $G_N(w) = G_N^f(w_1)$, $G_N(q) = G_N^f(q_1)$, $U_N(w) = U_N^f(w_1)$, $U_N(q) = U_N^f(q_1)$, $F_N(w) = F_N^f(w_1)$, $F_N(q) = F_N^f(q_1)$.

Now,

(i) $T_N(w + q)$

$= T_N(f(w_1) + f(q_1))$

$= T_N(f(w_1 + q_1))$

$= T_N^f(w_1 + q_1)$

$\geq \min\{T_N^f(w_1), T_N^f(q_1)\}$

$= \min\{T_N(w), T_N(q)\}$,

$C_N(w + q)$

$= C_N\big(f(w_1) + f(q_1)\big)$

$= C_N\big(f(w_1 + q_1)\big)$

$= C_N^f(w_1 + q_1)$

$\geq \min\{C_N^f(w_1), C_N^f(q_1)\}$

$= \min\{C_N(w), C_N(q)\},$

$G_N(w + q)$

$= G_N\big(f(w_1) + f(q_1)\big)$

$= G_N\big(f(w_1 + q_1)\big)$

$= G_N^f(w_1 + q_1)$

$\leq \max\{G_N^f(w_1), G_N^f(q_1)\}$

$= \max\{G_N(w), G_N(q)\},$

$U_N(w + q)$

$= U_N\big(f(w_1) + f(q_1)\big)$

$= U_N\big(f(w_1 + q_1)\big)$

$= U_N^f(w_1 + q_1)$

$\leq \max\{U_N^f(w_1), U_N^f(q_1)\}$

$= \max\{U_N(w), U_N(q)\},$

$F_N(w + q)$

$= F_N\big(f(w_1) + f(q_1)\big)$

$= F_N\big(f(w_1 + q_1)\big)$

$= F_N^f(w_1 + q_1)$

$\leq \max\{F_N^f(w_1), F_N^f(q_1)\}$

$= \max\{F_N(w), F_N(q)\}.$

(ii) $T_N(\alpha w)=T_N\big(\alpha f(w_1)\big)=T_N\big(f(\alpha w_1)\big)=T_N^f\big(f(\alpha w_1)\big)\geq T_N^f(w_1)=T_N(w),$

$C_N(\alpha w)=C_N\big(\alpha f(w_1)\big)=C_N\big(f(\alpha w_1)\big)=C_N^f\big(f(\alpha w_1)\big)\geq C_N^f(w_1)=C_N(w),$

$G_N(\alpha w)=G_N\big(\alpha f(w_1)\big)=G_N\big(f(\alpha w_1)\big)=G_N^f\big(f(\alpha w_1)\big)\leq G_N^f(w_1)=G_N(w),$

$U_N(\alpha w)=U_N\big(\alpha f(w_1)\big)=U_N\big(f(\alpha w_1)\big)=U_N^f\big(f(\alpha w_1)\big)\leq U_N^f(w_1)=U_N(w),$

$F_N(\alpha w)=F_N\big(\alpha f(w_1)\big)=F_N\big(f(\alpha w_1)\big)=F_N^f\big(f(\alpha w_1)\big)\leq F_N^f(w_1)=F_N(w).$

(iii) $T_N([w,q])=T_N([f(w_1),f(q_1)])=T_N\big(f([w_1,q_1])\big)=T_N^f([w_1,q_1])\geq T_N(w_1)=T_N(w),$

$C_N([w,q])=C_N([f(w_1),f(q_1)])=C_N\big(f([w_1,q_1])\big)=C_N^f([w_1,q_1])\geq C_N(w_1)=C_N(w),$

$G_N([w,q])=G_N([f(w_1),f(q_1)])=G_N\big(f([w_1,q_1])\big)=G_N^f([w_1,q_1])\leq G_N(w_1)=G_N(w),$

$U_N([w,q])=U_N([f(w_1),f(q_1)])=U_N\big(f([w_1,q_1])\big)=U_N^f([w_1,q_1])\leq U_N(w_1)=U_N(w),$

$F_N([w,q])=F_N([f(w_1),f(q_1)])=F_N\big(f([w_1,q_1])\big)=F_N^f([w_1,q_1])\leq F_N(w_1)=F_N(w),$

Therefore, $L^f = \{(w, T_M^f(w), C_M^f(w), G_M^f(w), U_M^f(w), F_M^f(w)) : w \in L_1\}$ satisfies all the conditions for being an SVPN-Lie ideal of L_2.

Novelty:

Conclusions:

In this article, we introduced the notion of SVPN-Lie ideal of SVPN-Lie algebra. Besides, we formulated several interesting results on SVPN-Lie ideal and SVPN-Lie algebra. Further, we furnish few illustrative examples.

In the future, we hope that based on the current study many new notions namely single-valued pentapartitioned neutrosophic anti-Lie ideal, single-valued pentapartitioned neutrosophic Lie topology can also be introduce.

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