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Some fundamental Operations for multi-Polar Interval-Valued Neutrosophic Soft Set and a Decision-Making Approach to Solve MCDM Problem

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Abstract:

The main purpose of this research is to propose an m-polar interval-valued neutrosophic soft set (mPIVNSSs) by merging the m-polar fuzzy set and interval-valued neutrosophic soft set. The mPIVNSSs is the most generalized form of interval-valued neutrosophic soft set. It can accommodate the truthiness, indeterminacy, and falsity in intervals form. We develop some fundamental operations for mPIVNSS such as AND Operator, OR Operator, Truth-favorite, and False-favorite Operators with their properties. The weighted aggregation operator for mPIVNSS is also established with its properties. Furthermore, the developed mPIVNSWA operator has demonstrated a novel decision-making methodology for mPIVNSS to solve the multi-criteria decision-making (MCDM) problem. Finally, the comparative analysis of the developed algorithm is given with the prevailing techniques.

Keywords: multipolar interval-valued neutrosophic set; multipolar interval-valued neutrosophic soft set; mPIWNSWA operator; MCDM.

1. Introduction

Uncertainty plays a dynamic role in many areas of life (such as modeling, medicine, engineering, etc.). However, people have raised a common problem: how do we express and use the concept of uncertainty in mathematical modeling. Many researchers plan and endorse different methods to solve the difficulties that involve hesitation. First, Zadeh proposed the idea of a Fuzzy Set (FS) [1] to solve uncertain complications. But in some cases, fuzzy sets cannot handle this situation. To overcome this situation, Turksen [2] proposed the idea of interval-valued fuzzy sets (IVFS). In some cases, we must consider the non-member value of the object, which neither FS nor IVFS can handle. Atanasov planned the Intuitionistic Fuzzy Set (IFS) [3] to overcome these problems. The ideas proposed by Atanassov only involve under-considered data and member and non-member values.

However, the IFS theory cannot handle the overall incompatibility and inaccurate information. To solve the problem of incompatibility and incorrect information, Smarandache [4] proposed the idea of NS. Molodtsov [5] proposed a general mathematical tool for solving uncertain environments, called soft sets (SS). Maggie et al. [6] Expanded the concept of SS and presented basic operations with ideal properties. Maggie et al. [7] A decision-making technique was established using the operations they developed and used for decision-making. Ali et al. [8] Expanded the concept of SS and developed some new operations using their characteristics. The author [9] proved De Morgan's law by using different operators on the SS theory. Çağman developed the concept of soft matrix and Enginoglu [10]. They also introduced some basic operations of soft matrices and studied their required properties.

Çağman and Enginoglu [11] extended the soft set (SS) concept with basic operations and attributes. They also established a decision-making (DM) technology to use the methods they developed to solve decision-making complexity. In [12], the authors proposed some new operations on soft matrices, such as soft difference product, soft finite-difference product, soft extended difference product, and soft extended difference product and their properties. Maji [13] put forward the idea of NSS with necessary operations and attributes. The concept of Possibility NSS was proposed by Karaaslan [14]. He also established a DM technology that uses the And product based on the possibility of NSS to solve the DM problem. Broumi [15] developed a generalized NSS with some operations and properties and applied the proposed concept to DM. Deli and Subas [16] extended the Single Valued neutrosophic number (SVNN) concept and provided a DM method to solve the MCDM problem. They also developed the idea of SVNN cut sets. Wang et al. [17] proposed the correlation coefficient (CC) of single-valued neutrosophic sets (SVNS) and constructed the DM method using the correlation measurement they developed. Ye [18] proposed the idea of a simplified neutrosophic set (NS), developed an aggregation operator (AO) for the simplified NS, and established a DM method to solve the MCDM problem using the AO he developed. Masooma et al. [19] combined multipolar fuzzy sets, and NS proposed multipolar neutrosophic sets and established various representations and operations based on examples. Zulqarnain et al. [20] introduced some AO and correlation coefficients for the interval value IFSS. They also extended the TOPSIS technology to solve the MADM problem with the relevant metrics they developed. Zulqarnain et al. [21] introduced Pythagorean fuzzy soft number (PFSN) operational laws. They developed AO using defined operational laws, such as Pythagorean fuzzy soft weighted average and geometric operators. They also planned a DM method to solve the MADM problem with the help of the provided operator. Zulqarnain et al. [22] planned the TOPSIS method in the PFSS environment based on the correlation coefficient. They also established a DM method to solve the MCGDM problem and used the developed method in green supply chain management.

Many mathematicians have developed various similarity measures, correlation coefficients, aggregation operators, and decision-making applications in the past few years. Garg [23] introduced a weighted cosine similarity measure for intuitionistic fuzzy sets. He also constructed the MCDM method based on his proposed technology and used the developed method for pattern recognition and medical diagnosis. Garg and Kumar [24] proposed some new similarity measures to measure the relative strength of IFS. They also formulated the number of connections for set pair analysis (SPA) and developed a new similarity measure based on the defined SPA. Ruan et al. [25] Some similarity measures have been developed for PFS by using exponential membership and non-membership and their attributes and relationships. Peng and Garg [26] proposed various PFS similarity measures with multiple parameters. Zulqarnain et al. [27, 28] offered the generalized TOPSIS and integrated TOPSIS models for NS and used their proposed techniques for supplier selection in the production industry. Said et al. [29] Established the concept of mPNSS with attributes and operators. They also developed a distance-based similarity measure and used the proposed similarity measure for decision-making and medical diagnosis.

1.1 Motivation

In this era, professionals believe that real life is moving towards multi-polarity. Therefore, there is no doubt that the multi-polarization of information has played a vital role in the prosperity of many scientific and technological fields. In neurobiology, multipolar neurons accumulate a lot of information from other neurons. The motivation for expanding and mixing this research work is gradually given throughout the manuscript. We prove that different hybrid structures containing fuzzy sets will be converted into mPIVNSS special permissions under any appropriate circumstances. The concept of the neutrosophic environment of the multipolar neutrosophic soft set is novel. We discuss the effectiveness, flexibility, quality, and advantages of planning work and algorithms. This research will be the most versatile form that can be used to incorporate data from the complications of daily life. In the future, current work may be extended to different types of hybrid structures and decision-making techniques in many areas of life.

The structure of the following paper is organized as follows: In Section 2, we reviewed some basic definitions used in subsequent sequels, such as NS, SS, NSS, multi-polar neutrosophic set, and interval value neutrosophic soft set. Section 3 puts forward the new idea of mPIVNSS by combining m-pole fuzzy sets (mPFS) with interval-valued neutral soft sets, their attributes, and operations. This section also developed Truth-Favorite, False-Favorite, AND, and OR operators. In Section 4, the multi-polar interval value Neutral Soft Weighted Aggregation (mPIVNSWA) operator was developed using its decision-making technique. Section 5 uses the developed decision-making method and gives a numerical example. Finally, in Section 6, a brief comparison between the method we developed and the existing technology. In addition, superiority, practicality, and flexibility are also introduced in the same section.

2. Preliminaries

This section recollects some basic concepts such as the neutrosophic set, soft set, neutrosophic soft set, and m-polar neutrosophic soft set used in the following sequel.

Definition 2.1 [4] Let $\boldsymbol{\mathcal{U}}$ be a universe and $\boldsymbol{\mathcal{A}}$ be an NS on $\boldsymbol{\mathcal{U}}$ is defined as $\boldsymbol{\mathcal{A}} = \{<\boldsymbol{u}, \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) >: \boldsymbol{u} \in \boldsymbol{\mathcal{U}}\}$,where $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}: \boldsymbol{\mathcal{U}} \to]\boldsymbol{0}^-, \boldsymbol{1}^+[$ and $\boldsymbol{0}^- \le \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) \le \boldsymbol{3}^+$.

Definition 2.2 [19] Let $\boldsymbol{\mathcal{U}}$ be the universal set and $\wp_{\mathfrak{R}}$ is said to multipolar neutrosophic set if $\wp_{\mathfrak{R}} = \{(\boldsymbol{u}, \boldsymbol{u}_{\alpha}(\boldsymbol{u}), \boldsymbol{v}_{\alpha}(\boldsymbol{u}), \boldsymbol{w}_{\alpha}(\boldsymbol{u})): \boldsymbol{u} \in \boldsymbol{\mathcal{U}}, \boldsymbol{\alpha} = \boldsymbol{1}, \boldsymbol{2}, \boldsymbol{3}, \ldots, \boldsymbol{m}\}$, where $\boldsymbol{u}_{\alpha}(\boldsymbol{u})$, $\boldsymbol{v}_{\alpha}(\boldsymbol{u})$, and $\boldsymbol{w}_{\alpha}(\boldsymbol{u})$ represents the truthiness, indeterminacy, and falsity respectively, "$\boldsymbol{u}_{\alpha}(\boldsymbol{u}), \boldsymbol{v}_{\alpha}(\boldsymbol{u}), \boldsymbol{w}_{\alpha}(\boldsymbol{u}) \subseteq [\boldsymbol{0}, \boldsymbol{1}]$ and $0 \le \boldsymbol{u}_{\alpha}(\boldsymbol{u}) + \boldsymbol{v}_{\alpha}(\boldsymbol{u}) + \boldsymbol{w}_{\alpha}(\boldsymbol{u}) \le 3$, for all $\boldsymbol{\alpha} = 1, 2, 3, \ldots, \boldsymbol{m}$; and $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$.

Definition 2.3 [5] Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a soft set over $\mathcal{U}$, and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \; if \; e \neq \mathcal{A}\}"$$

2.4 Definition [5]

Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the power set of $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called an SS over $\mathcal{U}$, and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

It is also defined as:

$$(\mathcal{F}, \mathcal{A}) = \{\mathcal{F}(e) \in \mathcal{P}(\mathcal{U}): e \in \mathcal{E}, \mathcal{F}(e) = \emptyset \text{ if } e \neq \mathcal{A}\}$$

Definition 2.5 [13] Let $\mathcal{U}$ be the universal set and $\mathcal{E}$ be the set of attributes concerning $\mathcal{U}$. Let $\mathcal{P}(\mathcal{U})$ be the set of neutrosophic sets over $\mathcal{U}$ and $\mathcal{A} \subseteq \mathcal{E}$. A pair $(\mathcal{F}, \mathcal{A})$ is called a neutrosophic soft set over $\mathcal{U}$ and its mapping is given as

$$\mathcal{F}: \mathcal{A} \to \mathcal{P}(\mathcal{U})$$

Definition 2.6 [30] Let $\boldsymbol{\mathcal{U}}$ be a universal set, then interval valued neutrosophic set can be expressed by the set $\boldsymbol{\mathcal{A}} = \{\boldsymbol{u}, (\boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u})): \boldsymbol{u} \in \boldsymbol{\mathcal{U}}\}$, where $\boldsymbol{u}_{\mathcal{A}}$, $\boldsymbol{v}_{\mathcal{A}}$, and $\boldsymbol{w}_{\mathcal{A}}$ are truth, indeterminacy and falsity membership functions for $\boldsymbol{\mathcal{A}}$ respectively, $\boldsymbol{u}_{\mathcal{A}}$, $\boldsymbol{v}_{\mathcal{A}}$, and $\boldsymbol{w}_{\mathcal{A}} \subseteq [0, 1]$ for each $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$. Where

$\boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{u}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{u}^{U}_{\mathcal{A}}(\boldsymbol{u})]$,
$\boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{v}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{v}^{U}_{\mathcal{A}}(\boldsymbol{u})]$, and
$\boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) = [\boldsymbol{w}^{L}_{\mathcal{A}}(\boldsymbol{u}), \boldsymbol{w}^{U}_{\mathcal{A}}(\boldsymbol{u})]$

For each point $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$, $0 \leq \boldsymbol{u}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{v}_{\mathcal{A}}(\boldsymbol{u}) + \boldsymbol{w}_{\mathcal{A}}(\boldsymbol{u}) \leq 3$ and IVN($\boldsymbol{\mathcal{U}}$) represent the family of all interval valued neutrosophic sets on $\boldsymbol{\mathcal{U}}$.

Definition 2.7 [31] Let $\boldsymbol{\mathcal{U}}$ be a universe of discourse and $\boldsymbol{\mathcal{E}}$ be a set of attributes, and m-polar neutrosophic soft set (mPNSS) $\wp_{\mathfrak{R}}$ over $\boldsymbol{\mathcal{U}}$ defined as

$\wp_{\mathfrak{R}} = \{(\boldsymbol{e}, \{(\boldsymbol{u}, \boldsymbol{u}_{\alpha}(\boldsymbol{u}), \boldsymbol{v}_{\alpha}(\boldsymbol{u}), \boldsymbol{w}_{\alpha}(\boldsymbol{u})): \boldsymbol{u} \in \boldsymbol{\mathcal{U}}, \boldsymbol{\alpha} = \mathbf{1}, \mathbf{2}, \mathbf{3}, \ldots, \boldsymbol{m}\}): \boldsymbol{e} \in \boldsymbol{\mathcal{E}}\}$,

where $\boldsymbol{u}_{\alpha}(\boldsymbol{u})$, $\boldsymbol{v}_{\alpha}(\boldsymbol{u})$, and $\boldsymbol{w}_{\alpha}(\boldsymbol{u})$ represent the truthiness, indeterminacy, and falsity respectively, $\boldsymbol{u}_{\alpha}(\boldsymbol{u}), \boldsymbol{v}_{\alpha}(\boldsymbol{u}), \boldsymbol{w}_{\alpha}(\boldsymbol{u}) \subseteq [\mathbf{0}, \mathbf{1}]$ and $0 \leq \boldsymbol{u}_{\alpha}(\boldsymbol{u}) + \boldsymbol{v}_{\alpha}(\boldsymbol{u}) + \boldsymbol{w}_{\alpha}(\boldsymbol{u}) \leq 3$, for all $\boldsymbol{\alpha} = 1, 2, 3, \ldots, \boldsymbol{m}$; $\boldsymbol{e} \in \boldsymbol{\mathcal{E}}$ and $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$. Simply an m-polar neutrosophic number (mPNSN) can be expressed as $\wp = \{\langle \boldsymbol{u}_{\alpha}, \boldsymbol{v}_{\alpha}, \boldsymbol{w}_{\alpha} \rangle\}$, where $0 \leq \boldsymbol{u}_{\alpha} + \boldsymbol{v}_{\alpha} + \boldsymbol{w}_{\alpha} \leq 3$ and $\boldsymbol{\alpha} = 1, 2, 3, \ldots, \boldsymbol{m}$.

Definition 2.8 [32] Let $\boldsymbol{\mathcal{U}}$ be a universe of discourse and $\boldsymbol{\mathcal{E}}$ be a set of attributes, an IVNSS $\wp_{\mathfrak{R}}$ over $\boldsymbol{\mathcal{U}}$ defined as

$\wp_{\mathfrak{R}} = \{(e, \{(u, u_{\mathfrak{R}}(u), v_{\mathfrak{R}}(u), w_{\mathfrak{R}}(u)): u \in \mathcal{U}, \alpha = 1, 2, 3, \ldots, m\}): e \in \mathcal{E}\}$,

where $\boldsymbol{u}_{\mathfrak{R}}(\boldsymbol{u}) = [\boldsymbol{u}^{\ell}_{\mathfrak{R}}(\boldsymbol{u}), \boldsymbol{u}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u})]$, $\boldsymbol{v}_{\mathfrak{R}}(\boldsymbol{u}) = [\boldsymbol{v}^{\ell}_{\mathfrak{R}}(\boldsymbol{u}), \boldsymbol{v}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u})]$, $\boldsymbol{w}_{\mathfrak{R}}(\boldsymbol{u}) = [\boldsymbol{w}^{\ell}_{\mathfrak{R}}(\boldsymbol{u}), \boldsymbol{w}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u})]$, represents the interval truthiness, indeterminacy, and falsity respectively, $\boldsymbol{u}_{\mathfrak{R}}(\boldsymbol{u}), \boldsymbol{v}_{\mathfrak{R}}(\boldsymbol{u}), \boldsymbol{w}_{\mathfrak{R}}(\boldsymbol{u}) \subseteq [\mathbf{0}, \mathbf{1}]$ and $0 \leq \boldsymbol{u}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u}) + \boldsymbol{v}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u}) + \boldsymbol{w}^{\mathfrak{u}}_{\mathfrak{R}}(\boldsymbol{u}) \leq 3$, for each $\boldsymbol{e} \in \boldsymbol{\mathcal{E}}$ and $\boldsymbol{u} \in \boldsymbol{\mathcal{U}}$.

3. Multi-Polar Interval Valued Neutrosophic Soft Set with Aggregate Operators and Properties

The idea of m-pole fuzzy sets (mPFS) was proposed by Chen et al. [33] In 2014, able to deal with ambiguous data and ambiguous multipolar information. Smarandache [34] proposed a three-pole, multi-pole neutrosophic set and its graph in 2016. The membership degree of mPFS is in the interval [0,1]^m, representing the m criteria of the object, but mPFS cannot deal with uncertainty and false objects. NS is bargaining with a single choice criterion of true, false, and uncertainty. But it cannot deal with the multi-standard, multi-source, and multi-polar information fusion that may be selected. Deli et al. [31] Combining the concepts of m-polar neutrosophic set and SS, a new model of mPNSS was introduced. The developed mPNSS can handle m standards for each alternative. mPNSS extends the bipolar Zhongzhi soft set proposed by Ali et al. [35]. Deli [32] established IVNSS, which is a combination of IVNS[30] and SS[5]. We constructed some basic concepts of mPNSS and extended mPNSS to mPIVSS with various operations and attributes.

Definition 3.1 Let $\boldsymbol{\mathcal{U}}$ be a universe of discourse and $\boldsymbol{\mathcal{E}}$ be a set of attributes, then m-polar interval-valued neutrosophic soft set (mPIVNSS) $\wp_{\mathfrak{R}}$ over $\boldsymbol{\mathcal{U}}$ defined as

$\wp_{\mathfrak{R}} = \{(e, \{(u, u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u)): u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m\}): e \in \mathcal{E}\}$,

where $u_{\alpha}(u) = [u^{\ell}_{\alpha}(u), u^{\mathfrak{u}}_{\alpha}(u)]$, $v_{\alpha}(u) = [v^{\ell}_{\alpha}(u), v^{\mathfrak{u}}_{\alpha}(u)]$, $w_{\alpha}(u) = [w^{\ell}_{\alpha}(u), w^{\mathfrak{u}}_{\alpha}(u)]$, represent the interval truthiness, indeterminacy, and falsity respectively, $u_{\alpha}(u), v_{\alpha}(u), w_{\alpha}(u) \subseteq [0, 1]$ and $0 \leq$

$u_\alpha^{u}(u)+v_\alpha^{u}(u)+w_\alpha^{u}(u) \leq 3$ for all $\alpha = 1, 2, 3, \ldots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$. Simply an m-polar interval-valued neutrosophic soft number (mPIVNSN) can be expressed as $\wp = \{[u_\alpha^{\ell}(u), u_\alpha^{u}(u)], [v_\alpha^{\ell}(u), v_\alpha^{u}(u)], [w_\alpha^{\ell}(u), w_\alpha^{u}(u)]\}$, where $0 \leq u_\alpha^{u}(u)+v_\alpha^{u}(u)+w_\alpha^{u}(u) \leq 3$ and $\alpha = 1, 2, 3, \ldots, m$.

Definition 3.2 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, $\wp_{\mathfrak{R}}$ is called an m-polar interval-valued neutrosophic soft subset of $\wp_{\mathcal{L}}$. If

$u_\alpha^{\ell\mathfrak{R}}(u) \leq u_\alpha^{\ell\mathcal{L}}(u)$, $u_\alpha^{u\mathfrak{R}}(u) \leq u_\alpha^{u\mathcal{L}}(u)$

$v_\alpha^{\ell\mathfrak{R}}(u) \geq v_\alpha^{\ell\mathcal{L}}(u)$, $v_\alpha^{u\mathfrak{R}}(u) \geq v_\alpha^{u\mathcal{L}}(u)$

$w_\alpha^{\ell\mathfrak{R}}(u) \geq w_\alpha^{\ell\mathcal{L}}(u)$, $w_\alpha^{u\mathfrak{R}}(u) \geq w_\alpha^{u\mathcal{L}}(u)$

for all $\alpha = 1, 2, 3, \cdots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$.

Definition 3.3 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, $\wp_{\mathfrak{R}} = \wp_{\mathcal{L}}$, if

$u_\alpha^{\ell\mathfrak{R}}(u) \leq u_\alpha^{\ell\mathcal{L}}(u)$, $u_\alpha^{\ell\mathcal{L}}(u) \leq u_\alpha^{\ell\mathfrak{R}}(u)$ and $u_\alpha^{u\mathfrak{R}}(u) \leq u_\alpha^{u\mathcal{L}}(u)$, $u_\alpha^{u\mathcal{L}}(u) \leq u_\alpha^{u\mathfrak{R}}(u)$

$v_\alpha^{\ell\mathfrak{R}}(u) \geq v_\alpha^{\ell\mathcal{L}}(u)$, $v_\alpha^{\ell\mathcal{L}}(u) \geq v_\alpha^{\ell\mathfrak{R}}(u)$ and $v_\alpha^{u\mathfrak{R}}(u) \geq v_\alpha^{u\mathcal{L}}(u)$, $v_\alpha^{u\mathcal{L}}(u) \geq v_\alpha^{u\mathfrak{R}}(u)$

$w_\alpha^{\ell\mathfrak{R}}(u) \geq w_\alpha^{\ell\mathcal{L}}(u)$, $w_\alpha^{\ell\mathcal{L}}(u) \geq w_\alpha^{\ell\mathfrak{R}}(u)$ and $w_\alpha^{u\mathfrak{R}}(u) \geq w_\alpha^{u\mathcal{L}}(u)$, $w_\alpha^{u\mathcal{L}}(u) \geq w_\alpha^{u\mathfrak{R}}(u)$

for all $\alpha = 1, 2, 3, \ldots, m$; $e \in \mathcal{E}$ and $u \in \mathcal{U}$.

Definition 3.4 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then,

$$\wp_{\mathfrak{R}} \cup \wp_{\mathcal{L}} = \left\{\left(e, \left\{\left(\begin{matrix} u, \left[sup\{u_\alpha^{\ell\mathfrak{R}}(u), u_\alpha^{\ell\mathcal{L}}(u)\}, sup\{u_\alpha^{u\mathfrak{R}}(u), u_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[inf\{v_\alpha^{\ell\mathfrak{R}}(u), v_\alpha^{\ell\mathcal{L}}(u)\}, inf\{v_\alpha^{u\mathfrak{R}}(u), v_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[inf\{w_\alpha^{\ell\mathfrak{R}}(u), w_\alpha^{\ell\mathcal{L}}(u)\}, inf\{w_\alpha^{u\mathfrak{R}}(u), w_\alpha^{u\mathcal{L}}(u)\}\right] \end{matrix}\right) : u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m \right\}\right) : e \in \mathcal{E}\right\}$$

Definition 3.5 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then,

$$\wp_{\mathfrak{R}} \cap \wp_{\mathcal{L}} = \left\{\left(e, \left\{\left(\begin{matrix} u, \left[inf\{u_\alpha^{\ell\mathfrak{R}}(u), u_\alpha^{\ell\mathcal{L}}(u)\}, inf\{u_\alpha^{u\mathfrak{R}}(u), u_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[sup\{v_\alpha^{\ell\mathfrak{R}}(u), v_\alpha^{\ell\mathcal{L}}(u)\}, sup\{v_\alpha^{u\mathfrak{R}}(u), v_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[sup\{w_\alpha^{\ell\mathfrak{R}}(u), w_\alpha^{\ell\mathcal{L}}(u)\}, sup\{w_\alpha^{u\mathfrak{R}}(u), w_\alpha^{u\mathcal{L}}(u)\}\right] \end{matrix}\right) : u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m \right\}\right) : e \in \mathcal{E}\right\}$$

3.6 Definition

Let $\wp_{\mathfrak{R}}$ be an mPIVNSS over $\mathcal{U}$. Then, the complement of mPIVNSS is defined as follows:

$$\wp_{\mathfrak{R}}^{c} = \{(e, \{(u, [w_\alpha^{\ell}(u), w_\alpha^{u}(u)], [1 - v_\alpha^{u}(u), 1 - v_\alpha^{\ell}(u)], [u_\alpha^{\ell}(u), u_\alpha^{u}(u)]) : u \in \mathcal{U}, \alpha = 1, 2, 3, \ldots, m\}) : e \in \mathcal{E}\}$$

Proposition 3.7 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then,

1. $(\wp_{\mathfrak{R}} \cup \wp_{\mathcal{L}})^{C} = {\wp_{\mathfrak{R}}}^{C} \cap {\wp_{\mathcal{L}}}^{C}$
2. $(\wp_{\mathfrak{R}} \cap \wp_{\mathcal{L}})^{C} = {\wp_{\mathfrak{R}}}^{C} \cup {\wp_{\mathcal{L}}}^{C}$

Proof 1 As we know that

$$\wp_{\mathfrak{R}} = \{(e, \{(u, [u_\alpha^{\ell\mathfrak{R}}(u), u_\alpha^{u\mathfrak{R}}(u)], [v_\alpha^{\ell\mathfrak{R}}(u), v_\alpha^{u\mathfrak{R}}(u)], [w_\alpha^{\ell\mathfrak{R}}(u), w_\alpha^{u\mathfrak{R}}(u)]) : u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m\}) : e \in \mathcal{E}\}$$

and

$$\wp_{\mathcal{L}} = \{(e, \{(u, [u_\alpha^{\ell\mathcal{L}}(u), u_\alpha^{u\mathcal{L}}(u)], [v_\alpha^{\ell\mathcal{L}}(u), v_\alpha^{u\mathcal{L}}(u)], [w_\alpha^{\ell\mathcal{L}}(u), w_\alpha^{u\mathcal{L}}(u)]) : u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m\}) : e \in \mathcal{E}\}$$

Then

$$\wp_{\mathfrak{R}} \cup \wp_{\mathcal{L}} = \left\{\left(e, \left\{\left(\begin{matrix} u, \left[sup\{u_\alpha^{\ell\mathfrak{R}}(u), u_\alpha^{\ell\mathcal{L}}(u)\}, sup\{u_\alpha^{u\mathfrak{R}}(u), u_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[inf\{v_\alpha^{\ell\mathfrak{R}}(u), v_\alpha^{\ell\mathcal{L}}(u)\}, inf\{v_\alpha^{u\mathfrak{R}}(u), v_\alpha^{u\mathcal{L}}(u)\}\right], \\ \left[inf\{w_\alpha^{\ell\mathfrak{R}}(u), w_\alpha^{\ell\mathcal{L}}(u)\}, inf\{w_\alpha^{u\mathfrak{R}}(u), w_\alpha^{u\mathcal{L}}(u)\}\right] \end{matrix}\right) : u \in \mathcal{U}, \alpha = 1,2,3, \ldots, m \right\}\right) : e \in \mathcal{E}\right\}$$

we get

$(\wp_{\Re} \cup \wp_{\mathcal{L}})^c$

$$=\left\{\left(e,\left\{\left(\begin{array}{c} u,\left[inf\left\{w_\alpha^{\ell\Re}(u), w_\alpha^{\ell\mathcal{L}}(u)\right\}, inf\left\{w_\alpha^{u\Re}(u), w_\alpha^{u\mathcal{L}}(u)\right\}\right], \\ \left[1-inf\left\{v_\alpha^{u\Re}(u), v_\alpha^{u\mathcal{L}}(u)\right\}, 1-inf\left\{v_\alpha^{\ell\Re}(u), v_\alpha^{\ell\mathcal{L}}(u)\right\}\right], \\ \left[sup\left\{u_\alpha^{\ell\Re}(u), u_\alpha^{\ell\mathcal{L}}(u)\right\}, sup\left\{u_\alpha^{u\Re}(u), u_\alpha^{u\mathcal{L}}(u)\right\}\right] \end{array}\right): u \in \mathcal{U}, \alpha = 1,2,3, \dots, m\right\}\right): e \in \mathcal{E}\right\}$$

Now

$\wp_{\Re}{}^C =$

$$\left\{\left(e,\left\{\left(u,\left[w_\alpha^{\ell\Re}(u), w_\alpha^{u\Re}(u)\right],\left[1-v_\alpha^{u\Re}(u), 1-v_\alpha^{\ell\Re}(u)\right],\left[u_\alpha^{\ell\Re}(u), u_\alpha^{u\Re}(u)\right]\right): u \in \mathcal{U}, \alpha = 1,2,3, \dots, m\right\}\right): e \in \mathcal{E}\right\}$$

$\wp_{\mathcal{L}}{}^C =$

$$\left\{\left(e,\left\{\left(u,\left[w_\alpha^{\ell\mathcal{L}}(u), w_\alpha^{u\mathcal{L}}(u)\right],\left[1-v_\alpha^{u\mathcal{L}}(u), 1-v_\alpha^{\ell\mathcal{L}}(u)\right],\left[u_\alpha^{\ell\mathcal{L}}(u), u_\alpha^{u\mathcal{L}}(u)\right]\right): u \in \mathcal{U}, \alpha = 1,2,3, \dots, m\right\}\right): e \in \mathcal{E}\right\}$$

By using definition 3.5

$\wp_{\Re}{}^C \cap \wp_{\mathcal{L}}{}^C =$

$$\left\{\left(e,\left\{\left(\begin{array}{c} u,\left[inf\left\{w_\alpha^{\ell\Re}(u), w_\alpha^{\ell\mathcal{L}}(u)\right\}, inf\left\{w_\alpha^{u\Re}(u), w_\alpha^{u\mathcal{L}}(u)\right\}\right], \\ \left[inf\left\{1-v_\alpha^{u\Re}(u), 1-v_\alpha^{u\mathcal{L}}(u)\right\}, inf\left\{1-v_\alpha^{\ell\Re}(u), 1-v_\alpha^{\ell\mathcal{L}}(u)\right\}\right], \\ \left[sup\left\{u_\alpha^{\ell\Re}(u), u_\alpha^{\ell\mathcal{L}}(u)\right\}, sup\left\{u_\alpha^{u\Re}(u), u_\alpha^{u\mathcal{L}}(u)\right\}\right] \end{array}\right): u \in \mathcal{U}, \alpha = 1,2,3, \dots, m\right\}\right): e \in \mathcal{E}\right\}$$

$\wp_{\Re}{}^C \cap \wp_{\mathcal{L}}{}^C =$

$$\left\{\left(e,\left\{\left(\begin{array}{c} u,\left[inf\left\{w_\alpha^{\ell\Re}(u), w_\alpha^{\ell\mathcal{L}}(u)\right\}, inf\left\{w_\alpha^{u\Re}(u), w_\alpha^{u\mathcal{L}}(u)\right\}\right], \\ \left[1-inf\left\{v_\alpha^{u\Re}(u), v_\alpha^{u\mathcal{L}}(u)\right\}, 1-inf\left\{v_\alpha^{\ell\Re}(u), v_\alpha^{\ell\mathcal{L}}(u)\right\}\right], \\ \left[sup\left\{u_\alpha^{\ell\Re}(u), u_\alpha^{\ell\mathcal{L}}(u)\right\}, sup\left\{u_\alpha^{u\Re}(u), u_\alpha^{u\mathcal{L}}(u)\right\}\right] \end{array}\right): u \in \mathcal{U}, \alpha = 1,2,3, \dots, m\right\}\right): e \in \mathcal{E}\right\}$$

Hence

$(\wp_{\Re} \cup \wp_{\mathcal{L}})^c = \wp_{\Re}{}^C \cap \wp_{\mathcal{L}}{}^C$.

Proof 2 Similar to assertion 1.

Definition 3.8 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their extended union is defined as

$$u(\wp_{\Re} \cup_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} \left[u_\alpha^{\ell\Re}(u), u_\alpha^{u\Re}(u)\right] & if\ e \in \Re - \mathcal{L} \\ \left[u_\alpha^{\ell\mathcal{L}}(u), u_\alpha^{u\mathcal{L}}(u)\right] & if\ e \in \mathcal{L} - \Re \\ \left[sup\left\{u_\alpha^{\ell\Re}(u), u_\alpha^{\ell\mathcal{L}}(u)\right\}, sup\left\{u_\alpha^{u\Re}(u), u_\alpha^{u\mathcal{L}}(u)\right\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

$$v(\wp_{\Re} \cup_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} \left[v_\alpha^{\ell\Re}(u), v_\alpha^{u\Re}(u)\right] & if\ e \in \Re - \mathcal{L} \\ \left[v_\alpha^{\ell\mathcal{L}}(u), v_\alpha^{u\mathcal{L}}(u)\right] & if\ e \in \mathcal{L} - \Re \\ \left[inf\left\{v_\alpha^{\ell\Re}(u), v_\alpha^{\ell\mathcal{L}}(u)\right\}, inf\left\{v_\alpha^{u\Re}(u), v_\alpha^{u\mathcal{L}}(u)\right\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

$$w(\wp_{\Re} \cup_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} \left[w_\alpha^{\ell\Re}(u), w_\alpha^{u\Re}(u)\right] & if\ e \in \Re - \mathcal{L} \\ \left[w_\alpha^{\ell\mathcal{L}}(u), w_\alpha^{u\mathcal{L}}(u)\right] & if\ e \in \mathcal{L} - \Re \\ \left[inf\left\{w_\alpha^{\ell\Re}(u), w_\alpha^{\ell\mathcal{L}}(u)\right\}, inf\left\{w_\alpha^{u\Re}(u), w_\alpha^{u\mathcal{L}}(u)\right\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

Example 3.9 Assume $\mathcal{U} = \{u_1, u_2\}$ be a universe of discourse and $E = \{e_1, e_2, e_3, e_4\}$ be a set of attributes and $\Re = \{e_1, e_2\} \subseteq E$ and $\mathcal{L} = \{e_2, e_3\} \subseteq E$. Consider 3-PIVNSSs $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ over $\mathcal{U}$ can be represented as follows:

$$\wp_{\Re} = \left\{\begin{array}{l} \left(e_1,\left\{\begin{array}{l}(u_1,([.5,.8],[.2,.5],[.1,.2]),([.3,.5],[.1,.3],[.2,.4]),([.6,.9],[.7,.8],[.8,1])) \\ (u_2,([.2,.4],[.3,.4],[.1,.3]),([.2,.5],[.1,.6],[.1,.3]),([.8,1],[.6,.9],[.6,.7]))\end{array}\right\}\right), \\ \left(e_2,\left\{\begin{array}{l}(u_1,([.3,.6],[.1,.6],[.3,.4]),([0,.2],[.1,.4],[.3,.5]),([.5,.9],[.3,.8],[.5,.8])) \\ (u_2,([.2,.5],[.2,.3],[.5,.6]),([.3,.5],[.1,.5],[.5,.8]),([.6,.9],[.5,.8],[.6,.9]))\end{array}\right\}\right) \end{array}\right\}$$

and

$$\wp_{\mathcal{L}} = \left\{ \begin{array}{l} \left(e_1, \left\{ \begin{array}{l} (u_1, ([.4,.8],[.3,.6],[.2,.5]),([.2,.7],[.3,.4],[.4,.6]),([.7,.8],[.4,.9],[.5,1])) \\ (u_2,([.1,.6],[.5,.7],[.1,.2]),([.3,.4],[.2,.5],[.2,.5]),([.5,.9],[.7,.8],[.4,.6])) \end{array} \right\} \right), \\ \left(e_2, \left\{ \begin{array}{l} (u_1,([.2,.7],[.3,.5],[.2,.6]),([.1,.3],[.2,.5],[.2,.7]),([.4,.9],[.4,.7],[.5,.8])) \\ (u_2,([.1,.6],[.1,.5],[.4,.8]),([.3,.6],[.3,.4],[1,1]),([.5,.9],[.3,.7],[.1,.8])) \end{array} \right\} \right) \end{array} \right\}$$

Then

$$\wp_{\Re} \cup_{\varepsilon} \wp_{\mathcal{L}} = \left\{ \begin{array}{l} \left(e_1, \left\{ \begin{array}{l} (u_1,([.5,.8],[.2,.5],[.1,.2]),([.3,.5],[.1,.3],[.2,.4]),([.6,.9],[.7,.8],[.8,1])) \\ (u_2,([.2,.4],[.3,.4],[.1,.3]),([.2,.5],[.1,.6],[.1,.3]),([.8,1],[.6,.9],[.6,.7])) \end{array} \right\} \right), \\ \left(e_2, \left\{ \begin{array}{l} (u_1,([.4,.8],[.1,.6],[.2,.4]),([.2,.7],[.1,.4],[.3,.5]),([.7,.9],[.3,.8],[.5,.8])) \\ (u_2,([.2,.6],[.2,.3],[.1,.2]),([.3,.5],[.1,.5],[.2,.5]),([.6,.9],[.5,.8],[.4,.6])) \end{array} \right\} \right), \\ \left(e_2, \left\{ \begin{array}{l} (u_1,([.2,.7],[.3,.5],[.2,.6]),([.1,.3],[.2,.5],[.2,.7]),([.4,.9],[.4,.7],[.5,.8])) \\ (u_2,([.1,.6],[.1,.5],[.4,.8]),([.3,.6],[.3,.4],[1,1]),([.5,.9],[.3,.7],[.1,.8])) \end{array} \right\} \right) \end{array} \right\}$$

Definition 3.10 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their extended intersection is defined as

$$u\ (\wp_{\Re} \cap_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} [u_{\alpha}^{\ell\Re}(u), u_{\alpha}^{u\Re}(u)] & if\ e \in \Re - \mathcal{L} \\ [u_{\alpha}^{\ell\mathcal{L}}(u), u_{\alpha}^{u\mathcal{L}}(u)] & if\ e \in \mathcal{L} - \Re \\ \left[inf\{u_{\alpha}^{\ell\Re}(u), u_{\alpha}^{\ell\mathcal{L}}(u)\}, inf\{u_{\alpha}^{u\Re}(u), u_{\alpha}^{u\mathcal{L}}(u)\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

$$v\ (\wp_{\Re} \cap_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} [v_{\alpha}^{\ell\Re}(u), v_{\alpha}^{u\Re}(u)] & if\ e \in \Re - \mathcal{L} \\ [v_{\alpha}^{\ell\mathcal{L}}(u), v_{\alpha}^{u\mathcal{L}}(u)] & if\ e \in \mathcal{L} - \Re \\ \left[sup\{v_{\alpha}^{\ell\Re}(u), v_{\alpha}^{\ell\mathcal{L}}(u)\}, sup\{v_{\alpha}^{u\Re}(u), v_{\alpha}^{u\mathcal{L}}(u)\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

$$w\ (\wp_{\Re} \cap_{\varepsilon} \wp_{\mathcal{L}}) = \begin{cases} [w_{\alpha}^{\ell\Re}(u), w_{\alpha}^{u\Re}(u)] & if\ e \in \Re - \mathcal{L} \\ [w_{\alpha}^{\ell\mathcal{L}}(u), w_{\alpha}^{u\mathcal{L}}(u)] & if\ e \in \mathcal{L} - \Re \\ \left[sup\{w_{\alpha}^{\ell\Re}(u), w_{\alpha}^{\ell\mathcal{L}}(u)\}, sup\{w_{\alpha}^{u\Re}(u), w_{\alpha}^{u\mathcal{L}}(u)\}\right] & if\ e \in \Re \cap \mathcal{L} \end{cases}$$

Remark 3.1 Generally, if $\wp_{\Re} \neq \wp_{\breve{0}}$ and $\wp_{\Re} \neq \wp_{\grave{\mathrm{E}}}$, then the law of contradiction $\wp_{\Re} \cap {\wp_{\Re}}^{C} = \wp_{\breve{0}}$ and the law of the excluded middle $\wp_{\Re} \cup {\wp_{\Re}}^{C} = \wp_{\grave{\mathrm{E}}}$ does not hold in mPIVNSS. But in classical set theory law of contradiction and excluded middle always hold.

Definition 3.11 Let $\wp_{\Re}$ be an mPIVNSS over $\mathcal{U}$. Then, Truth-Favorite operator on $\wp_{\Re}$ is denoted by $\widetilde{\Delta}\wp_{\Re}$ and defined as follow:

$\widetilde{\Delta}\wp_{\Re}$ =

$$\left\{ \left(e, \left\{ \left(\begin{array}{c} u, \left[inf\{u_{\alpha}^{\ell\Re}(u) + v_{\alpha}^{\ell\Re}(u), 1\}, inf\{u_{\alpha}^{u\Re}(u) + v_{\alpha}^{u\Re}(u), 1\}\right], \\ [0,0],[0,0],\dots,[0,0], \left[w_{\alpha}^{\ell\Re}(u), w_{\alpha}^{u\Re}(u)\right] \end{array} \right) : u \in \mathcal{U}, \alpha = 1,2,3,\dots,m \right\} \right) : e \in \mathcal{E} \right\}$$

Proposition 3.12 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then,

1. $\widetilde{\Delta}\widetilde{\Delta}\wp_{\Re} = \widetilde{\Delta}\wp_{\Re}$
2. $\widetilde{\Delta}(\wp_{\Re} \cup \wp_{\mathcal{L}}) \subseteq \widetilde{\Delta}\wp_{\Re} \cup \widetilde{\Delta}\wp_{\mathcal{L}}$
3. $\widetilde{\Delta}(\wp_{\Re} \cap \wp_{\mathcal{L}}) \subseteq \widetilde{\Delta}\wp_{\Re} \cap \widetilde{\Delta}\wp_{\mathcal{L}}$

Proof of the above proposition is easily obtained by using definitions 3.4, 3.5, 3.11.

Definition 3.13 Let $\wp_{\Re}$ be an mPIVNSS over $\mathcal{U}$. Then, the False-Favorite operator on $\wp_{\Re}$ denoted by $\widetilde{\Delta}\wp_{\Re}$ and is defined as follows:

$\widetilde{\nabla}\wp_{\Re}$ =

$$\left\{ \left(e, \left\{ \left(\begin{array}{c} u, [u_{\alpha}^{\ell\Re}(u), u_{\alpha}^{u\Re}(u)], [0,0],[0,0],\dots,[0,0], \\ \left[inf\{w_{\alpha}^{\ell\Re}(u) + v_{\alpha}^{\ell\Re}(u), 1\}, inf\{w_{\alpha}^{u\Re}(u) + v_{\alpha}^{u\Re}(u), 1\}\right] \end{array} \right) : u \in \mathcal{U}, \alpha = 1,2,3,\dots,m \right\} \right) : e \in \mathcal{E} \right\}$$

Proposition 3.14 Let $\wp_{\Re}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then,

1. $\tilde{\nabla}\tilde{\nabla}\wp_{\mathfrak{R}} = \tilde{\nabla}\wp_{\mathfrak{R}}$
2. $\tilde{\nabla}(\wp_{\mathfrak{R}} \cup \wp_{\mathcal{L}}) \subseteq \tilde{\nabla}\wp_{\mathfrak{R}} \cup \tilde{\nabla}\wp_{\mathcal{L}}$
3. $\tilde{\nabla}(\wp_{\mathfrak{R}} \cap \wp_{\mathcal{L}}) \subseteq \tilde{\nabla}\wp_{\mathfrak{R}} \cap \tilde{\nabla}\wp_{\mathcal{L}}$

Proof of the above proposition is easily obtained using definitions 3.4, 3.5, 3.13.

Definition 3.15 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their AND-Operator is represented by $\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}}$ and defined as follows:

$\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}} = \daleth_{\mathfrak{R}\times\mathcal{L}}$, where

$\daleth_{\mathfrak{R}\times\mathcal{L}}(x,y) = \wp_{\mathfrak{R}}(x) \cap \wp_{\mathcal{L}}(y)$ for all $(x,y) \in \mathfrak{R}\times\mathcal{L}$.

Definition 3.16 Let $\wp_{\mathfrak{R}}$ and $\wp_{\mathcal{L}}$ be two mPIVNSSs over $\mathcal{U}$. Then, their OR-Operator is represented by $\wp_{\mathfrak{R}} \vee \wp_{\mathcal{L}}$ and defined as follows:

$\wp_{\mathfrak{R}} \vee \wp_{\mathcal{L}} = \daleth_{\mathfrak{R}\times\mathcal{L}}$, where

$\daleth_{\mathfrak{R}\times\mathcal{L}}(x,y) = \wp_{\mathfrak{R}}(x) \cup \wp_{\mathcal{L}}(y)$ for all $(x,y) \in \mathfrak{R}\times\mathcal{L}$.

Example 3.17 Reconsider example 3.9

$$\wp_{\mathfrak{R}} = \left\{\begin{array}{l} \left(e_1, \left\{\begin{array}{l}(u_1, ([.5,.8],[.2,.5],[.1,.2]),([.3,.5],[.1,.3],[.2,.4]),([.6,.9],[.7,.8],[.8,1]))\\ (u_2, ([.2,.4],[.3,.4],[.1,.3]),([.2,.5],[.1,.6],[.1,.3]),([.8,1],[.6,.9],[.6,.7]))\end{array}\right\}\right),\\ \left(e_2, \left\{\begin{array}{l}(u_1, ([.3,.6],[.1,.6],[.3,.4]),([0,.2],[.1,.4],[.3,.5]),([.5,.9],[.3,.8],[.5,.8]))\\ (u_2, ([.2,.5],[.2,.3],[.5,.6]),([.3,.5],[.1,.5],[.5,.8]),([.6,.9],[.5,.8],[.6,.9]))\end{array}\right\}\right)\end{array}\right\}$$

and

$$\wp_{\mathcal{L}} = \left\{\begin{array}{l} \left(e_1, \left\{\begin{array}{l}(u_1, ([.4,.8],[.3,.6],[.2,.5]),([.2,.7],[.3,.4],[.4,.6]),([.7,.8],[.4,.9],[.5,1]))\\ (u_2, ([.1,.6],[.5,.7],[.1,.2]),([.3,.4],[.2,.5],[.2,.5]),([.5,.9],[.7,.8],[.4,.6]))\end{array}\right\}\right),\\ \left(e_2, \left\{\begin{array}{l}(u_1, ([.2,.7],[.3,.5],[.2,.6]),([.1,.3],[.2,.5],[.2,.7]),([.4,.9],[.4,.7],[.5,.8]))\\ (u_2, ([.1,.6],[.1,.5],[.4,.8]),([.3,.6],[.3,.4],[1,1]),([.5,.9],[.3,.7],[.1,.8]))\end{array}\right\}\right)\end{array}\right\}$$

$$\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}} = \left\{\begin{array}{c} (e_1,e_2),(u_1,([.4,.8],[.3,.6],[.2,.5]),([.2,.5],[.3,.4],[.4,.6]),([.6,.8],[.7,.9],[.8,1]),\\ (u_2,([.1,.4],[.5,.7],[.1,.3]),([.2,.4],[.2,.6],[.2,.5]),([.5,.9],[.7,.9],[.6,.7]),\\ (e_1,e_3),(u_1,([.2,.7],[.3,.5],[.2,.6]),([.1,.3],[.2,.5],[.2,.7]),([.4,.9],[.7,.8],[.8,1]),\\ (u_2,([.1,.4],[.3,.5],[.4,.8]),([.2,.5],[.3,.6],[1,1]),([.5,.9],[.6,.9],[.6,.8]),\\ (e_2,e_2),(u_1,([.3,.6],[.1,.6],[.3,.4]),([0,.2],[.1,.4],[.3,.5]),([.5,.9],[.3,.8],[.5,.8]),\\ (u_2,([.2,.5],[.2,.3],[.5,.6]),([.3,.5],[.1,.5],[.5,.8]),([.6,.9],[.5,.8],[.6,.9])),\\ (e_2,e_3),(u_1,([.2,.6],[.1,.6],[.3,.6]),([0,.2],[.2,.5],[.3,.7]),([.4,.9],[.4,.8],[.5,.8]),\\ (u_2,([.1,.5],[.2,.5],[.5,.8]),([.3,.5],[.3,.5],[.5,.8]),([.5,.9],[.5,.9],[.6,.9])) \end{array}\right\}.$$

Proposition 3.18 Let $\wp_{\mathfrak{R}}$, $\wp_{\mathcal{L}}$, and $\wp_{\mathcal{H}}$ be three mPIVNSSs over $\mathcal{U}$. Then,

1. $\wp_{\mathfrak{R}} \vee \wp_{\mathcal{L}} = \wp_{\mathcal{L}} \vee \wp_{\mathfrak{R}}$
2. $\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}} = \wp_{\mathcal{L}} \wedge \wp_{\mathfrak{R}}$
3. $\wp_{\mathfrak{R}} \vee (\wp_{\mathcal{L}} \vee \wp_{\mathcal{H}}) = (\wp_{\mathfrak{R}} \vee \wp_{\mathcal{L}}) \vee \wp_{\mathcal{H}}$
4. $\wp_{\mathfrak{R}} \wedge (\wp_{\mathcal{L}} \wedge \wp_{\mathcal{H}}) = (\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}}) \wedge \wp_{\mathcal{H}}$
5. $(\wp_{\mathfrak{R}} \vee \wp_{\mathcal{L}})^c = \wp^c(\mathfrak{R}) \wedge \wp^c(\mathcal{L})$
6. $(\wp_{\mathfrak{R}} \wedge \wp_{\mathcal{L}})^c = \wp^c(\mathfrak{R}) \vee \wp^c(\mathcal{L})$

Proof We can prove easily by using definitions 3.15, 3.16.

4. Weighted Aggregation Operator for m-Polar Interval Valued Neutrosophic Soft set

Many mathematicians developed various methodologies to solve MCDM problems in the past few years, such as aggregation operators for different hybrid structures, CC, similarity measures, and decision-making applications.

Definition 4.1 Let $\wp_{\mathfrak{R}} = \left\langle [\boldsymbol{u}_\alpha^\ell(\boldsymbol{u}), \boldsymbol{u}_\alpha^{\mathrm{u}}(\boldsymbol{u})], [\boldsymbol{v}_\alpha^\ell(\boldsymbol{u}), \boldsymbol{v}_\alpha^{\mathrm{u}}(\boldsymbol{u})], [\boldsymbol{w}_\alpha^\ell(\boldsymbol{u}), \boldsymbol{w}_\alpha^{\mathrm{u}}(\boldsymbol{u})] \right\rangle$, $\wp_{\mathfrak{R}_1} = \left\langle [\boldsymbol{u}_\alpha^{\ell\mathfrak{R}_1}(\boldsymbol{u}), \boldsymbol{u}_\alpha^{\mathrm{u}\mathfrak{R}_1}(\boldsymbol{u})], [\boldsymbol{v}_\alpha^{\ell\mathfrak{R}_1}(\boldsymbol{u}), \boldsymbol{v}_\alpha^{\mathrm{u}\mathfrak{R}_1}(\boldsymbol{u})], [\boldsymbol{w}_\alpha^{\ell\mathfrak{R}_1}(\boldsymbol{u}), \boldsymbol{w}_\alpha^{\mathrm{u}\mathfrak{R}_1}(\boldsymbol{u})] \right\rangle$, and $\wp_{\mathfrak{R}_2} = \left\langle [\boldsymbol{u}_\alpha^{\ell\mathfrak{R}_2}(\boldsymbol{u}), \boldsymbol{u}_\alpha^{\mathrm{u}\mathfrak{R}_2}(\boldsymbol{u})], [\boldsymbol{v}_\alpha^{\ell\mathfrak{R}_2}(\boldsymbol{u}), \boldsymbol{v}_\alpha^{\mathrm{u}\mathfrak{R}_2}(\boldsymbol{u})], [\boldsymbol{w}_\alpha^{\ell\mathfrak{R}_2}(\boldsymbol{u}), \boldsymbol{w}_\alpha^{\mathrm{u}\mathfrak{R}_2}(\boldsymbol{u})] \right\rangle$ are three mPIVNSNs, the basic operators for mPIVNSNs are defined as when $\boldsymbol{\delta} > 0$

1. $\wp_{\mathfrak{R}_1} \oplus \wp_{\mathfrak{R}_2} =$
$$\left\langle \begin{matrix} [u_\alpha^{\ell\mathfrak{R}_1}(u) + u_\alpha^{\ell\mathfrak{R}_2}(u) - u_\alpha^{\ell\mathfrak{R}_1}(u)u_\alpha^{\ell\mathfrak{R}_2}(u), u_\alpha^{\mathrm{u}\mathfrak{R}_1}(u) + u_\alpha^{\mathrm{u}\mathfrak{R}_2}(u) - u_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)u_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)], \\ [v_\alpha^{\ell\mathfrak{R}_1}(u)v_\alpha^{\ell\mathfrak{R}_2}(u), v_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)\, v_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)], [w_\alpha^{\ell\mathfrak{R}_1}(u)w_\alpha^{\ell\mathfrak{R}_2}(u), w_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)\, w_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)] \end{matrix} \right\rangle$$
2. $\wp_{\mathfrak{R}_1} \otimes \wp_{\mathfrak{R}_2} =$
$$\left\langle \begin{matrix} [u_\alpha^{\ell\mathfrak{R}_1}(u)u_\alpha^{\ell\mathfrak{R}_2}(u), u_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)u_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)], \\ [v_\alpha^{\ell\mathfrak{R}_1}(u) + v_\alpha^{\ell\mathfrak{R}_2}(u) - v_\alpha^{\ell\mathfrak{R}_1}(u)v_\alpha^{\ell\mathfrak{R}_2}(u), v_\alpha^{\mathrm{u}\mathfrak{R}_1}(u) + v_\alpha^{\mathrm{u}\mathfrak{R}_2}(u) - v_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)v_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)], \\ [w_\alpha^{\ell\mathfrak{R}_1}(u) + w_\alpha^{\ell\mathfrak{R}_2}(u) - w_\alpha^{\ell\mathfrak{R}_1}(u)w_\alpha^{\ell\mathfrak{R}_2}(u), w_\alpha^{\mathrm{u}\mathfrak{R}_1}(u) + w_\alpha^{\mathrm{u}\mathfrak{R}_2}(u) - w_\alpha^{\mathrm{u}\mathfrak{R}_1}(u)w_\alpha^{\mathrm{u}\mathfrak{R}_2}(u)] \end{matrix} \right\rangle$$
3. $\delta\wp_{\mathfrak{R}} =$
$$\left\langle \left[1 - \left(1 - u_\alpha^{\ell\mathfrak{R}}(u)\right)^\delta, 1 - \left(1 - u_\alpha^{\mathrm{u}\mathfrak{R}}(u)\right)^\delta\right], \left[\left(v_\alpha^{\ell\mathfrak{R}}(u)\right)^\delta, \left(v_\alpha^{\mathrm{u}\mathfrak{R}}(u)\right)^\delta\right], \left[\left(w_\alpha^{\ell\mathfrak{R}}(u)\right)^\delta, \left(w_\alpha^{\mathrm{u}\mathfrak{R}}(u)\right)^\delta\right] \right\rangle$$
4. $(\wp_{\mathfrak{R}})^\delta = \left\langle \begin{matrix} \left[\left(\boldsymbol{u}_\alpha^\ell(\boldsymbol{u})\right)^{\boldsymbol{\delta}}, \left(\boldsymbol{u}_\alpha^{\mathrm{u}}(\boldsymbol{u})\right)^{\boldsymbol{\delta}}\right], \left[\mathbf{1} - \left(\mathbf{1} - \boldsymbol{v}_\alpha^{\ell\mathfrak{R}}(\boldsymbol{u})\right)^{\boldsymbol{\delta}}, \mathbf{1} - \left(\mathbf{1} - \boldsymbol{v}_\alpha^{\mathrm{u}\mathfrak{R}}(\boldsymbol{u})\right)^{\boldsymbol{\delta}}\right], \\ \left[\mathbf{1} - \left(\mathbf{1} - \boldsymbol{w}_\alpha^{\ell\mathfrak{R}}(\boldsymbol{u})\right)^{\boldsymbol{\delta}}, \mathbf{1} - \left(\mathbf{1} - \boldsymbol{w}_\alpha^{\mathrm{u}\mathfrak{R}}(\boldsymbol{u})\right)^{\boldsymbol{\delta}}\right] \end{matrix} \right\rangle$

Definition 4.3 Let $\wp_{\mathfrak{R}_{e_{ij}}}$ = " $\left\langle \left[u_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), u_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right], \left[v_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), v_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right], \left[w_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), w_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right] \right\rangle$ be a collection of mPIVNSNs, Ω_i and γ_j are weight vector for expert's and parameters respectively with given conditions $\Omega_i > 0$, $\sum_{i=1}^n \Omega_i = 1$, $\gamma_j > 0$, $\sum_{j=1}^m \gamma_j = 1$, where $(i = 1, 2, \ldots, n, and\ j = 1, 2, \ldots, m)$. Then mPIVNSWA operator defined as
mPIVNSWA: $\Delta^n \rightarrow \Delta$ defined as follows

$$mPIVNSWA\left(\wp_{\mathfrak{R}_{e_{11}}}, \wp_{\mathfrak{R}_{e_{12}}}, \ldots, \wp_{\mathfrak{R}_{e_{nk}}}\right) = \oplus_{j=1}^k \gamma_j \left(\oplus_{i=1}^n \Omega_i \wp_{\mathfrak{R}_{e_{ij}}}\right). \tag{4.1}$$

Theorem 4.4 Let $\wp_{\mathfrak{R}_{e_{ij}}} = \left\langle \left[u_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), u_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right], \left[v_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), v_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right], \left[w_{\alpha_{ij}}^{\ell\mathfrak{R}}(u), w_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(u)\right] \right\rangle$ be a collection of mPIVNSNs, where $(i = 1, 2, \ldots, n, and\ j = 1, 2, \ldots, k)$, the aggregated value is also an interval-valued neutrosophic soft number, such as
$mPIVNSWA\left(\wp_{\mathfrak{R}_{e_{11}}}, \wp_{\mathfrak{R}_{e_{12}}}, \ldots, \wp_{\mathfrak{R}_{e_{nk}}}\right)$

$$= \left\langle \begin{matrix} \left[\mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{u}_{\alpha_{ij}}^{\ell\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}, \mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{u}_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}\right], \\ \left[\mathbf{1} - \left(\mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{v}_{\alpha_{ij}}^{\ell\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}\right), \mathbf{1} - \left(\mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{v}_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}\right)\right], \\ \left[\mathbf{1} - \left(\mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{w}_{\alpha_{ij}}^{\ell\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}\right), \mathbf{1} - \left(\mathbf{1} - \prod_{j=1}^m \left(\prod_{i=1}^n \left(\mathbf{1} - \boldsymbol{w}_{\alpha_{ij}}^{\mathrm{u}\mathfrak{R}}(\boldsymbol{u})\right)^{\Omega_i}\right)^{\gamma_j}\right)\right] \end{matrix} \right\rangle. \tag{4.2}$$

Definition 4.5 Let $\wp_{\mathfrak{R}} = \langle [u_\alpha^\ell(u), u_\alpha^{\mathrm{u}}(u)], [v_\alpha^\ell(u), v_\alpha^{\mathrm{u}}(u)], [w_\alpha^\ell(u), w_\alpha^{\mathrm{u}}(u)] \rangle$ be an mPIVNSN, then the score, accuracy, and certainty functions for an mPIVNSN respectively defined as follows:

1. $\mathbb{S}(\wp_{\mathfrak{R}}) = \frac{1}{6m}\left(u_\alpha^\ell(u) + u_\alpha^{\mathrm{u}}(u) + 1 - v_\alpha^\ell(u) + 1 - v_\alpha^{\mathrm{u}}(u) + 1 - w_\alpha^\ell(u) + 1 - w_\alpha^{\mathrm{u}}(u)\right)$
2. $\mathbb{A}(\wp_{\mathfrak{R}}) = \frac{1}{4m}\left(4 + u_\alpha^\ell(u) + u_\alpha^{\mathrm{u}}(u) - w_\alpha^\ell(u) - w_\alpha^{\mathrm{u}}(u)\right)$
3. $\mathbb{C}(\wp_{\mathfrak{R}}) = \frac{1}{2m}\left(2 + u_\alpha^\ell(u) + u_\alpha^{\mathrm{u}}(u)\right)$, where $\alpha = 1, 2, \cdots, m$.

Definition 4.6 Let $\wp_{\Re}$ and $\wp_{\Re_1}$ be two mPIVNSSs. Then, the comparison approach is presented as follows:

1. If $\mathbb{S}(\wp_{\Re}) > \mathbb{S}(\wp_{\Re_1})$, then $\wp_{\Re}$ is superior to $\wp_{\Re_1}$.
2. If $\mathbb{S}(\wp_{\Re}) = \mathbb{S}(\wp_{\Re_1})$ and $\mathbb{A}(\wp_{\Re}) > \mathbb{A}(\wp_{\Re_1})$, then $\wp_{\Re}$ is superior to $\wp_{\Re_1}$.
3. If $\mathbb{S}(\wp_{\Re}) = \mathbb{S}(\wp_{\Re_1})$, $\mathbb{A}(\wp_{\Re}) = \mathbb{A}(\wp_{\Re_1})$, and $\mathbb{C}(\wp_{\Re}) > \mathbb{C}(\wp_{\Re_1})$, then $\wp_{\Re}$ is superior to $\wp_{\Re_1}$.
4. If $\mathbb{S}(\wp_{\Re}) = \mathbb{S}(\wp_{\Re_1})$, $\mathbb{A}(\wp_{\Re}) > \mathbb{A}(\wp_{\Re_1})$, and $\mathbb{C}(\wp_{\Re}) = \mathbb{C}(\wp_{\Re_1})$, then $\wp_{\Re}$ is indifferent to $\wp_{\Re_1}$, can be denoted as $\wp_{\Re} \sim \wp_{\Re_1}$.

5. Decision-making approach based mPIVNSWA for mPIVNSS

Assume a set of "s" alternatives such as $\beta = \{\beta^1, \beta^2, \beta^3, \dots, \beta^s\}$ for assessment under the team of experts such as $\mathcal{U} = \{u_1, u_2, u_3, \dots, u_n\}$ with weights $\Omega = (\Omega_1, \Omega_1, \dots, \Omega_n)^T$ such that $\Omega_i > 0$, $\sum_{i=1}^{n} \Omega_i = 1$. Let $\mathcal{E} = \{e_1, e_2, \dots, e_m\}$ be a set of attributes with weights $\gamma = (\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_m)^T$ be a weight vector for parameters such as $\gamma_j > 0$, $\sum_{j=1}^{m} \gamma_j = 1$. The team of experts $\{u_i$: $i = 1, 2, \dots, n\}$ evaluate the alternatives $\{\beta^{(z)}$: $z = 1, 2, \dots, s\}$ under the considered parameters $\{e_j$: $j = 1, 2, \dots, m\}$ given in the form of mPIVNSNs $\mathcal{L}_{ij}^{(z)} = \left(u_{\alpha_{ij}}^{(z)}, v_{\alpha_{ij}}^{(z)}, w_{\alpha_{ij}}^{(z)}\right)$, where $u_{\alpha_{ij}}^{(z)} = \left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, $v_{\alpha_{ij}}^{(z)} = \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, and $w_{\alpha_{ij}}^{(z)} = \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u)\right]$, here $0 \leq u_{\alpha}^{\ell}(u), u_{\alpha}^{\mathrm{u}}(u)$, $v_{\alpha}^{\ell}(u), v_{\alpha}^{\mathrm{u}}(u)$, $w_{\alpha}^{\ell}(u), w_{\alpha}^{\mathrm{u}}(u) \leq 1$ and $0 \leq u_{\alpha_{ij}}^{\mathrm{u}}(u) + v_{\alpha_{ij}}^{\mathrm{u}}(u) + w_{\alpha_{ij}}^{\mathrm{u}}(u) \leq 3$. So $\Delta_k = \left(\left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u)\right]\right)$ for all i, j. Experts give their preferences for each alternative in terms of mPIVNSNs by using the mPIVNSWA operator in the form of $\Delta_k = \left(\left[u_{\alpha_{ij}}^{\ell}(u), u_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[v_{\alpha_{ij}}^{\ell}(u), v_{\alpha_{ij}}^{\mathrm{u}}(u)\right], \left[w_{\alpha_{ij}}^{\ell}(u), w_{\alpha_{ij}}^{\mathrm{u}}(u)\right]\right)$. Compute the score values for each alternative and analyze the ranking of the alternatives.

5.1 Algorithm for mPIVNSWA operator

Step 1. Develop the m-polar interval-valued neutrosophic soft matrix for each alternative.
Step 2. Aggregate the mPIVNSNs for each alternative into a collective decision matrix Δ_k by using the mPIVNSWA operator.
Step 3. Compute the score value for each alternative Δ_k, where $k = 1, 2, \cdots, s$."
Step 4. Choose the best alternative $\beta^{(k)}$.
Step 5. Alternatives ranking.
A flow chart of the above-presented model is given in the following Figure 1.

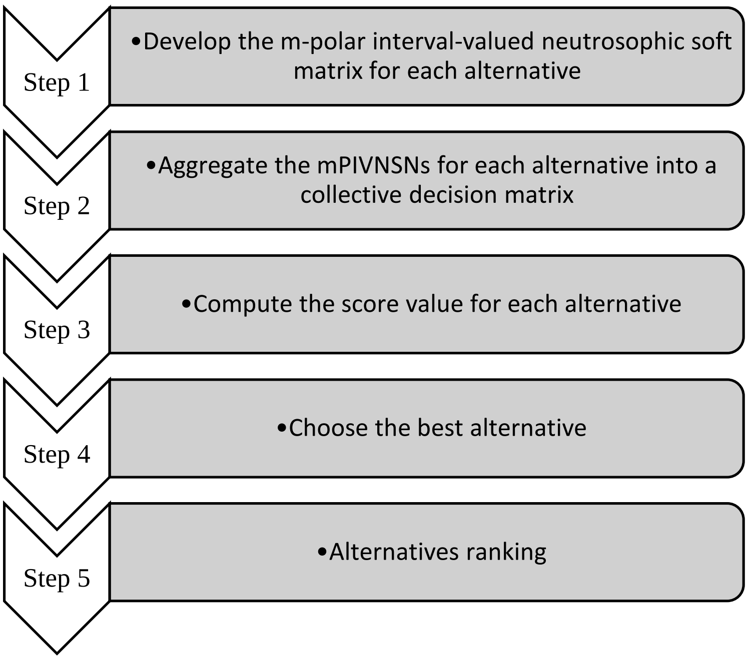

Figure 1: Flowchart of the proposed model

5.2. Application of the Proposed Model in Decision Making

This section utilized the developed approach based on the mPIVNSWA operator for decision-making.

5.2.1. Numerical Example

A university calls for the appointment of a vacant position of associate professor. For further assessment, four candidates (alternatives) chooses after preliminary review such as $\{\beta^{(1)}, \beta^{(2)}, \beta^{(3)}, \beta^{(4)}\}$. The president of the institution {has hired a team of three experts$\mathcal{u}_1, \mathcal{u}_2, \mathcal{u}_3\}$ with weights $(0.25, 0.30, 0.45)^T$ for final scrutiny. First of all, the group of experts decides the parameters for the selection of the candidate, such as e_1 = experience, e_2 = publications, and e_3 = research quality with weights $(0.35, 0.25, 0.40)^T$. Each expert gives preferences for each alternative in mPIVNSNs under the considered parameters. The developed methods to find the best alternative for the position of associate professor are presented in 5.1.

5.2.2. Applications of proposed approaches.

Assume $\{\beta^{(1)}, \beta^{(2)}, \beta^{(3)}, \beta^{(4)}\}$ be a set of alternatives which are shortlisted for interview and $\mathcal{E}$ = $\{e_1$ = experience, e_2 = publications, e_3= research quality$\}$ be a set of parameters for the selection of associate professor. Let $\mathfrak{R}$ and $\mathcal{L} \subseteq \mathcal{E}$. Then we construct the 3-PIVNSS $\wp_{\mathfrak{R}}(e)$ according to the requirement of university management such as follows:

Step 1. The experts will evaluate the condition in the case of mPIVNSNs. There are just four alternatives; parameters and a summary of their scores given in Tables 2, 3, 4, 5.

Table 1. Construction of 3-PIVNSS of Alternatives According to Management Requirement

$\wp_{\mathfrak{R}}(e)$	e_1	e_2	e_3

u_1	([.3,.5],[.2,.4],[.2,.6]), ([.2,.3],[.5,.7],[.1,.3]), ([.5,.6],[.1,.3],[.4,.6])	([.2,.4],[.3,.5],[.3,.6]), ([.2,.3],[.2,.4],[.4,.5]), ([.4,.6],[.1,.3],[.2,.4])	([.6,.7],[.2,.3],[.3,.4]), ([.4,.5],[.5,.8],[.1,.2]), ([.1,.2],[.5,.8],[.2,.4])
u_2	([.5,.7],[.1,.2],[.4,.6]), ([.2,.4],[.3,.4],[.2,.5]) ([.6,.8],[.1,.2],[.3,.5])	([.5,.6],[.2,.3],[.3,.4]), ([.4,.6],[.4,.5],[.3,.5]), ([.3,.5],[.4,.5],[.1,.3])	([.5,.7],[.1,.2],[.5,.6]), ([.2,.4],[.5,.6],[.4,.6]), ([.2,.4],[.3,.4],[.2,.5])
u_3	([.4 .6],[.2,.3],[.1,.4]), ([.2,.5],[.2,.3],[.1,.6]), ([.3,.4],[.2,.5],[.5,.7])	([.3,.5],[.4,.5],[.1,.3]), ([.2,.4],[.7,.8],[.1,.2]), ([.1,.2],[.7,.8],[.2,.3])	([.2,.3],[.5,.7],[.1,.3]), ([.3,.4],[.2,.5],[.5,.7]), ([.2,.4],[.3,.5],[.3,.6])

Construct the 3-PIVNSS $\wp_{\mathcal{L}}^{(t)}(e)$ for each alternative according to experts, where t = 1, 2, 3, 4.

Table 2. Evaluation Report for Alternative $\beta^{(1)}$

$\wp_{\mathcal{L}}^{(1)}(e)$	e_1	e_2	e_3
u_1	([.2,.4],[.4,.5],[.3,.4]), ([.6,.7],[.1,.2],[.2,.3]), ([.3,.4],[.4,.5],[.2,.4])	([.3,.4],[.4,.5],[.2,.5]), ([.3,.6],[.2,.3],[.1,.2]), ([.4,.6],[.2,.3],[.4,.5])	([.2,.4],[.4,.6],[.1,.2]), ([.1,.3],[.6,.7],[.2,.3]), ([.4,.5],[.2,.5],[.2,.3])
u_2	([.5,.7],[.1,.2],[.2,.4]), ([.7,.8],[.1,.2],[.2,.4]) ([.1,.3],[.1,.5],[.2,.5])	([.1,.4],[.2,.4],[.1,.2]), ([.2,.5],[.2,.4],[.3,.5]), ([.3,.5],[.2,.4],[.4,.6])	([.5,.7],[.1,.2],[.5,.6]), ([.3,.5],[.3,.4],[.6,.7]), ([.2,.4],[.3,.4],[.2,.5])
u_3	([.4,.5],[.2,.5],[.1,.2]), ([.4,.7],[.1,.2],[.1,.2]), ([.3,.4],[.2,.5],[.5,.7])	([.6,.8],[.1,.2],[.1,.5]), ([.2,.4],[.7,.8],[.1,.2]), ([.5,.7],[.1,.2],[.2,.4])	([.5,.6],[.2,.3],[.4,.5]), ([.3,.4],[.4,.5],[.2,.4]), ([.2,.4],[.3,.5],[.3,.6])

Table 3. Evaluation Report for Alternative $\beta^{(2)}$

$\wp_{\mathcal{L}}^{(2)}(e)$	e_1	e_2	e_3
u_1	([.2,.4],[.4,.6],[.4,.5]), ([.2,.3],[.4,.6],[.3,.5]), ([.1,.2],[.6,.8],[.2,.5])	([.4,.5],[.2,.5],[.1,.2]), ([.2,.3],[.4,.6],[.3,.5]), ([.1,.2],[.6,.8],[.2,.5])	([.7,.8],[.1,.2],[.2,.3]), ([.1,.3],[.6,.7],[.2,.5]), ([.4,.5],[.2,.5],[.1,.2])
u_2	([.5,.7],[.1,.2],[.2,.4]), ([.1,.3],[.6,.7],[.2,.6]) ([.1,.4],[.2,.5],[.4,.6])	([.1,.4],[.2,.4],[.1,.2]), ([.1,.2],[.2,.5],[.4,.6]), ([.1,.4],[.2,.5],[.4,.6])	([.1,.4],[.2,.5],[.4,.6]), ([.3,.4],[.2,.6],[.4,.6]), ([.2,.4],[.3,.4],[.2,.5])
u_3	([.4,.5],[.2,.5],[.1,.2]), ([.1,.2],[.2,.5],[.4,.6]), ([.3,.5],[.3,.5],[.6,.7])	([.3,.5],[.3,.5],[.6,.7]), ([.1,.2],[.2,.5],[.4,.6]), ([.5,.7],[.1,.2],[.2,.4])	([.2,.4],[.4,.5],[.6,.8]), ([.3,.5],[.3,.5],[.6,.7]), ([.1,.2],[.2,.5],[.4,.6])

Table 4. Evaluation Report for Alternative $\beta^{(3)}$

$\wp_{\mathcal{L}}^{(3)}(e)$	e_1	e_2	e_3

u_1	([.6,.7], [.1,.2], [.3,.5]), ([.6,.8], [.1,.2], [.2,.3]), ([.6,.7], [.3,.5], [.1,.2])	([.7,.8], [.1,.2], [.2,.5]), ([.6,.7], [.1,.2], [.1,.2]), ([.5,.8], [.1,.2], [.2,.4])	([.1,.3], [.6,.7], [.2,.5]), ([.7,.8], [.1,.2], [.2,.3]), ([.5,.7], [.3,.4], [.2,.3])
u_2	([.5,.7], [.2,.5], [.2,.3]), ([.7,.8], [.3,.5], [.1,.3]) ([.4,.7], [.2,.3], [.3,.7])	([.5,.6], [.3,.4], [.1,.2]), ([.1,.2], [.2,.5], [.4,.6]), ([.4,.6], [.2,.3], [.1,.2])	([.1,.4], [.2,.5], [.4,.6]), ([.4,.6], [.2,.3], [.1,.2]]), ([.2,.4], [.3,.4], [.2,.5])
u_3	([.4,.6], [.2,.3], [.1,.2]), ([.1,.2], [.2,.5], [.4,.6]), ([.6,.8], [.1,.2], [.1,.3])	([.3,.5], [.3,.5], [.6,.7]), ([.6,.8], [.1,.2], [.1,.2]), ([.7,.8], [.1,.2], [.2,.4])	([.6,.8], [.3,.4], [.1,.2]), ([.5,.7], [.1,.2], [.4,.5]), ([.1,.2], [.2,.5], [.4,.6])

Table 5. Evaluation Report for Alternative $\beta^{(4)}$

$\wp_{\mathcal{L}}^{(4)}(e)$	e_1	e_2	e_3
u_1	([.3,.5], [.2,.4], [.1,.2]), ([.3,.6], [.1,.2], [.4,.7]), ([.4,.7], [.3,.4], [.2,.3])	([.7,.8], [.2,.4], [.3,.5]), ([.5,.7], [.3,.4], [.2,.4]), ([.4,.6], [.2,.5], [.3,.4])	([.2,.3], [.5,.7], [.2,.4]), ([.5,.7], [.2,.4], [.3,.5]), ([.4,.5], [.5,.7], [.2,.4])
u_2	([.4,.7], [.3,.5], [.2,.4]), ([.5,.8], [.3,.6], [.2,.3]) ([.4,.6], [.2,.3], [.3,.5])	([.5,.8], [.3,.4], [.2,.3]), ([.2,.4], [.2,.3], [.4,.5]), ([.3,.5], [.2,.3], [.3,.5])	([.2,.4], [.2,.3], [.3,.6]), ([.4,.6], [.2,.3], [.1,.2]]), ([.2,.4], [.3,.4], [.2,.5])
u_3	([.3,.5], [.3,.5], [.1,.2]), ([.1,.2], [.2,.5], [.4,.6]), ([.5,.7], [.2,.4], [.1,.3])	([.3,.5], [.4,.6], [.6,.7]), ([.5,.7], [.1,.2], [.4,.5]), ([.3,.5], [.2,.5], [.1,.3])	([.4,.6], [.3,.5], [.1,.2]), ([.6,.7], [.1,.2], [.3,.5]), ([.2,.5], [.2,.3], [.4,.6])

Step 2. The opinion of the experts for each alternative are aggregated by using equation 4.2. Hence, we get $\Delta_1 = \langle[.3144\ .5379], [.1819, .3711], [.2437, .3752]\rangle$, $\Delta_2 = \langle[.4569\ .6073], [.2813, .3947], [.2988, .4815]\rangle$, $\Delta_3 = \langle[.3303\ .4884], [.3018, .4429], [.4296, .5670]\rangle$, and $\Delta_4 = \langle[.3530\ .5200], [.2815, .4420], [.3546, .5037]\rangle$.
Step 3. Compute the score values for each alternative by using Definition 4.5 (1). $\mathbb{S}(\Delta_1) = .2045$, $\mathbb{S}(\Delta_2) = .2004$, $\mathbb{S}(\Delta_3) = .1709$, and $\mathbb{S}(\Delta_4) = .1828$.
Step 4. Therefore, the ranking of the alternatives is as follows $\mathbb{S}(\Delta_1) > \mathbb{S}(\Delta_2) > \mathbb{S}(\Delta_4) > \mathbb{S}(\Delta_3)$. So, $\beta^{(1)} > \beta^{(2)} > \beta^{(4)} > \beta^{(3)}$, hence, the alternative $\beta^{(1)}$ is the most suitable alternative for the position of associate professor.

6. Discussion and Comparative Analysis:

In the next section, we will discuss the proposed method's effectiveness, simplicity, flexibility, and good location. A brief comparative analysis of our proposed method and popular method.

6.1 Comparative Studies

This manuscript develops a new DM technology based on the mPIVNSWA operator using mPIVNSS. Compared with existing technologies, the developed method is more operative and provides appropriate results in MCDM problems. Through this scientific research and comparison, we realize that the results of the proposed method are more versatile than traditional methods. However, the DM process contains more information to deal with uncertain data than the current

DM method. Except that the hybrid structure of multiple FS becomes a particular case of mPIVNSS adds some appropriate conditions. Among them, the information related to the object can be displayed accurately and analytically, so mPIVNSS is an effective power tool to deal with inaccurate and uncertain information in the DM process. Therefore, our method is more suitable, flexible, and better than FS's unique and accessible hybrid structure.

Table 6: Comparative analysis between some existing techniques and the proposed approach

	Set	Truthiness	Indeterminacy	Falsity	Multi-polarity	Loss of information
Chen et al. [33]	mPFS	✓	×	×	✓	×
Xu et al. [38]	IFS	✓	×	✓	×	×
Zhang et al. [39]	IFS	✓	×	✓	×	✓
Talebi et al. [42]	mPIVIFS	✓	×	✓	✓	✓
Yager [40, 41]	PFS	✓	×	✓	×	×
Naeem et al. [43]	mPyFS	✓	×	✓	✓	×
Zhang et al. [44]	INSs	✓	✓	✓	×	×
Ali et al. [35]	BPNSS	✓	✓	✓	×	×
Proposed approach	mPIVNSS	✓	✓	✓	✓	×

7. Conclusion

This manuscript establishes a new hybrid structure, mPIVNSS, by combining two independent structure m-pole fuzzy sets and interval-valued neutrosophic soft sets. Several basic operations have been introduced for mPIVNSS, and their ideal characteristics have been discussed. In addition, we developed the algorithm of mPIVNSS and used the proposed algorithm to establish a neutrosophic weighted aggregation operator for m-polar interval-valued. A decision-making method was developed to solve the MCDM problem by using our mPIVNSWA operator. A comparative analysis was also carried out to prove the proposed method. Finally, the proposed technique shows higher stability and practicality for decision-makers in the decision-making process. Based on the results obtained, it can be concluded that this method is most suitable for solving the MCDM problem in today's life. We will apply this technique to other fields in future work, such as mathematical programming, cluster analysis, etc.

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