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A Novel Approach by using Interval-Valued Trapezoidal Neutrosophic Numbers in Transportation Problem

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Abstract: In today's scenario transportation problem [TP] is the prominent area of optimization. In the present paper, a TP in a neutrosophic environment, known as a neutrosophic transportation problem [NTP] is introduced with interval-valued trapezoidal neutrosophic numbers [IVTrNeNs]. To maintain physical distance among the industrialists and researchers during the covid-19 pandemic, the interval-valued fuzzy numbers [IVFNs] in place of crisp numbers are very much essential to address the hesitation and uncertainty in real-life situations. IVTrNeN is the generalization of single-valued neutrosophic numbers [SVNeN], which are used as the cost, the demand, and the supply to transport the necessary equipment, medicines, food products, and other relevant items from one place to another to save the human lives in a covid-19 pandemic. A Neutrosophic set, which has uncertainty, inconsistency, and incompleteness information is the abstract principle of crisp, fuzzy, and intuitionistic fuzzy sets. Here we suggest some numerical problems for better execution of the neutrosophic transportation problem [NTP], to understand the practical applications of interval-valued neutrosophic numbers [IVNeNs]. In the last, we compare our results and a conclusion is given in support of our proposed result methodology with IVTrNeNs.

Keywords: Interval–valued trapezoidal neutrosophic number, De-neutrosophication, neutrosophic transportation problem.

1. Introduction

In the current scenario of covid-19, the role of a neutrosophic optimization technique in TP has fascinated awareness of their high efficiency, accuracy, and adaptability that gives high standard real-life outcomes. Neutrosophic optimization has been extremely searched in industrial, management, engineering, and health sectors. Zadeh in 1965 introduced the mathematical formula of fuzzy set FS [1] by which the researchers try to check the ambiguity or uncertainty in engineering, industrial, and management problems [2, 3]. In realistic problems, the FS was not perfect to observe the uncertainty and hesitation. To encompass this problem, Atanassov extended the FS and introduced a set with membership and non-membership degrees, called an intuitionistic fuzzy set IFS [4]. For more detailed applications of IFS, please (see [5-10] and references therein). Atanassov and Gargov generalized the IFS by introducing the interval-valued IFS to strengthen the attitude of grasp uncertainty and hesitation in IFS [11]. To solve the real-world problems with inconsistent information or contain indeterminacy in data the FS and IFS are not sufficient. To rectify such problems, Smarandache in 1988 introduced the neutrosophic set [NS] [12], by which the inconsistent information is in the form of truth-membership, indeterminacy-membership, and falsity-membership degrees respectively. For practical applications and some technical references in real-world problems, NSs are difficult to apply, so the notion of a single-valued neutrosophic set [SVNeS] was imported by Wang et. al. in 2010 [13]. The idea of SVNeS is more suitable and effective in solving many real-life problems of decision-makers that contain uncertainty in data by using fuzzy numbers and intuitionistic fuzzy numbers. Since in the real world, there exists stipulated and non-stipulated knowledge, so to overcome such problems Samaranache introduced the neutrosophic number [NN] [14, 15]. In 2016, Ye. proposed de-neutrosophic and possibility degree ranking methods for the application of NNs [16]. Samrandache in 2015, proposed the interval function to describe the stipulated and non-stipulated issues in real-world problems [17]. For more uncertain linear programming problems (see [18-29] and references therein).

In real-life optimization problems, the TP shows high execution and due to its clarity and minimum cost, it is a noted optimization technique in the current scenario. The basic theme of a TP is to find a direct connection between source and destination in minimum time with minimum cost. Hitchcock introduced the initial basic structure of TP and developed a special mathematical module for the basic results of TP by the simplex method [30]. For more recent development in fuzzy transportation problem [FTP] (see [31-47] and references therein).

The IFS theory can handle the problems of incomplete information but not the indeterminate and inconsistent information that exists in the transportation modal. The TP with inconsistent information or indeterminate data i.e. in fuzzy numbers or intuitionistic fuzzy numbers cannot be handled in the current structure. To resolve such issues, the NTP is the best option with indeterminacy and inconsistent information by truth, indeterminacy, and falsity membership degree function. Many researchers formulated efficient mathematical models in various uncertain environments. We proposed the NTP of type-4, with all entries such as cost, demand, and supply termed as IVTrNeNs, which include membership, indeterminacy, and non-memberships degree function. The more real-world developments in the field of neutrosophic optimization problems (see [48-63] and references therein).

For the solution of NTP, the first one will change it into a crisp transportation problem [CTP] by converting the cost, demand, and supply, which are in IVTrNeNs into crisp values with the help of the introduced ranking method. For unbalanced CTP or NTP, here we use Vogel's approximation method [VAM] and minimum row-column method [MRCM] to solve these by excel solver and then compare our results [46]. The paper is well organized in several sections such as the introduction of the present paper with some earlier research are given, the basics concepts of FS, IFS, and NS are discussed and reviewed, introduce the ranking function, score function, and de-neutrosophication to convert neutrosophic values into crisp values and vice-versa. Here we proposed CTP & NTP of type-4, their solution by existing and MRCM, comparison, and the conclusion for future aspects of research work.

2. Preliminaries

Definition 2.1 ([39]): A FS $\tilde{A}$ of a non empty set X is defined as $\tilde{A}=\left\{\left\langle x,\mu_{\tilde{A}}(x)\right\rangle / x\in X\right\}$ where $\mu_{\tilde{A}}(x): X\rightarrow[0,1]$ is the membership function.

Definition 2.2: A fuzzy number on the universal set $\boldsymbol{R}$ is a convex, normalized fuzzy set $\tilde{A}$, where the membership function $\mu_{\tilde{A}}(x): X\rightarrow[0,1]$ is continuous, strictly increasing on [a, b] and strictly decreasing on [c, d] , $\mu_{\tilde{A}}(x)=1,$ for all $x\in\left[b,c\right]$, where $a\le b\le c\le d$ and $\mu_{\tilde{A}}(x)=0,$ for all $x\in\left(-\infty,a\right]\cup\left[d,\infty\right)$.

Definition 2.3 ([52]): A trapezoidal fuzzy number (TrFN) denoted as $\tilde{A}=(a,b,c,d)$, with its membership function $\mu_{\tilde{A}}(x)$ on $\boldsymbol{R}$, is given by

$$\mu_{\tilde{A}}(x)=\begin{cases}(x-a)/(b-a), & \text{for } a\le x<b\\ 1, & \text{for } b\le x<c\\ (d-x)/(d-c), & \text{for } c<x\le d\\ 0, & \text{otherwise}\end{cases}$$

If $b=c$ in TrFN $\tilde{A}=(a,b,c,d)$, then it becomes TFN $\tilde{A}=(a,b,c)$.

Definition 2.4: An IFS in a non-empty set X is denoted by $\tilde{A}^I$ and defined as $\tilde{A}^I=\left\{\left\langle x,\mu_{\tilde{A}^I},\nu_{\tilde{A}^I}\right\rangle : x\in X\right\}$, where $\mu_{\tilde{A}^I},\nu_{\tilde{A}^I}: X\to[0,1]$, are denoted as degree of membership and degree of non-membership functions respectively. The function $h(x)=1-\mu_{\tilde{A}^I}-\nu_{\tilde{A}^I}\le 1, \forall\, x\in X$ called the degree of hesitancy in $\tilde{A}^I$.

The single valued neutrosophic numbers [SVNN] introduced by Deli and Suba [64] in 2014.

Definition 2.5: A SVNS is denoted and defined as $\tilde{A}_N=\{x,T_{\tilde{A}_N}(x),I_{\tilde{A}_N}(x),F_{\tilde{A}_N}(x)/x\in X\}$, where for each generic point x in X, $T_{\tilde{A}_N}(x)$ called truth membership function, $I_{\tilde{A}_N}(x)$ called indeterminacy membership function and $F_{\tilde{A}_N}(x)$ called falsity membership function in [0,1] and $0\le T_{\tilde{A}_N}(x)+I_{\tilde{A}_N}(x)+F_{\tilde{A}_N}(x)\le 3$. For continuous SVNS $\tilde{A}_N=\int_{\tilde{A}_N}\left\langle T_{\tilde{A}_N}(x),I_{\tilde{A}_N}(x),F_{\tilde{A}_N}(x)\right\rangle/x,\ x\in X$. For discrete values, SVNS can be written as $\tilde{A}_N=\sum_{i=1}^{n}\left\langle T_{\tilde{A}_N}(x_i),I_{\tilde{A}_N}(x_i),F_{\tilde{A}_N}(x_i)\right\rangle/x_i,\ x_i\in X.$

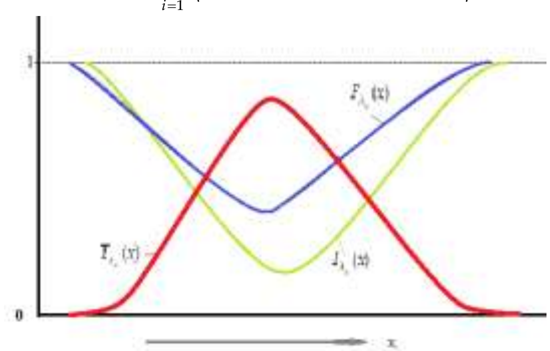

Fig. 1: Neutrosophic set

Definition 2.6 ([15]): Let x be a generic element of a non empty set X. A neutrosophic number $\tilde{A}_N$ in X is defined as $\tilde{A}_N=\{\langle x,T_{\tilde{A}_N}(x),I_{\tilde{A}_N}(x),F_{\tilde{A}_N}(x)\rangle/x\in X\}$, $\forall\ T_{\tilde{A}_N}(x)$, $I_{\tilde{A}_N}(x)$ and $F_{\tilde{A}_N}(x)$ belongs $]^-0,1^+[$ where $T_{\tilde{A}_N}: X\to]0^-,1^+[$, $I_{\tilde{A}_N}: X\to]0^-,1^+[$ and $F_{\tilde{A}_N}: X\to]0^-,1^+[$ are functions of truth-membership, indeterminacy membership and falsity-membership in $\tilde{A}_N$ respectively with $^-0\le T_{\tilde{A}_N}(x)+I_{\tilde{A}_N}(x)+F_{\tilde{A}_N}(x)\le 3^+$.

Definition 2.7 ([17]): Let X be a nonempty set. Then an interval-valued neutrosophic set [IVNS] $\tilde{A}_N^{IV}$ of X is defined as:

$$\tilde{A}_N^{IV}=\left\{\left\langle x;\left[T^L_{\tilde{A}_N},T^U_{\tilde{A}_N}\right],\left[I^L_{\tilde{A}_N},I^U_{\tilde{A}_N}\right],\left[F^L_{\tilde{A}_N},F^U_{\tilde{A}_N}\right]\right\rangle :\ x\in X\right\}$$

where $\left[T^L_{\tilde{A}_N}, T^U_{\tilde{A}_N}\right], \left[I^L_{\tilde{A}_N}, I^U_{\tilde{A}_N}\right]$ and $\left[F^L_{\tilde{A}_N}, F^U_{\tilde{A}_N}\right] \subset [0,1]$ $\forall x \in X$. $T^L_{\tilde{A}_N} = \inf\left(T_{\tilde{A}_N}\right)$, $T^U_{\tilde{A}_N} = \sup\left(T_{\tilde{A}_N}\right)$; $I^L_{\tilde{A}_N} = \inf\left(I_{\tilde{A}_N}\right)$, $I^U_{\tilde{A}_N} = \sup\left(I_{\tilde{A}_N}\right)$ and $F^L_{\tilde{A}_N} = \inf\left(F_{\tilde{A}_N}\right)$, $F^U_{\tilde{A}_N} = \sup\left(F_{\tilde{A}_N}\right)$.

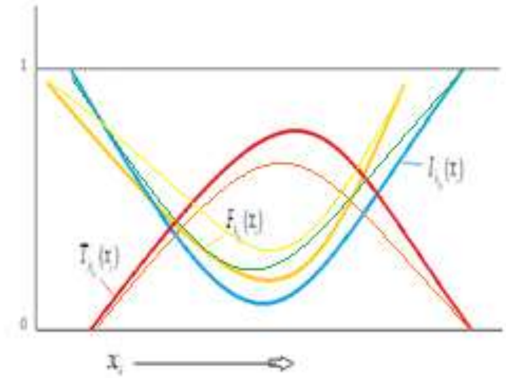

Fig. 2: Interval-valued neutrosophic set

Definition 2.8: Let $\tilde{A}^{IV}_N = \left\{\left\langle x; \left[T^L_{\tilde{A}_N}, T^U_{\tilde{A}_N}\right], \left[I^L_{\tilde{A}_N}, I^U_{\tilde{A}_N}\right], \left[F^L_{\tilde{A}_N}, F^U_{\tilde{A}_N}\right]\right\rangle : x \in X\right\}$ be IVNS, then

(i) $\tilde{A}^{IV}_N$ is empty if $T^L_{\tilde{A}_N} = T^U_{\tilde{A}_N} = 0, I^L_{\tilde{A}_N} = I^U_{\tilde{A}_N} = 1, F^L_{\tilde{A}_N} = F^U_{\tilde{A}_N} = 1$, $\forall x \in X$.

(ii) let $\underline{0} = \langle x, 0, 1, 1\rangle$ and $\langle x, 1, 0, 0\rangle$.

The interval-valued numbers and their operational properties are most valuable to survey for interval-valued neutrosophic numbers [IVNeNs]. Here we are given some impotent operations & facts about Interval valued numbers.

Definition 2.9 ([65]): An interval on R is defined as $A = [a^L, a^R] = \{a : a^L \le a \le a^R, a \in R\}$, where a^L in left limit and a^R is the right limit of *A*, it may also be defined as $A = \langle a_c, a_w\rangle = \{a : a_c - a_w \le a \le a_c + a_w, a \in R\}$, where $a_c = \dfrac{(a^R + a^L)}{2}$ in centre $a_w = \dfrac{(a^R - a^L)}{2}$ is width of *A*.

Definition 2.10 ([66, 67): Let $A(x) = [a^L, a^U] = \{x : a^L \le x \le a^U\}$, then $A(x)$ is called an interval number. $A(x)$ is positive interval if $0 \le a^L \le x \le a^U$. Let $A(x) = [a^L, a^U]$ and $B(x) = [b^L, b^U]$ be two interval numbers, then the following operational properties are holds:

(i) $A = B \Leftrightarrow a^L = b^L,\ a^U = b^U$;

(ii) $A + B = [a^L + b^L,\ a^U + b^U]$; $A - B = [a^L - b^U,\ a^U - b^L]$;

(iii) $A \times B \Leftrightarrow [min\{a^L b^L,\ a^L b^U, a^U b^L,\ a^U b^U\}; \max\{a^L b^L,\ a^L b^U, a^U b^L,\ a^U b^U\}]$;

(iv) $\mu A = [\mu a^L, \mu a^U],\ \mu > 0$.

Definition 2.11 ([35]): The interval-valued trapezoidal neutrosophic number [IVTrNeN]) is a special case of NS on the real line **R.** Let $a_1, a_2, a_3, a_4 \in R$ such that $a_1 \le a_2 \le a_3 \le a_4$ then

$$\tilde{a}^{IV}_N = \left\langle (a_1, a_2, a_3, a_4); \left[u^L_{\tilde{a}^{IV}_N}, u^U_{\tilde{a}^{IV}_N}\right], \left[v^L_{\tilde{a}^{IV}_N}, v^U_{\tilde{a}^{IV}_N}\right], \left[w^L_{\tilde{a}^{IV}_N}, w^U_{\tilde{a}^{IV}_N}\right]\right\rangle,$$

is IVTrNeN, where $\left[u^L_{\tilde{a}^{IV}_N}, u^U_{\tilde{a}^{IV}_N}\right]$ are upper and lower bound of the truth-membership degree function $u_{\tilde{a}^{IV}_N}$, $\left[v^L_{\tilde{a}^{IV}_N}, v^U_{\tilde{a}^{IV}_N}\right]$ are upper and lower bound of the indeterminacy-membership degree function $v_{\tilde{a}^{IV}_N}$ and

$\left[w^L_{\tilde{a}^{IV}_N}, w^U_{\tilde{a}^{IV}_N}\right]$ are the upper and lower bound of the falsity-membership degree function $w_{\tilde{a}^{IV}_N}$ in [0,1] respectively, whose truth-membership $T_{\tilde{a}^{IV}_N}(x)$, indeterminacy-membership $I_{\tilde{a}^{IV}_N}(x)$, and a falsity-membership $F_{\tilde{a}^{IV}_N}(x)$ are defined as follows:

$$T_{\tilde{a}^{IV}_N}(x)=\begin{cases} u_{\tilde{a}^{IV}_N}\left(\dfrac{x-a_1}{a_2-a_1}\right), & \text{for } a_1 \le x \le a_2, \\ u_{\tilde{a}^{IV}_N}, & \text{for } a_2 \le x \le a_3, \\ u_{\tilde{a}^{IV}_N}\left(\dfrac{a_4-x}{a_4-a_3}\right), & \text{for } a_3 \le x \le a_4, \\ 0, & \text{for } x<a_1 \text{ and } x>a_4. \end{cases} \qquad I_{\tilde{a}^{IV}_N}(x)=\begin{cases} \dfrac{a_2-x+v_{\tilde{a}^{IV}_N}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \le x \le a_2, \\ v_{\tilde{a}^{IV}_N}, & \text{for } a_2 \le x \le a_3, \\ \dfrac{x-a_3+v_{\tilde{a}^{IV}_N}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \le x \le a_4, \\ 1, & \text{for } x<a_1 \text{ and } x>a_4. \end{cases}$$

$$F_{\tilde{a}^{IV}_N}(x)=\begin{cases} \dfrac{a_2-x+w_{\tilde{a}^{IV}_N}(x-a_1)}{a_2-a_1}, & \text{for } a_1 \le x \le a_2, \\ w_{\tilde{a}^{IV}_N}, & \text{for } a_2 \le x \le a_3, \\ \dfrac{x-a_3+w_{\tilde{a}^{IV}_N}(a_4-x)}{a_4-a_3}, & \text{for } a_3 \le x \le a_4, \\ 1, & \text{for } x<a_1 \text{ and } x>a_4. \end{cases}$$

when $a_1>0$, $\tilde{a}^{IV}_N=\left\langle (a_1,a_2,a_3,a_4);\left[u^L_{\tilde{a}^{IV}_N},u^U_{\tilde{a}^{IV}_N}\right],\left[v^L_{\tilde{a}^{IV}_N},v^U_{\tilde{a}^{IV}_N}\right],\left[w^L_{\tilde{a}^{IV}_N},w^U_{\tilde{a}^{IV}_N}\right]\right\rangle$, is called positive IVTrNeN, denoted by $\tilde{a}^{IV}_N>0$, and if $a_4 \le 0$, then $\tilde{a}^{IV}_N$ becomes a negative IVTrNeN, denoted by $\tilde{a}^{IV}_N>0$. If $a_2=a_3$, then IVTrNeN is reduces interval-valued triangular neutrosophic number [IVTriNeN], denoted as $\tilde{a}^{IV}_N=\left\langle (a_1,a_2,a_3);\left[u^L_{\tilde{a}^{IV}_N},u^U_{\tilde{a}^{IV}_N}\right],\left[v^L_{\tilde{a}^{IV}_N},v^U_{\tilde{a}^{IV}_N}\right],\left[w^L_{\tilde{a}^{IV}_N},w^U_{\tilde{a}^{IV}_N}\right]\right\rangle$.

On the basis of [8, 41, 58], we will take here the twelve components of IVTrNeNs i.e. $\tilde{a}^{IV}_{N_1}=\left\langle\left[\left\{(a_1,b_1,c_1,d_1);u_{\tilde{a}^{IV}_{N_1}}\right\},\left\{(e_1,f_1,g_1,h_1);v_{\tilde{a}^{IV}_{N_1}}\right\},\left\{(l_1,m_1,n_1,p_1);w_{\tilde{a}^{IV}_{N_1}}\right\}\right]\right\rangle$ guided as $l_1 \le e_1 \le a_1 \le m_1 \le f_1 \le b_1 \le n_1 \le g_1 \le c_1 \le p_1 \le h_1 \le d_1$.

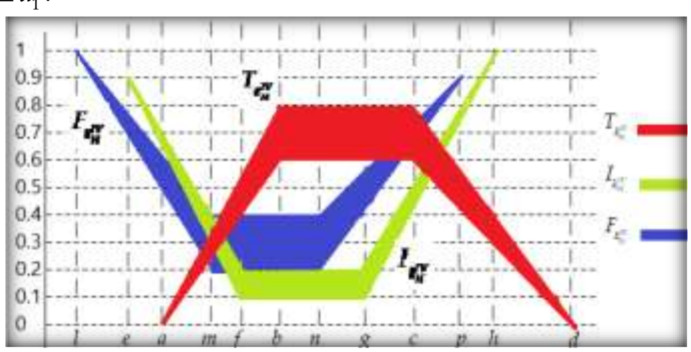

Fig. 3: Interval-valued trapezoidal neutrosophic number

2.1. Operational Laws on IVTrNeNs

Let $\tilde{a}^{IV}_{N_1}=\left\langle\left[\left\{(a_1,b_1,c_1,d_1);u_{\tilde{a}^{IV}_{N_1}}\right\},\left\{(e_1,f_1,g_1,h_1);v_{\tilde{a}^{IV}_{N_1}}\right\},\left\{(l_1,m_1,n_1,p_1);w_{\tilde{a}^{IV}_{N_1}}\right\}\right]\right\rangle$ and

$\tilde{a}_{N_2}^{IV} = \left\langle \left[\left\{ (a_2, b_2, c_2, d_2); u_{\tilde{a}_{N_2}^{IV}} \right\}, \left\{ (e_2, f_2, g_2, h_2); v_{\tilde{a}_{N_2}^{IV}} \right\}, \left\{ (l_2, m_2, n_2, p_2); w_{\tilde{a}_{N_2}^{IV}} \right\} \right] \right\rangle$ be two IVTrNeNs with twelve components, where $u_{\tilde{a}_{N_1}^{IV}} = \left[u^L_{\tilde{a}_{N_1}^{LIV}}, u^U_{\tilde{a}_{N_1}^{UIV}} \right]$; $u_{\tilde{a}_{N_2}^{IV}} = \left[u^L_{\tilde{a}_{N_2}^{LIV}}, u^U_{\tilde{a}_{N_2}^{UIV}} \right]$; $v_{\tilde{a}_{N_1}^{IV}} = \left[v^L_{\tilde{a}_{N_1}^{LIV}}, v^U_{\tilde{a}_{N_1}^{UIV}} \right]$; $v_{\tilde{a}_{N_2}^{IV}} = \left[v^L_{\tilde{a}_{N_2}^{LIV}}, v^U_{\tilde{a}_{N_2}^{UIV}} \right]$; and $w_{\tilde{a}_{N_1}^{IV}} = \left[w^L_{\tilde{a}_{N_1}^{LIV}}, w^U_{\tilde{a}_{N_1}^{UIV}} \right]$; $w_{\tilde{a}_{N_2}^{IV}} = \left[w^L_{\tilde{a}_{N_2}^{LIV}}, w^U_{\tilde{a}_{N_2}^{UIV}} \right]$, then the following operations hold:

Addition of IVTrNeNs:

$$\tilde{a}_{N_1}^{IV} + \tilde{a}_{N_2}^{IV} = \left\langle \begin{bmatrix} \left\{ (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); u_{\tilde{a}_{N_1}^{IV}} \wedge u_{\tilde{a}_{N_2}^{IV}} \right\}, \\ \left\{ (e_1 + e_2, f_1 + f_2, g_1 + g_2, h_1 + h_2); v_{\tilde{a}_{N_1}^{IV}} \vee v_{\tilde{a}_{N_2}^{IV}} \right\}, \\ \left\{ (l_1 + l_2, m_1 + m_2, n_1 + n_2, p_1 + p_2); w_{\tilde{a}_{N_1}^{IV}} \vee w_{\tilde{a}_{N_2}^{IV}} \right\} \end{bmatrix} \right\rangle$$

Negative of IVTrNeN:

$$-\tilde{a}_{N_2}^{IV} = \left\langle \left[\left\{ (-d_2, -c_2, -b_2, -a_2); u_{\tilde{a}_{N_2}^{IV}} \right\}, \left\{ (-h_2, -g_2, -f_2, -e_2); v_{\tilde{a}_{N_2}^{IV}} \right\}, \left\{ (-p_2, -n_2, -m_2, -l_2); w_{\tilde{a}_{N_2}^{IV}} \right\} \right] \right\rangle$$

Subtraction of IVTrNeNs:

$$\tilde{a}_{N_1}^{IV} - \tilde{a}_{N_2}^{IV} = \left\langle \begin{bmatrix} \left\{ (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2); u_{\tilde{a}_{N_1}^{IV}} \wedge u_{\tilde{a}_{N_2}^{IV}} \right\}, \\ \left\{ (e_1 - h_2, f_1 - g_2, g_1 - f_2, h_1 - e_2); v_{\tilde{a}_{N_1}^{IV}} \vee v_{\tilde{a}_{N_2}^{IV}} \right\}, \\ \left\{ (l_1 - p_2, m_1 - n_2, n_1 - m_2, p_1 - l_2); w_{\tilde{a}_{N_1}^{IV}} \vee w_{\tilde{a}_{N_2}^{IV}} \right\} \end{bmatrix} \right\rangle$$

Scalar multiplication of SVTrNeN:

$$\lambda.\tilde{a}_{N_1}^{IV} = \begin{Bmatrix} \left\langle \left\{ (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1); u_{\tilde{a}_{N_1}^{IV}} \right\}, \left\{ (\lambda e_1, \lambda f_1, \lambda g_1, \lambda h_1); v_{\tilde{a}_{N_1}^{IV}} \right\}, \left\{ (\lambda l_1, \lambda m_1, \lambda n_1, \lambda p_1); w_{\tilde{a}_{N_1}^{IV}} \right\} \right\rangle \text{ if } \lambda > 0 \\ \left\langle \left\{ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1); u_{\tilde{a}_{N_1}^{IV}} \right\}, \left\{ (\lambda h_1, \lambda g_1, \lambda f_1, \lambda e_1); v_{\tilde{a}_{N_1}^{IV}} \right\}, \left\{ (\lambda p_1, \lambda n_1, \lambda m_1, \lambda l_1); w_{\tilde{a}_{N_1}^{IV}} \right\} \right\rangle \text{if } \lambda < 0 \end{Bmatrix}$$

Multiplication of IVTrNeNs:

$$\tilde{a}_{N_1}^{IV}.\tilde{a}_{N_2}^{IV} = \begin{Bmatrix} \left\langle \begin{matrix} (a_1.a_2, b_1.b_2, c_1.c_2, d_1.d_2); (e_1.e_2, f_1.f_2, g_1.g_2, h_1.h_2); (l_1.l_2, m_1.m_2, n_1.n_2, p_1.p_2) \\ \left[u_{\tilde{a}_{N_1}^{IV}} \wedge u_{\tilde{a}_{N_2}^{IV}} \right], \left[v_{\tilde{a}_{N_1}^{IV}} \vee v_{\tilde{a}_{N_2}^{IV}} \right], \left[w_{\tilde{a}_{N_1}^{IV}} \vee w_{\tilde{a}_{N_2}^{IV}} \right] \text{ if } d_1 > 0, d_2 > 0, h_1 > 0, h_2 > 0, p_1 > 0, p_2 > 0 \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} (a_1.d_2, b_1.c_2, c_1.b_2, d_1.a_2); (e_1.h_2, f_1.g_2, g_1.f_2, h_1.e_2); (l_1.p_2, m_1.n_2, n_1.m_2, p_1.l_2) \\ \left[u_{\tilde{a}_{N_1}^{IV}} \wedge u_{\tilde{a}_{N_2}^{IV}} \right], \left[v_{\tilde{a}_{N_1}^{IV}} \vee v_{\tilde{a}_{N_2}^{IV}} \right], \left[w_{\tilde{a}_{N_1}^{IV}} \vee w_{\tilde{a}_{N_2}^{IV}} \right] \text{ if } d_1 < 0, d_2 > 0, h_1 < 0, h_2 > 0, p_1 < 0, p_2 > 0 \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} (d_1.d_2, c_1.c_2, b_1.b_2, a_1.a_2); (h_1.h_2, g_1.g_2, f_1.f_2, e_1.e_2); (p_1.p_2, n_1.n_2, m_1.m_2, l_1.l_2) \\ \left[u_{\tilde{a}_{N_1}^{IV}} \wedge u_{\tilde{a}_{N_2}^{IV}} \right], \left[v_{\tilde{a}_{N_1}^{IV}} \vee v_{\tilde{a}_{N_2}^{IV}} \right], \left[w_{\tilde{a}_{N_1}^{IV}} \vee w_{\tilde{a}_{N_2}^{IV}} \right] \text{ if } d_1 < 0, d_2 < 0, h_1 < 0, h_2 < 0, p_1 < 0, p_2 < 0 \end{matrix} \right\rangle, \end{Bmatrix}$$

Inverse of SVTrNeN:

$$(\tilde{a}_{N_1}^{IV})^{-1} = \frac{1}{\tilde{a}_{N_1}^{IV}} = \begin{Bmatrix} \left\langle \left(\frac{1}{d_1}, \frac{1}{c_1}, \frac{1}{b_1}, \frac{1}{a_1} \right); \left(\frac{1}{h_1}, \frac{1}{g_1}, \frac{1}{f_1}, \frac{1}{e_1} \right); \left(\frac{1}{p_1}, \frac{1}{n_1}, \frac{1}{m_1}, \frac{1}{l_1} \right); u_{\tilde{a}_{N_1}^{IV}}, v_{\tilde{a}_{N_1}^{IV}}, w_{\tilde{a}_{N_1}^{IV}} \right\rangle, \\ \text{if } a_1 > 0, b_1 > 0, c_1 > 0, d_1 > 0, e_1 > 0, f_1 > 0, g_1 > 0, h_1 > 0, l_1 > 0, m_1 > 0, n_1 > 0, p_1 > 0, \\ \left\langle \left(\frac{1}{a_1}, \frac{1}{b_1}, \frac{1}{c_1}, \frac{1}{d_1} \right); \left(\frac{1}{e_1}, \frac{1}{f_1}, \frac{1}{g_1}, \frac{1}{h_1} \right); \left(\frac{1}{l_1}, \frac{1}{m_1}, \frac{1}{n_1}, \frac{1}{p_1} \right); u_{\tilde{a}_{N_1}^{IV}}, v_{\tilde{a}_{N_1}^{IV}}, w_{\tilde{a}_{N_1}^{IV}} \right\rangle, \\ \text{if } a_1 < 0, b_1 < 0, c_1 < 0, d_1 < 0, e_1 < 0, f_1 < 0, g_1 < 0, h_1 < 0, l_1 < 0, m_1 < 0, n_1 < 0, p_1 < 0. \end{Bmatrix}$$

Division of SVTrNeNs:

$$\frac{\tilde{a}_{N_1}^{IV}}{\tilde{a}_{N_2}^{IV}}=\begin{cases}\left\langle\left(\frac{a_1}{d_1},\frac{b_1}{c_1},\frac{c_1}{b_1},\frac{d_1}{a_1}\right);\left(\frac{e_1}{h_1},\frac{f_1}{g_1},\frac{g_1}{f_1},\frac{h_1}{e_1}\right);\left(\frac{l_1}{p_1},\frac{m_1}{n_1},\frac{n_1}{m_1},\frac{p_1}{l_1}\right);\left[u_{\tilde{a}_{N_1}^{IV}}\wedge u_{\tilde{a}_{N_2}^{IV}}\right],\left[v_{\tilde{a}_{N_1}^{IV}}\vee v_{\tilde{a}_{N_2}^{IV}}\right],\left[w_{\tilde{a}_{N_1}^{IV}}\vee w_{\tilde{a}_{N_2}^{IV}}\right]\right\rangle, \\ \qquad \text{if } d_1>0,d_2>0,h_1>0,h_2>0,p_1>0,p_2>0, \\ \left\langle\left(\frac{d_2}{d_1},\frac{c_2}{c_1},\frac{b_2}{b_1},\frac{a_2}{a_1}\right);\left(\frac{h_2}{h_1},\frac{g_2}{g_1},\frac{f_2}{f_1},\frac{e_2}{e_1}\right);\left(\frac{p_2}{p_1},\frac{n_2}{n_1},\frac{m_2}{m_1},\frac{l_2}{l_1}\right);\left[u_{\tilde{a}_{N_1}^{IV}}\wedge u_{\tilde{a}_{N_2}^{IV}}\right],\left[v_{\tilde{a}_{N_1}^{IV}}\vee v_{\tilde{a}_{N_2}^{IV}}\right],\left[w_{\tilde{a}_{N_1}^{IV}}\vee w_{\tilde{a}_{N_2}^{IV}}\right]\right\rangle, \\ \qquad \text{if } d_1<0,d_2>0,h_1<0,h_2>0,p_1<0,p_2>0, \\ \left\langle\left(\frac{d_2}{a_1},\frac{c_2}{b_1},\frac{b_2}{c_1},\frac{a_2}{d_1}\right);\left(\frac{h_2}{e_1},\frac{g_2}{f_1},\frac{f_2}{g_1},\frac{e_2}{h_1}\right);\left(\frac{p_2}{l_1},\frac{n_2}{m_1},\frac{m_2}{n_1},\frac{l_2}{p_1}\right);\left[u_{\tilde{a}_{N_1}^{IV}}\wedge u_{\tilde{a}_{N_2}^{IV}}\right],\left[v_{\tilde{a}_{N_1}^{IV}}\vee v_{\tilde{a}_{N_2}^{IV}}\right],\left[w_{\tilde{a}_{N_1}^{IV}}\vee w_{\tilde{a}_{N_2}^{IV}}\right]\right\rangle, \\ \qquad \text{if } d_1<0,d_2<0,h_1<0,h_2<0,p_1<0,p_2<0\end{cases}$$

wher $u_{\tilde{a}_{N_1}^{IV}}\wedge u_{\tilde{a}_{N_2}^{IV}}=\left[\min\left(u^L_{\tilde{a}_{N_1}^{LIV}},u^L_{\tilde{a}_{N_2}^{LIV}}\right),\min\left(u^U_{\tilde{a}_{N_1}^{UIV}},u^U_{\tilde{a}_{N_2}^{UIV}}\right)\right]$, $v_{\tilde{a}_{N_1}^{IV}}\vee v_{\tilde{a}_{N_2}^{IV}}=\left[\max\left(v^L_{\tilde{a}_{N_1}^{LIV}},v^L_{\tilde{a}_{N_2}^{LIV}}\right),\max\left(v^U_{\tilde{a}_{N_1}^{UIV}},v^U_{\tilde{a}_{N_2}^{UIV}}\right)\right]$

and $w_{\tilde{a}_{N_1}^{IV}}\vee w_{\tilde{a}_{N_2}^{IV}}=\left[\max\left(w^L_{\tilde{a}_{N_1}^{LIV}},w^L_{\tilde{a}_{N_2}^{LIV}}\right),\max\left(w^U_{\tilde{a}_{N_1}^{UIV}},w^U_{\tilde{a}_{N_2}^{UIV}}\right)\right]$.

Example 2.1.1: let $\tilde{a}_{N_1}^{IV}=\left\langle\begin{matrix}(7,11,16,21);[0.6,0.8]\\(6,10,15,20);[0.3,0.4]\\(5,9,14,19);[0.4,0.6]\end{matrix}\right\rangle$ and $\tilde{a}_{N_2}^{IV}=\left\langle\begin{matrix}(6,11,13,20);[0.7,0.8]\\(5,10,12,18);[0.4,0.5]\\(3,8,11,16);[0.5,0.6]\end{matrix}\right\rangle$ be two IVTrNeNs,

then $\tilde{a}_{N_1}^{IV}+\tilde{a}_{N_2}^{IV}=\left\langle\begin{matrix}(13,22,29,41);[0.6,0.8]\\(11,20,27,38);[0.4,0.5]\\(8,17,25,35);[0.5,0.6]\end{matrix}\right\rangle$, $\tilde{a}_{N_1}^{IV}-\tilde{a}_{N_2}^{IV}=\left\langle\begin{matrix}(-13,-2,5,15);[0.6,0.8]\\(-13,-2,5,15);[0.4,0.5]\\(-11,-2,6,16);[0.5,0.6]\end{matrix}\right\rangle$

$$\tilde{a}_{N_1}^{IV}.\tilde{a}_{N_2}^{IV}=\left\langle\begin{matrix}(42,121,208,420);[0.6,0.8]\\(30,100,180,360);[0.4,0.5]\\(15,72,154,304);[0.5,0.6]\end{matrix}\right\rangle,\quad \frac{\tilde{a}_{N_1}^{IV}}{\tilde{a}_{N_2}^{IV}}=\left\langle\begin{matrix}(0.35,0.85,1.45,3.50);[0.6,0.8]\\(0.33,0.83,1.50,4.00);[0.4,0.5]\\(0.31,0.81,1.75,6.33);[0.5,0.6]\end{matrix}\right\rangle$$

$$5\tilde{a}_{N_1}^{IV}=\left\langle\begin{matrix}(35,55,80,105);[0.6,0.8]\\(30,50,75,100);[0.3,0.4]\\(25,45,70,95);[0.4,0.6]\end{matrix}\right\rangle$$

3. **Score and Accuracy functions of** IVTrNeNs

Definition 3.1: Sahin [69] used the score function concept to find comparison between two IVTrNeNs. Greater of score function value demonstrate the greater of IVTrNeN. According the base of [70] the score and accuracy functions of an IVTrNeN $\tilde{a}_N^{IV}$ can be defined as follows:

$$S(\tilde{a}_N^{IV})=\frac{1}{12}\begin{pmatrix}8+(a_1+b_1+c_1+d_1)-(e_1+f_1+g_1+h_1)\\-(l_1+m_1+n_1+p_1)\end{pmatrix}\times\left(2+u^L_{\tilde{a}_N^{LIV}}+u^U_{\tilde{a}_N^{UIV}}-v^L_{\tilde{a}_N^{LIV}}-v^U_{\tilde{a}_N^{UIV}}-w^L_{\tilde{a}_N^{LIV}}-w^U_{\tilde{a}_N^{UIV}}\right)$$

$S(\tilde{a}_N^{IV})\in[0,1]$. The accuracy function $A(\tilde{a}_N^{IV})\in[-1,1]$ is defined as:

$$A(\tilde{a}_N^{IV})=\frac{1}{4}\left(a_1+b_1+c_1+d_1-l_1-m_1-n_1-p_1\right)\times\left(2+u^L_{\tilde{a}_N^{LIV}}+u^U_{\tilde{a}_N^{UIV}}-v^L_{\tilde{a}_N^{LIV}}-v^U_{\tilde{a}_N^{UIV}}-w^L_{\tilde{a}_N^{LIV}}-w^U_{\tilde{a}_N^{UIV}}\right)$$

Definition 3.2 Let $\tilde{a}_{N_1}^{IV}$ and $\tilde{a}_{N_2}^{IV}$ be any two IVTrNeNs, then one has the following comparison:

(a) If $S(\tilde{a}_{N_1}^{IV})<S(\tilde{a}_{N_2}^{IV})\Rightarrow\ \tilde{a}_{N_1}^{IV}<\tilde{a}_{N_2}^{IV}$

(b) If $S(\tilde{a}_{N_1}^{IV})=S(\tilde{a}_{N_2}^{IV})$ with $A(\tilde{a}_{N_1}^{IV})<A(\tilde{a}_{N_2}^{IV})\Rightarrow\tilde{a}_{N_1}^{IV}<\tilde{a}_{N_2}^{IV}$, $A(\tilde{a}_{N_1}^{IV})>A(\tilde{a}_{N_2}^{IV})\Rightarrow\tilde{a}_{N_1}^{IV}>\tilde{a}_{N_2}^{IV}$ and

$A(\tilde{a}_{N_1}^{IV}) = A(\tilde{a}_{N_2}^{IV})$ then $\tilde{a}_{N_1}^{IV} = \tilde{a}_{N_2}^{IV}$.

Example 3.1. Let $\tilde{a}_{N_1}^{IV} = \langle (7,11,16,21),(6,10,15,20),(5,9,14,19);[0.6,0.8],[0.3,0.4],[0.4,0.6]\rangle$ and $\tilde{a}_{N_2}^{IV} = \langle (6,11,13,20),(5,10,12,18),(3,8,11,16);[0.7,0.8],[0.4,0.5],[0.5,0.6]\rangle$ be two SVTrNeNs, then the score and accuracy function $S(\tilde{a}_{N_1}^{IV}) = -4.95833$, $A(\tilde{a}_{N_1}^{IV}) = 5.1$ and $S(\tilde{a}_{N_2}^{IV}) = -9.375$, $A(\tilde{a}_{N_2}^{IV}) = 4.875$.

Here $S(\tilde{a}_{N_1}^{IV}) > S(\tilde{a}_{N_2}^{IV})$ and $A(\tilde{a}_{N_1}^{IV}) > A(\tilde{a}_{N_2}^{IV})$ implies that $\tilde{a}_{N_1}^{IV} > \tilde{a}_{N_2}^{IV}$.

4. Neutrosophic Transportation Problem [NTP] and its Mathematical formulation

In a TP, if at least one parameter such as cost, demand, or supply is in form of neutrosophic numbers, then TP is termed as NTP. An NTP has neutrosophic availabilities and neutrosophic demand but the crisp cost is classified as NTP of type-1, if NTP has crisp availabilities and crisp demand but neutrosophic cost, is classified as NTP of type-2. If all the specifications of TP such as cost, demand, and availabilities are a combination of crisp, triangular, or trapezoidal neutrosophic numbers, then it is classified as NTP of type-3. In last if all the specifications of TP must be in neutrosophic numbers form, then TP is said to be NTP of type-4 or fully NTP.

4.1 Mathematical Formulation of NTP

In TP if uncertainty occurs in cost, demand or supply then it is more difficult to find the strict way and time. During the current scenario of covid-19, it is very important for transporting the drugs and medical equipment from one source to another destination in an unchallenging way. Keeping in mind for social distancing the IVTrNeS has a deep concern and special features. To maintain this type of impreciseness in cost to a transferred product from ith sources to jth destination or uncertainty in supply and demand, the decision-maker introduces NTP with IVTrNeNs. Here we discuss NTP of type-4 with IVTrNeNs in cost, supply and demand.

Let the cost and number of units and assumptions and constraints in NTP be defined as IVTrNeNs that are transported from ith sources to jth destination. In the formulation of NTP the following assumptions and constraints are required:

m	**total number of source point**
n	total number of destination point
i	table of source (for all m)
j	table of destination (for all n)
$\tilde{x}_{ij}^{IV}$	number of transported neutrosophic unites from i^{th} source to j^{th} destination
$\tilde{c}_{ij}^{IV}$	Neutrosophic cost of one unit transported from i^{th} source to j^{th} destination
$\tilde{a}_{ij}^{IV}$	available neutrosophic supply quantity from i^{th} source
$\tilde{b}_{ij}^{IV}$	required neutrosophic demand quantity to j^{th} destination
$c_{ij}^{IV} = \left[c_{ij}^{L}, c_{ij}^{U}\right]$	crisp cost of one unit quantity
$x_{ij}^{IV} = \left[x_{ij}^{L}, x_{ij}^{U}\right]$	number of transported crisp unites from i^{th} source to j^{th} destination
$a_{i}^{IV} = \left[a_{i}^{L}, a_{i}^{U}\right]$	available crisp supply quantity from i^{th} source
$b_{j}^{IV} = \left[b_{j}^{L}, b_{j}^{U}\right]$	required crisp demand quantity to j^{th} destination

For balance of NTP $\sum_{i=0}^{m}\tilde{a}_i^{IV}=\sum_{j=0}^{n}\tilde{a}_j^{IV}$ i.e. total supply is equal to total demand. The objective of this NTP model is to minimize the cost of transported product. The mathematical formulation of NTP with uncertain transported units, cost, demand and supply is as follows:

$$\text{Minimum } \tilde{Z}^{IV}=\sum_{i=0}^{m}\sum_{j=0}^{n}\tilde{x}_{ij}^{IV}\tilde{c}_{ij}^{IV}$$

$$\text{Subject to } \sum_{j=0}^{n}\tilde{x}_{ij}^{IV}\approx\tilde{a}_i^{IV},\ \forall i=1,2,3,\ldots,m(\text{sources}),$$

$$\sum_{j=0}^{n}\tilde{x}_{ij}^{IV}\approx\tilde{b}_j^{IV},\forall\ j=1,2,3,\ldots,n(\text{destination}),$$

$$\tilde{x}_{ij}^{IV}\geq\tilde{0},\ \forall\ i=1,2,3,\ldots,m, j=1,2,3,\ldots,n.$$

where

$$\tilde{c}_{ij}^{IV}=\begin{Bmatrix}\left(\tilde{c}(a)_{ij}^{IV},\tilde{c}(b)_{ij}^{IV},\tilde{c}(c)_{ij}^{IV},\tilde{c}(d)_{ij}^{IV}\right);\left[u_{\tilde{c}_{ij}^{IV}}^{L},u_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{c}(e)_{ij}^{IV},\tilde{c}(f)_{ij}^{IV},\tilde{c}(g)_{ij}^{IV},\tilde{c}(h)_{ij}^{IV}\right)\left[v_{\tilde{c}_{ij}^{IV}}^{L},v_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{c}(l)_{ij}^{IV},\tilde{c}(m)_{ij}^{IV},\tilde{c}(n)_{ij}^{IV},\tilde{c}(p)_{ij}^{IV}\right)\left[w_{\tilde{c}_{ij}^{IV}}^{L},w_{\tilde{c}_{ij}^{IV}}^{U}\right]\end{Bmatrix},\quad \tilde{x}_{ij}^{IV}=\left[\begin{Bmatrix}\left(\tilde{x}(a)_{ij}^{IV},\tilde{x}(b)_{ij}^{IV},\tilde{x}(c)_{ij}^{IV},\tilde{x}(d)_{ij}^{IV}\right);\left[u_{\tilde{c}_{ij}^{IV}}^{L},u_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{x}(e)_{ij}^{IV},\tilde{x}(f)_{ij}^{IV},\tilde{x}(g)_{ij}^{IV},\tilde{x}(h)_{ij}^{IV}\right)\left[v_{\tilde{c}_{ij}^{IV}}^{L},v_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{x}(l)_{ij}^{IV},\tilde{x}(m)_{ij}^{IV},\tilde{x}(n)_{ij}^{IV},\tilde{x}(p)_{ij}^{IV}\right)\left[w_{\tilde{c}_{ij}^{IV}}^{L},w_{\tilde{c}_{ij}^{IV}}^{U}\right]\end{Bmatrix}\right]^{t}$$

$$\tilde{a}_{i}^{IV}=\begin{Bmatrix}\left(\tilde{a}(a)_{ij}^{IV},\tilde{a}(b)_{ij}^{IV},\tilde{a}(c)_{ij}^{IV},\tilde{a}(d)_{ij}^{IV}\right);\left[u_{\tilde{c}_{ij}^{IV}}^{L},u_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{a}(e)_{ij}^{IV},\tilde{a}(f)_{ij}^{IV},\tilde{a}(g)_{ij}^{IV},\tilde{a}(h)_{ij}^{IV}\right)\left[v_{\tilde{c}_{ij}^{IV}}^{L},v_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{a}(l)_{ij}^{IV},\tilde{a}(m)_{ij}^{IV},\tilde{a}(n)_{ij}^{IV},\tilde{a}(p)_{ij}^{IV}\right)\left[w_{\tilde{c}_{ij}^{IV}}^{L},w_{\tilde{c}_{ij}^{IV}}^{U}\right]\end{Bmatrix},\quad \tilde{b}_{j}^{IV}=\left[\begin{Bmatrix}\left(\tilde{b}(a)_{ij}^{IV},\tilde{b}(b)_{ij}^{IV},\tilde{b}(c)_{ij}^{IV},\tilde{b}(d)_{ij}^{IV}\right);\left[u_{\tilde{c}_{ij}^{IV}}^{L},u_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{b}(e)_{ij}^{IV},\tilde{b}(f)_{ij}^{IV},\tilde{b}(g)_{ij}^{IV},\tilde{b}(h)_{ij}^{IV}\right)\left[v_{\tilde{c}_{ij}^{IV}}^{L},v_{\tilde{c}_{ij}^{IV}}^{U}\right],\\ \left(\tilde{b}(l)_{ij}^{IV},\tilde{b}(m)_{ij}^{IV},\tilde{b}(n)_{ij}^{IV},\tilde{b}(p)_{ij}^{IV}\right)\left[w_{\tilde{c}_{ij}^{IV}}^{L},w_{\tilde{c}_{ij}^{IV}}^{U}\right]\end{Bmatrix}\right]^{t}$$

4.2. Steps for Balancing of NTP by Existing Method

The total transportation cost does not depends on the mode of transportation and distance, also the framework of the problem will be denoted by either crisp or IVTrNeNs. For solution of NTP, first we convert all IVTrNeNs into crisp values by using score function and so the NTP converted into simple TP. After balancing by existing method, the following steps are required for solution of NTP:

Step 4.2.1: To change the each neutrosophic cost $\tilde{c}_{ij}^{IV}$ neutrosophic supply $\tilde{a}_i^{IV}$ and neutrosophic demand $\tilde{b}_j^{IV}$ of NTP in cost matrix into crisp values by using score function $S(\tilde{a}_N^{IV})$.

Step 4.2.2: For balance TP, verify that the sum of demands is equal to the sum of supply i.e. If $\sum_{i=0}^{m}\tilde{a}_i^{IV}<\sum_{j=0}^{n}\tilde{b}_j^{IV}$ or $\sum_{i=0}^{m}\tilde{a}_i^{IV}>\sum_{j=0}^{n}\tilde{b}_j^{IV}\ \forall i,j$ the one can make sure to balance the TP, as $\sum_{i=0}^{m}\tilde{a}_i^{IV}=\sum_{j=0}^{n}\tilde{b}_j^{IV},\ \forall i,j$, by adding a row or column with zero entries in cost matrix.

Step 4.2.3: Verify that the sum of demands is greater than the supply in each row and the sum of supplies are greater than the demand in each column, if ok go on step 4.2.4, otherwise go on step 4.2.2

Step 4.2.4: Here we use the excel solver to solve the TP and obtained optimal solution.

4.3. Steps for Balancing of NTP by MRCM

For balance the unbalance NTP, we use minimum row column method [MRCM] introduced by Saini [45] as follows:

Step 4.3.1. Convert neutrosophic cost $\tilde{c}_{ij}^{IV}$ neutrosophic supply $\tilde{a}_i^{IV}$ and neutrosophic demand $\tilde{b}_j^{IV}$ of NTP in cost matrix to crisp values by using score function $S(\tilde{a}_N^{IV})$.

Step 4.3.2 If NTP is unbalance i.e. $\sum_{i=0}^{m} \tilde{a}_i^{IV} <$ or $> \sum_{j=0}^{n} \tilde{b}_j^{IV}$, $\forall i, j$ than we find

$$\tilde{a}_{i(m+1)}^{IV} = \sum_{i=0}^{m} \tilde{a}_i^{IV} \text{ and } \tilde{b}_{j(n+1)}^{IV} = \sum_{j=0}^{n} \tilde{b}_j^{IV} \oplus \text{excess supply},$$

or

$$\tilde{b}_{j(n+1)}^{IV} = \sum_{j=0}^{n} \tilde{b}_j^{IV} \text{ and } \tilde{a}_{i(m+1)}^{IV} = \sum_{i=1}^{m} \tilde{a}_i^{IV} \oplus \text{excess demand}.$$

The unit transportation costs are taken as follows:

$$\tilde{c}_{i(n+1)}^{IV} = \min_{1\le j\le n} \tilde{c}_{ij}^{IV},\ 1 \le i \le m, \quad \tilde{c}_{(m+1)j}^{IV} = \min_{1\le i\le m} \tilde{c}_{ij}^{IV},\ 1 \le j \le n,$$

$$\tilde{c}_{ij}^{IV} = \tilde{c}_{ji}^{IV},\ 1 \le i \le m,\ 1 \le j \le n, \text{ and } \tilde{c}_{(m+1)(n+1)}^{IV} = 0.$$

Step 4.3.3 Obtain optimal solution of NTP by excel solver. Let the neutrosophic optimal solution obtained be $\tilde{x}_{ij}^{IV}$, $1 \le i \le m+1$, $1 \le j \le n+1$.

Step 4.3.4 By assuming $\tilde{\omega}_{m+1}^{'IV} = 0$ and using the relation $\tilde{\omega}_i^{'IV} \oplus \tilde{v}_j^{'IV} = \tilde{\sigma}_{ij}^{'IV}$ for basic variables, find the values of all the dual variables $\tilde{\omega}_i^{'IV}$, $1 \le i \le m$ and $\tilde{v}_j^{'IV}$, $1 \le j \le n+1$,

Step 4.3.5. According to MRCM, $\tilde{\omega}_i^{'IV} = \tilde{\omega}_i^{IV}$ and $\tilde{v}_j^{'IV} = \tilde{v}_j^{IV}$ for $1 \le i \le m, 1 \le j \le n$, obtain only central rank zero duals.

5. Numerical Example

Let us consider a NTP of type-4 with three container (sources) say M_1, M_2, M_3 in which medical equipment are initially stored and ready to transport in three different destinations (cities), say C_1, C_2, C_3 with unit transportation cost, demand and supply are as IVTrNeN. The input data of NTP with IVTrNeNs is given in table 1:

Table 1	M_1	M_2	M_3	*Supply*
C_1	$\langle(7,12,21.5,28);[0.7,0.9], (4,10,17.5,25);[0.4,0.5], (2,8,15.5,22);[0.3,0.4]\rangle$	$\langle(7,11,15,19);[0.6,0.8] (5,8,12,15);[0.4,0.5] (3,5,7,11);[0.2,0.3]\rangle$	$\langle 5,10,14,19.5);[0.5,0.6], (3,7.5,10,15);[0.4,0.6], (1,4,7,10);[0.3,0.4]\rangle$	$\langle(9,19,28,34);[0.7,0.8], (7,12,19,24);[0.4,0.5], (3,8,11,16);[0.2,0.4]\rangle$
C_2	$\langle(5,11,16.5,21);[0.6,0.7], (3,8,12,16);[0.3,0.5], (0,3,9.5,12);[0.2,0.4]\rangle$	$\langle(2,4,7.5,10);[0.6,0.7], (1,3,6.5,9);[0.3,0.5], (-1,2,5,7);[0.2,0.3]\rangle$	$\langle(3,6,11,16);[0.7,0.9], (1,5,9,14);[0.4,0.5], (-3,2,6,12);[0.3,0.4]\rangle$	$\langle 8,14,25,35);[0.7,0.8], (4,10,18,28);[0.3,0.5], (1,8,14,22);[0.3,0.4]\rangle$
C_3	$\langle(6,14,21,28);[0.8,0.9], (4,11,18,25);[0.4,0.6], (2,8,15,22);[0.3,0.4]\rangle$	$\langle(4,8.5,14,17);[0.6,0.8], (2,6.5,11,15);[0.4,0.5], (-2,2,7,11);[0.2,0.3]\rangle$	$\langle(5,10,14,20);[0.7,0.9], (3,8,9,15);[0.3,0.5], (-1,5,7,12);[0.2,0.4]\rangle$	$\langle(14,22,30,39);[0.6,0.8], (12,18,25,34);[0.3,0.5], (8,15,23,31);[0.2,0.4]\rangle$
Demand	$\langle(10,20,28,35);[0.7,0.9], (4,12,22.5,29);[0.4,0.5], (-1,7,12,19);[0.3,0.4],\rangle$	$\langle 6,12,23,33);[0.7,0.8], (4,10,19,28);[0.4,0.5], (2,8,15,24);[0.2,0.4]\rangle$	$\langle(12,18,25,33);[0.7,0.9], (9,16,23,30);[0.4,0.5], (5,14,20,27);[0.3,0.4]\rangle$	

With the help of score function, the cost, demand and supply of NTP i.e. in IVTrNeNs are convert into the crisp numbers as follows:

$S(\tilde{c}_{11}^{IV}) = -4.58333$, $S(\tilde{c}_{12}^{IV}) = -1$, $S(\tilde{c}_{13}^{IV}) = -0.11667$, $S(\tilde{a}_{1}^{IV}) = -0.33333$, $S(\tilde{c}_{21}^{IV}) = -0.31667$, $S(\tilde{c}_{22}^{IV}) = -0.16667$, $S(\tilde{c}_{23}^{IV}) = -0.333333$, $S(\tilde{a}_{2}^{IV}) = -2.50$, $S(\tilde{c}_{31}^{IV}) = -4.66667$, $S(\tilde{c}_{32}^{IV}) = -0.16667$, $S(\tilde{c}_{33}^{IV}) = -0.183333$,

$S(\tilde{a}_{3}^{IV}) = -8.83334$, $S(\tilde{b}_{1}^{IV}) = -0.58333$, $S(\tilde{b}_{2}^{IV}) = -4.66667$, $S(\tilde{b}_{3}^{IV}) = -8.0$

The unbalance TP with crisp values shown in table 2:

Table 2	M_1	M_2	M_3	***Supply***
C_1	-4.58333	-1	-0.11667	***-0.33333***
C_2	-0.31667	-0.16667	-0.33333	***-2.50***
C_3	-4.66667	-0.16667	-0.18333	***-8.83334***
Demand	***-0.58333***	***-4.66667***	***-8.0***	

In table 2, $\sum_{i=0}^{m} \tilde{a}_i^{IV} = -11.6667$, $\sum_{j=0}^{n} \tilde{b}_j^{IV} = -13.25$, i.e. $\sum_{i=0}^{m} \tilde{b}_j^{IV} - \sum_{j=0}^{n} \tilde{a}_i^{IV} = 1.58300$, this shows that NTP is unbalanced. The balance TP and the solution of NTP in crisp form by excel solver shown in table 3 and table 4 respectively as follows:

Table 3	M_1	M_2	M_3	M_4	*Supply*
C_1	-4.58333	-1	-0.11667	0	***-0.33333***
C_2	-0.31667	-0.16667	-0.33333	0	***-2.50***
C_3	-4.66667	-0.16667	-0.18333	0	***-8.83334***
Demand	***-0.58333***	***-4.66667***	***-8.0***	***1.58300***	

Table 4	M_1	M_2	M_3	M_4	*Supply*
C_1	1.91667	-3.83333	-	1.583	***-0.33333***
C_2	-2.5	-	-	-	***-2.50***
C_3	-	-0.83334	-8	-	***-8.83334***
Demand	***-0.58333***	***-4.66667***	***-8.0***	***1.58300***	

The optimal solution of NTP in crisp form is $Z_{CTP} = -2.55419$. The solution of NTP with IVTrNeNs shown in table 5:

Table 5	M_1	M_2	M_3	M_4	*Supply*
C_1	$\left\langle \begin{matrix}(-25,-5,14,27);[0.7,0.9],\\(-24,-6,12.5,25);[0.3,0.5],\\(-23,-7,4,18);[0.3,0.4]\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(-71,-14,49,104);[0.7,0.8]\\(-62,-12,43.5,88);[0.3,0.5]\\(-60,-11,27,74);[0.2,0.4]\end{matrix} \right\rangle$	-	$\left\langle \begin{matrix}(-70,-21,33,80);[0.6,0.8],\\(-64,-24.5,24,69);[0.3,0.5],\\(-58,-16,19,63);[0.2,0.4]\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(9,19,28,34);[0.7,0.8],\\(7,12,19,24);[0.4,0.5],\\(3,8,11,16);[0.2,0.4]\end{matrix} \right\rangle$
C_2	$\left\langle \begin{matrix}8,14,25,35);[0.7,0.8],\\(4,10,18,28);[0.3,0.5],\\(1,8,14,22);[0.3,0.4]\end{matrix} \right\rangle$	-	-	-	$\left\langle \begin{matrix}8,14,25,35);[0.7,0.8],\\(4,10,18,28);[0.3,0.5],\\(1,8,14,22);[0.3,0.4]\end{matrix} \right\rangle$
C_3	-	$\left\langle \begin{matrix}(-19,-3,12,27);[0.7,0.9],\\(-18,-5,9,25);[0.3,0.5],\\(-19,-5,9,26);[0.2,0.4]\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(12,18,25,33);[0.7,0.9],\\(9,16,23,30);[0.4,0.5],\\(5,14,20,27);[0.3,0.4]\end{matrix} \right\rangle$	-	$\left\langle \begin{matrix}(14,22,30,39);[0.6,0.8],\\(12,18,25,34);[0.3,0.5],\\(8,15,23,31);[0.2,0.4]\end{matrix} \right\rangle$
Demand	$\left\langle \begin{matrix}(10,20,28,35);[0.7,0.9],\\(4,12,22.5,29);[0.4,0.5],\\(-1,7,12,19);[0.3,0.4],\end{matrix} \right\rangle$	$\left\langle \begin{matrix}6,12,23,33);[0.7,0.8],\\(4,10,19,28);[0.4,0.5],\\(2,8,15,24);[0.2,0.4]\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(12,18,25,33);[0.7,0.9],\\(9,16,23,30);[0.4,0.5],\\(5,14,20,27);[0.3,0.4]\end{matrix} \right\rangle$	$\left\langle \begin{matrix}(-70,-21,33,80);[0.6,0.8],\\(-64,-24.5,24,69);[0.3,0.5],\\(-58,-16,19,63);[0.2,0.4]\end{matrix} \right\rangle$	

$$Z_{NTP} = \left\langle \begin{matrix} (7,12,21.5,28);[0.7,0.9], \\ (4,10,17.5,25);[0.4,0.5], \\ (2,8,15.5,22);[0.3,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (-25,-5,14,27);[0.7,0.9], \\ (-24,-6,12.5,25);[0.3,0.5], \\ (-23,-7,4,18);[0.3,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} (7,11,15,19);[0.6,0.8] \\ (5,8,12,15);[0.4,0.5] \\ (3,5,7,11);[0.2,0.3] \end{matrix} \right\rangle . \left\langle \begin{matrix} (-71,-14,49,104);[0.7,0.8] \\ (-62,-12,43.5,88);[0.3,0.5] \\ (-60,-11,27,74);[0.2,0.4] \end{matrix} \right\rangle$$

$$+ \left\langle \begin{matrix} (-70,-21,33,80);[0.6,0.8], \\ (-64,-24.5,24,69);[0.3,0.5], \\ (-58,-16,19,63);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (0,0,0,0);[0.6,0.8], \\ (0,0,0,0);[0.3,0.5], \\ (0,0,0,0);[0.2,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} (5,11,16.5,21);[0.6,0.7], \\ (3,8,12,16);[0.3,0.5], \\ (0,3,9.5,12);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} 8,14,25,35);[0.7,0.8], \\ (4,10,18,28);[0.3,0.5], \\ (1,8,14,22);[0.3,0.4] \end{matrix} \right\rangle$$

$$+ \left\langle \begin{matrix} (4,8.5,14,17);[0.6,0.8], \\ (2,6.5,11,15);[0.4,0.5], \\ (-2,2,7,11);[0.2,0.3] \end{matrix} \right\rangle . \left\langle \begin{matrix} (-19,-3,12,27);[0.7,0.9], \\ (-18,-5,9,25);[0.3,0.5], \\ (-19,-5,9,26);[0.2,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} (5,10,14,20);[0.7,0.9], \\ (3,8,9,15);[0.3,0.5], \\ (-1,5,7,12);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (12,18,25,33);[0.7,0.9], \\ (9,16,23,30);[0.4,0.5], \\ (5,14,20,27);[0.3,0.4] \end{matrix} \right\rangle$$

i.e. $$Z_{NTP} = \left\langle \begin{matrix} (-648,94.5,1966.5,4586);[0.6,0.9], \\ (-403,19.5,1262.75,3218);[0.3,0.5], \\ (-193,-27,587,2084);[0.2,0.3] \end{matrix} \right\rangle \approx -7.07292$$

Now we balance the unbalance CTP in table 2 by MRCM, the balance CTP with crisp numbers shown in table 6 as follows:

Table 6	M_1	M_2	M_3	M_4	*Supply*
C_1	-4.58333	-1	-0.11667	-4.58333	***-0.33333***
C_2	-0.31667	-0.16667	-0.33333	-0.33333	***-2.50***
C_3	-4.66667	-0.16667	-0.18333	-4.66667	***-8.83334***
C_4	-4.66667	-1	-0.33333	***0***	***-11.6667***
Demand	***-0.58333***	***-4.66667***	***-8.0***	***-10.0834***	

The solution of balance CTP as in table 7

Table 7	M_1	M_2	M_3	M_4	*Supply*
C_1	10.75001	-4.66667	-8	1.5833	***-0.33333***
C_2	-2.5	0	0	0	***-2.50***
C_3	-8.83334	0	0	0	***-8.83334***
C_4	0	0	0	-11.6667	***-11.6667***
Demand	***-0.58333***	***-4.66667***	***-8.0***	***-10.0834***	

The cost $Z_{CTP(MRCM)} = -8.91364$.

The solution of corresponding balanced NTP shown in table 8 as follows:

Table 8	M_1	M_2	M_3	M_4	*Supply*
C_1	$\left\langle \begin{matrix} (-64,-35,-8,13);[0.7,0.9], \\ (-48,-31,-5.5,13);[0.4,0.5], \\ (-54,-30,-11,10);[0.3,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 6,12,23,33);[0.7,0.8], \\ (4,10,19,28);[0.4,0.5], \\ (2,8,15,24);[0.2,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (12,18,25,33);[0.7,0.9], \\ (9,16,23,30);[0.4,0.5], \\ (5,14,20,27);[0.3,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (-147,-49,61,157);[0.7,0.9], \\ (-127,-46.5,46,132);[0.4,0.5], \\ (-115,-33,36,120);[0.3,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (9,19,28,34);[0.7,0.8], \\ (7,12,19,24);[0.4,0.5], \\ (3,8,11,16);[0.2,0.4] \end{matrix} \right\rangle$
C_2	$\left\langle \begin{matrix} 8,14,25,35);[0.7,0.8], \\ (4,10,18,28);[0.3,0.5], \\ (1,8,14,22);[0.3,0.4] \end{matrix} \right\rangle$	-	-	-	$\left\langle \begin{matrix} 8,14,25,35);[0.7,0.8], \\ (4,10,18,28);[0.3,0.5], \\ (1,8,14,22);[0.3,0.4] \end{matrix} \right\rangle$
C_3	$\left\langle \begin{matrix} (14,22,30,39);[0.6,0.8], \\ (12,18,25,34);[0.3,0.5], \\ (8,15,23,31);[0.2,0.4] \end{matrix} \right\rangle$	-	-	-	$\left\langle \begin{matrix} (14,22,30,39);[0.6,0.8], \\ (12,18,25,34);[0.3,0.5], \\ (8,15,23,31);[0.2,0.4] \end{matrix} \right\rangle$
C_4	-	-	-	$\left\langle \begin{matrix} (31,55,83,108);[0.6,0.8], \\ (23,40,62,86);[0.3,0.5], \\ (12,31,48,69);[0.2,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (31,55,83,108);[0.6,0.8], \\ (23,40,62,86);[0.3,0.5], \\ (12,31,48,69);[0.2,0.4] \end{matrix} \right\rangle$
Demand	$\left\langle \begin{matrix} (10,20,28,35);[0.7,0.9], \\ (4,12,22.5,29);[0.4,0.5], \\ (-1,7,12,19);[0.3,0.4], \end{matrix} \right\rangle$	$\left\langle \begin{matrix} 6,12,23,33);[0.7,0.8], \\ (4,10,19,28);[0.4,0.5], \\ (2,8,15,24);[0.2,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (12,18,25,33);[0.7,0.9], \\ (9,16,23,30);[0.4,0.5], \\ (5,14,20,27);[0.3,0.4] \end{matrix} \right\rangle$	$\left\langle \begin{matrix} (-39,34,116,188);[0.7,0.9], \\ (-41,15.5,86,155);[0.4,0.5], \\ (-46,15,67,132);[0.3,0.4] \end{matrix} \right\rangle$	

$$
\begin{aligned}
Z_{NTP(MRCM)} = {} & \left\langle \begin{matrix} (-64,-35,-8,13);[0.7,0.9], \\ (-48,-31,-5.5,13);[0.4,0.5], \\ (-54,-30,-11,10);[0.3,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (7,12,21.5,28);[0.7,0.9], \\ (4,10,17.5,25);[0.4,0.5], \\ (2,8,15.5,22);[0.3,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} 6,12,23,33);[0.7,0.8], \\ (4,10,19,28);[0.4,0.5], \\ (2,8,15,24);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (7,11,15,19);[0.6,0.8] \\ (5,8,12,15);[0.4,0.5] \\ (3,5,7,11);[0.2,0.3] \end{matrix} \right\rangle + \left\langle \begin{matrix} (12,18,25,33);[0.7,0.9], \\ (9,16,23,30);[0.4,0.5], \\ (5,14,20,27);[0.3,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} 5,10,14,19.5);[0.5,0.6], \\ (3,7.5,10,15);[0.4,0.6], \\ (1,4,7,10);[0.3,0.4] \end{matrix} \right\rangle \\
& + \left\langle \begin{matrix} (-147,-49,61,157);[0.7,0.9], \\ (-127,-46.5,46,132);[0.4,0.5], \\ (-115,-33,36,120);[0.3,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (7,12,21.5,28);[0.7,0.9], \\ (4,10,17.5,25);[0.4,0.5], \\ (2,8,15.5,22);[0.3,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} 8,14,25,35);[0.7,0.8], \\ (4,10,18,28);[0.3,0.5], \\ (1,8,14,22);[0.3,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (5,11,16.5,21);[0.6,0.7], \\ (3,8,12,16);[0.3,0.5], \\ (0,3,9.5,12);[0.2,0.4] \end{matrix} \right\rangle + \left\langle \begin{matrix} (14,22,30,39);[0.6,0.8], \\ (12,18,25,34);[0.3,0.5], \\ (8,15,23,31);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (6,14,21,28);[0.8,0.9], \\ (4,11,18,25);[0.4,0.6], \\ (2,8,15,22);[0.3,0.4] \end{matrix} \right\rangle \\
& + \left\langle \begin{matrix} (31,55,83,108);[0.6,0.8], \\ (23,40,62,86);[0.3,0.5], \\ (12,31,48,69);[0.2,0.4] \end{matrix} \right\rangle . \left\langle \begin{matrix} (0,0,0,0);[0.6,0.8], \\ (0,0,0,0);[0.3,0.5], \\ (0,0,0,0);[0.2,0.4] \end{matrix} \right\rangle
\end{aligned}
$$

$$
Z_{NTP(MRCM)} = \left\langle \begin{matrix} (-1230,-234,3221,7857.5);[0.6,0.9], \\ (-593,-297,1832.75,5793);[0.3,0.5], \\ (-311,-264,1110.5,4340);[0.2,0.3] \end{matrix} \right\rangle \approx -13.810
$$

6. Comparative Study

To maintain physical distance during Covid-19 pandemic, we introduced here some advanced version of neutrosophic numbers such as IVTrNeNs, which provides the better results in real life for uncertainty and hesitation in place of crisp numbers. For practical application of NTP type-4, the minimum cost of unbalanced CTP and NTP obtained by VAM and MRCM is summarized in table 9. It is also clear from the table 9, that minimum cost of unbalanced CTP and NTP obtained by using MRCM is far better than the existing method VAM. In figure 4, the bar graph represents the minimum cost of CTP and NTP and their comparison for better one.

Table 9: **Comparative Study**

Balance of CTP by existing method	**Balance of CTP by MRCM**
$Z_{CTP} = -2.55419$	$Z_{CTP(MRCM)} = -8.91364$
Balance of NTP by existing method	**Balance of NTP by MRCM**
$Z_{NTP} = \left\langle \begin{matrix} (-648,94.5,1966.5,4586);[0.6,0.9], \\ (-403,19.5,1262.75,3218);[0.3,0.5], \\ (-193,-27,587,2084);[0.2,0.3] \end{matrix} \right\rangle$	$Z_{NTP(MRCM)} = \left\langle \begin{matrix} (-1230,-234,3221,7857.5);[0.6,0.9], \\ (-593,-297,1832.75,5793);[0.3,0.5], \\ (-311,-264,1110.5,4340);[0.2,0.3] \end{matrix} \right\rangle$
≈ -7.07292	≈ -13.810

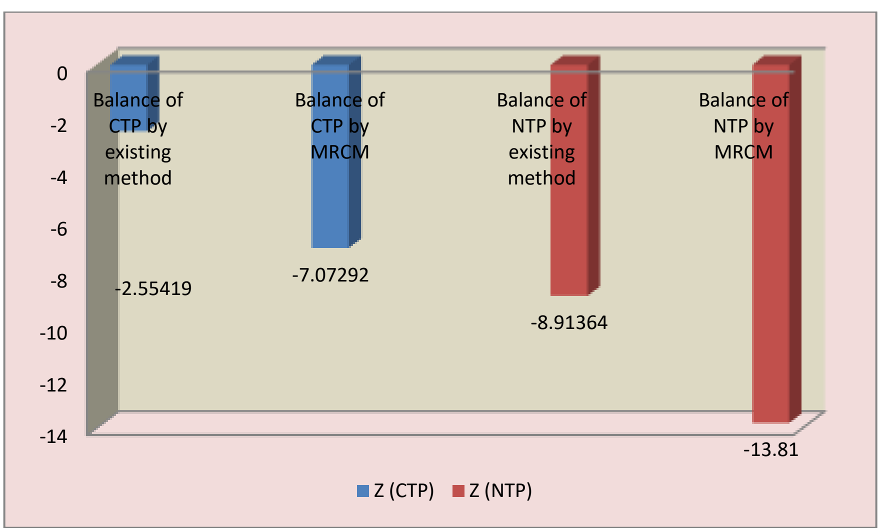

Figure 3: Comparison of results by chart

8. Result and discussion

In this present study the optimal transportation crisp cost and optimal transportation neutrosophic cost of unbalanced NTP using MRCM is minimum than the existing method in [30]. It is also verified that in de-neutrosophication, the crisp values before and after conversion from neutrosophic to crisp and crisp to neutrosophic are different. For the real life applications one can find the degree of result. The best of minimum neutrosophic cost of unbalanced NTP is $Z_{NTP(MRCM)} = \left\langle \begin{array}{l} (\text{-}1230, \text{-}234, 3221, 7857.5); [0.6, 0.9], \\ (\text{-}593, \text{-}297, 1832.75, 5793); [0.3, 0.5], \\ (\text{-}311, \text{-}264, 1110.5, 4340); [0.2, 0.3] \end{array} \right\rangle$ i.e. total minimum transportation cost lies between -1230 to 7857.5 in the interval [0.6, 0.9] for level of truthfulness, -593 to 5793 in the interval [0.3, 0.5] for level of indeterminacy and -311 to 4340 in the interval [0.2, 0.3] for level of falsity. $u_{T_{\tilde{a}_N^{IV}}} \times 100$, $v_{I_{\tilde{a}_N^{IV}}} \times 100$, and $w_{F_{\tilde{a}_N^{IV}}} \times 100$ represents the degree of truthfulness, degree of indeterminacy and degree of falsity respectively. Thus

$$u_{T_{\tilde{a}_N^{IV}}}(x) = \begin{cases} \left(\dfrac{x + 1230}{1230 - 234} \right) [0.6, 0.9], & \text{for } -1230 \le x \le -234, \\ [0.6, 0.9], & \text{for } -234 \le x \le 3221, \\ \left(\dfrac{7857.5 - x}{7857.5 - 3221} \right) [0.6, 0.9], & \text{for } 3221 \le x \le 7857.5, \\ 0, & \text{for otherwise.} \end{cases}$$

$$v_{I_{\tilde{a}_N^{IV}}}(x) = \begin{cases} \dfrac{(-297 - x) + (x + 593)[0.3, 0.5]}{593 - 297}, & \text{for } -593 \le x \le -297, \\ [0.3, 0.5], & \text{for } -297 \le x \le 1832.75, \\ \dfrac{(x - 1832.75) + (5793 - x)[0.3, 0.5]}{5793 - 1832.75}, & \text{for } 1832.75 \le x \le 5793, \\ 0, & \text{for otherwise.} \end{cases}$$

$$w_{F_{\tilde{a}_N^{IV}}}(x) = \begin{cases} \dfrac{(-264 - x) + (x + 311)[0.2, 0.3]}{311 - 264}, & \text{for } -311 \le x \le \text{-}264, \\ [0.2, 0.3], & \text{for } -264 \le x \le 1110.5, \\ \dfrac{(x - 1110.5) + (4340 - x)[0.2, 0.3]}{4340 - 1110.5}, & \text{for } 1110.5 \le x \le 4340, \\ 0, & \text{for otherwise.} \end{cases}$$

where x denotes the total cost.

Table 10

$x \rightarrow$ Degree $\downarrow$	-500	0	2000	3000	4000	5000	7000
$u_{T_{\tilde{a}_N^{IV}}} \times 100$	[43.976, 5.964]	[60, 90]	[60, 90]	[60, 90]	[49.919, 74.879]	[36.978, 55.467]	[11.097, 16.645]
$v_{I_{\tilde{a}_N^{IV}}} \times 100$	[78.007, 84.290]	[30, 50]	[32.969, 52.124]	[50.632, 64.737]	[68.307, 77.362]	[85.983, 89.987]	-
$w_{F_{\tilde{a}_N^{IV}}} \times 100$	-	[20, 30]	[42.034, 49.280]	[66.806, 70.956]	[91.577, 92.630]	-	-

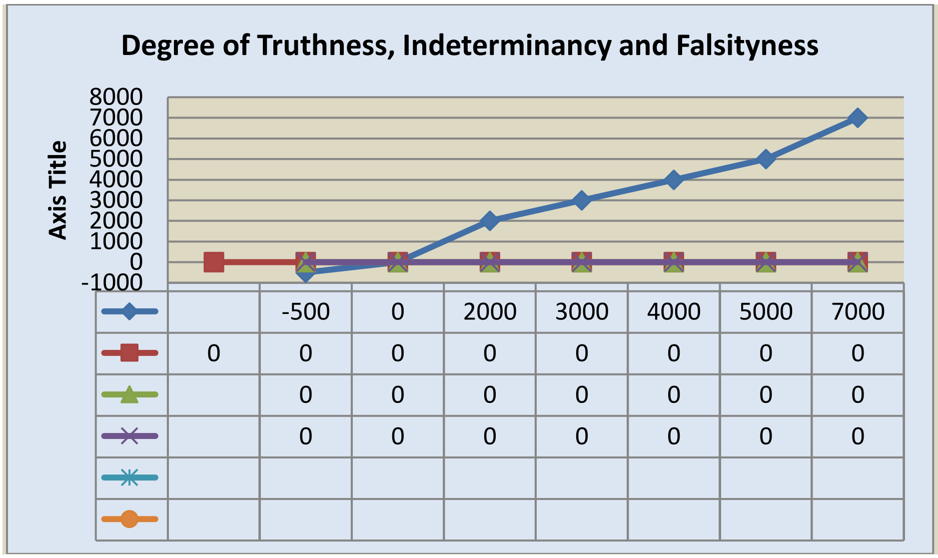

Figure 3: Degree of truthfulness, indeterminacy and falsity

The total neutrosophic cost from the range of -1230 to 7857.5 for truthfulness, -593 to 5793 for indeterminacy and -311to 4340 for falsity are concluded by degree of truthfulness, degree of indeterminacy and degree of falsity to schedule the transportation cost and budget allocation.

9. Conclusions and Novelty

Today in society, the concept of neutrosophic numbers is well linked to handling uncertainty or vagueness in applied mathematical modeling. The current research paper is the study of unbalanced CTP & NTP by introducing a new balancing approach MRCM to obtain an optimal solution where all parameters and values of TP are as IVTrNeNs. The proposed ranking function provides a more practical structure and considers the various characteristics of TP in a neutrosophic environment. Such a type of transportation problem with IVTrNeNs and their comparison between the two methods are not introduced earlier, and we hope that in the future, the proposed MRCM will be more applicable to the multilevel programming problem, unbalanced multi-attribute transportation problem, and multi-level assignment problems. The existing analysis will be a landmark for TP's with generalization by considering the pick value of truth, indeterminacy, and falsity functions and for schedule transportation cost and budget allocation for the total neutrosophic cost, that concluded by a degree of truthfulness, degree of indeterminacy, and degree of falsity.

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