Neutrosophic Sets and Systems, Vol. 51, 2022
University of New Mexico

# The Neutrosophic Axial Set theory

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**Abstract:** We presented in this paper a new concept of sets that we launched it neutrosophic axial sets . These sets are considered as generalization of neutrosophic sets . The union relationships , intersection, union , belonging and other concepts were built on these sets , then we created two different concepts of points . Also we studied many important properties and basic theories about axial sets theory.

**Keywords:** neutrosophic sets; fuzzy sets; SƝĄ-points ; ƝĄ-sets ; union relationships.

## 1. Introduction

The neutrosophic sets [1] are the important and influential topic in human life in direct way. It's considered to be one of the applied and pure topics at the same time .

Also it contributes to quantum leaps in the field of electronics , software and other sciences as well as in various engineering branches . Where Salama, A., FlorentinSmarandache, and Valeri Kromov who were the researchers first to know these sets in [2,3]. Researchers and scientists have taken it upon them and solves to develop and work on it . On the other hand , these sets are considered to be a development to the second type of fuzzy sets , which the researcher Zadeh , L. A. know in 1965 [ 4 ] introduced. At the same time , the fuzzy sets are generalized into so-called the soft sets which were defined in [ 5- 7] and attributed to Molodtsov [ 8 ]. There are many researchers who worked in this field and remind them of [ 9- 12 ].

In 2019 Abdulsada, D.A., Al-Swidi, L.A.A. defined a new concept of sets and called it the center sets , for more information , you can review the papers [ 13- 15] , and the pillar of construction is proximity spaces by . A. Naimpally and . D. Warrack [16], where we combined the proximity space with the i-topological space by Al Talkany, A.Y.K.M., AL-Swidi, L.A.A [ 17] to produce the i-topological proximity space in 2020 [ 18, 19 ]. These ideas can be generalized on the topic of neutrosophic.

## 2. The Neutrosophic Axial Set theory

**2.1. Definition** Let X be any set , the set of the form $ɈȦA = \{ < A, A_1, A_2 > ;\ A \cap A_i = \emptyset , i = 1,2 \}$ is called neutrosophic axial set , where A be any subset of X , and the sets $A_1, A_2$ are called the parts of $< A, A_1, A_2 >$ For example , if we take $X = R$ the real numbers ,then $ɈȦA = \{ < (1,2), A1, A2 > ;\ (1,2) \cap A_i = \emptyset , i = 1,2 \}$ where

$$A_i = \begin{cases} \emptyset \ or\ discret\ set\ in\ {}^{R}/_{(1,2)} \\ [2, x) & for\ x \geq 2 \\ (y, 1] & for\ y \leq 1 \end{cases}$$

**2.2. Definition**

I- Let $X$ be any non-empty set , the neutrosophic point (ɈȦ-point ) are of the forms $ɈP_A^{\emptyset} = < A, \emptyset, \emptyset >$, $ɈPA = < \emptyset, A, \emptyset >$ and

$ɈPA = < \emptyset, \emptyset, A >$ , for any proper non-empty subset $A$ of .

II- The singular neutrosophic point ($S$ɈȦ-point ) of any ɈȦ-set $ɈȦA$ is denoted by $SA = < A, A_1, A_2 >$ , where $A \cap A_i = \emptyset$ where $i = 1,2$ , so $SA \in ɈȦA$ ( where $\in$ is the notion of classical belongs to ) .

So we can claim the number of ɈȦ-points of any non-empty universal set $X$ is $|X|.(|P(X)/\{\emptyset, X\}|$ where $|X|$ the number of elements of $X$.

Also from $|X|$ of above definition any ɈȦ-set is the classical union of its $S$ɈȦ-points and any $S$ɈȦ-point of any ɈȦ-set is neutrosophic set , but the converse is not true .

**2.3. Definition**

I- The empty ɈȦ-set with respect to the subset A of X is denoted by $ɈȦ_A^{\emptyset}$ is of the form

$ɈȦ_A^{\emptyset} = \{ < \emptyset, A_1, A_2 > ;\ where\ A \cap A_i = \emptyset\ \ i = 1,2 \}$ .

For example , if $X = \{ a, b, c \}$ , then $ɈȦ_{\{a,b\}}^{\emptyset} = \{ < \emptyset, \emptyset, \emptyset > , < \emptyset, \{c\}, \{c\} > , < \emptyset, \{c\}, \emptyset > , < \emptyset, \emptyset, \{c\} > \}$.

II- The null ɈȦ-set with respect to a subset $A$ of , which denoted by $ɈɈȦA$ is of form $ɈɈȦA = \{ < A, \emptyset, \emptyset > \}$ .

**2.4. Definition**

I- The ɈȦ-sum between two $S$ɈȦ-points $SA$ and $SB$ is denoted by the notion $\oplus_N$ which is defined by $S_A \oplus_N S_B = \{ < A \cup B, C_1 \cup D_1, C_2 \cup D_2 >$ , where $C_i \cap A = \emptyset\ \ and\ D_i \cap B = \emptyset , i = 1,2 \}$ . So from this definition we claim that every $S$ɈȦ-point is ɈȦ-sum of two or more than two , but every ɈȦ-point is $S$ɈȦ-point .

II- The $S$ƝĄ-point $SA$ is called interlaced with respect to ƝĄ-set Ɲ$ĄB$ , if $SA \in$ Ɲ$ĄB$ there exist $SB \in$ Ɲ$ĄB$ such that $A_i \sqsubseteq B_i$ , $S_A =< A, A_1, A_2 >$ , $S_B =< B, B_1, B_2 >$ $i = 1, 2$ .If any ƝĄ-point of the forms $ƝP_A^{\emptyset}$ , Ɲ$PA$ and Ɲ$PA$ belong to ƝĄ-set Ɲ$ĄB$ , if $A$ is a part of some $S$ƝĄ-point of Ɲ$ĄB$ .So that we easily show that , $SB \in$ Ɲ$ĄB$ iff $SB \in_{Ɲ}$ Ɲ$ĄB$ .

III- The ƝĄ-set Ɲ$ĄA$ is said to be interlaced set with one of the ƝĄ-set Ɲ$ĄB$ which is denoted by Ɲ$ĄA <_{Ɲ}$ Ɲ$ĄB$ iff for each $S_A =< A, A_1, A_2 >$ $\in$ Ɲ$ĄB$ there exist $SB =< B, B_1, B_2 > \in$ Ɲ$ĄB$ with the condition $A_i \sqsubseteq B_i, i = 1, 2$. Clearly every ƝĄ-set is interlaced set of $ƝĄ_{\emptyset}$ , also Ɲ$ĄX$ is interlaced set of any ƝĄ-set.

**2.5. Note**

If $A \sqsubset B$ , then $ƝĄ_B <_{Ɲ} ƝĄ_A$ . Because , for every $S$ƝĄ-point$SB =< B, B_1, B_2 > \in$ƝĄB that is $B \cap B_i = \emptyset$ and $A \sqsubset B$ imply that $A \cap B_i = \emptyset$ for $i = 1, 2$ , thus $SB \in$ Ɲ$ĄA$ , which satisfy the condition of interlaced set .Two ƝĄ-sets $ƝĄ_A$ and $ƝĄ_B$are called intertwind sets which is denoted by $ƝĄ_A \approx_{Ɲ} ƝĄ_B$ iff Ɲ$ĄA <_{Ɲ}$ Ɲ$ĄB$ and Ɲ$ĄB <_{Ɲ}$ Ɲ$ĄA$ .

**2.6. Proposition**

Let $X$ be any sets and $A, B$ are subsets of $X$. $A = B$ iff $ƝĄ_A \approx_{Ɲ} ƝĄ_B$ .

**Proof .**

Assume that $= B$ , so by (Note 2.5) , we get that $ƝĄ_A \approx_{Ɲ} ƝĄ_B$. Conversely , if possible that $A \neq B$ .

**Case 1 .** If $A \cap B = \emptyset$ , then each $S_A =< A, A_1, A_2 >$ $\in$ Ɲ$ĄA$ and since each subset $C$ of $X$ with $C \cap B = \emptyset$ there is no $S$ƝĄ-points in $ƝĄ_B$ which satisfy the condition of interlaced set , so Ɲ$ĄA$ is not interlaced set of $ƝĄ_B$ . Similarly that $ƝĄ_B$ is not interlaced set of Ɲ$ĄA$ , which contradiction with $ƝĄ_A \approx_{Ɲ} ƝĄ_B$ .

**Case 2.** If $A \cap B \neq \emptyset$ , that is there exist a point $x$ in $A$ and not in $B$ or the point $y$ in $B$ but not in $A$ , so $SB =< B, \{x\}, \{x\} > \in$ Ɲ$ĄB$ imply that no $S$ƝĄ-points in Ɲ$ĄA$ which satisfy the condition of interlaced set , hence Ɲ$ĄB$ , similarly if we take $y \in B$ and $y$ is not in $A$ , which contradiction with $ƝĄ_A \approx_{Ɲ} ƝĄ_B$ . Therefore we get $= B$ .

**2.7.Definition**

The ƝĄ- complement of any ƝĄ-set $ƝĄ_A$ which is denoted by $(ƝĄ_A)c$ and of the form $(ƝĄ_A)c = ƝĄ_{A^c}$. Now we give the notions of union , intersection of ƝĄ - sets .

**2. 8. Definition**

The ƝĄ – union of two ƝĄ-sets $\text{ƝĄ}_{A}$ and $\text{ƝĄ}_{B}$ , which is denoted by $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B}$ and is of the form $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B} = \{ < A \cup B, A_1 \cap B_1, A_2 \cap B_2 > ; \forall < A, A_1, A_2 > \in \text{ƝĄ}_{A}, < B, B_1, B_2 > \in \text{ƝĄ}_{B} \}$ .Also for the same away we defined that for any collection$\{\text{ƝĄ}_{A_i}; i \in II \}$ of ƝĄ - sets , the ƝĄ-union of this collection is of form $(\cup_{i \in II} A_i)_N = \{ < \cup_{i \in II} A_i, C_{j_1} \cap C_{j_2}, D_{j_1} \cap D_{j_2} >; \forall j_1, j_2 \in II\}$ where $S_{A_{j_1}} =< A_{j_1}, C_{j_1}, D_{j_1} >$and $S_{B_{j_2}} =< B_{j_2}, C_{j_2}, D_{j_2} >$ .

The ƝĄ – intersection of two ƝĄ-sets $\text{ƝĄ}_{A}$ and $\text{ƝĄ}_{B}$ , which is denoted by $\text{ƝĄ}_{A} \cap_{N} \text{ƝĄ}_{B}$ and is of the form $\text{ƝĄ}_{A} \cap_{N} \text{ƝĄ}_{B} = \{ < A \cap B, A_1 \cap B_1, A_2 \cap B_2 > ; \forall < A, A_1, A_2 > \in \text{ƝĄ}_{A}, < B, B_1, B_2 > \in \text{ƝĄ}_{B} \}$ .
Also for the same away we defined that for any collection$\{\text{ƝĄ}_{A_i}; i \in II \}$ of ƝĄ – sets , the ƝĄ-intersection of this collection is of form $(\cap_{i \in II} A_i)_N = \{ < \cap_{i \in II} A_i, C_{i_1} \cap C_{i_2}, D_{i_1} \cap D_{i_2} >; \forall i_1, i_2 \in II\}$ where $S_{A_{i_1}} =< A_{i_1}, C_{i_1}, D_{i_1} >$and $S_{B_{i_2}} =< B_{i_2}, C_{i_2}, D_{i_2} >$ .
The ƝĄ –participating of two ƝĄ-sets $\text{ƝĄ}_{A}$ and $\text{ƝĄ}_{B}$ , which is denoted by $\text{ƝĄ}_{A} \Cap \text{ƝĄ}_{B} = \text{ƝĄ}_{A \cap B}$.

It is easy to show that the commutative and associative properties for the ƝĄ – union, ƝĄ – intersection and ƝĄ –participating are satisfied .

**2.9. Proposition**

For any set $X$ and any subsets $A , B$ , that is $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B} = \text{ƝĄ}_{A \cup B}$.

**Proof .**

For any $S$ƝĄ-point $SD \in \text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B}$ , which of the form $SD =< A \cap B, A_1 \cap B_1, A_2 \cap B_2 >$ , with $S_A =< A, A_1, A_2 > \in \text{ƝĄ}_{A}$, $S_B =< B, B_1, B_2 > \in \text{ƝĄ}_{B}$ . Thus $SD \in \text{ƝĄ}_{A \cup B}$ , because that $(A \cup B) \cap (A_i \cap B_i) =( A \cap (A_i \cap B_i)) \cup( B \cap (A_i \cap B_i)) = \emptyset$ .
Conversely , now let $S_{A \cup B} \in \text{ƝĄ}_{A \cup B}$ , if possible that $S_{A \cup B} \notin \text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B}$ , since $S_{A \cup B} =< A \cup B, D1 , D2 >$ with $(A \cup B) \cap Di = \emptyset$ , $for\ i = 1 , 2$. But $\emptyset =(A \cup B) \cap Di = ( A \cap Di ) \cup (B \cap Di)$ , imply that $A \cap Di = \emptyset$ , $B \cap Di = \emptyset$, so we have $< A , D1 , D2 > \in \text{ƝĄ}_{A}$ and $< B , D1 , D2 > \in \text{ƝĄ}_{B}$ , also $S_{A \cup B} =< A \cup B , D1 , D2 > = < A \cup B , D1 \cap D1 , D2 \cap D2 > \in \text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{B}$ , which is a contradiction .

From this proposition we can prove easily the following corollary.

**2.10 .Corollary**

1. $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{A^c} = \text{ƝĄ}X$ .
2. $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}_{A} = \text{ƝĄ}_{A}$ .
3. $\text{ƝĄ}_{A} \cup_{N} \text{ƝĄ}X = \text{ƝĄ}X$.

**2.11. Remark**

For any set $X$ and subset $A$ of $X$ we have $\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_X = \mathrm{ȠȠA}_A = \{ \mathrm{ȠP}_A^{\emptyset}\}$, because $\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_X = \{< A \cap X, \emptyset \cap A_1, \emptyset \cap A_2 >$ ; for each $< A, A_1, A_2 > \in \mathrm{ȠA}_A\} = \{< A, \emptyset, \emptyset >\} = \{\mathrm{ȠP}_A^{\emptyset}\} = \mathrm{ȠȠA}_A$.

**2.12. Remark**

Let $X$ be any set with $\mathrm{ȠA}_A$ and $\mathrm{ȠA}_B$ are $\mathrm{ȠA}$ – sets on $X$. If $\mathrm{ȠA}_A <_N \mathrm{ȠA}_B$ , then $(\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B) <_N \mathrm{ȠA}_A$ also $(\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B) <_N \mathrm{ȠA}_B$ because for any $< A \cap B, C_1 \cap D_1, C_2 \cap D_2 > \in \mathrm{ȠA}_A$ from the fact $< A, C1, C2 > \in \mathrm{ȠA}_A$ and $A \cap C_i \cap D_i = \emptyset$ , for $i = 1, 2$.from above (Remark 2.12.) and( Note 2.5.) we have the following proposition .

**2. 13. proposition**

1- $(\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B) <_N \mathrm{ȠA}_{A\cap B}$ .

2- $(\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B) \approx_N \mathrm{ȠA}_A$.

3- If $\mathrm{ȠA}_A <_N \mathrm{ȠA}_B$ and $\mathrm{ȠA}_B <_N \mathrm{ȠA}_C$ , then $\mathrm{ȠA}_A <_N \mathrm{ȠA}_C$ .

4- If $\sqsubseteq B$ , then $\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B \approx_N \mathrm{ȠA}_B$ .

**Proof (4).**

By (Note 2 . 5) and Remark (2 .12) we have $(\mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B) <_N \mathrm{ȠA}_B$ . Now let us $< B, D_1, D_2 > \in \mathrm{ȠA}_B$ , so $B \cap D_i = \emptyset$ , but $A \sqsubseteq B$ , then $A \cap D_i = \emptyset$ for $i = 1,2$ , then $< A, D_1, D_2 > = < A \cap B, D_1, D_2 > \in \mathrm{ȠA}_A \cap_N \mathrm{ȠA}_B$ . So we get the result .

**2. 14. Proposition**

For any $\mathrm{ȠA}$ – points $\mathrm{ȠP}_D, \mathrm{Ƞp}^D \in_N \mathrm{ȠA}_A$ which satisfy that , there exist a part $C$ of some $S\,\mathrm{ȠA}$ – point of $\mathrm{ȠA}_B$ such that $D \sqsubseteq C$ iff $\mathrm{ȠA}_A <_N \mathrm{ȠA}_B$ .

**Proof.**

Let $SA = < A, C1, C2 > \in \mathrm{ȠA}_A$ , then $\mathrm{ȠA}$ – points $\mathrm{ȠP}_{C_1}$ and $\mathrm{ȠP}^{C_2}$ are in $\mathrm{ȠA}_A$ , so by assumption , there exist parts $D1$ , $D2$ of $S\,\mathrm{ȠA}$ – points in $\mathrm{ȠA}_B$ with $Ci \sqsubseteq Di$ and $B \cap D_i = \emptyset$ , $i = 1, 2$ , so $< B, D_1, D_2 > \in \mathrm{ȠA}_B$ , this imply that $\mathrm{ȠA}_A <_N \mathrm{ȠA}_B$ .

Conversely , let $\mathrm{ȠA}_A <_N \mathrm{ȠA}_B$ and let $\mathrm{ȠP}_D \in_N \mathrm{ȠA}_A$ , then the $S\,\mathrm{ȠA}$ – point $< A, D, \emptyset >, < A, \emptyset, D >$ or $< A, D, D >$ are in $\mathrm{ȠA}_A$ , then there exist $< B, C1, \emptyset >$. Such that $D \sqsubseteq C1$ with $\mathrm{ȠP}_{C_1} \in \mathrm{ȠA}_B$ or $< B, \emptyset, C2 >$ such that $D \sqsubseteq C2$ with $\mathrm{ȠP}^{C_2} \in \mathrm{ȠA}_B$ or $< B, H1, H2 > \in \mathrm{ȠA}_B$ with $D \sqsubseteq Hi$ , so $\mathrm{ȠP}_{D_1}$ and $\mathrm{ȠP}^{D_2} \in \mathrm{ȠA}_B$ .

## 3. Conclusions

1. After an extensive study of these sets and spaces , we did this research establishing the basic structures for generalizing the neutrosophic sets , and under the name neutrosophicsets . Therefore , we can put the identification of the topological spaces on it , by taking a family of these ƝĄ – sets that achieve the following ; $ƝĄ_X$ , $ƝĄ_\emptyset$ belong it , second is closed under the finite ƝĄ – intersection , finally is must be closed under ƝĄ – union for any subfamily of it .

2. Also we can study their properties and characteristics , as well as define the functions on there to give as a good suggestions to work . Then , we can modify the various open sets and further study can be continued with this concept. For example , we can modify in the papers [ 20- 28] .

**Acknowledgments:**The authors remain thankful to the referee for his helpful suggestions and comments.

## References


1. F. Smarandache. A Unifying Field in Logics: Neutrosophic Logic. Neutrosophy, Neutrosophic Set, Neutrosophic Probability, American Re-search Press, Rehoboth, NM, **1999**.

2. Salama, A., FlorentinSmarandache, and Valeri Kromov. "Neutrosophic Closed Set and Neutrosophic Continuous Functions." Neutrosophic Sets and Systems, **2014**, 4.1.

3. Salama, A. A., Alblowi, S. A., Smarandache, F."Neutrosophic crisp open set and neutrosophic crisp continuity via neutrosophic crisp ideals". I.J. Information Engineering and Electronic Business, **2014**, vol. 3, pp.1 –8.

4. Zadeh , L. A. , Fuzzy sets , Inform. Control , **1965**, Vol. 8 , PP 338-353 .

5. Almohammed, R., AL-Swidi, L.A. , "New concepts of fuzzy local function " Baghdad Science Journal , **2020**, 17(2), pp. 515-522.

6. Al-Abbasi, H., Al-Swidi, L.A.A. , "On free-open sets " Journal of Advanced Research in Dynamical and Control Systems", **2020**, 12(7), pp. 782-787.

7. Al-Swidi, L.A., Al-Rubaye, M.S. , " New classes of separation axiom via special case of local function ," International Journal of Mathematical Analysis, **2014**, 8(21-24), pp. 1119-1131.

8. Molodtsov , D. , Soft set theory first results , Comput. Math. Appl., **1999**, Vol. 37 , PP 19-31.

9. Al-Swidi, L.A., Al-Ethary, M.A. , "Compactness with "Gem-Set" ," International Journal of Mathematical Analysis,**2014**, 8(21-24), pp. 1105-1117.

10. Al-Swidi, L.A., Awad, F.S.S. , "Analysis on the Soft Bench Points ," ICOASE 2018 - International Conference on Advanced Science and Engineering, **2018** , pp. 330-335 ,.

11. Al Razzaq, A.S.A., Al Swidi, L.A.A.H. , "Soft generalized vague sets: An application in medical diagnosis", Indian Journal of Public Health Research and Development, **2019** ,10(2), pp. 901-907.

12. Hadi, M.H., AL-Yaseen, M.A.A.-K., Al-Swidi, L.A. , "Forms weakly continuity using weak ω–open sets " Journal of Interdisciplinary Mathematics **2020**.

13. Abdulsada, D.A., Al-Swidi, L.A.A. , "Separation axioms of center topological space " Journal of Advanced Research in Dynamical and Control Systems , **2020**, 12(5), pp. 186-192.

14. Abdulsada, D.A., Al-Swidi, L.A.A. , " Some Properties of C-Topological Space " ,1st International Scientific Conference of Computer and Applied Sciences, CAS,**2019**, pp. 52-56.

15. Ali Abdulsada, D., Al-Swidi, L.A.A. " Compatibility of Center Ideals with Center Topology," IOP Conference Series: Materials Science and Engineering, **2020**, 928(4).

16. S. A. Naimpally and . D. Warrack , Proximity spaces , Cambridge Tracts No. 59, **1970** , Cambridge.

17. Mahdi Altalkany, Y.K., Al Swidi, L.A.A. , " Focal Function in i-Topological Spaces via Proximity Spaces " Journal of Physics: Conference Series, **2020**, 1591(1).

18. Al Talkany, A.Y.K.M., AL-Swidi, L.A.A. ," Ψ- operator proximity in i-Topological space ".Turkish Journal of Computer and Mathematics Education Research Article , **2021**, vol. 12, no. 1S, pp. 679-684.

19. Al Talkany, Y.K.M., AL-Swidi, L.A.A. ," New concepts of dense set in i-topological space and proximity space ". Turkish Journal of Computer and Mathematics Education Research Article, **2021**, vol. 12, no. 1S, pp. 685–690.

20. Abdulsada, D.A., Al-Swidi, L.A.A. , "Center Set Theory of Proximity Space" Journal of Physics: Conference Series, **2021**, 1804(1).

21. Al-Abbasi, H., Al-Swidi, L. , "Measurable SB-Functions Space and Confine MSB-Function Topology " Journal of Physics: Conference Series , **2021**, 1804(1).

22. Al-Abbasi, H., Al-Swidi, L.A.A., "Comparison between Confine MO-Connectedness and Connectedness, Confine MO-Countability and Countability" Journal of Physics: Conference Series , **2020**, 1591(1).

23. Al-Swidi, L.A., Mustafa, H.H. , "Characterizations of continuity and compactness with respect to weak forms of ω -Open sets ," European Journal of Scientific Research, , **2011**, 57(4), pp. 577-582.

24. Alswidi, L.A., Alhosaini, A.M.A. , " Weak forms of ω-open sets in bitopological spaces and connectedness ," European Journal of Scientific Research , **2011**, 52(2), pp. 204-212.

25. Abd Al-Haine Al-Swidi, L., Al-Fatlawe, I.J. , "On semi paracompactness and z-paracompactness in bitopological spaces ", European Journal of Scientific Research, **2010**, 47(4), pp. 554-561.

26. Al-Swidi, L.A.H. , "On preparacompactness in bitopological spaces ", European Journal of Scientific Research , **2010**, 46(2), pp. 165-171.

27. Hadi, M.H., Al-Yaseen, M.A.A.-K. , "Study of Hpre -open sets in topological spaces ," AIP Conference Proceedings 2292 , **2020**.

28. Al-Swidi, L.A., Awad, F.S.S. , " On soft turning points " Baghdad Science Journal, **2018** ,15(3), pp. 352-360.

Received: July 5, 2022. Accepted: September 19, 2022.