Neutrosophic Sets and Systems, Vol. 51, 2022
University of New Mexico

Geometric Programming in Imprecise Domain with Application

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Abstract: The paper aims to obtain a computational algorithm to solve a geometric Programming Problem by weighted sum method with equal priority in imprecise condition i.e. in Fuzzy, Intuitionistic Fuzzy and Neutrosophic field. A contrasting study of optimal solution among three has been prescribed to show the efficiency of this method. Numerical example and an application Gravel Box Design Problem is presented to compare different designs. Proposed method is determined by maximizing the truth and indeterminacy membership degree and minimizing the negative membership degree.

Keywords: Geometric Programming, Fuzzy, Intuitionistic Fuzzy, Neutrosophic sets, Gravel Box Design Problem.

1. Introduction

Geometric programming is an advanced method to solve a nonlinear programming problem. It has certain benefits over the other optimization methods. The concept of fuzzy sets (FS) was launched by Zadeh in 1965 **[1]**. Since the fuzzy sets and fuzzy logic have been applied in many real applications to maintain uncertainty. The conventional fuzzy sets uses single real value $\mu_A(x) \in [0, 1]$ to represents the truth membership function of a fuzzy set. In some applications we should consider not only the truth membership supported by the evident but also the falsity membership against by the evident. That is out of the scope of fuzzy sets and interval valued fuzzy sets. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to 1 minus the membership degree because there may be some negative degree. In 1986, Atanassov **[3]**, **[5]** introduced the intuitionistic fuzzy sets (IFS) which is a modification of fuzzy sets. The intuitionistic fuzzy sets consider both truth membership and falsity membership. Intuitionistic fuzzy sets can only handle incomplete information not the indeterminate information and inconsistent information. In IFS, sum of membership-degree and non-membership degree of a vague parameter is less than unity. Therefore a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems. Hence further modification of fuzzy set as well as intuitionistic fuzzy sets are need. In neutrosophic sets (NS) indeterminacy is clarified explicitly and truth membership, indeterminacy membership and falsity membership are not dependent. Neutrosophy was launched by Florentin Smarandache in 1995 **[4]** which is actually generalization of different types of FS and IFS. The term "neutrosophy" means advance information of neutral thought. This neutral concepts make the differece between NS and other sets like FS, IFS.

Fuzzy representation is analyzed by a single variable: degree of truth μ, while the degree of falsity ν has a defect value calculated by negative formula: $\nu = 1- \mu$, and the degree of neutrality has a defect value that is $\sigma =0$.

Intuitionistic fuzzy representation is described by two explicit variables: degree of truth μ and degree of falsity ν, while the degree of neutrality has a defect value that is $\sigma = 0$. Atanassov considered the incomplete variant taking into account that $\mu + \nu \leq 1$.

Neutrosophic representation of information is described by three parameters: degree of truth μ, degree of falsity ν, and degree of neutrality σ.

Intuitionistic fuzzy set is a device in formating real life problem like sale analysis, new product marketing, financial services, negotiation process, portfolio optimization, psychological investigation etc. Since there is a fair chance of the existence of a non-null hesitation part at each moment of evaluation of an unknown object (Szmidt and Kacprzyk, 1997, 2001). Atanassov (1999, 2012) carried out rigorous research based on the theory and applications of intuitionistic fuzzy sets. Geometric programming has been applied to simple riser problems by R.C. Creese [13] using Chvorinov's rule. In the last 20 yrs fuzzy geometric programming has received rapid development in the theory and application. In 2002, B.Y. Cao [11] published the first monograph of fuzzy geometric programming as applied optimization series (vol 76), fuzzy geometric programming by Kluwer academy publishing (the present spinger), the book gives a detailed exposition to theory and application of fuzzy geometric programming. In 1990 R. k. verma [14] has studied fuzzy programming technique to solve geometric programming problems. Recently a paper multi-objective geometric programming problem based on intuitionistic fuzzy geometric programming technique is published by Pintu Das et al. [**15**]. Multi-objective non-linear programming problem based on Neutrosophic Optimization Technique and its application in Riser Design Problem is published by Pintu Das et al. **[16]**. In our uncertain life a decision-maker has to allow to handle indeterminacy or neutral thoughts in decision-making process. Neutrosophic optimization technique is limited in application to design optimization. The motivation of the present study is to explain computational procedure for solving Geometric Programming Problem in imprecise environment (i.e. Fuzzy, Intuitionistic Fuzzy, Neutrosophic) and as an application "Gravel Box Design" problem is represented. A contrasting study of optimal solution among three has been prescribed to show the efficiency of this method. Numerical example and an application Gravel Box Design Problem is presented to compare different designs. Proposed method is determined by maximizing the truth and indeterminacy membership degree and minimizing the negative membership degree.

2. Geometric Programming

Geometric programming (GP) is an advanced method to solve the special class of non-linear programming problems subject to linear or non-linear restriction. The original mathematical development of this method used the arithmetic–geometric mean inequality relationship between sums and products of real numbers. In 1967 Duffin, Peterson and Zener made a beginning stone to solve vast range of engineering problems by basic theories of geometric programming in the book "**Geometric Programming" [12]**. Beightler and Phillips gave a full account of whole modern theory of geometric programming and numerous examples of successful applications of geometric programming to real-world problems in their book "**Applied Geometric Programming" [6]**. The study of GP by Duffin et al. (1967) deals with the problem associating only a positive coefficient for the component cost terms. However, many real world problems comprise of positive as well as negative coefficients for the cost terms. GP method has some advantages. The advantage is that it is sometimes simple to solve the dual problem than primal.

3. Posynomial Geometric Programming Problem

Primal Problem

A single objective posynomial geometric programming problem can be written as

Minimize $f_0(x)$ (1)

subject to

$f_j(x) \leq 1$ (j=1,2,……..,m)

$x_i > 0$ (i=1,2,……...,n)

Where $f_j(x) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} {x_i}^{a_{jki}}$

Where c_{jk} (> 0) and a_{jki} i =1, 2,……,n; k=1,2,………N_j ;

j = 0, 1, 2, ……,m ; are real.

$x \equiv (x_1, x_2, \dots \dots \dots . x_n)^T$.

Dual Problem

The dual programming of (1) is as follows

Maximize d (w) = $\prod_{j=0}^{m} \prod_{k=1}^{N_j} (\frac{c_{jk} w_{j0}}{w_{jk}})^{w_{jk}}$ (2)

subject to

$\sum_{k=1}^{N_j} w_{0k} = 1$ (Normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} a_{jki} w_{jk} = 0$ (Orthogonality condition)

$w_{j0} = \sum_{k=1}^{N_j} w_{jk} \geq 0,\ w_{jk} \geq 0$

i = 1,2,…………n; k= 1,2,……………N_j, $w_{00} = 1$

4. Signomial Geometric Programming Problem

Primal Problem

A single objective signomial geometric programming problem can be formulated as

Min $f_0(x)$ (3)

Subject to

$f_j(x) \leq \delta_j$ (j=1,2,……..,m)

$x_i > 0$ (i=1,2,……...,n)

Where $f_j(x) = \sum_{k=1}^{N_j} \delta_{jk} c_{jk} \prod_{i=1}^{n} x_i^{a_{jki}}$ (j = 0,1,2,……….,m)

$\delta_j = \pm 1$ (j = 2,3, ………..m.), $\delta_{jk} = \pm 1$ (j=0,1,2,……….,m); k=1,2,………,N_j

$x \equiv (x_1, x_2, \dots \dots \dots, x_n)^T$.

Dual Problem

The dual problem of (3) is as follows

Maximize d (w)=$\delta_0 [\prod_{j=0}^{m} \prod_{k=1}^{N_j} (\frac{c_{jk} w_{j0}}{w_{jk}})^{\delta_{jk} w_{jk}}]^{\delta_0}$ (4)

subject to

$\sum_{k=1}^{N_0} \delta_{0k} w_{0k} = \delta_0$ (Normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \delta_{jk} a_{jki} w_{jk} = 0$ (Orthogonality condition)

i = 1,2,……..n.

Where $\delta_j = \pm 1$ (j = 2,3, ………..m.), $\delta_{jk} = \pm 1$ (j=1,2,……….m);

K= 1,2,………..N_j and $w_{00} = 1$

$\delta_0 = \pm 1$

$w_{j0} = \delta_j \sum_{k=1}^{N_j} \delta_{jk} w_{jk} \geq 0, \ \delta_{jk} \geq 0$

j = 1,2,…………m; k= 1,2…………N_j .

5. Fuzzy Geometric Programming (FGP)

A geometric programming problem with fuzzy objective can be written as

$$\widetilde{Minimize} f_0(x) \tag{5}$$

Subject to $f_j(x) \lesssim b_j$ j=1,2,…………..,m

$x \geq 0$

Here the symbol "$\widetilde{Minimize}$" denotes a flexible version of "Minimize". Similarly the symbol "≲" denotes a fuzzy version of "≤". These fuzzy requirements may be determined by taking membership functions $\mu_j(f_j(x))$ (j= 0,1,2,……..m) from the decision maker for all functions $f_j(x)$ (j=0,1,2,……..m) by taking account of the rate of increased membership functions. It is, in general strictly monotone decreasing linear or non-linear functions with respect to $f_j(x)$ (j = 0, 1, 2,……..,m). Here for simplicity, linear membership functions are considered. The linear membership functions can be presented by

$$\mu_j(f_j(x)) = \begin{cases} 1 & if \ f_j(x) \leq f_j^0 \\ \frac{f_j' - f_j(x)}{f_j' - f_j^0} & if \ f_j^0 \leq f_j(x) \leq f_j' \\ 0 & if \ f_j(x) \geq f_j' \end{cases}$$

for j= 0,1,2,3,…………m.

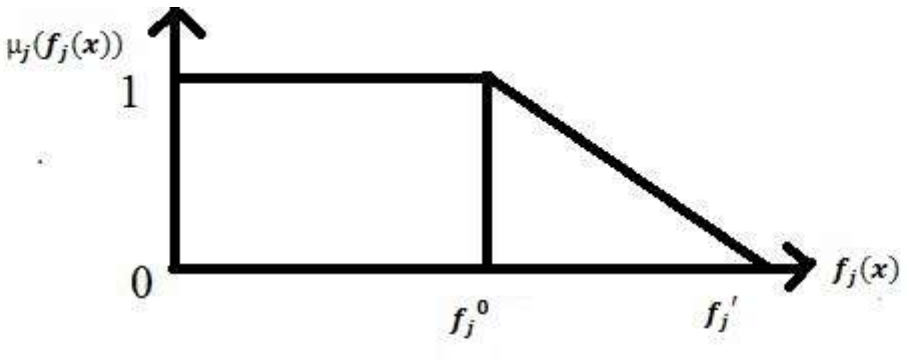

Figure-1: Membership function of a minimization-type objective function

The problem (5) reduces to FGP when f_0(t) and $f_j(x)$ are signomial and posynomial functions.

Based on fuzzy decision making of bellman and zadeh (1972), we may write

i) $\mu_D(x^*) = \max(\min \mu_j(f_j(x)))$ (Max-min operator) (6)

subject to $\mu_j(f_j(x)) = \frac{f_j' - f_j(x)}{f_j' - f_j^0}$

(j= 0,1,2,3,…………,m.)

$x > 0$

ii) $\mu_D(x^*) = \max(\sum_{j=0}^{m} \lambda_j \mu_j(f_j(x)))$ (Max-additive operator) (7)

subject to $\mu_j(f_j(x)) = \frac{f_j' - f_j(x)}{f_j' - f_j^0}$

(j= 0,1,2,3,…………m.)

$$x > 0$$

iii) $\mu_D(x^*) = \max\left(\prod_{j=0}^{m}\left(\mu_j\left(f_j(x)\right)\right)^{\lambda_j}\right)$ (Max - product operator) (8)

subject to $\mu_j(f_j(x)) = \frac{f_j{}' - f_j(x)}{f_j{}' - f_j{}^0}$

(j= 0, 1, 2, 3,...........,m.)

$$x > 0$$

Here for λ_j (j=0,1,2,...........m) are numerical weights determined by a decision making unit . For normalized weights $\sum_{j=0}^{m}\lambda_j = 1$

For equal priority of objective and constraint goals, λ_j =1 and $\lambda_j \in [0, 1]$. For equal priority of objective and constraint goals, $\lambda_j = 1$ (j=0,1,2,...........,m).

6. Numerical Example

Let us take a fuzzy posynomial geometric programming problem as

$$\widetilde{Minimize}\ f_0(x_1,x_2) = 2x_1{}^{-2}x_2{}^{-3} \quad (9)$$

Here objective goal is 57.87 with tolerance 2.91

$f_1(x_1,x_2) = x_1{}^{-1}x_2{}^{-2} \leq 6.75$ (with tolerance 0.19)

$f_2(x_1,x_2) = x_1 + x_2 \leq 1$

$x_1, x_2 > 0$.

Here, linear membership functions for the fuzzy objective and constraint goals are

$$\mu_1(f_1(x_1,x_2)) = \begin{cases} 1 & if\ x_1{}^{-1}x_2{}^{-2} \leq 6.75 \\ \frac{6.94 - x_1{}^{-1}x_2{}^{-2}}{0.19} & if\ 6.75 \leq x_1{}^{-1}x_2{}^{-2} \leq 6.94 \\ 0 & if\ x_1{}^{-1}x_2{}^{-2} \geq 6.94 \end{cases}$$

$$\mu_0\,(f_0(x_1,x_2)) = \begin{cases} 1 & if\ 2x_1^{-2}x_2^{-3} \le 57.87 \\ \frac{60.78-2x_1^{-2}x_2^{-3}}{2.91} & if\ 57.87 \le 2x_1^{-2}x_2^{-3} \le 60.78 \\ 0 & if\ 2x_1^{-2}x_2^{-3} \ge 60.78 \end{cases}$$

Based on the max-additive operator (7), FGP (9) reduces to

Maximize $V_A(x_1,x_2) = \frac{6.94-x_1^{-1}x_2^{-2}}{0.19} + \frac{60.78-2x_1^{-2}x_2^{-3}}{2.91}$

subject to $x_1 + x_2 \le 1$

$x_1, x_2 > 0$

Neglecting the constant term in the above model we have the following crisp geometric programming

Minimize V $(x_1,x_2) = 5.263x_1^{-1}x_2^{-2} + 0.687x_1^{-2}x_2^{-3}$

subject to $x_1 + x_2 \le 1$

$x_1, x_2 > 0$

Here D.D = 4−(2+1) = 1

The DP of this GP is

Max d $(w) = \left(\frac{5.263}{w_{01}}\right)^{w_{01}} \left(\frac{0.687}{w_{02}}\right)^{w_{02}} \left(\frac{1}{w_{11}}\right)^{w_{11}} \left(\frac{1}{w_{12}}\right)^{w_{12}} \times$

$(w_{11} + w_{12})^{(w_{11}+w_{12})}$

Such that $w_{01} + w_{02} = 1$,

$-\ W_{01} - 2w_{02} + w_{11} = 0$,

$-\ 2w_{01} - 3w_{02} + w_{12} = 0$,

So $w_{02} = 1 - w_{01}, w_{11} = 2 - w_{01}, w_{12} = 3 - w_{01}$

Max d (w_{01}) $=\left(\frac{5.263}{w_{01}}\right)^{w_{01}} \left(\frac{0.687}{1-w_{01}}\right)^{1-w_{01}} \left(\frac{1}{2-w_{01}}\right)^{2-w_{01}} \left(\frac{1}{3-w_{01}}\right)^{3-w_{01}} \times$

$(5 - 2w_{01})^{(5-2w_{01})}$

Subject to $0 < w_{01} < 1$

For optimality, $\frac{d(\mathrm{d}(w_{01}))}{dw_{01}} = 0$

$5.263(1- w_{01})(2 - w_{01})(3-w_{01}) = 0.687w_{01}(5 - 2w_{01})^2$

$w_{01}{}^{*}=0.7035507$, $w_{02}{}^{*}= 0.2964493$, $w_{11}{}^{*}= 1.296449$, $w_{12}{}^{*}= 2.296449$.

$x_1{}^{*}= 0.360836$, $x_2{}^{*}= 0.6391634$

$f_0{}^{*}(x_1{}^{*}, x_2{}^{*}) = 58.82652$, $f_1{}^{*}(x_1{}^{*}, x_2{}^{*}) = 6.783684$.

7. Intuitionistic Fuzzy Geometric Programming

Let us consider the intuitionistic fuzzy geometric programming problem as

$$\widetilde{Min}^{i} f_0(x) \quad (10)$$

Subject to $f_j(x) \lesssim^{i} b_j$ j=1,2,…………..,m

$x \geq 0$

Here the symbol "≲i" denotes the intuitionistic fuzzy type of "≤".

Now for Intuitionistic fuzzy geometric programming linear membership and non-membership functions can be prescribed as follows.

$$\mu_j\ (f_j(x)) = \begin{cases} 1 & if\ f_j(x) \le f_j^{\,0} \\ \frac{f_j{}' - f_j(x)}{f_j{}' - f_j^{\,0}} & if\ f_j^{\,0} \le f_j(x) \le f_j{}' \\ 0 & if\ f_j(x) \ge f_j{}' \end{cases}$$

for j= 0,1,2,3,…………,m.

$$\nu_j\ (f_j\ (x)) = \begin{cases} 0 & if\ f_j(x) \le f_j{}' - f_j{}'' \\ \frac{f_j(x) - (f_j{}' - f_j{}'')}{f_j{}''} & if\ f_j{}' - f_j{}'' \le f_j(x) \le f_j{}' \\ 1 & if\ f_j(x) \ge f_j{}' \end{cases}$$

for j= 0,1,2,3,…………,m.

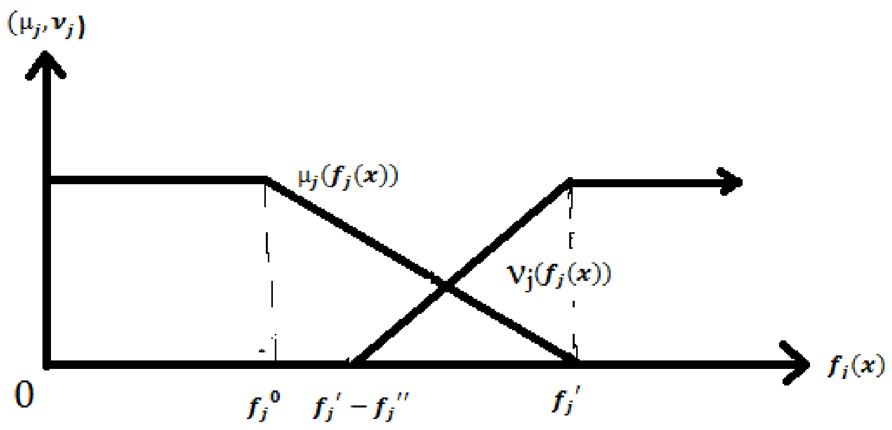

Figure-2: Membership and non-membership functions of a minimization-type objective function.

Now an intuitionistic fuzzy geometric programming problem (10) with membership and non-membership function can be written as

Maximize $\mu_j\ (f_j\ (x))$ (11)

Minimize $\nu_j\ (f_j\ (x))$

for j = 0,1,2,………..,m.

Considering equal priority on all membership and non-membership functions of (11) and using weighted sum method the above optimization problem reduces to

Maximize $V_A\ = \sum_{j=0}^{m}\{\mu_j(f_j\ (x)) - \nu_j\ (f_j\ (x))\}$

subject to $x\ \geq 0$

The above problem is equivalent to

Min $V_{A1} = \sum_{j=0}^{m}\left\{\left(\frac{1}{f_j{}'-f_j{}^0}+\frac{1}{f_j{}''}\right) f_j\ (x) - \left(\frac{f_j{}'-g_j{}''}{f''}+\frac{f_j{}'}{f_j{}'-f_j{}^0}\right)\right\}$

subject to $x\ \geq 0$ (12)

Where $f_j\ (x) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} x_i{}^{a_{jki}}$

Where c_{jk} (> 0) and a_{jki} (i=1,2,…….n; k=1,2,………,N_j ; j=0,1,2,…….,m ;) are real.

$x \equiv (x_1,x_2,\dots\dots\dots.x_n)^T$.

The posynomial geometric programming problem (12) can be solved by usual geometric programming technique.

8. Numerical Example

Let us consider an intuitionistic geometric programming problem with intuitionistic fuzzy goal as

$\widetilde{Min}^i\ f_0\ (x_1,x_2) = \ 2x_1{}^{-2}x_2{}^{-3}$

Here objective goal is 57.87 with tolerance 2.91

$f_1\ (x_1,x_2) = x_1{}^{-1}x_2{}^{-2} \leq\ 6.75$ (with tolerance 0.19)

$f_2\ (x_1,x_2) = x_1 + x_2 \leq\ 1$

$x_1, x_2 > 0$

Here, linear membership and non-membership functions for the fuzzy objective and constraint goals are

$$\mu_0 (f_0(x_1,x_2)) = \begin{cases} 1 & if\ 2x_1^{-2}x_2^{-3} \leq 57.87 \\ \frac{60.78-2x_1^{-2}x_2^{-3}}{2.91} & if\ 57.87 \leq 2x_1^{-2}x_2^{-3} \leq 60.78 \\ 0 & if\ 2x_1^{-2}x_2^{-3} \geq 60.78 \end{cases}$$

$$\mu_1 (f_1 (x_1,x_2)) = \begin{cases} 1 & if\ x_1^{-1}x_2^{-2} \leq 6.75 \\ \frac{6.94-x_1^{-1}x_2^{-2}}{0.19} & if\ 6.75 \leq x_1^{-1}x_2^{-2} \leq 6.94 \\ 0 & if x_1^{-1}x_2^{-2} \geq 6.94 \end{cases}$$

$$\nu_0 (f_0(x_1,x_2)) = \begin{cases} 0 & if\ 2x_1^{-2}x_2^{-3} \leq 59.03 \\ \frac{2x_1^{-2}x_2^{-3}-59.03}{1.75} & if\ 59.03 \leq 2x_1^{-2}x_2^{-3} \leq 60.78 \\ 1 & if\ 2x_1^{-2}x_2^{-3} \geq 60.78 \end{cases}$$

$$\nu_1 (f_1 (x_1,x_2)) = \begin{cases} 0 & if\ x_1^{-1}x_2^{-2} \leq 6.83 \\ \frac{x_1^{-1}x_2^{-2}-6.83}{0.11} & if\ 6.83 \leq x_1^{-1}x_2^{-2} \leq 6.94 \\ 1 & if x_1^{-1}x_2^{-2} \geq 6.94 \end{cases}$$

Minimize $\left(\frac{1}{0.19}+\frac{1}{0.11}\right) x_1^{-1}x_2^{-2} + \left(\frac{1}{2.91}+\frac{1}{1.75}\right) 2x_1^{-2}x_2^{-3}$

subject to $x_1 + x_2 \leq 1$

$x_1, x_2 > 0$

Minimize V $(x_1,x_2) = 14.354x_1^{-1}x_2^{-2} + 1.828x_1^{-2}x_2^{-3}$

Subject to $x_1 + x_2 \leq 1$

$x_1, x_2 > 0$

Here DD = 4−(2+1) = 1

The DP of this GP is

Max d (w) = $\left(\frac{14.354}{w_{01}}\right)^{w_{01}} \left(\frac{1.828}{w_{02}}\right)^{w_{02}} \left(\frac{1}{w_{11}}\right)^{w_{11}} \left(\frac{1}{w_{12}}\right)^{w_{12}} \times$

$(w_{11} + w_{12})^{(w_{11}+w_{12})}$

such that $w_{01} + w_{02} = 1$,

$-\ w_{01} - 2w_{02} + w_{11} = 0$,

$-\ 2w_{01} - 3w_{02} + w_{12} = 0$,

So $w_{02} = 1 - w_{01}$, $w_{11} = 2 - w_{01}$, $w_{12} = 3 - w_{01}$

Max d (w_{01}) = $\left(\frac{14.354}{w_{01}}\right)^{w_{01}} \left(\frac{1.828}{1-w_{01}}\right)^{1-w_{01}} \left(\frac{1}{2-w_{01}}\right)^{2-w_{01}} \left(\frac{1}{3-w_{01}}\right)^{3-w_{01}} \times$

$(5 - 2w_{01})^{(5-2w_{01})}$

Subject to $0 < w_{01} < 1$

For optimality, $\frac{d(\mathrm{d}(w01))}{dw_{01}} = 0$

$14.354(1 - w_{01})(2 - w_{01})(3 - w_{01}) = 1.828 w_{01}(5 - 2w_{01})^2$

$w_{01}{}^{*} = 0.6454384$, $w_{02}{}^{*} = 0.3545616$, $w_{11}{}^{*} = 1.3545616$, $w_{12}{}^{*} = 2.3545616$

$x_1{}^{*} = 0.365197$, $x_2{}^{*} = 0.63348027$

$f_0{}^{*}(x_1{}^{*}, x_2{}^{*}) = 58.62182$, $f_1{}^{*}(x_1{}^{*}, x_2{}^{*}) = 6.795091$

9. Neutrosophic Geometric Programming

Let us consider a neutrosophic geometric programming problem with neutrosophic objective goal as

$$\widetilde{Min}^{n} f_0(x) \qquad (13)$$

Subject to $f_j(x) \lesseqq n\ b_j$ j=1,2,…………..,m

$x \geq 0$

Here the symbol "≦n" denotes the Neutrosophic variant of "≤". Now for Neutrosophic geometric programming linear Truth membership (simply membership), Falsity membership (simply non-membership) and Indeterminacy membership functions can be presented as follows.

$$\mu_j\ (f_j\ (x)) = \begin{cases} 1 & if\ f_j(x) \leq f_j^{\,0} \\ \frac{f_j{}' - f_j(t)}{f_j{}' - f_j{}^0} & if\ f_j^{\,0} \leq f_j(x) \leq f_j{}' \\ 0 & if\ f_j(x) \geq f_j{}' \end{cases}$$

for j= 0,1,2,3,…………,m.

$$\nu_j\ (f_j\ (x)) = \begin{cases} 0 & if\ f_j(x) \leq f_j{}' - f_j{}'' \\ \frac{f_j(x) - (f_j{}' - f_j{}'')}{f_j{}''} & if\ f_j{}' - f_j{}'' \leq f_j(x) \leq f_j{}' \\ 1 & if\ f_j(x) \geq f_j{}' \end{cases}$$

for j= 0,1,2,3,…………m.

$$\sigma_j\ (f_j\ (x)) = \begin{cases} 1 & if\ f_j(x) \leq f_j^{\,0} \\ \frac{(f_j{}' - f_j{}''') - f_j(x)}{f_j{}' - f_j{}''' - f_j{}^0} & if\ f_j^{\,0} \leq f_j(x) \leq f_j{}' - f_j{}''' \\ 0 & if\ f_j(x) \geq f_j{}' - f_j{}''' \end{cases}$$

for j= 0,1,2,3,…………,m.

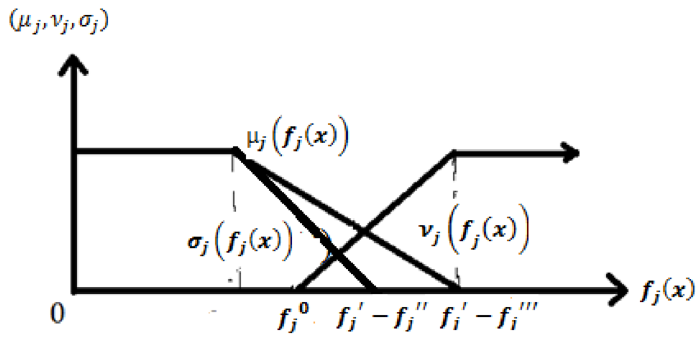

Figure-3: Truth membership, Falsity membership and Indeterminacy membership functions of a minimization-type objective function.

Now Neutrosophic geometric programming problem (13) with Truth membership, Falsity membership and Indeterminacy membership functions can be written as

Maximize $\mu_j\ (f_j\ (x))$ (14)

Minimize $\nu_j\ (f_j\ (x))$

Maximize $\sigma_j\ (f_j\ (x))$

subject to $x \geq 0$

for j = 0,1,2,………..,m.

Using weighted-sum method and giving equal priority on all Truth membership, Falsity membership and Indeterminacy membership functions the above problem (14) becomes

Maximize $V_A = \sum_{j=0}^{m}\{\mu_j(f_j\ (x)) - \nu_j(f_j\ (x)) + \sigma_j\ (f_j\ (x))\}$

subject to $x \geq 0$

The above problem is similar to

Min $V_{A1} = \sum_{j=0}^{m} \left\{ \left(\frac{1}{f_j{}' - f_j{}^{o}} + \frac{1}{f_j{}''} + \frac{1}{f_j{}' - f_j{}''' - f_j{}^{o}} \right) f_j\,(x) - \left(\frac{f_j{}' - f_j{}''}{f_j{}''} + \frac{f_j{}'}{f_j{}' - f_j{}^{o}} + \frac{f_j{}' - f_j{}'''}{f_j{}' - f_j{}''' - f_j{}^{o}} \right) \right\}$

Subject to $x \geq 0$ (15)

Where $f_j(x) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} x_i{}^{a_{jki}}$

Where c_{jk} (> 0) and a_{jki} (i=1,2,…….,n; k=1,2,………,N_j ; j=0,1,2,…….,m ;) are real.

$x \equiv (x_1, x_2, \dots \dots \dots, x_n)^T.$

By usual geometric programming technique the posynomial geometric programming problem (15) can be solved

10. Numerical Example

Let us take a neutrosophic geometric programming problem with neutrosophic objective goal as

$\widetilde{Min}^n\ f_0\ (x_1, x_2) = 2x_1{}^{-2} x_2{}^{-3}$

Here objective goal is 57.87 with tolerance 2.91

$f_1\ (x_1, x_2) = x_1{}^{-1} x_2{}^{-2} \leq$ 6.75 (with tolerance 0.19)

$f_2\ (x_1, x_2) = x_1 + x_2 \leq 1$

$x_1, x_2 > 0.$

Here, linear Truth membership, Falsity membership and Indeterminacy membership functions for the fuzzy objective and constraint goals are

$$\mu_0 (f_0(x_1, x_2)) = \begin{cases} 1 & if\ 2x_1{}^{-2} x_2{}^{-3} \leq 57.87 \\ \frac{60.78 - 2x_1{}^{-2} x_2{}^{-3}}{2.91} & if\ 57.87 \leq 2x_1{}^{-2} x_2{}^{-3} \leq 60.78 \\ 0 & if\ 2x_1{}^{-2} x_2{}^{-3} \geq 60.78 \end{cases}$$

$$\mu_1(f_1(x_1,x_2)) = \begin{cases} 1 & if\ x_1^{-1}x_2^{-2} \leq 6.75 \\ \frac{6.94 - x_1^{-1}x_2^{-2}}{0.19} & if\ 6.75 \leq x_1^{-1}x_2^{-2} \leq 6.94 \\ 0 & if x_1^{-1}x_2^{-2} \geq 6.94 \end{cases}$$

$$\nu_0(f_0(x_1,x_2)) = \begin{cases} 0 & if\ 2x_1^{-2}x_2^{-3} \leq 59.03 \\ \frac{2x_1^{-2}x_2^{-3} - 59.03}{1.75} & if\ 59.03 \leq 2x_1^{-2}x_2^{-3} \leq 60.78 \\ 1 & if\ 2x_1^{-2}x_2^{-3} \geq 60.78 \end{cases}$$

$$\nu_1(f_1(x_1,x_2)) = \begin{cases} 0 & if\ x_1^{-1}x_2^{-2} \leq 6.83 \\ \frac{x_1^{-1}x_2^{-2} - 6.83}{0.11} & if\ 6.83 \leq x_1^{-1}x_2^{-2} \leq 6.94 \\ 1 & if x_1^{-1}x_2^{-2} \geq 6.94 \end{cases}$$

$$\sigma_0(f_0(x_1,x_2)) = \begin{cases} 1 & if\ 2x_1^{-2}x_2^{-3} \leq 57.87 \\ \frac{59.50 - 2x_1^{-2}x_2^{-3}}{1.63} & if\ 57.87 \leq 2x_1^{-2}x_2^{-3} \leq 59.50 \\ 0 & if\ 2x_1^{-2}x_2^{-3} \geq 59.50 \end{cases}$$

$$\sigma_1(f_1(x_1,x_2)) = \begin{cases} 1 & if\ x_1^{-1}x_2^{-2} \leq 6.75 \\ \frac{6.88 - x_1^{-1}x_2^{-2}}{0.13} & if\ 6.75 \leq x_1^{-1}x_2^{-2} \leq 6.88 \\ 0 & if x_1^{-1}x_2^{-2} \geq 6.88 \end{cases}$$

using truth, indeterminacy, falsity membership functions above problem can be formulated as

Minimize V $(x_1,x_2) = 22.046x_1^{-1}x_2^{-2} + 3.057132x_1^{-2}x_2^{-3}$

subject to $x_1 + x_2 \leq 1$

$x_1,x_2 > 0$

Here DD = 4−(2+1) = 1

The DP of this GP is

Max d (w) $=\left(\frac{22.046}{w_{01}}\right)^{w_{01}}\left(\frac{3.057132}{w_{02}}\right)^{w_{02}}\left(\frac{1}{w_{11}}\right)^{w_{11}}\left(\frac{1}{w_{12}}\right)^{w_{12}}\times$

$(w_{11}+w_{12})^{(w_{11}+w_{12})}$

such that $w_{01}+w_{02}=1$,

$-w_{01}-2w_{02}+w_{11}=0$,

$-2w_{01}-3w_{02}+w_{12}=0$,

So $w_{02}=1-w_{01}, w_{11}=2-w_{01}, w_{12}=3-w_{01}$

Max d (w_{01}) $=\left(\frac{22.046}{w_{01}}\right)^{w_{01}}\left(\frac{3.057132}{1-w_{01}}\right)^{1-w_{01}}\left(\frac{1}{2-w_{01}}\right)^{2-w_{01}}\left(\frac{1}{3-w_{01}}\right)^{3-w_{01}}\times(5-2w_{01})^{(5-2w_{01})}$

Subject to $0 < w_{01} < 1$

For optimality, $\frac{d(\mathrm{d}(w_{01}))}{dw_{01}}=0$

$22.046(1-w_{01})(2-w_{01})(3-w_{01})=3.057132w_{01}(5-2w_{01})^2$

$w_{01}{}^{*}=0.6260958$, $w_{02}{}^{*}=0.3739042$, $w_{11}{}^{*}=1.3739042$, $w_{12}{}^{*}=2.3739042$

$x_1{}^{*}=0.366588$, $x_2{}^{*}=0.633411$

$f_0{}^{*}(x_1{}^{*},x_2{}^{*})=58.56211$, $f_1{}^{*}(x_1{}^{*},x_2{}^{*})=6.799086$

11. Application of Neutrosophic Geometric Programming in Gravel Box Design Problem

Gravel Box Problem: A sum of 800 cubic-meters of gravel is to be carried across a river on a barrage. A box (with an open top) is to be made for this occasion. After the whole gravel has been carried, the box is to be rejected. The transport cost per round trip of barrage of box is Rs 1 and the cost of substances of the ends of the box are Rs20/m^2 and the cost of substances of other two sides and bottom are Rs 10/m^2 and Rs 80/m^2. Find the size of the box that is to be made for this occasion and the total optimal cost.

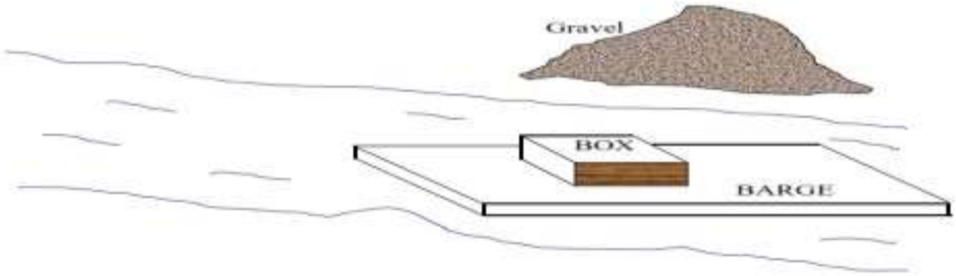

Figure -4: Gravel box design

Let length =x_1 m, breadth = x_2 m, height = x_3 m. The area of the end of the gravel box =x_2x_3 m^2. Area of the sides =x_1x_3 m^2. Area of the bottom =x_1x_2 m^2 .The volume of the gravel box=$x_1x_2x_3$ m^3. Transport cost: Rs $\frac{80}{x_1x_2x_3}$. Material cost: 40x_2x_3.

So the geometric programming problem is

Min $f_0(x_1,x_2,x_3) = \frac{80}{x_1x_2x_3} + 40x_2x_3$

such that $f_1(x_1,x_2,x_3) = x_1x_2 + 2x_1x_3 \leq 4.$

$x_1,x_2,x_3 > 0.$

Here objective goal is 90 (with truth-flexibility 8, falsity-flexibility 5, and indeterminacy-flexibility 5)

and constrained goal

$f_1\,(x_1,x_2,x_3) \leq 4$ (with truth- flexibility 0.9, falsity- flexibility 0.5, indeterminacy-flexibility 0.6)

x_1^*= 2.4775, x_2^*= 1.1271, x_3^*= 0.5635

$f_0^*(x_1^*,x_2^*,x_3^*)$ = 76.237, $f_1^*(x_1^*,x_2^*,x_3^*)$ = 4.5856.

12. Conclusion:

In respect of contrasting the Neutrosophic geometric programming method with Fuzzy, Intuitionistic fuzzy geometric programming method, we also got the solution of the given numerical problem by Fuzzy and Intuitionistic fuzzy optimization method. The aims of the present study is to give the constructive algorithm for geometric programming method in imprecise conditions for obtaining optimal solutions to a single-objective non-linear programming problem.

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Received: July 16, 2022. Accepted: September 21, 2022.