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Single Valued Neutrosophic Similarity Measures for Multiple Attribute Decision-Making

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Abstract. Similarity measures play an important role in data mining, pattern recognition, decision making, machine learning, image process etc. Then, single valued neutrosophic sets (SVNSs) can describe and handle the indeterminate and inconsistent information, which fuzzy sets and intuitionistic fuzzy sets cannot describe and deal with. Therefore, the paper proposes new similarity meas-ures between SVNSs based on the minimum and maxi-mum operators. Then a multiple attribute decision-making method based on the weighted similarity measure of SVNSs is established in which attribute values for al-ternatives are represented by the form of single valued neutrosophic values (SVNVs) and the attribute weights and the weights of the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNV are considered in the decision-making method. In the decision making, we utilize the single-valued neutrosophic weighted similarity measure between the ideal alternative and an alternative to rank the alternatives corresponding to the measure values and to select the most desirable one(s). Finally, two practical examples are provided to demonstrate the applications and effectiveness of the single valued neutrosophic multiple attribute decision-making method.

Keywords: *Neutrosophic set, single valued neutrosophic set, similarity measure, decision making.*

1 Introduction

Since fuzzy sets [1], intuitionistic fuzzy sets (IFSs) [2], interval-valued intuitionistic fuzzy sets (IVIFSs) [3] were introduced, they have been widely applied in data mining, pattern recognition, information retrieval, decision making, machine learning, image process and so on. Although they are very successful in their respective domains, fuzzy sets, IFSs, and IVIFSs cannot describe and deal with the indeterminate and inconsistent information that exists in real world. To handle uncertainty, imprecise, incomplete, and inconsistent information, Smarandache [4] proposed the concept of a neutrosophic set. The neutrosophic set is a powerful general formal framework which generalizes the concepts of the classic set, fuzzy set, IFS, IVIFS etc. [4]. In the neutrosophic set, truth-membership, indeterminacy-membership, and falsity-membership are represented independently. However, the neutrosophic set generalizes the above mentioned sets from philosophical point of view and its functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$, i.e., $T_A(x): X \to]^-0, 1^+[$, $I_A(x): X \to]^-0, 1^+[$, and $F_A(x): X \to]^-0, 1^+[$. Thus, it is difficult to apply in real scientific and engineering areas. Therefore, Wang et al. [5, 6] introduced a single valued neutrosophic set (SVNS) and an interval neutrosophic set (INS), which are the subclass of a neutrosophic set. They can describe and handle indeterminate information and inconsistent information, which fuzzy sets, IFSs, and IVIFSs cannot describe and deal with. Recently, Ye [7-9] proposed the correlation coefficients of SVNSs and the cross-entropy measure of SVNSs and applied them to single valued neutrosophic decision-making problems. Then, Ye [10] introduced similarity measures based on the distances between INSs and applied them to multicriteria decision-making problems with interval neutrosophic information. Chi and Liu [11] proposed an extended TOPSIS method for the multiple attribute decision making problems with interval neutrosophic information. Furthermore, Ye [12] introduced the concept of simplified neutrosophic sets and presented simplified neutrosophic weighted aggregation operators, and then he applied them to multicriteria decision-making problems with simplified neutrosophic information. Majumdar and Samanta [13] introduced several similarity measures between SVNSs based on distances, a matching function, membership grades, and then proposed an entropy measure for a SVNS. Broumi and Smarandache [14] defined the distance between neutrosophic sets on the basis of the Hausdorff distance and some similarity measures based on the distances, set theoretic approach, and matching function to calculate the similarity degree between neutrosophic sets.

Because the concept of similarity is fundamentally important in almost every scientific field and SVNSs can describe and handle the indeterminate and inconsistent information, this paper proposes new similarity measures between SVNSs based on the minimum and maximum operators and establishes a multiple attribute decision-making method based on the weighted similarity measure of SVNSs under single valued neutrosophic environment. To do so, the rest of the article is organized as follows. Section 2 introduces some basic concepts of SVNSs. Section 3 proposes new similarity measures between SVNSs based on the minimum and maximum operators and investigates their properties. In Section 4, a single valued neutrosophic decision-making approach is proposed based on the weighted similarity measure of SVNSs. In Section 5, two practical examples are given to demonstrate the applications and the effectiveness of the proposed decision-making approach. Conclusions and further research are contained in Section 6.

2 Some basic concepts of SVNSs

Smarandache [4] originally introduced the concept of a neutrosophic set from philosophical point of view, which generalizes that of fuzzy set, IFS, and IVIFS etc..

Definition 1 [4]. Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or nonstandard subsets of $]^-0, 1^+[$. That is $T_A(x)$: $X \to\ ]^-0, 1^+[$, $I_A(x)$: $X \to\ ]^-0, 1^+[$, and $F_A(x)$: $X \to\ ]^-0, 1^+[$. Thus, there is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+$.

Obviously, it is difficult to apply in real scientific and engineering areas. Hence, Wang et al. [6] introduced the definition of a SVNS.

Definition 2 [6]. Let X be a universal set. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity-membership function $F_A(x)$. Then, a SVNS A can be denoted by

$$A = \left\{\left\langle x, T_A(x), I_A(x), F_A(x)\right\rangle \mid x \in X\right\},$$

where $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$ for each point x in X. Therefore, the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$ satisfies the condition $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$.

Definition 3 [6]. The complement of a SVNS A is denoted by A^c and is defined as $T_A{}^c(x) = F_A(x)$, $I_A{}^c(x) = 1 - I_A(x)$, $F_A{}^c(x) = T_A(x)$ for any x in X. Then, it can be denoted by

$$A^c = \left\{\left\langle x, F_A(x), 1 - I_A(x), T_A(x)\right\rangle \mid x \in X\right\}.$$

Definition 4 [6]. A SVNS A is contained in the other SVNS B, $A \subseteq B$, if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$ for any x in X.

Definition 5 [6]. Two SVNSs A and B are equal, i.e., $A = B$, if and only if $A \subseteq B$ and $B \subseteq A$.

3 Similarity measures of SVNSs

This section proposes several similarity measures of SVNSs based on the minimum and maximum operators and investigates their properties.

In general, a similarity measure between two SVNSs A and B is a function defined as S: $N(X)^2 \to [0, 1]$ which satisfies the following properties:

(SP1) $0 \le S(A, B) \le 1$;

(SP2) $S(A, B) = 1$ if $A = B$;

(SP3) $S(A, B) = S(B, A)$;

(SP4) $S(A, C) \le S(A, B)$ and $S(A, C) \le S(B, C)$ if $A \subseteq B \subseteq C$ for a SVNS C.

Let two SVNSs A and B in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ be $A = \left\{\left\langle x_i, T_A(x_i), I_A(x_i), F_A(x_i)\right\rangle \mid x_i \in X\right\}$ and $B = \left\{\left\langle x_i, T_B(x_i), I_B(x_i), F_B(x_i)\right\rangle \mid x_i \in X\right\}$, where $T_A(x_i)$, $I_A(x_i)$, $F_A(x_i)$, $T_B(x_i)$, $I_B(x_i)$, $F_B(x_i) \in [0, 1]$ for every $x_i \in X$. Based on the minimum and maximum operators, we present the following similarity measure between A and B:

$$S_1(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{\min\left(T_A(x_i), T_B(x_i)\right)}{\max\left(T_A(x_i), T_B(x_i)\right)} + \frac{\min\left(I_A(x_i), I_B(x_i)\right)}{\max\left(I_A(x_i), I_B(x_i)\right)} + \frac{\min\left(F_A(x_i), F_B(x_i)\right)}{\max\left(F_A(x_i), F_B(x_i)\right)}\right). \quad (1)$$

The similarity measure has the following proposition.

Proposition 1. Let A and B be two SVNSs in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. The single valued neutrosophic similarity measure $S_1(A, B)$ should satisfy the following properties:

(SP1) $0 \le S_1(A, B) \le 1$;

(SP2) $S_1(A, B) = 1$ if $A = B$;

(SP3) $S_1(A, B) = S_1(B, A)$;

(SP4) $S_1(A, C) \le S_1(A, B)$ and $S_1(A, C) \le S_1(B, C)$ if $A \subseteq B \subseteq C$ for a SVNS C.

Proof. It is easy to remark that $S_1(A, B)$ satisfies the properties (SP1)-(SP3). Thus, we must prove the property (SP4).

Let $A \subseteq B \subseteq C$, then, $T_A(x_i) \le T_B(x_i) \le T_C(x_i)$, $I_A(x_i) \ge I_B(x_i) \ge I_C(x_i)$, and $F_A(x_i) \ge F_B(x_i) \ge F_C(x_i)$ for every $x_i \in X$. According to these inequalities, we have the following similarity measures:

$$S_1(A,B) = \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{T_A(x_i)}{T_B(x_i)} + \frac{I_B(x_i)}{I_A(x_i)} + \frac{F_B(x_i)}{F_A(x_i)}\right),$$

$$S_1(A,C) = \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{T_A(x_i)}{T_C(x_i)} + \frac{I_C(x_i)}{I_A(x_i)} + \frac{F_C(x_i)}{F_A(x_i)}\right)$$

$$S_1(B,C) = \frac{1}{3n}\sum_{i=1}^{n}\left(\frac{T_B(x_i)}{T_C(x_i)} + \frac{I_C(x_i)}{I_B(x_i)} + \frac{F_C(x_i)}{F_B(x_i)}\right).$$

Since there are $\frac{T_A(x_i)}{T_B(x_i)} \ge \frac{T_A(x_i)}{T_c(x_i)}$, $\frac{I_B(x_i)}{I_A(x_i)} \ge \frac{I_C(x_i)}{I_A(x_i)}$, and $\frac{F_B(x_i)}{F_A(x_i)} \ge \frac{F_C(x_i)}{F_A(x_i)}$, we can obtain that $S_1(A, C) \le S_1(A, B)$.

Similarly, there are $\frac{T_B(x_i)}{T_C(x_i)} \ge \frac{T_A(x_i)}{T_C(x_i)}$, $\frac{I_C(x_i)}{I_B(x_i)} \ge \frac{I_C(x_i)}{I_A(x_i)}$, and $\frac{F_C(x_i)}{F_B(x_i)} \ge \frac{F_C(x_i)}{F_A(x_i)}$. Then, we can obtain that $S_1(A, C) \le S_1(B, C)$.

Thus $S_1(A, B)$ satisfies the property (SP4).

Therefore, we finish the proof. □

If the important differences are considered in the three independent elements (i.e., truth-membership, indeterminacy-membership, and falsity-membership) in a SVNS, we need to take the weights of the three independent terms in Eq.(1) into account. Therefore, we develop another similarity measure between SVNSs:

$$S_2(A,B) = \frac{1}{n}\sum_{i=1}^{n}\left(\alpha\frac{\min\left(T_A(x_i),T_B(x_i)\right)}{\max\left(T_A(x_i),T_B(x_i)\right)} + \beta\frac{\min\left(I_A(x_i),I_B(x_i)\right)}{\max\left(I_A(x_i),I_B(x_i)\right)} + \gamma\frac{\min\left(F_A(x_i),F_B(x_i)\right)}{\max\left(F_A(x_i),F_B(x_i)\right)}\right), \quad (2)$$

where α, β, γ are the weights of the three independent elements (i.e., truth-membership, indeterminacy-membership, and falsity-membership) in a SVNS and $\alpha + \beta + \gamma = 1$. Especially, when $\alpha = \beta = \gamma = 1/3$, Eq. (2) reduces to Eq. (1).

Then, the similarity measure of $S_2(A, B)$ also has the following proposition:

Proposition 2. Let A and B be two SVNSs in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. The single valued neutrosophic similarity measure $S_2(A, B)$ should satisfy the following properties:

(SP1) $0 \le S_2(A, B) \le 1$;

(SP2) $S_2(A, B) = 1$ if $A = B$;

(SP3) $S_2(A, B) = S_2(B, A)$;

(SP4) $S_2(A, C) \le S_2(A, B)$ and $S_2(A, C) \le S_2(B, C)$ if $A \subseteq B \subseteq C$ for a SVNS C.

By the similar proof method in Proposition 1, we can prove that the similarity measure of $S_2(A, B)$ also satisfies the properties (SP1)-(SP4) (omitted).

Furthermore, if the important differences are considered in the elements in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, we need to take the weight of each element x_i ($i = 1, 2, \ldots, n$) into account. Therefore, we develop a weighted similarity measure between SVNSs.

Let w_i be the weight for each element x_i ($i = 1, 2, \ldots, n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$, and then we have the following weighted similarity measure:

$$S_3(A,B) = \sum_{i=1}^{n} w_i\left(\alpha\frac{\min\left(T_A(x_i),T_B(x_i)\right)}{\max\left(T_A(x_i),T_B(x_i)\right)} + \beta\frac{\min\left(I_A(x_i),I_B(x_i)\right)}{\max\left(I_A(x_i),I_B(x_i)\right)} + \gamma\frac{\min\left(F_A(x_i),F_B(x_i)\right)}{\max\left(F_A(x_i),F_B(x_i)\right)}\right). \quad (3)$$

Similarly, the weighted similarity measure of $S_3(A, B)$ also has the following proposition:

Proposition 3. Let A and B be two SVNSs in a universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$. Then, the single valued neutrosophic similarity measure $S_3(A, B)$ should satisfy the following properties:

(SP1) $0 \le S_3(A, B) \le 1$;

(SP2) $S_3(A, B) = 1$ if $A = B$;

(SP3) $S_3(A, B) = S_3(B, A)$;

(SP4) $S_3(A, C) \le S_3(A, B)$ and $S_3(A, C) \le S_3(B, C)$ if $A \subseteq B \subseteq C$ for a SVNS C.

Similar to the proof method in Proposition 1, we can prove that the weighted similarity measure of $S_3(A, B)$ also satisfies the properties (SP1)–(SP4) (omitted).

If $w = (1/n, 1/n,\ldots, 1/n)^T$, then Eq. (3) reduces to Eq. (2).

For Example, Assume that we have the following three SVNSs in a universe of discourse $X = \{x_1, x_2\}$:

$A = \{<x_1, 0.3, 0.6, 0.7>, <x_2, 0.4, 0.4, 0.6>\}$,

$B = \{<x_1, 0.5, 0.4, 0.5>, <x_2, 0.5, 0.3, 0.4>\}$,

$C = \{<x_1, 0.7, 0.2, 0.3>, <x_2, 0.8, 0.2, 0.2>\}$.

Then, there are $A \subseteq B \subseteq C$, with $T_A(x_i) \le T_B(x_i) \le T_C(x_i)$, $I_A(x_i) \ge I_B(x_i) \ge I_C(x_i)$, and $F_A(x_i) \ge F_B(x_i) \ge F_C(x_i)$ for each x_i in $X = \{x_1, x_2\}$.

By using Eq. (1), the similarity measures between the SVNSs are as follows:

$S_1(A, B) = 0.6996$, $S_1(B, C) = 0.601$, and $S_1(A, C) = 0.4206$.

Thus, there are $S_1(A, C) \le S_1(A, B)$ and $S_1(A, C) \le S_1(B, C)$.

If the weight values of the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNS are $\alpha = 0.25$, $\beta = 0.35$, and $\gamma = 0.4$, by applying Eq. (2) the similarity measures between the SVNSs are as follows:

$S_2(A, B) = 0.6991$, $S_2(B, C) = 0.5916$, and $S_2(A, C) = 0.4143$.

Then, there are $S_2(A, C) \le S_2(A, B)$ and $S_2(A, C) \le S_2(B, C)$.

Assume that the weight vector of the two attributes is $w = (0.4, 0.6)^T$ and the weight values of the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNS are $\alpha = 0.25$, $\beta = 0.35$, and $\gamma = 0.4$. By applying Eq. (3), the weighted similarity measures between the SVNSs are as follows:

$S_3(A, B) = 0.7051$, $S_3(B, C) = 0.4181$, and $S_3(A, C) = 0.5912$.

Hence, there are $S_3(A, C) \le S_3(A, B)$ and $S_3(A, C) \le S_3(B, C)$.

4 Decisions making method using the weighted similarity measure of SVNSs

In this section, we propose a multiple attribute decision-making method based on the weighted similarity measures between SVNSs under single valued neutrosophic environment.

Let $A = \{A_1, A_2,\ldots, A_m\}$ be a set of alternatives and $C = \{C_1, C_2,\ldots, C_n\}$ be a set of attributes. Assume that the weight of the attribute C_j ($j = 1, 2,\ldots, n$) is w_j with $w_j \in [0, 1]$, $\sum_{j=1}^{n} x_j = 1$ and the weights of the three independent elements (i.e., truth-membership, indeterminacy-membership, and falsity-membership) in a SVNS are α, β, and γ and $\alpha + \beta + \gamma = 1$, which are entered by the decision-maker. In this case, the characteristic of the alternative A_i ($i = 1, 2,\ldots, m$) on an attribute C_j ($j = 1, 2,\ldots, n$) is represented by a SVNS form:

$$A_i = \{\langle C_j, T_{A_i}(C_j), I_{A_i}(C_j), F_{A_i}(C_j)\rangle \mid C_j \in C\},$$

where $F_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j) \in [0, 1]$ and $0 \le T_{A_i}(C_j) + I_{A_i}(C_j) + F_{A_i}(C_j) \le 3$ for $C_j \in C$, $j = 1, 2, \ldots, n$, and $i = 1, 2, \ldots, m$.

For convenience, the three elements $T_{A_i}(C_j)$, $I_{A_i}(C_j)$, $F_{A_i}(C_j)$ in the SVNS are denoted by a single valued neutrosophic value (SVNV) $a_{ij} = \langle t_{ij}, i_{ij}, f_{ij}\rangle$ ($i = 1, 2, \ldots, m;\ j = 1, 2,\ldots, n$), which is usually derived from the evaluation of an alternative A_i with respect to an attribute C_j by the expert or decision maker. Hence, we can establish a single valued neutrosophic decision matrix $D = (a_{ij})_{m\times n}$:

$$D = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}.$$

In multiple attribute decision making environments, the concept of ideal point has been used to help identify the best alternative in the decision set [7, 8]. Generally, the evaluation attributes can be categorized into two kinds: benefit attributes and cost attributes. Let H be a collection of benefit attributes and L be a collection of cost attributes. In the presented decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes to determine the best value of each attribute among all alternatives. Therefore, we define an ideal SVNV for a benefit attribute in the ideal alternative A^* as

$$a_j^* = \left\langle t_j^*, i_j^*, f_j^* \right\rangle = \left\langle \max_i (t_{ij}), \min_i (i_{ij}), \min_i (f_{ij}) \right\rangle \text{ for } j \in H;$$

while for a cost attribute, we define an ideal SVNV in the ideal alternative A^* as

$$a_j^* = \left\langle t_j^*, i_j^*, f_j^* \right\rangle = \left\langle \min_i (t_{ij}), \max_i (i_{ij}), \max_i (f_{ij}) \right\rangle \text{ for } j \in L.$$

Thus, by applying Eq. (3), the weighted similarity measure between an alternative A_i and the ideal alternative A^* are written as

$$S_4(A_i, A^*) = \sum_{j=1}^{n} w_j \left(\alpha \frac{\min(t_{ij}, t_j^*)}{\max(t_{ij}, t_j^*)} + \beta \frac{\min(i_{ij}, i_j^*)}{\max(i_{ij}, i_j^*)} + \gamma \frac{\min(f_{ij}, f_j^*)}{\max(f_{ij}, f_j^*)} \right), \quad (4)$$

which provides the global evaluation for each alternative regarding all attributes. According to the weighted similarity measure between each alternative and the ideal alternative, the bigger the measure value $S_4(A_i, A^*)$ (i = 1, 2, 3, 4), the better the alternative A_i. Hence, the ranking order of all alternatives can be determined and the best one can be easily selected as well.

5 Practical examples

This section provides two practical examples for multiple attribute decision-making problems with single valued neutrosophic information to demonstrate the applications and effectiveness of the proposed decision-making method.

Example 1. Let us consider the decision-making problem adapted from [7, 8]. There is an investment company, which wants to invest a sum of money in the best option. There is a panel with four possible alternatives to invest the money: (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; (4) A_4 is an arms company. The investment company must take a decision according to the three attributes: (1) C_1 is the risk; (2) C_2 is the growth; (3) C_3 is the environmental impact, where C_1 and C_2 are benefit attributes and C_3 is a cost attribute. The weight vector of the three attributes is given by w = $(0.35, 0.25, 0.4)^T$. The four possible alternatives are to be evaluated under the above three attributes by the form of SVNVs.

For the evaluation of an alternative A_i (i =1, 2, 3, 4) with respect to an attribute C_j (j =1, 2, 3), it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to an attribute C_1, he/she may say that the possibility in which the statement is good is 0.4 and the statement is poor is 0.3 and the degree in which he/she is not sure is 0.2. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.4, 0.2, 0.3 \rangle$. Thus, when the four possible alternatives with respect to the above three attributes are evaluated by the expert, we can obtain the following single valued neutrosophic decision matrix D:

$$D = \begin{bmatrix} \langle 0.4,0.2,0.3 \rangle & \langle 0.4,0.2,0.3 \rangle & \langle 0.8,0.2,0.5 \rangle \\ \langle 0.6,0.1,0.2 \rangle & \langle 0.6,0.1,0.2 \rangle & \langle 0.5,0.2,0.8 \rangle \\ \langle 0.3,0.2,0.3 \rangle & \langle 0.5,0.2,0.3 \rangle & \langle 0.5,0.3,0.8 \rangle \\ \langle 0.7,0.0,0.1 \rangle & \langle 0.6,0.1,0.2 \rangle & \langle 0.6,0.3,0.8 \rangle \end{bmatrix}.$$

Without loss of generality, let the weight values of the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNV be $\alpha = \beta = \gamma = 1/3$. Then we utilize the developed approach to obtain the most desirable alternative(s).

Firstly, from the single valued neutrosophic decision matrix we can yield the following ideal alternative:

$$A^* = \{\langle C_1, 0.7, 0.0, 0.1 \rangle, \langle C_2, 0.6, 0.1, 0.2 \rangle, \langle C_3, 0.5, 0.3, 0.8 \rangle\}.$$

Then, by using Eq. (4) we can obtain the values of the weighted similarity measure $S_4(A_i, A^*)$ (i =1, 2, 3, 4):

$S_4(A_1, A^*) = 0.6595$, $S_4(A_2, A^*) = 0.9805$,

$S_4(A_3, A^*) = 0.7944$, and $S_4(A_4, A^*) = 0.9828$.

Thus, the ranking order of the four alternatives is $A_4 \succ A_2 \succ A_3 \succ A_1$. Therefore, the alternative A_4 is the best choice among the four alternatives.

From the above results we can see that the ranking order of the alternatives and best choice are in agreement with the results (i.e., the ranking order is $A_4 \succ A_2 \succ A_3 \succ A_1$ and the best choice is A_4.) in Ye's method [8], but not in agreement with the results (i.e., the ranking order is $A_2 \succ A_4 \succ A_3 \succ A_1$ and the best choice is A_2.) in Ye's method [7]. The reason is that different measure methods may yield different ranking orders of the alternatives in the decision-making process.

Example 2. A multi-criteria decision making problem adopted from Ye [9] is concerned with a manufacturing company which wants to select the best global supplier according to the core competencies of suppliers. Now suppose that there are a set of four suppliers $A = \{A_1, A_2, A_3, A_4\}$ whose core competencies are evaluated by means of the four attributes (C_1, C_2, C_3, C_4): (1) the level of technology innovation (C_1), (2) the control ability of flow (C_2), (3) the ability of management (C_3), (4) the level of service (C_4), which are all benefit attributes. Then, the weight vector for the four attributes is $w = (0.3, 0.25, 0.25, 0.2)^T$. The four possible alternatives are to be evaluated under the above four attributes by the form of SVNVs.

For the evaluation of an alternative A_i (i =1, 2, 3, 4) with respect to an attribute C_j (j =1, 2, 3, 4), by the similar evaluation method in Example 1 it is obtained from the questionnaire of a domain expert. For example, when we ask the opinion of an expert about an alternative A_1 with respect to an attribute C_1, he/she may say that the possibility in which the statement is good is 0.5 and the statement is poor is 0.3 and the degree in which he/she is not sure is 0.1. For the neutrosophic notation, it can be expressed as $a_{11} = \langle 0.5, 0.1, 0.3\rangle$. Thus, when the four possible alternatives with respect to the above four attributes are evaluated by the similar method from the expert, we can establish the following single valued neutrosophic decision matrix D:

$$D = \begin{pmatrix} \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.1,0.4\rangle & \langle 0.7,0.1,0.2\rangle & \langle 0.3,0.2,0.1\rangle \\ \langle 0.4,0.2,0.3\rangle & \langle 0.3,0.2,0.4\rangle & \langle 0.9,0.0,0.1\rangle & \langle 0.5,0.3,0.2\rangle \\ \langle 0.4,0.3,0.1\rangle & \langle 0.5,0.1,0.3\rangle & \langle 0.5,0.0,0.4\rangle & \langle 0.6,0.2,0.2\rangle \\ \langle 0.6,0.1,0.2\rangle & \langle 0.2,0.2,0.5\rangle & \langle 0.4,0.3,0.2\rangle & \langle 0.7,0.2,0.1\rangle \end{pmatrix}.$$

Without loss of generality, let the weight values of the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNV be $\alpha = \beta = \gamma = 1/3$. Then the proposed decision-making method is applied to solve this problem for selecting suppliers.

From the single valued neutrosophic decision matrix, we can yield the following ideal alternative:

$$A^* = \{\langle C_1, 0.6, 0.1, 0.1\rangle, \langle C_2, 0.5, 0.1, 0.3\rangle, \langle C_3, 0.9, 0.0, 0.1\rangle, \langle C_4, 0.7, 0.2, 0.1\rangle\}.$$

By applying Eq. (4), the weighted similarity measure values between an alternative A_i (i = 1, 2, 3, 4) and the ideal alternative A^* are as follows:

$S_4(A_1, A^*) = 0.7491$, $S_4(A_2, A^*) = 0.7433$,

$S_4(A_3, A^*) = 0.7605$, and $S_4(A_4, A^*) = 0.6871$.

According to the measure values, the ranking order of the four suppliers is $A_3 \succ A_1 \succ A_2 \succ A_4$. Hence, the best supplier is A_3. From the results we can see that the ranking order of the alternatives and best choice are in agreement with the results in Ye's method [9].

From the above two examples, we can see that the proposed single valued neutrosophic multiple attribute decision-making method is more suitable for real scientific and engineering applications because it can handle not only incomplete information but also the indeterminate information and inconsistent information which exist commonly in real situations. Especially, in the proposed decision-making method we consider the important differences in the three independent elements (i.e., truth-membership degree, indeterminacy-membership degree, and falsity-membership degree) in a SVNV and can adjust the weight values of the three independent elements. Thus, the proposed single valued neutrosophic decision-making method is more flexible and practical than the existing decision-making methods [7-9]. The technique proposed in this paper extends the existing decision-making methods and provides a new way for decision-makers.

6 Conclusion

This paper has developed three similarity measures between SVNSs based on the minimum and maximum operators and investigated their properties. Then the proposed weighted similarity measure of SVNSs has been applied to multiple attribute decision-making problems under single valued neutrosophic environment. The proposed method differs from previous approaches for single valued neutrosophic multiple attribute decision making not only due to the fact that the proposed method use the weighted similarity measure of SVNSs, but also due to considering the weights of the truth-membership, indeterminacy-membership, and falsity-membership in SVNSs, which makes it have more flexible and practical than existing decision making methods [7-9] in real decision making problems. Through the weighted similarity measure between each alternative and the ideal alternative, we can obtain the ranking order of all alternatives and the best alternative. Finally, two practical examples demonstrated the applications and effectiveness of the decision-making approach under single valued neutrosophic environments. The proposed decision-making method can effectively deal with decision-making problems with the incomplete, indeterminate, and inconsistent information which exist commonly in real situations. Furthermore, by the similar method we can easily extend the proposed weighted similarity measure of SVNSs and its decision-making method to that of INSs. In the future, we shall investigate similarity measures between SVNSs and between INSs in the applications of other domains, such as pattern recognition, clustering analysis, image process, and medical diagnosis.

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