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# Connectedness on Hypersoft Topological Spaces

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**Abstract**. Connectedness (resp. disconnectedness), which reflects the key characteristic of topological spaces and helps in the differentiation of two topologies, is one of the most significant and fundamental concept in topological spaces. In light of this, we introduce hypersoft connectedness (resp. hypersoft disconnectedness) in hypersoft topological spaces and investigate its properties in details. Furthermore, we present the concepts of disjoint hypersoft sets, separated hypersoft sets, and hypersoft hereditary property. Also, some examples are provided for the better understanding of these ideas.

**Keywords:** hypersoft connected (resp. hypersoft disconnected); hypersoft topology; hypersoft sets; disjoint hypersoft sets; separated hypersoft sets; hypersoft hereditary property.

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## 1. **Introduction**

Some mathematical concepts, such as theory of fuzzy sets, theory of rough sets, and theory of vague sets, can be considered as mathematical tools for dealing with uncertainties. However, each of these theories has its own difficulties. Molodtsov [1] first proposed the concept of soft sets as a general mathematical tool for dealing with uncertain objects. He successfully applied soft set theory to a variety of fields, including the smoothness of functions, game theory, operation research, Riemann integration, and elsewhere [1,2]. Applications have been made to decision-making, business competitive capacity information systems, classification of natural textures, optimization problems, data analysis, similarity measures, algebraic structures of soft sets, soft matrix theory, parameter reduction in soft set theory, classification of natural textures, and soft sets and their relation to rough and fuzzy sets. In 2003, Maji et al. [3]

presented the basic operations of soft sets. After that, the properties and applications of soft set theory have been studied increasingly [4–7]. In 2011, Shabir and Naz [8] and Çağman et al. [9] introduced and studied the notion of soft topological spaces in different ways. Then, some authors has began to study some of basic concepts and properties of soft topological spaces [10–20]. Moreover, the concept of connectedness attracted the interest of researchers [21, 22].

In 2018, Smarandache [23] proposed the notion of hypersoft set as a generalization of soft set. Then, Saeed et al. [24] put forward the basic concepts of hypersoft set theory. They defined the operators of the intersection, union, and difference between two hypersoft sets as well as a complement of a hypersoft set. In [25], Saeed et al. modified some operators in [24] and presented some new types. Abbas et al. [26], in a unique approach, presented new types of these operators as well as they introduced the concept of hypersoft points.

The concept of bipolar hypersoft sets (a hybridization of hypersoft set and bipolarity) was introduced and discussed in detail by Musa and Asaad [27]. In [28], they initiated the study of bipolar hypersoft topological spaces and studied some topological structures via bipolar hypersoft sets. Musa and Asaad [29] continued studying bipolar hypersoft topological spaces by presenting the notion of bipolar hypersoft connected (resp. bipolar hypersoft disconnected) spaces. The concepts of separated bipolar hypersoft sets and bipolar hypersoft hereditary property were also investigated by them.

Recently, Musa and Asaad [30] initiated the study of hypersoft topological spaces. They defined hypersoft topology as a collection $\mathcal{T}_{\mathcal{H}}$ of hypersoft sets over the universe $\mathcal{U}$ with a fixed set of parameters $\mathcal{E}$. Consequently, they defined basic concepts of hypersoft neighborhood, hypersoft limit point, and hypersoft subspace and investigated their several properties. Furthermore, Musa and Asaad explored and studied in detail hypersoft closure, hypersoft interior, hypersoft exterior, and hypersoft boundary, as well as the relationship between them were discussed.

In this work, we introduce a new concept in hypersoft topological spaces called hypersoft connected (resp. hypersoft disconnected) spaces. Preliminaries on basic notions related to hypersoft sets and hypersoft topological spaces are presented in Section 2. Section 3 gives the concepts of disjoint hypersoft sets, separated hypersoft sets, hypersoft connected (resp. hypersoft disconnected) spaces, and hypersoft hereditary property as well as some examples are given for the better understanding of these ideas. A summary of the recent work and an idea for additional research are provided in Section 4.

## 2. Preliminaries

In this section, we present the necessary concepts and results that are related to hypersoft set and hypersoft topology.

### 2.1. *Hypersoft Sets*

Let $\mathcal{U}$ be an initial universe, $\mathcal{P}(\mathcal{U})$ the power set of $\mathcal{U}$, and $E_1, E_2, ..., E_n$ the pairwise of disjoint sets of parameters. Let $A_i, B_i \subseteq E_i$ for $i = 1, 2, ..., n$.

**Definition 2.1.** [23] A pair $(\mathbb{F}, A_1 \times A_2 \times ... \times A_n)$ is called a hypersoft set over $\mathcal{U}$, where $\mathbb{F}$ is a mapping given by $\mathbb{F} : A_1 \times A_2 \times ... \times A_n \to \mathcal{P}(\mathcal{U})$.

From now on, we write the symbol $\mathcal{E}$ for $E_1 \times E_2 \times ... \times E_n$, $\mathcal{A}$ for $A_1 \times A_2 \times ... \times A_n$, and $\mathcal{B}$ for $B_1 \times B_2 \times ... \times B_n$ where $\mathcal{A}$, $\mathcal{B} \subseteq \mathcal{E}$. Clearly, each element in $\mathcal{A}, \mathcal{B}$ and $\mathcal{E}$ is an $n$-tuple element.

Moreover, we represent hypersoft set $(\mathbb{F}, \mathcal{A})$ as an ordered pair,

$$(\mathbb{F}, \mathcal{A}) = \{(\alpha, \mathbb{F}(\alpha)) : \alpha \in \mathcal{A}\}.$$

**Definition 2.2.** [24] For two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$, we say that $(\mathbb{F}, \mathcal{A})$ is a hypersoft subset of $(\mathbb{G}, \mathcal{B})$ if

(1) $\mathcal{A} \subseteq \mathcal{B}$, and
(2) $\mathbb{F}(\alpha) \subseteq \mathbb{G}(\alpha)$ for all $\alpha \in \mathcal{A}$.

We write $(\mathbb{F}, \mathcal{A}) \,\tilde{\sqsubseteq}\, (\mathbb{G}, \mathcal{B})$.

$(\mathbb{F}, \mathcal{A})$ is said to be a hypersoft superset of $(\mathbb{G}, \mathcal{B})$, if $(\mathbb{G}, \mathcal{B})$ is a hypersoft subset of $(\mathbb{F}, \mathcal{A})$. We denote it by $(\mathbb{F}, \mathcal{A}) \,\tilde{\sqsupseteq}\, (\mathbb{G}, \mathcal{B})$.

**Definition 2.3.** [24] Two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$ are said to be hypersoft equal if $(\mathbb{F}, \mathcal{A})$ is a hypersoft subset of $(\mathbb{G}, \mathcal{B})$ and $(\mathbb{G}, \mathcal{B})$ is a hypersoft subset of $(\mathbb{F}, \mathcal{A})$.

**Definition 2.4.** [24] The complement of a hypersoft set $(\mathbb{F}, \mathcal{A})$ is denoted by $(\mathbb{F}, \mathcal{A})^c$ and is defined by $(\mathbb{F}, \mathcal{A})^c = (\mathbb{F}^c, \mathcal{A})$ where $\mathbb{F}^c : \mathcal{A} \to \mathcal{P}(\mathcal{U})$ is a mapping given by $\mathbb{F}^c(\alpha) = \mathcal{U} \setminus \mathbb{F}(\alpha)$ for all $\alpha \in \mathcal{A}$ .

**Definition 2.5.** [25] A hypersoft set $(\mathbb{F}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative null hypersoft set, denoted by $(\Phi, \mathcal{A})$, if for all $\alpha \in \mathcal{A}$, $\mathbb{F}(\alpha) = \phi$.

The relative null hypersoft set with respect to the universe set of parameters $\mathcal{E}$ is called the null hypersoft set over $\mathcal{U}$ and is denoted by $(\Phi, \mathcal{E})$.

A hypersoft set $(\mathbb{F}, \mathcal{E})$ over $\mathcal{U}$ is said to be a non-null hypersoft set if $\mathbb{F}(\alpha) \neq \phi$ for some $\alpha \in \mathcal{E}$.

**Definition 2.6.** [25] A hypersoft set $(\mathbb{F}, \mathcal{A})$ over $\mathcal{U}$ is said to be a relative whole hypersoft set, denoted by $(\Psi, \mathcal{A})$, if for all $\alpha \in \mathcal{A}$, $\mathbb{F}(\alpha) = \mathcal{U}$.

The relative whole hypersoft set with respect to the universe set of parameters $\mathcal{E}$ is called the whole hypersoft set over $\mathcal{U}$ and is denoted by $(\Psi, \mathcal{E})$.

A hypersoft set $(\mathbb{F}, \mathcal{E})$ over $\mathcal{U}$ is said to be a non-whole hypersoft set if $\mathbb{F}(\alpha) \neq \mathcal{U}$ for some $\alpha \in \mathcal{E}$.

**Definition 2.7.** [25] Difference of two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$, is a hypersoft set $(\mathbb{H}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and for all $\alpha \in C$, $\mathbb{H}(\alpha) = \mathbb{F}(\alpha) \backslash \mathbb{G}(\alpha)$. We write $(\mathbb{F}, \mathcal{A}) \setminus (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

**Definition 2.8.** [25] Union of two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$, is a hypersoft set $(\mathbb{H}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and for all $\alpha \in C$, $\mathbb{H}(\alpha) = \mathbb{F}(\alpha) \cup \mathbb{G}(\alpha)$. We write $(\mathbb{F}, \mathcal{A}) \,\widetilde{\sqcup}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

**Definition 2.9.** [25] Intersection of two hypersoft sets $(\mathbb{F}, \mathcal{A})$ and $(\mathbb{G}, \mathcal{B})$ over a common universe $\mathcal{U}$, is a hypersoft set $(\mathbb{H}, C)$, where $C = \mathcal{A} \cap \mathcal{B}$ and for all $\alpha \in C$, $\mathbb{H}(\alpha) = \mathbb{F}(\alpha) \cap \mathbb{G}(\alpha)$. We write $(\mathbb{F}, \mathcal{A}) \,\widetilde{\sqcap}\, (\mathbb{G}, \mathcal{B}) = (\mathbb{H}, C)$.

**Definition 2.10.** [30] Let $\Upsilon$ be a non-empty subset of $\mathcal{U}$. Then $(\mathcal{Y}, \mathcal{A})$ denotes the hypersoft set over $\mathcal{U}$ defined by $\mathcal{Y}(\alpha) = \Upsilon$ for all $\alpha \in \mathcal{A}$.

**Definition 2.11.** [30] Let $(\mathbb{F}, \mathcal{A})$ be a hypersoft set over $\mathcal{U}$ and $\Upsilon$ be a non-empty subset of $\mathcal{U}$. Then the sub hypersoft set of $(\mathbb{F}, \mathcal{A})$ over $\Upsilon$ denoted by $(\mathbb{F}_\Upsilon, \mathcal{A})$ is defined as $\mathbb{F}_\Upsilon(\alpha) = \Upsilon \cap \mathbb{F}(\alpha)$ for all $\alpha \in \mathcal{A}$.

In other words, $(\mathbb{F}_\Upsilon, \mathcal{E}) = (\mathcal{Y}, \mathcal{A}) \,\widetilde{\sqcap}\, (\mathbb{F}, \mathcal{A})$.

The following results are obvious.

**Proposition 2.12.** *Let $(\mathbb{F}_1, \mathcal{A})$ and $(\mathbb{F}_2, \mathcal{A})$ be two hypersoft sets over a universe $\mathcal{U}$. Then the following holds.*

*(1) $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqcap}\, (\mathbb{F}_2, \mathcal{A}) = (\Phi, \mathcal{A})$ if and only if $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2, \mathcal{A})^c$ and $(\mathbb{F}_2, \mathcal{A}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_1, \mathcal{A})^c$;*
*(2) If $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2, \mathcal{A})$ then $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqcap}\, (\mathbb{F}_2, \mathcal{A}) = (\mathbb{F}_1, \mathcal{A})$;*
*(3) If $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2, \mathcal{A})$ then $(\mathbb{F}_1, \mathcal{A}) \,\widetilde{\sqcup}\, (\mathbb{F}_2, \mathcal{A}) = (\mathbb{F}_2, \mathcal{A})$.*

### 2.2. *Hypersoft Topological Spaces*

Let $\mathcal{U}$ be an initial universe and $\mathcal{E}$ be a set of parameters.

**Definition 2.13.** [30] Let $\mathcal{T}_\mathcal{H}$ be the collection of hypersoft sets over $\mathcal{U}$, then $\mathcal{T}_\mathcal{H}$ is said to be a hypersoft topology on $\mathcal{U}$ if

(1) $(\Phi, \mathcal{E})$, $(\Psi, \mathcal{E})$ belong to $\mathcal{T}_{\mathcal{H}}$,

(2) the intersection of any two hypersoft sets in $\mathcal{T}_{\mathcal{H}}$ belongs to $\mathcal{T}_{\mathcal{H}}$,

(3) the union of any number of hypersoft sets in $\mathcal{T}_{\mathcal{H}}$ belongs to $\mathcal{T}_{\mathcal{H}}$.

We call $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ a hypersoft topological space over $\mathcal{U}$.

**Definition 2.14.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$, then the members of $\mathcal{T}_{\mathcal{H}}$ are said to be hypersoft open sets in $\mathcal{U}$.

**Definition 2.15.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$. A hypersoft set $(\mathbb{F}, \mathcal{E})$ over $\mathcal{U}$ is said to be a hypersoft closed set in $\mathcal{U}$, if its complement $(\mathbb{F}, \mathcal{E})^c$ belongs to $\mathcal{T}_{\mathcal{H}}$.

**Proposition 2.16.** *[30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$. Then*

*(1) $(\Phi, \mathcal{E})$, $(\Psi, \mathcal{E})$ are hypersoft closed set over $\mathcal{U}$,*

*(2) the union of any two hypersoft closed sets is a hypersoft closed set over $\mathcal{U}$,*

*(3) the intersection of any number of hypersoft closed sets is a hypersoft closed set over $\mathcal{U}$.*

**Definition 2.17.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$ and $\Upsilon$ be a non-empty subset of $\mathcal{U}$. Then

$$\mathcal{T}_{\mathcal{H}_{\Upsilon}} = \{(\mathbb{F}_{\Upsilon}, \mathcal{E}) \mid (\mathbb{F}, \mathcal{E}) \in \mathcal{T}_{\mathcal{H}}\}$$

is said to be the relative hypersoft topology on $\Upsilon$ and $(\Upsilon, \mathcal{T}_{\mathcal{H}_{\Upsilon}}, \mathcal{E})$ is called a hypersoft subspace of $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$.

The following results are obvious.

**Proposition 2.18.** *Let $(\Upsilon, \mathcal{T}_{\mathcal{H}_{\Upsilon}}, \mathcal{E})$ be a hypersoft subspace of hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E})$ be a hypersoft set over $\mathcal{U}$, then*

*(1) $(\mathbb{F}, \mathcal{E})$ is hypersoft open in $\Upsilon$ if and only if $(\mathbb{F}, \mathcal{E}) = (\mathcal{Y}, \mathcal{E}) \,\tilde{\sqcap}\, (\mathbb{G}, \mathcal{E})$ for some $(\mathbb{G}, \mathcal{E}) \in \mathcal{T}_{\mathcal{H}}$;*

*(2) $(\mathbb{F}, \mathcal{E})$ is hypersoft closed in $\Upsilon$ if and only if $(\mathbb{F}, \mathcal{E}) = (\mathcal{Y}, \mathcal{E}) \,\tilde{\sqcap}\, (\mathbb{G}, \mathcal{E})$ for some hypersoft closed set $(\mathbb{G}, \mathcal{E})$ in $\mathcal{U}$.*

**Definition 2.19.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space and $(\mathbb{F}, \mathcal{E})$ be a hypersoft set over $\mathcal{U}$. The intersection of all hypersoft closed supersets of $(\mathbb{F}, \mathcal{E})$ is called the hypersoft closure of $(\mathbb{F}, \mathcal{E})$ and is denoted by $\overline{(\mathbb{F}, \mathcal{E})}$.

**Definition 2.20.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$. Then hypersoft interior of hypersoft set $(\mathbb{F}, \mathcal{E})$ over $\mathcal{U}$ is denoted by $(\mathbb{F}, \mathcal{E})^o$ and is defined as the union of all hypersoft open set contained in $(\mathbb{F}, \mathcal{E})$.

**Definition 2.21.** [30] Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$, then hypersoft boundary of hypersoft set $(\mathbb{F}, \mathcal{E})$ over $\mathcal{U}$ is denoted by $(\mathbb{F}, \mathcal{E})^b$ and is defined as $(\mathbb{F}, \mathcal{E})^b = \overline{(\mathbb{F}, \mathcal{E})} \,\tilde{\sqcap}\, \overline{(\mathbb{F}, \mathcal{E})^c}$.

## 3. **Hypersoft Connected (resp. Hypersoft Disconnected ) Spaces**

In this section, we introduce and characterize one of the most important property of hypersoft topological spaces called hypersoft connectedness (resp. hypersoft disconnectedness).

**Definition 3.1.** Two hypersoft sets $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are said to be disjoint hypersoft sets if $(\mathbb{F}_1, \mathcal{E}) \mathbin{\widetilde{\sqcap}} (\mathbb{F}_2, \mathcal{E}) = (\Phi, \mathcal{E})$, that is, $F_1(\alpha) \cap \mathbb{F}_2(\alpha) = \phi$ for all $\alpha \in \mathcal{E}$.

**Definition 3.2.** Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft topological space over $\mathcal{U}$. Two non-null hypersoft sets $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are said to be separated hypersoft sets if and only if $(\mathbb{F}_1, \mathcal{E}) \mathbin{\widetilde{\sqcap}} \overline{(\mathbb{F}_2, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}_1, \mathcal{E})} \mathbin{\widetilde{\sqcap}} (\mathbb{F}_2, \mathcal{E}) = (\Phi, \mathcal{E})$.

Note that any two separated hypersoft sets are disjoint hypersoft sets.

**Remark 3.3.** The following example shows that two disjoint hypersoft sets are not necessarily separated hypersoft sets.

**Example 3.4.** Let $\mathcal{U} = \{u_1, u_2, u_3, u_4\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_1, \mathcal{E}), (\mathbb{F}_2, \mathcal{E}), (\mathbb{F}_3, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$ where $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E})$, and $(\mathbb{F}_3, \mathcal{E})$ are hypersoft sets over $\mathcal{U}$, defined as follows

$$(\mathbb{F}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2, u_3\}), ((e_2, e_3, e_4), \{u_3, u_4\})\}.$$

$$(\mathbb{F}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \{u_3\})\}.$$

$$(\mathbb{F}_3, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1, u_4\}), ((e_2, e_3, e_4), \{u_1, u_2, u_3\})\}.$$

Suppose that $(\mathbb{G}_1, \mathcal{E})$ and $(\mathbb{G}_2, \mathcal{E})$ are two hypersoft sets over $\mathcal{U}$, defined as follows

$$(\mathbb{G}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2, u_3, u_4\}), ((e_2, e_3, e_4), \phi)\}.$$

$$(\mathbb{G}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1\}), ((e_2, e_3, e_4), \mathcal{U})\}.$$

It is easy to see that the two hypersoft sets $(\mathbb{G}_1, \mathcal{E})$ and $(\mathbb{G}_2, \mathcal{E})$ are disjoint hypersoft sets but they are not separated hypersoft sets.

**Proposition 3.5.** *If $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are separated hypersoft sets over $\mathcal{U}$ and $(\mathbb{G}_1, \mathcal{E}) \mathbin{\widetilde{\sqsubseteq}} (\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{G}_2, \mathcal{E}) \mathbin{\widetilde{\sqsubseteq}} (\mathbb{F}_2, \mathcal{E})$, then $(\mathbb{G}_1, \mathcal{E})$ and $(\mathbb{G}_2, \mathcal{E})$ are also separated hypersoft sets.*

*Proof.* We are given that $(\mathbb{F}_1, \mathcal{E}) \mathbin{\widetilde{\sqcap}} \overline{(\mathbb{F}_2, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}_1, \mathcal{E})} \mathbin{\widetilde{\sqcap}} (\mathbb{F}_2, \mathcal{E}) = (\Phi, \mathcal{E})$. Also, $(\mathbb{G}_1, \mathcal{E}) \mathbin{\widetilde{\sqsubseteq}} (\mathbb{F}_1, \mathcal{E})$ implies $\overline{(\mathbb{G}_1, \mathcal{E})} \mathbin{\widetilde{\sqsubseteq}} \overline{(\mathbb{F}_1, \mathcal{E})}$ and $(\mathbb{G}_2, \mathcal{E}) \mathbin{\widetilde{\sqsubseteq}} (\mathbb{F}_2, \mathcal{E})$ implies $\overline{(\mathbb{G}_2, \mathcal{E})} \mathbin{\widetilde{\sqsubseteq}} \overline{(\mathbb{F}_2, \mathcal{E})}$.

It follows that $(\mathbb{G}_1,\mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{G}_2,\mathcal{E})} = (\Phi,\mathcal{E})$ and $\overline{(\mathbb{G}_1,\mathcal{E})} \widetilde{\sqcap} (\mathbb{G}_2,\mathcal{E}) = (\Phi,\mathcal{E})$. Hence, $(\mathbb{G}_1,\mathcal{E})$ and $(\mathbb{G}_2,\mathcal{E})$ are separated hypersoft sets. □

**Proposition 3.6.** *Two hypersoft closed (resp. hypersoft open) sets $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ of a hypersoft topological space are separated hypersoft sets if and only if they are disjoint hypersoft sets.*

*Proof.* Since any two separated hypersoft sets are disjoint hypersoft sets, we need only show that two disjoint hypersoft closed (resp. hypersoft open) sets are separated hypersoft sets. If $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are both disjoint hypersoft sets and hypersoft closed sets, then $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$, $\overline{(\mathbb{F}_1,\mathcal{E})} = (\mathbb{F}_1,\mathcal{E})$, $\overline{(\mathbb{F}_2,\mathcal{E})} = (\mathbb{F}_2,\mathcal{E})$ so that $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}_2,\mathcal{E})} = (\Phi,\mathcal{E})$ and $\overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$ showing that $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are separated hypersoft sets. If $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are both disjoint hypersoft sets and hypersoft open sets, then $(\mathbb{F}_1,\mathcal{E})^c$ and $(\mathbb{F}_2,\mathcal{E})^c$ are hypersoft closed so that $\overline{(\mathbb{F}_1,\mathcal{E})^c} = (\mathbb{F}_1,\mathcal{E})^c$, $\overline{(\mathbb{F}_2,\mathcal{E})^c} = (\mathbb{F}_2,\mathcal{E})^c$. Also, $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$ then $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_2,\mathcal{E})^c$ and $(\mathbb{F}_2,\mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_1,\mathcal{E})^c$. Implies that $\overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqsubseteq} \overline{(\mathbb{F}_2,\mathcal{E})^c} = (\mathbb{F}_2,\mathcal{E})^c$ and $\overline{(\mathbb{F}_2,\mathcal{E})} \widetilde{\sqsubseteq} \overline{(\mathbb{F}_1,\mathcal{E})^c} = (\mathbb{F}_1,\mathcal{E})^c$. It follows that $\overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$ and $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}_2,\mathcal{E})} = (\Phi,\mathcal{E})$. Hence, $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are separated hypersoft sets. □

**Proposition 3.7.** *Let $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are separated hypersoft sets of a hypersoft topological space $(\mathcal{U},\mathcal{T}_{\mathcal{H}},\mathcal{E})$. If $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2,\mathcal{E})$ is a hypersoft closed, then $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are hypersoft closed.*

*Proof.* Suppose that $(\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2,\mathcal{E})$ is a hypersoft closed so that $\overline{(\mathbb{F}_1,\mathcal{E})\widetilde{\sqcup}(\mathbb{F}_2,\mathcal{E})} = (\mathbb{F}_1,\mathcal{E})\widetilde{\sqcup}(\mathbb{F}_2,\mathcal{E})$. To prove that $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ are hypersoft closed, we have to prove that $\overline{(\mathbb{F}_1,\mathcal{E})} = (\mathbb{F}_1,\mathcal{E})$ and $\overline{(\mathbb{F}_2,\mathcal{E})} = (\mathbb{F}_2,\mathcal{E})$. Since we have $\overline{(\mathbb{F}_1,\mathcal{E})\widetilde{\sqcup}(\mathbb{F}_2,\mathcal{E})} = \overline{(\mathbb{F}_1,\mathcal{E})}\widetilde{\sqcup}\overline{(\mathbb{F}_2,\mathcal{E})}$, then $\overline{(\mathbb{F}_1,\mathcal{E})}\widetilde{\sqcup}\overline{(\mathbb{F}_2,\mathcal{E})} = (\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2,\mathcal{E})$. Evidently, $\overline{(\mathbb{F}_1,\mathcal{E})} = \overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} (\overline{(\mathbb{F}_1,\mathcal{E})}\widetilde{\sqcup}\overline{(\mathbb{F}_2,\mathcal{E})})$ $= \overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} ((\mathbb{F}_1,\mathcal{E})\widetilde{\sqcup}(\mathbb{F}_2,\mathcal{E})) = (\overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_1,\mathcal{E})) \widetilde{\sqcup} (\overline{(\mathbb{F}_1,\mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_2,\mathcal{E})) = (\mathbb{F}_1,\mathcal{E}) \widetilde{\sqcup} (\Phi,\mathcal{E}) = (\mathbb{F}_1,\mathcal{E})$. Thus, $\overline{(\mathbb{F}_1,\mathcal{E})} = (\mathbb{F}_1,\mathcal{E})$. Similarly, we can prove that $\overline{(\mathbb{F}_2,\mathcal{E})} = (\mathbb{F}_2,\mathcal{E})$. □

**Definition 3.8.** A hypersoft topological space $(\mathcal{U},\mathcal{T}_{\mathcal{H}},\mathcal{E})$ is said to be hypersoft disconnected if and only if $(\Psi,\mathcal{E})$ can be expressed as the union of two non-null separated hypersoft sets. Otherwise, $(\mathcal{U},\mathcal{T}_{\mathcal{H}},\mathcal{E})$ is said to be hypersoft connected.

**Example 3.9.** Let $\mathcal{U} = \{u_1,u_2\}$, $E_1 = \{e_1,e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}} = \{(\Phi,\mathcal{E}),\ (\Psi,\mathcal{E}),\ (\mathbb{F},\mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$ where $(\mathbb{F},\mathcal{E}) = \{((e_1,e_3,e_4),\mathcal{U}),((e_2,e_3,e_4),\phi)\}$. Then, it is easy to see that $(\mathcal{U},\mathcal{T}_{\mathcal{H}},\mathcal{E})$ is a hypersoft connected space.

**Example 3.10.** Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_1, \mathcal{E}), (\mathbb{F}_2, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$ where $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are hypersoft sets over $\mathcal{U}$, defined as follows

$$(\mathbb{F}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1, u_2\}), ((e_2, e_3, e_4), \{u_1\})\}.$$

$$(\mathbb{F}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_3\}), ((e_2, e_3, e_4), \{u_2, u_3\})\}.$$

Then, obviously $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is a hypersoft disconnected space.

**Proposition 3.11.** *A hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected if and only if any one of the following statements holds.*

*i.* $(\Psi, \mathcal{E})$ *is the union of two non-null disjoint hypersoft open sets;*
*ii.* $(\Psi, \mathcal{E})$ *is the union of two non-null disjoint hypersoft closed sets.*

*Proof.* Follows from Definition 3.8 and Proposition 3.6. □

**Corollary 3.12.** *A hypersoft subspace $(\Upsilon, \mathcal{T}_{\mathcal{H}_\Upsilon}, \mathcal{E})$ of a hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected if and only if $(\mathcal{Y}, \mathcal{E})$ is the union of two non-null disjoint hypersoft sets both hypersoft open (resp. hypersoft closed) sets. Thus, $(\Upsilon, \mathcal{T}_{\mathcal{H}_\Upsilon}, \mathcal{E})$ is hypersoft disconnected if and only if there exist two non-null hypersoft sets $(\mathbb{F}, \mathcal{E})$ and $(\mathbb{G}, \mathcal{E})$ in $\mathcal{U}$ both hypersoft open (resp. hypersoft closed) sets such that $(\mathbb{F}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E}) \neq (\Phi, \mathcal{E})$, $(\mathbb{G}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E}) \neq (\Phi, \mathcal{E})$, $((\mathbb{F}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E})) \tilde{\sqcap} ((\mathbb{G}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E})) = (\Phi, \mathcal{E})$, and $((\mathbb{F}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E})) \tilde{\sqcup} ((\mathbb{G}, \mathcal{E}) \tilde{\sqcap} (\mathcal{Y}, \mathcal{E})) = (\mathcal{Y}, \mathcal{E})$.*

**Proposition 3.13.** *Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ be two hypersoft connected spaces on $\mathcal{U}$, then $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is a hypersoft connected space over $\mathcal{U}$.*

*Proof.* Suppose to the contrary that $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is not a hypersoft connected space. By Proposition 3.11, there exist two non-null disjoint hypersoft sets $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E}) \tilde{\in} \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}$ such that their union is $(\Psi, \mathcal{E})$ in $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$. Since $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E}) \tilde{\in} \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}$ then $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E}) \tilde{\in} \mathcal{T}_{\mathcal{H}_1}$ and $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E}) \tilde{\in} \mathcal{T}_{\mathcal{H}_2}$. This implies that $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E})$ are two non-null disjoint hypersoft sets such that their union is $(\Psi, \mathcal{E})$ in $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E})$ are two non-null disjoint hypersoft sets such that their union is $(\Psi, \mathcal{E})$ in $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ which is a contradiction to given hypothesis. Thus, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is a hypersoft connected space over $\mathcal{U}$. □

**Remark 3.14.** The following example shows that the union of two hypersoft connected spaces over the same universe need not be a hypersoft connected.

**Example 3.15.** Let $\mathcal{U} = \{u_1, u_2\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}_1} = \mathcal{T}_{\mathcal{H}}$ as in Example 3.9 and $\mathcal{T}_{\mathcal{H}_2} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_2, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$ where $(\mathbb{F}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \phi), ((e_2, e_3, e_4), \mathcal{U})\}$. Then, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ are hypersoft connected spaces.
Now, $\mathcal{T}_{\mathcal{H}_1} \tilde{\sqcup} \mathcal{T}_{\mathcal{H}_2} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_1, \mathcal{E}), (\mathbb{F}_2, \mathcal{E})\}$ then $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcup} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is not a hypersoft connected space since the two hypersoft open sets $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are disjoint hypersoft sets and their union is $(\Psi, \mathcal{E})$ in $\mathcal{T}_{\mathcal{H}_1} \tilde{\sqcup} \mathcal{T}_{\mathcal{H}_2}$.

**Proposition 3.16.** *Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ be two hypersoft disconnected spaces on $\mathcal{U}$, then $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcup} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is a hypersoft disconnected space over $\mathcal{U}$.*

*Proof.* This is straightforward. □

**Remark 3.17.** The following example shows that the intersection of two hypersoft disconnected spaces over the same universe need not be a hypersoft disconnected.

**Example 3.18.** Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}_1} = \mathcal{T}_{\mathcal{H}}$ as in Example 3.10 and $\mathcal{T}_{\mathcal{H}_2} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{H}_1, \mathcal{E}), (\mathbb{H}_2, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$ where

$$(\mathbb{H}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1, u_3\}), ((e_2, e_3, e_4), \{u_2\})\}.$$

$$(\mathbb{H}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2\}), ((e_2, e_3, e_4), \{u_1, , u_3\})\}.$$

Then, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ are hypersoft disconnected spaces over $\mathcal{U}$. Now, $\mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E})\}$ then $(\mathcal{U}, \mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is not a hypersoft disconnected since there do not exist two non-null disjoint hypersoft open sets such that their union is $(\Psi, \mathcal{E})$ in $\mathcal{T}_{\mathcal{H}_1} \tilde{\sqcap} \mathcal{T}_{\mathcal{H}_2}$.

**Proposition 3.19.** *A hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected if and only if there exists non-null, non-whole hypersoft set which is both hypersoft open and hypersoft closed.*

*Proof.* Let $(\mathbb{F}, \mathcal{E})$ be a non-null, non-whole hypersoft set which is both hypersoft open and hypersoft closed. We have to show that $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected. Let $(\mathbb{G}, \mathcal{E}) = (\mathbb{F}, \mathcal{E})^c$. Then, $(\mathbb{G}, \mathcal{E})$ is non-null since $(\mathbb{F}, \mathcal{E})$ is non-whole hypersoft set. Moreover, $(\mathbb{F}, \mathcal{E}) \tilde{\sqcup} (\mathbb{G}, \mathcal{E}) = (\Psi, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E}) \tilde{\sqcap} (\mathbb{G}, \mathcal{E}) = (\Phi, \mathcal{E})$. Since $(\mathbb{F}, \mathcal{E})$ is both hypersoft open and hypersoft closed, then $(\mathbb{G}, \mathcal{E})$ is also hypersoft open and hypersoft closed. Hence, $\overline{(\mathbb{F}, \mathcal{E})} = (\mathbb{F}, \mathcal{E})$, $\overline{(\mathbb{G}, \mathcal{E})} = (\mathbb{G}, \mathcal{E})$. It follows that $(\mathbb{F}, \mathcal{E}) \tilde{\sqcap} \overline{(\mathbb{G}, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}, \mathcal{E})} \tilde{\sqcap} (\mathbb{G}, \mathcal{E}) =$

$(\Phi, \mathcal{E})$. Thus, $(\Psi, \mathcal{E})$ has been expressed as the union of two separated hypersoft sets and so $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is a hypersoft disconnected.

Conversely, let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft disconnected. Then, there exists non-null hypersoft sets $(\mathbb{F}, \mathcal{E})$ and $(\mathbb{G}, \mathcal{E})$ such that $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{G}, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}, \mathcal{E})} \widetilde{\sqcap} (\mathbb{G}, \mathcal{E}) = (\Phi, \mathcal{E})$ and $(\Psi, \mathcal{E}) = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcup} (\mathbb{G}, \mathcal{E})$. Since $(\mathbb{F}, \mathcal{E}) \widetilde{\sqsubseteq} \overline{(\mathbb{F}, \mathcal{E})}$, $\overline{(\mathbb{F}, \mathcal{E})} \widetilde{\sqcap} (\mathbb{G}, \mathcal{E}) = (\Phi, \mathcal{E})$ then $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{G}, \mathcal{E}) = (\Phi, \mathcal{E})$. Hence, $(\mathbb{F}, \mathcal{E}) = (\mathbb{G}, \mathcal{E})^c$. Since $(\mathbb{G}, \mathcal{E})$ is non-null and $(\mathbb{G}, \mathcal{E}) \widetilde{\sqcup} (\mathbb{G}, \mathcal{E})^c = (\Psi, \mathcal{E})$, it follows that $(\mathbb{G}, \mathcal{E}) = (\mathbb{F}, \mathcal{E})^c$ is a non-whole hypersoft set. Now, $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcup} \overline{(\mathbb{G}, \mathcal{E})} = (\Psi, \mathcal{E})$. Also, $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{G}, \mathcal{E})} = (\Phi, \mathcal{E})$ then $(\mathbb{F}, \mathcal{E}) = [\overline{(\mathbb{G}, \mathcal{E})}]^c$ and similarly $(\mathbb{G}, \mathcal{E}) = [\overline{(\mathbb{F}, \mathcal{E})}]^c$. Since $\overline{(\mathbb{F}, \mathcal{E})}$ and $\overline{(\mathbb{G}, \mathcal{E})}$ are hypersoft closed sets, it follows that $(\mathbb{F}, \mathcal{E})$ and $(\mathbb{G}, \mathcal{E})$ are hypersoft open sets. Since $(\mathbb{F}, \mathcal{E}) = (\mathbb{G}, \mathcal{E})^c$, $(\mathbb{F}, \mathcal{E})$ is also hypersoft closed set. Thus, $(\mathbb{F}, \mathcal{E})$ is non-null, non-whole hypersoft set which is both hypersoft open and hypersoft closed. We have shown incidentally that $(\mathbb{G}, \mathcal{E})$ is also a non-null, non-whole hypersoft set which is both hypersoft open and hypersoft closed. □

**Corollary 3.20.** *A hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft connected if and only if the only hypersoft sets which are both hypersoft open and hypersoft closed are $(\Phi, \mathcal{E})$ and $(\Psi, \mathcal{E})$.*

**Corollary 3.21.** *Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft topological space and let $\Upsilon$ be a non-empty subset of $\mathcal{U}$. Then, $(\Upsilon, \mathcal{T}_{\mathcal{H}_\Upsilon}, \mathcal{E})$ is hypersoft disconnected if and only if there exists non-null, non-whole hypersoft set, say, $(\mathbb{F}_\Upsilon, \mathcal{E})$ which is both hypersoft open and hypersoft closed. That is, if and only if there exists a hypersoft open set, say, $(\mathbb{F}, \mathcal{E})$ in $\mathcal{U}$ and a hypersoft closed set, say, $(\mathbb{G}, \mathcal{E})$ in $\mathcal{U}$ such that $(\mathbb{F}_\Upsilon, \mathcal{E}) = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathcal{Y}, \mathcal{E})$ and $(\mathbb{F}_\Upsilon, \mathcal{E}) = (\mathbb{G}, \mathcal{E}) \widetilde{\sqcap} (\mathcal{Y}, \mathcal{E})$.*

**Proposition 3.22.** *A hypersoft topological space $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft connected if and only if every non-null, non-whole hypersoft set has a non-null hypersoft boundary.*

*Proof.* Let every non-null, non-whole hypersoft set has a non-null hypersoft boundary. To show that $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft connected. Suppose, if possible $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected. Then there exist non-null disjoint hypersoft sets $(\mathbb{F}, \mathcal{E})$ and $(\mathbb{G}, \mathcal{E})$ both hypersoft open and hypersoft closed sets such that $(\Psi, \mathcal{E}) = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcup} (\mathbb{G}, \mathcal{E})$. Therefore, $(\mathbb{F}, \mathcal{E}) = \overline{(\mathbb{F}, \mathcal{E})} = (\mathbb{F}, \mathcal{E})^o$. But, $(\mathbb{F}, \mathcal{E})^b = \overline{(\mathbb{F}, \mathcal{E})} \setminus (\mathbb{F}, \mathcal{E})^o$. Hence, $(\mathbb{F}, \mathcal{E})^b = (\mathbb{F}, \mathcal{E}) \setminus (\mathbb{F}, \mathcal{E}) = (\Phi, \mathcal{E})$, which is contrary to our hypothesis. Hence, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ must be hypersoft connected.

Conversely, let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be hypersoft connected and suppose, if possible, there exists a non-null, non-whole hypersoft set $(\mathbb{F}, \mathcal{E})$ such that $(\mathbb{F}, \mathcal{E})^b = (\Phi, \mathcal{E})$. Now, $\overline{(\mathbb{F}, \mathcal{E})} = (\mathbb{F}, \mathcal{E})^o \widetilde{\sqcup} (\mathbb{F}, \mathcal{E})^b = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcup} (\mathbb{F}, \mathcal{E})^b$. Hence, $\overline{(\mathbb{F}, \mathcal{E})} = (\mathbb{F}, \mathcal{E})^o = (\mathbb{F}, \mathcal{E})$ showing that $(\mathbb{F}, \mathcal{E})$ is both both hypersoft open and hypersoft closed set and therefore $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is hypersoft disconnected by Proposition 3.19. But this is a contradiction. Hence, every non-null, non-whole hypersoft set must have a non-null hypersoft boundary. □

**Proposition 3.23.** *Let $(\mathcal{U}_1, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}_2, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ be two hypersoft topological spaces. If $(\mathcal{U}_1, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ is hypersoft disconnected and $\mathcal{T}_{\mathcal{H}_1} \widetilde{\sqsubseteq} \mathcal{T}_{\mathcal{H}_2}$, then $(\mathcal{U}_2, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is hypersoft disconnected.*

*Proof.* Since $(\mathcal{U}_1, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ is hypersoft disconnected, then there exists non-null, non-whole hypersoft set $(\mathbb{F}, \mathcal{E})$ which is both hypersoft open and hypersoft closed in $\mathcal{U}_1$. Since $\mathcal{T}_{\mathcal{H}_2}$ is finer than $\mathcal{T}_{\mathcal{H}_1}$, then $(\mathbb{F}, \mathcal{E})$ is a hypersoft open set belonging to $\mathcal{U}_2$. Again, since $(\mathbb{F}, \mathcal{E})$ is a hypersoft closed set in $\mathcal{U}_1$, then $(\mathbb{F}, \mathcal{E})^c$ is a hypersoft open set. Since $\mathcal{T}_{\mathcal{H}_2}$ is finer than $\mathcal{T}_{\mathcal{H}_1}$, then $(\mathbb{F}, \mathcal{E})^c$ is a hypersoft open set belonging to $\mathcal{U}_2$ and consequently $(\mathbb{F}, \mathcal{E})$ is a hypersoft closed set in $\mathcal{U}_2$. Thus, $(\mathbb{F}, \mathcal{E})$ is a non-null, non-whole hypersoft set which is both hypersoft open and hypersoft closed in $\mathcal{U}_2$. It follows that $(\mathcal{U}_2, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is hypersoft disconnected. □

**Corollary 3.24.** *Let $(\mathcal{U}_1, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ and $(\mathcal{U}_2, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ be two hypersoft topological spaces. If $(\mathcal{U}_1, \mathcal{T}_{\mathcal{H}_1}, \mathcal{E})$ is hypersoft connected and $\mathcal{T}_{\mathcal{H}_2} \widetilde{\sqsubseteq} \mathcal{T}_{\mathcal{H}_1}$, then $(\mathcal{U}_2, \mathcal{T}_{\mathcal{H}_2}, \mathcal{E})$ is hypersoft connected.*

**Definition 3.25.** Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft space over $\mathcal{U}$. A hypersoft set $(\mathbb{F}, \mathcal{E})$ is said to be hypersoft disconnected if and only if it is the union of two non-null separated hypersoft sets, that is, if and only if there exists two non-null hypersoft sets $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ such that $(\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}_2, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}_1, \mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E}) = (\Phi, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E}) = (\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2, \mathcal{E})$. A hypersoft set $(\mathbb{F}, \mathcal{E})$ is said to be hypersoft connected if it is not hypersoft disconnected.

**Proposition 3.26.** *Let $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ be a hypersoft topological space and let $(\mathbb{F}, \mathcal{E})$ be a hypersoft connected set such that $(\mathbb{F}, \mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2, \mathcal{E})$ where $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are separated hypersoft sets. Then $(\mathbb{F}, \mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_1, \mathcal{E})$ or $(\mathbb{F}, \mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_2, \mathcal{E})$.*

*Proof.* Since $(\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}_2, \mathcal{E})$ are separated hypersoft sets, then $(\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}_2, \mathcal{E})} = (\Phi, \mathcal{E})$ and $\overline{(\mathbb{F}_1, \mathcal{E})} \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E}) = (\Phi, \mathcal{E})$. Now, $(\mathbb{F}, \mathcal{E}) \widetilde{\sqsubseteq} (\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2, \mathcal{E})$ then $(\mathbb{F}, \mathcal{E}) = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} ((\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcup} (\mathbb{F}_2, \mathcal{E})) = ((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E})) \widetilde{\sqcup} ((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E}))$. We claim that at least one of the hypersoft sets $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E})$ is null hypersoft set. For , if possible, suppose none of these hypersoft sets is null, that is, suppose that $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E}) \neq (\Phi, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E}) \neq (\Phi, \mathcal{E})$. Then, $((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E})) \widetilde{\sqcap} \overline{((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E}))} \widetilde{\sqsubseteq} ((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E})) \widetilde{\sqcap} (\overline{(\mathbb{F}, \mathcal{E})} \widetilde{\sqcap} \overline{(\mathbb{F}_2, \mathcal{E})}) = ((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}, \mathcal{E})}) \widetilde{\sqcap} ((\mathbb{F}_1, \mathcal{E}) \widetilde{\sqcap} \overline{(\mathbb{F}_2, \mathcal{E})}) = (\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\Phi, \mathcal{E}) = (\Phi, \mathcal{E})$. Similarly, $\overline{((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E}))} \widetilde{\sqcap} ((\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E})) = (\Phi, \mathcal{E})$. Hence, $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_1, \mathcal{E})$ and $(\mathbb{F}, \mathcal{E}) \widetilde{\sqcap} (\mathbb{F}_2, \mathcal{E})$ are separated hypersoft sets. Thus, $(\mathbb{F}, \mathcal{E})$ has been expressed as union of two separated hypersoft sets and consequently $(\mathbb{F}, \mathcal{E})$ is hypersoft disconnected.

But this is a contradiction. Hence, at least one of the hypersoft sets $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E})$ is null hypersoft set. If $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_1,\mathcal{E}) = (\Phi,\mathcal{E})$, then $(\mathbb{F},\mathcal{E}) = (\mathbb{F},\mathcal{E})$ $\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E})$ which implies that $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2,\mathcal{E})$. Similarly, if $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$, then $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_1,\mathcal{E})$. Hence, either $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_1,\mathcal{E})$ or $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2,\mathcal{E})$. □

**Proposition 3.27.** *Let $(\mathbb{F},\mathcal{E})$ be a hypersoft connected set and $(\mathbb{G},\mathcal{E})$ be any hypersoft set such that $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{G},\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F},\mathcal{E})}$, then $(\mathbb{G},\mathcal{E})$ is hypersoft connected. In particular, $\overline{(\mathbb{F},\mathcal{E})}$ is hypersoft connected.*

*Proof.* Suppose $(\mathbb{G},\mathcal{E})$ is hypersoft disconnected. Then, there exist non-null hypersoft sets $(\mathbb{F}_1,\mathcal{E})$ and $(\mathbb{F}_2,\mathcal{E})$ such that $(\mathbb{F}_1,\mathcal{E}) \,\widetilde{\sqcap}\, \overline{(\mathbb{F}_2,\mathcal{E})} = (\Phi,\mathcal{E})$ and $\overline{(\mathbb{F}_1,\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$ and $(\mathbb{G},\mathcal{E}) = (\mathbb{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathbb{F}_2,\mathcal{E})$. Since $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{G},\mathcal{E}) = (\mathbb{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathbb{F}_2,\mathcal{E})$, it follows from Proposition 3.26 that $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_1,\mathcal{E})$ or $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_2,\mathcal{E})$. Let $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{F}_1,\mathcal{E})$ which implies that $\overline{(\mathbb{F},\mathcal{E})} \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F}_1,\mathcal{E})}$ then $\overline{(\mathbb{F},\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F}_1,\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$, but $(\Phi,\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F},\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E})$, then we have $\overline{(\mathbb{F},\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$. Also, $(\mathbb{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathbb{F}_2,\mathcal{E})$ $= (\mathbb{G},\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F},\mathcal{E})}$ then $(\mathbb{F}_2,\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{G},\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F},\mathcal{E})}$ implies that $\overline{(\mathbb{F},\mathcal{E})} \,\widetilde{\sqcap}\, (\mathbb{F}_2,\mathcal{E}) = (\mathbb{F}_2,\mathcal{E})$. Hence, $(\mathbb{F}_2,\mathcal{E}) = (\Phi,\mathcal{E})$ which is a contradiction since $(\mathbb{F}_2,\mathcal{E})$ is non-null. Hence, $(\mathbb{G},\mathcal{E})$ must be hypersoft connected. Again, since $(\mathbb{F},\mathcal{E}) \,\widetilde{\sqsubseteq}\, (\mathbb{G},\mathcal{E}) \,\widetilde{\sqsubseteq}\, \overline{(\mathbb{F},\mathcal{E})}$, we see that $\overline{(\mathbb{F},\mathcal{E})}$ is hypersoft connected. □

**Proposition 3.28.** *Let $\{(\mathbb{F}_i,\mathcal{E}) \mid i \in I\}$ be the family of hypersoft connected sets such that $\widetilde{\sqcap}\{(\mathbb{F}_i,\mathcal{E}) \mid i \in I\} \neq (\Phi,\mathcal{E})$. Then $\widetilde{\sqcup}\{(\mathbb{F}_i,\mathcal{E}) \mid i \in I\}$ is hypersoft connected sets.*

*Proof.* Suppose $(\mathcal{F},\mathcal{E}) = \widetilde{\sqcup}\{(\mathbb{F}_i,\mathcal{E}) \mid i \in I\}$ is not hypersoft connected. Then, there exist two non-null disjoint hypersoft sets $(\mathcal{F}_1,\mathcal{E})$ and $(\mathcal{F}_2,\mathcal{E})$ both hypersoft open such that $(\mathcal{F},\mathcal{E}) = (\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathcal{F}_2,\mathcal{E})$. For each $i$, $(\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})$ and $(\mathcal{F}_2,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})$ are disjoint hypersoft sets both hypersoft open in $(\mathbb{F}_i,\mathcal{E})$ such that $((\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})) \,\widetilde{\sqcup}\, ((\mathcal{F}_2,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})) = ((\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathcal{F}_2,\mathcal{E})) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E}) = (\mathbb{F}_i,\mathcal{E})$. Since $(\mathbb{F}_i,\mathcal{E})$ is hypersoft connected, one of the hypersoft sets $(\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})$ and $(\mathcal{F}_2,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E})$ must be null hypersoft set, say, $(\mathcal{F}_1,\mathcal{E})$ $\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E}) = (\Phi,\mathcal{E})$. Then, we have $(\mathcal{F}_2,\mathcal{E}) \,\widetilde{\sqcap}\, (\mathbb{F}_i,\mathcal{E}) = (\mathbb{F}_i,\mathcal{E})$ which implies that $(\mathbb{F}_i,\mathcal{E})$ $\widetilde{\sqsubseteq}\, (\mathcal{F}_2,\mathcal{E})$ for all $i \in I$ and hence $\widetilde{\sqcup}\{(\mathbb{F}_i,\mathcal{E}) \mid i \in I\} \,\widetilde{\sqsubseteq}\, (\mathcal{F}_2,\mathcal{E})$, that is, $(\mathcal{F}_1,\mathcal{E}) \,\widetilde{\sqcup}\, (\mathcal{F}_2,\mathcal{E})$ $\widetilde{\sqsubseteq}\, (\mathcal{F}_2,\mathcal{E})$. This gives, $(\mathcal{F}_1,\mathcal{E}) = (\Phi,\mathcal{E})$ which is a contradiction since $(\mathcal{F}_1,\mathcal{E})$ is non-null. Hence, $(\mathcal{F},\mathcal{E})$ must be hypersoft connected. □

**Definition 3.29.** A property of a hypersoft topological space is said to be hypersoft hereditary if every hypersoft subspace of the space has that property.

**Remark 3.30.** The hypersoft disconnectedness (resp. hypersoft connectedness) is not a hypersoft hereditary.

**Example 3.31.** Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_1, \mathcal{E}), (\mathbb{F}_2, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$, where $(\mathbb{F}_1, \mathcal{E})$, and $(\mathbb{F}_2, \mathcal{E})$, are hypersoft sets over $\mathcal{U}$, defined as follows

$$(\mathbb{F}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2\}), ((e_2, e_3, e_4), \{u_1\})\}.$$

$$(\mathbb{F}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1, u_3\}), ((e_2, e_3, e_4), \{u_2, u_3\})\}.$$

Then, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is a hypersoft disconnected space.

Now, let $\Upsilon = \{u_3\}$, then $\mathcal{T}_{\mathcal{H}_\Upsilon} = \{(\Phi, \mathcal{E}), (\mathcal{Y}, \mathcal{E})\}$ is a hypersoft topology defined on $\mathcal{Y}$. Since $(\Phi, \mathcal{E})$ and $(\mathcal{Y}, \mathcal{E})$ are the only hypersoft open and hypersoft closed sets then by Corollary 3.20, $(\Upsilon, \mathcal{T}_{\mathcal{H}_\Upsilon}, \mathcal{E})$ is a hypersoft connected subspace of hypersoft disconnected space.

**Example 3.32.** Let $\mathcal{U} = \{u_1, u_2, u_3\}$, $E_1 = \{e_1, e_2\}$, $E_2 = \{e_3\}$, and $E_3 = \{e_4\}$. Let $\mathcal{T}_{\mathcal{H}} = \{(\Phi, \mathcal{E}), (\Psi, \mathcal{E}), (\mathbb{F}_1, \mathcal{E}), (\mathbb{F}_2, \mathcal{E}), (\mathbb{F}_3, \mathcal{E})\}$ be a hypersoft topology defined on $\mathcal{U}$, where $(\mathbb{F}_1, \mathcal{E})$, $(\mathbb{F}_2, \mathcal{E})$, and $(\mathbb{F}_3, \mathcal{E})$ are hypersoft sets over $\mathcal{U}$, defined as follows

$$(\mathbb{F}_1, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1\}), ((e_2, e_3, e_4), \{u_2\})\}.$$

$$(\mathbb{F}_2, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2\}), ((e_2, e_3, e_4), \{u_1\})\}.$$

$$(\mathbb{F}_3, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1, u_2\}), ((e_2, e_3, e_4), \{u_1, u_2\})\}.$$

Then, $(\mathcal{U}, \mathcal{T}_{\mathcal{H}}, \mathcal{E})$ is a hypersoft connected space.

Now, let $\Upsilon = \{u_1, u_2\}$, then $\mathcal{T}_{\mathcal{H}_\Upsilon} = \{(\Phi, \mathcal{E}), (\mathcal{Y}, \mathcal{E}), (\mathbb{F}_{1_\Upsilon}, \mathcal{E}), (\mathbb{F}_\Upsilon, \mathcal{E}), (\mathbb{F}_{3_\Upsilon}, \mathcal{E})\}$ is a hypersoft topology defined on $\mathcal{Y}$, where $(\mathbb{F}_{1_\Upsilon}, \mathcal{E})$, $(\mathbb{F}_{2_\Upsilon}, \mathcal{E})$, and $(\mathbb{F}_{3_\Upsilon}, \mathcal{E})$ are hypersoft sets over $\mathcal{Y}$, defined as follows

$$(\mathbb{F}_{1_\Upsilon}, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_1\}), ((e_2, e_3, e_4), \{u_2\})\}.$$

$$(\mathbb{F}_{2_\Upsilon}, \mathcal{E}) = \{((e_1, e_3, e_4), \{u_2\}), ((e_2, e_3, e_4), \{u_1\})\}.$$

$(\mathbb{F}_{3_\Upsilon}, \mathcal{E}) = \{((e_1, e_3, e_4), \Upsilon), ((e_2, e_3, e_4), \Upsilon)\} = (\mathcal{Y}, \mathcal{E})$.

It is easy to see that $(\Upsilon, \mathcal{T}_{\mathcal{H}_\Upsilon}, \mathcal{E})$ is a hypersoft disconnected subspace of hypersoft connected space.

## 4. Conclusions

In this paper, we have initiated the concept of hypersoft connected (resp. hypersoft disconnected) spaces. Then, some results of this concept were discussed. Furthermore, we have presented the concepts of disjoint hypersoft sets, separated hypersoft sets, and hypersoft hereditary property along with some illustrative examples. In future studies, we can define some other topological structures in the frame of hypersoft topological spaces such as hypersoft locally connected space, hypersoft component, hypersoft compact space, and hypersoft paracompact space. Moreover, we can define hypersoft separation axioms by using both ordinary points and hypersoft points.

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