NSS
University of New Mexico

Hybrid Binary Logarithm Similarity Measure for MAGDM Problems under SVNS Assessments

Kalyan Mondal[1], Surapati Pramanik[2], and Bibhas C. Giri[3]

[1]Department of Mathematics, Jadavpur University, Kolkata: 700032, West Bengal, India. E mail:kalyanmathematic@gmail.com
[2]Department of Mathematics, Nandalal Ghosh B.T. College, Panpur, P O - Narayanpur, and District: North 24 Parganas, Pin Code: 743126, West Bengal, India. Email: sura_pati@yahoo.co.in,
[3]Department of Mathematics, Jadavpur University, Kolkata: 700032, West Bengal, India. Email: bibhasc.giri@jadavpuruniversity.in

Abstract: Single valued neutrosophic set is an important mathematical tool for tackling uncertainty in scientific and engineering problems because it can handle situation involving indeterminacy. In this research, we introduce new similarity measures for single valued neutrosophic sets based on binary logarithm function. We define two type of binary logarithm similarity measures and weighted binary logarithm similarity measures for single valued neutrosophic sets. Then we define hybrid binary logarithm similarity measure and weighted hybrid binary logarithm similarity measure for single valued neutrosophic sets. We prove the basic properties of the proposed measures. Then, we define a new entropy function for determining unknown attribute weights. We develop a novel multi attribute group decision making strategy for single valued neutrosophic sets based on the weighted hybrid binary logarithm similarity measure. We present an illustrative example to demonstrate the effectiveness of the proposed strategy. We conduct a sensitivity analysis of the developed strategy. We also present a comparison analysis between the obtained results from proposed strategy and different existing strategies in the literature.

Keywords: single valued neutrosophic set; binary logarithm function; similarity measure; entropy function; ideal solution; MAGDM

1 Introduction

Smarandache [1] introduced neutrosophic sets (NSs) to pave the way to deal with problems involving uncertainty, indeterminacy and inconsistency. Wang et al. [2] grounded the concept of single valued neutrosophic sets (SVNSs), a subclass of NSs to tackle engineering and scientific problems. SVNSs have been applied to solve various problems in different fields such as medical problems [3–5], decision making problems [6–18], conflict resolution [19], social problems [20–21] engineering problems [22-23], image processing problems [24–26] and so on.

The concept of similarity measure is very significant in studying almost every practical field. In the literature, few studies have addressed similarity measures for SNVSs [27–30]. Peng et al. [31] developed SVNSs based multi attribute decision making (MADM) strategy employing MABAC (Multi-Attributive Border Approximation area Comparison and similarity measure), TOPSIS (Technique for Order Preference by Similarity to an Ideal Solution) and a new similarity measure.

Ye [32] proposed cosine similarity measure based neutrosophic multiple attribute decision making (MADM) strategy. In order to overcome some disadvantages in the definition of cosine similarity measure, Ye [33] proposed 'improved cosine similarity measures' based on cosine function. Biswas et al. [34] studied cosine similarity measure based MCDM with trapezoidal fuzzy neutrosophic numbers. Pramanik and Mondal [35] proposed weighted fuzzy similarity measure based on tangent function. Mondal and Pramanik [36] proposed intuitionistic fuzzy similarity measure based on tangent function. Mondal and Pramanik [37] developed tangent similarity measure of SVNSs and applied it to MADM. Ye and Fu [38] studied medical diagnosis problem using a SVNSs similarity measure based on tangent function. Can and Ozguven [39] studied a MADM problem for adjusting the proportional-integral-derivative (PID) coefficients based on neutrosophic Hamming, Euclidean, set-theoretic, Dice, and Jaccard similarity measures.

Several studies [40–42] have been reported in the literature for multi-attribute group decision making (MAGDM) in neutrosophic environment. Ye [43] studied the similarity measure based on distance function of SVNSs and applied it to MAGDM. Ye [44] developed several clustering methods using distance-based similarity measures for SVNSs.

Mondal et al. [45] proposed sine hyperbolic similarity measure for solving MADM problems. Mondal et al. [46] also proposed tangent similarity measure to deal with MADM problems for interval neutrosophic environment.

Lu and Ye [47] proposed logarithmic similarity measure for interval valued fuzzy set [48] and applied it in fault diagnosis strategy.

Research gap:

MAGDM strategy using similarity measure based on binary logarithm function under single valued neutrosophic environment is yet to appear.

Research questions:

- Is it possible to define a new similarity measure between single valued neutrosophic sets using binary logarithm function?
- Is it possible to define a new entropy function for single valued neutrosophic sets for determining unknown attribute weights?
- Is it possible to develop a new MAGDM strategy based on the proposed similarity measures in single valued neutrosophic environment?

The objectives of the paper:

- To define binary logarithm similarity measures for SVNS environment and prove the basic properties.
- To define a new entropy function for determining unknown weight of attributes.
- To develop a multi-attribute droup decision making model based on proposed similarity measures.
- To present a numerical example for the efficiency and effectiveness of the proposed strategy.

Having motivated from the above researches on neutrosophic similarity measures, we introduce the concept of binary logarithm similarity measures for SVNS environment. The properties of binary logarithm similarity measures are established. We also propose a new entropy function to determine unknown attribute weights. We develope a MAGDM strategy using the proposed hybrid binary logarithm similarity measures. The proposed similarity measure is applied to a MAGDM problem.

The structure of the paper is as follows. Section 2 presents basic concepts of NSs, operations on NSs, SVNSs and operations on SVNSs. Section 3 proposes binary logarithm similarity measures and weighted binary logarithm similarity measures, hybrid binary logarithm similarity measure (HBLSM), weighted hybrid binary logarithm similarity measure (WHBLSM) in SVNSs environment. Section 4 proposes a new entropy measure to calculate unknown attribute weights and proves basic properties of entropy function. Section 5 presents a MAGDM strategy based weighted hybrid binary logarithm similarity measure. Section 6 presents an illustrative example to demonstrate the applicability and feasibility of the proposed strategies. Section 7 presents a sensitivity analysis for the results of the numerical example. Section 8 conducts a comparative analysis with the other existing strategies. Section 9 presents the key contribution of the paper. Section 10 summarizes the paper and discusses future scope of research.

2 Preliminaries

In this section, the concepts of NSs, SVNSs, operations on NSs and SVNSs and binary logarithm function are outlined.

2.1 Neutrosophic set (NS)

Assume that X be an universe of discourse. Then a neutrosophic sets [1] N can be defined as follows:

$$N = \{< x: T_N(x), I_N(x), F_N(x) > | \, x \in X\}.$$

Here the functions T, I and F define respectively the membership degree, the indeterminacy degree, and the non-membership degree of the element $x \in X$ to the set N. The three functions T, I and F satisfy the following the conditions:

- $T, I, F: X \rightarrow]^-0,1^+[$
- $^-0 \leq \sup T_N(x) + \sup I_N(x) + \sup F_N(x) \leq 3^+$

For two neutrosophic sets $M = \{< x: T_M(x), I_M(x), F_M(x) > | \, x \in X\}$ and $N = \{< x, T_N(x), I_N(x), F_N(x) > | \, x \in X\}$, the two relations are defined as follows:

- $M \subseteq N$ if and only if $T_M(x) \leq T_N(x)$, $I_M(x) \geq I_N(x)$, $F_M(x) \geq F_N(x)$
- $M = N$ if and only if $T_M(x) = T_N(x)$, $I_M(x) = I_N(x)$, $F_M(x) = F_N(x)$.

2.2. Single valued Neutrosophic sets (SVNSs)

Assume that X be an universe of discourse. A SVNS [2] P in X is formed by a truth-membership function $T_P(x)$, an indeterminacy membership function $I_P(x)$, and a falsity membership function $F_P(x)$. For each point x in X, $T_P(x)$, $I_P(x)$, and $F_P(x) \in [0, 1]$.

For continuous case, a SVNS P can be expressed as follows:

$$P = \int_x \frac{< T_P(x), I_P(x), F_P(x) >}{x} : x \in X \; ,$$

For discrete case, a SVNS P can be expressed as follows:

$$P=\sum_{i=1}^{n}\frac{<T_P(x_i), I_P(x_i), F_P(x_i)>}{x_i}: x_i\in X$$

For two SVNSs $P = \{< x: T_P(x), I_P(x), F_P(x)> \mid x\in X\}$ and $Q = \{< x: T_Q(x), I_Q(x), F_Q(x)> \mid x\in X\}$, some definitions are stated below:

- $P\subseteq Q$ if and only if $T_P(x) \le T_Q(x)$, $I_P(x) \ge I_Q(x)$, and $F_P(x) \ge F_Q(x)$.
- $P\supseteq Q$ if and only if $T_P(x) \ge T_Q(x)$, $I_P(x) \le I_Q(x)$, and $F_P(x) \le F_Q(x)$.
- $P = Q$ if and only if $T_P(x) = T_Q(x)$, $I_P(x) = I_Q(x)$, and $F_P(x) = F_Q(x)$ for any $x\in X$.
- Complement of P i.e. $P^c = \{< x: F_P(x), 1- I_P(x), T_P(x)> \mid x\in X \}$.

2.3. Some arithmetic operations on SVNSs

Definition 1 [49]

Let $P=\langle T_P(x), I_P(x), F_P(x)\rangle$ and $Q=\langle T_Q(x), I_Q(x), F_Q(x)\rangle$ be any two SVNSs in a universe of discourse then arithmetic operations are stated as follows.

- $P\oplus Q=\begin{pmatrix} T_P(x)+T_Q(x)-T_P(x)T_Q(x),\ I_P(x)I_Q(x), \\ F_P(x)F_Q(x) \end{pmatrix}$
- $P\otimes Q=\begin{pmatrix} T_P(x)T_Q(x),\ I_P(x)+I_Q(x)-I_P(x)I_Q(x), \\ F_P(x)+F_Q(x)-F_P(x)F_Q(x) \end{pmatrix}$
- $\alpha P=\left(1-\left(1-T_P(x)^{\alpha}\right), (I_P(x))^{\alpha}, (F_P(x))^{\alpha}\right);\ \alpha>0$
- $(P)^{\alpha}=\left((T_P(x))^{\alpha}, 1-\left(1-I_P(x)^{\alpha}\right), 1-\left(1-F_P(x)^{\alpha}\right)\right);\ \alpha>0$

2.4. Binary logarithm function

In mathematics, the logarithm of the form $\log_2{}^x$, $x > 0$ is called binary logarithm function [50]. For example, the binary logarithm of 1 is 0, the binary logarithm of 4 is 2, the binary logarithm of 16 is 4, and the binary logarithm of 64 is 6.

3. Binary logarithm similarity measures for SVNSs

In this section, we define two types of binary logarithm similarity measures and their hybrid and weighted hybrid similarity measures.

3.1. Binary logarithm similarity measures of SVNSs (type-I)

Definition 2. Let $A = <x(T_A(x_i), I_P(x_i), F_P(x_i))>$ and $B = <x(T_B(x_i), I_B(x_i), F_B(x_i))>$ be any two SVNSs. The binary logarithm similarity measure (type-I) between SVNSs A and B are defined as follows:

$$\mathrm{BL}_1(A,B)=$$

$$\frac{1}{n}\sum_{i=1}^{n}\log_2\left(2-\left(\frac{1}{3}\begin{pmatrix}\left|T_A(x_i)-T_B(x_i)\right|+\left|I_A(x_i)-I_B(x_i)\right| \\ +\left|F_A(x_i)-F_B(x_i)\right|\end{pmatrix}\right)\right) \tag{1}$$

Theorem 1. The binary logarithm similarity measure $\mathrm{BL}_1(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0\le \mathrm{BL}_1(A,B)\le 1$

P2. $\mathrm{BL}_1(A,B)=1$, if and only if $A = B$

P3. $\mathrm{BL}_1(A,B)=\mathrm{BL}_1(B,A)$

P4. If C is a SVNS in X and $A\subseteq B\subseteq C$ then $\mathrm{BL}_1(A,C)\le \mathrm{BL}_1(A,B)$ and $\mathrm{BL}_1(A,C)\le \mathrm{BL}_1(B,C)$.

Proof 1.

From the definition of SVNS, we write,

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and

$0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$\Rightarrow$

$0\le \left|T_A(x_i)-T_B(x_i)\right|+\left|I_A(x_i)-I_B(x_i)\right|+\left|F_A(x_i)-F_B(x_i)\right|\le 3,$

$$0\le \max\begin{pmatrix}\left|T_A(x_i)-T_B(x_i)\right|, \left|I_A(x_i)-I_B(x_i)\right|, \\ \left|F_A(x_i)-F_B(x_i)\right|\end{pmatrix}\le 1$$

$\Rightarrow\ 0\le \mathrm{BL}_1(A,B)\le 1$.

Proof 2.

For any two SVNSs A and B,

$A = B$

$\Rightarrow T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$

$\Rightarrow \left|T_A(x)-T_B(x)\right|=0, \left|I_A(x)-I_B(x)\right|=0,$

$\left|F_A(x)-F_B(x)\right|=0$

$\Rightarrow \mathrm{BL}_1(A,B)=1$.

Conversely,

for $\mathrm{BL}_1(A,B)=1$, we have,

$\Rightarrow \left|T_A(x)-T_B(x)\right|=0,\ \left|I_A(x)-I_B(x)\right|=0,$

$|F_A(x)-F_B(x)|=0$

$\Rightarrow T_A(x)=T_B(x)$, $I_A(x)=I_B(x)$, $F_A(x)=F_B(x)$

$\Rightarrow A = B.$

Proof 3.

We have,

$|T_A(x)-T_B(x)|=|T_B(x)-T_A(x)|$,

$|I_A(x)-I_B(x)|=|I_B(x)-I_A(x)|$,

$|F_A(x)-F_B(x)|=|F_B(x)-F_A(x)|$

$\Rightarrow \mathrm{BL}_1(A,B) = \mathrm{BL}_1(B,A)$.

Proof 4.

For $A\subseteq B\subseteq C$, we have,

$T_A(x) \le T_B(x) \le T_C(x)$, $I_A(x) \ge I_B(x) \ge I_C(x)$,
$F_A(x) \ge F_B(x) \ge F_C(x)$ for $x\in X$.

$\Rightarrow |T_A(x)-T_B(x)| \le |T_A(x)-T_C(x)|$,

$|T_B(x)-T_C(x)| \le |T_A(x)-T_C(x)|$;

$|I_A(x)-I_B(x)| \le |I_A(x)-I_C(x)|$,

$|I_B(x)-I_C(x)| \le |I_A(x)-I_C(x)|$;

$|F_A(x)-F_B(x)| \le |F_A(x)-F_C(x)|$,

$|F_B(x)-F_C(x)| \le |F_A(x)-F_C(x)|$.

$\Rightarrow \mathrm{BL}_1(A,C) \le \mathrm{BL}_1(A,B)$ and $\mathrm{BL}_1(A,C) \le \mathrm{BL}_1(B,C)$.

3.2. Binary logarithm similarity measures of SVNSs (type-II)

Definition 3. [51] Let A = <$x(T_A(x_i)$, $I_P(x_i)$, $F_P(x_i))$> and B = <$x(T_B(x_i)$, $I_B(x_i)$, $F_B(x_i))$> be any two SVNSs. The binary logarithm similarity measure (type-II) between SVNSs A and B are defined as follows:

$\mathrm{BL}_2(A,B)$ =

$$\frac{1}{n}\sum_{i=1}^{n}\log_2\left(2-\max\begin{pmatrix}|T_A(x_i)-T_B(x_i)|, |I_A(x_i)-I_B(x_i)|,\\ |F_A(x_i)-F_B(x_i)|\end{pmatrix}\right) \quad (2)$$

Theorem 2. The binary logarithm similarity measure $\mathrm{BL}_2(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0 \le \mathrm{BL}_2(A,B) \le 1$

P2. $\mathrm{BL}_2(A,B)=1$, if and only if $A = B$

P3. $\mathrm{BL}_2(A,B) = \mathrm{BL}_2(B,A)$

P4. If C is a SVNS in X and $A\subseteq B\subseteq C$ then $\mathrm{BL}_2(A,C) \le \mathrm{BL}_2(A,B)$ and $\mathrm{BL}_2(A,C) \le \mathrm{BL}_2(B,C)$.

Proof.
Proofs of the properties are shown in [51].

3.3. Weighted binary logarithm similarity measures of SVNSs for type-I

Definition 4. Let A = <$x(T_A(x_i)$, $I_P(x_i)$, $F_P(x_i))$> and B = <$x(T_B(x_i)$, $I_B(x_i)$, $F_B(x_i))$> be any two SVNSs. Then the weighted binary logarithm similarity measure for type-I between SVNSs A and B are defined as follows:

$\mathrm{BL}_1^w(A,B)$ =

$$\sum_{i=1}^{n} w_i \log_2\left(2-\left(\frac{1}{3}\begin{pmatrix}|T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|\\ +|F_A(x_i)-F_B(x_i)|\end{pmatrix}\right)\right)$$

(3)

Here, $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i=1$.

Theorem 3. The weighted binary logarithm similarity measures $\mathrm{BL}_1^w(A,B)$ between SVNSs A and B satisfy the following properties:

P1. $0\le \mathrm{BL}_1^w(A,B) \le 1$

P2. $\mathrm{BL}_1^w(A,B)=1$, if and only if $A = B$

P3. $\mathrm{BL}_1^w(A,B) = \mathrm{BL}_1^w(B,A)$

P4. If C is a SVNS in X and $A\subseteq B\subseteq C$, then $\mathrm{BL}_1^w(A,C) \le \mathrm{BL}_1^w(A,B)$ and $\mathrm{BL}_1^w(A,C) \le \mathrm{BL}_1^w(B,C)$;

$\sum_{i=1}^{n} w_i=1$.

Proof 1.

From the definition of SVNSs A and B, we write,
$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and
$0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$$\Rightarrow 0\le \max\begin{pmatrix}|T_A(x_i)-T_B(x_i)|, |I_A(x_i)-I_B(x_i)|,\\ |F_A(x_i)-F_B(x_i)|\end{pmatrix} \le 1$$

$\Rightarrow 0\le |T_A(x_i)-T_B(x_i)|+|I_A(x_i)-I_B(x_i)|+|F_A(x_i)-F_B(x_i)| \le 3$,

$\Rightarrow 0 \le \mathrm{BL}_1^w(A,B) \le 1$. since, $\sum_{i=1}^{n} w_i=1$.

Proof 2.

For any two SVNSs A and B if $A = B$, then we have,
$T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

$\Rightarrow |T_A(x)-T_B(x)|=0$, $|I_A(x)-I_B(x)|=0$,

$|F_A(x)-F_B(x)|=0$

$\Rightarrow \ \mathrm{BL}_1^w(A,B) = 1$, (t = 1, 2), since $\sum_{i=1}^{n} w_i = 1$.

Conversely,

For $\mathrm{BL}_1^w(A,B) = 1$, then we have,

$\Rightarrow \ |T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,

$|F_A(x) - F_B(x)| = 0$

$\Rightarrow \ T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

$\Rightarrow \ A = B$, since $\sum_{i=1}^{n} w_i = 1$.

Proof 3.

For any two SVNSs A and B, we have,

$|T_A(x) - T_B(x)| = |T_B(x) - T_A(x)|$,

$|I_A(x) - I_B(x)| = |I_B(x) - I_A(x)|$,

$|F_A(x) - F_B(x)| = |F_B(x) - F_A(x)|$

$\Rightarrow \ \mathrm{BL}_1^w(A,B) = \mathrm{BL}_1^w(B,A)$ for.

Proof 4.

For $A \subseteq B \subseteq C$, we have,

$T_A(x) \le T_B(x) \le T_C(x)$, $I_A(x) \ge I_B(x) \ge I_C(x)$,

$F_A(x) \ge F_B(x) \ge F_C(x)$ for $x \in X$.

$\Rightarrow \ |T_A(x) - T_B(x)| \le |T_A(x) - T_C(x)|$,

$|T_B(x) - T_C(x)| \le |T_A(x) - T_C(x)|$;

$|I_A(x) - I_B(x)| \le |I_A(x) - I_C(x)|$,

$|I_B(x) - I_C(x)| \le |I_A(x) - I_C(x)|$;

$|F_A(x) - F_B(x)| \le |F_A(x) - F_C(x)|$,

$|F_B(x) - F_C(x)| \le |F_A(x) - F_C(x)|$.

$\Rightarrow \ \mathrm{BL}_1^w(A,C) \le \mathrm{BL}_1^w(A,B)$ and $\mathrm{BL}_1^w(A,C) \le \mathrm{BL}_1^w(B,C)$ since $\sum_{i=1}^{n} w_i = 1$.

3.4. Weighted binary logarithm similarity measures of SVNSs for type-II

Definition 5. [51] Let A = <$x(T_A(x_i), I_P(x_i), F_P(x_i))$> and B = <$x(T_B(x_i), I_B(x_i), F_B(x_i))$> be any two SVNSs. Then the weighted binary logarithm similarity measure (type-II between SVNSs A and B is defined as follows:

$\mathrm{BL}_2^w(A,B) =$

$$\sum_{i=1}^{n} w_i \log_2\left(2 - \max\begin{pmatrix}|T_A(x_i) - T_B(x_i)|, |I_A(x_i) - I_B(x_i)|, \\ |F_A(x_i) - F_B(x_i)|\end{pmatrix}\right)$$

(4)

Here, $0 \le w_i \le 1$ and $\sum_{i=1}^{n} w_i = 1$.

Proof.

For proof, see [51].

3.3. Hybrid binary logarithm similarity measures (HBLSM) for SVNSs

Definition 6. Let A = <$x(T_A(x_i), I_P(x_i), F_P(x_i))$> and B = <$x(T_B(x_i), I_B(x_i), F_B(x_i))$> be any two SVNSs. The hybrid binary logarithm similarity measure between SVNSs A and B is defined as follows:

$\mathrm{BL}_{Hyb}(A,B) =$

$$\frac{1}{n}\left[\begin{array}{l}\lambda\left\{\sum_{i=1}^{n}\log_2\left(2 - \left(\frac{1}{3}\begin{pmatrix}|T_A(x_i) - T_B(x_i)| \\ + |I_A(x_i) - I_B(x_i)| \\ + |F_A(x_i) - F_B(x_i)|\end{pmatrix}\right)\right)\right\} \\ + (1-\lambda)\sum_{i=1}^{n}\log_2\left(2 - \max\begin{pmatrix}|T_A(x_i) - T_B(x_i)|, \\ |I_A(x_i) - I_B(x_i)|, \\ |F_A(x_i) - F_B(x_i)|\end{pmatrix}\right)\end{array}\right] \quad (5)$$

Here, $0 \le \lambda \le 1$.

Theorem 4. The hybrid binary logarithm similarity measure $\mathrm{BL}_{Hyb}(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0 \le \mathrm{BL}_{Hyb}(A,B) \le 1$

P2. $\mathrm{BL}_{Hyb}(A,B) = 1$, if and only if $A = B$

P3. $\mathrm{BL}_{Hyb}(A,B) = \mathrm{BL}_{Hyb}(B,A)$

P4. If C is a SVNS in X and $A \subseteq B \subseteq C$ then $\mathrm{BL}_{Hyb}(A,C) \le \mathrm{BL}_{Hyb}(A,B)$ and $\mathrm{BL}_{Hyb}(A,C) \le \mathrm{BL}_{Hyb}(B,C)$.

Proof 1.

From the definition of SVNS, we write,

$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and

$0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$$\Rightarrow\ 0 \le \max\begin{pmatrix}|T_A(x_i)-T_B(x_i)|, |I_A(x_i)-I_B(x_i)|,\\ |F_A(x_i)-F_B(x_i)|\end{pmatrix} \le 1$$

$$\Rightarrow \begin{matrix}0 \le |T_A(x_i)-T_B(x_i)| + |I_A(x_i)-I_B(x_i)|\\ +|F_A(x_i)-F_B(x_i)| \le 3\end{matrix};$$

$$\Rightarrow\ 0 \le \mathrm{BL}_{Hyb}(A,B) \le 1.$$

Proof 2.

For any two SVNSs A and B,
for $A = B$, we have,
$\Rightarrow T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$
$\Rightarrow |T_A(x)-T_B(x)| = 0$, $|I_A(x)-I_B(x)| = 0$,
$|F_A(x)-F_B(x)| = 0$
$\Rightarrow \mathrm{BL}_{Hyb}(A,B) = 1$.

Conversely,

for $\mathrm{BL}_{Hyb}(A,B) = 1$, we have,
$|T_A(x)-T_B(x)| = 0$, $|I_A(x)-I_B(x)| = 0$,
$|F_A(x)-F_B(x)| = 0$
$\Rightarrow T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$
$\Rightarrow A = B$.

Proof 3.

For any two SVNSs A and B, we have,
$|T_A(x)-T_B(x)| = |T_B(x)-T_A(x)|$,
$|I_A(x)-I_B(x)| = |I_B(x)-I_A(x)|$,
$|F_A(x)-F_B(x)| = |F_B(x)-F_A(x)|$
$\Rightarrow \mathrm{BL}_{Hyb}(A,B) = \mathrm{BL}_{Hyb}(B,A)$.

Proof 4.

For $A \subseteq B \subseteq C$, we have,
$T_A(x) \le T_B(x) \le T_C(x)$, $I_A(x) \ge I_B(x) \ge I_C(x)$,
$F_A(x) \ge F_B(x) \ge F_C(x)$ for $x \in X$.
$\Rightarrow |T_A(x)-T_B(x)| \le |T_A(x)-T_C(x)|$,
$|T_B(x)-T_C(x)| \le |T_A(x)-T_C(x)|$;
$|I_A(x)-I_B(x)| \le |I_A(x)-I_C(x)|$,
$|I_B(x)-I_C(x)| \le |I_A(x)-I_C(x)|$;
$|F_A(x)-F_B(x)| \le |F_A(x)-F_C(x)|$,
$|F_B(x)-F_C(x)| \le |F_A(x)-F_C(x)|$.
$\Rightarrow \mathrm{BL}_{Hyb}(A,C) \le \mathrm{BL}_{Hyb}(A,B)$

and $\mathrm{BL}_{Hyb}(A,C) \le \mathrm{BL}_{Hyb}(B,C)$.

3.4. Weighted hybrid binary logarithm similarity measures (WHBLSM) for SVNSs

Definition 7. Let A = <$x(T_A(x_i), I_P(x_i), F_P(x_i))$> and B = <$x(T_B(x_i), I_B(x_i), F_B(x_i))$> be any two SVNSs. The weighted hybrid binary logarithm similarity measure between SVNSs A and B is defined as follows:

$$\mathrm{BL}_{wHyb}(A,B) = \left[\begin{matrix}\lambda\left\{\sum_{i=1}^{n} w_i \log_2\left(2-\left(\frac{1}{3}\begin{pmatrix}|T_A(x_i)-T_B(x_i)|\\ +|I_A(x_i)-I_B(x_i)|\\ +|F_A(x_i)-F_B(x_i)|\end{pmatrix}\right)\right)\right\}\\ +(1-\lambda)\sum_{i=1}^{n} w_i \log_2\left(2-\max\begin{pmatrix}|T_A(x_i)-T_B(x_i)|,\\ |I_A(x_i)-I_B(x_i)|,\\ |F_A(x_i)-F_B(x_i)|\end{pmatrix}\right)\end{matrix}\right] \quad (6)$$

Here, $0 \le \lambda \le 1$.

Theorem 5. The weighted hybrid binary logarithm similarity measure $\mathrm{BL}_{wHyb}(A,B)$ between any two SVNSs A and B satisfy the following properties:

P1. $0 \le \mathrm{BL}_{wHyb}(A,B) \le 1$

P2. $\mathrm{BL}_{wHyb}(A,B) = 1$, if and only if $A = B$

P3. $\mathrm{BL}_{wHyb}(A,B) = \mathrm{BL}_{wHyb}(B,A)$

P4. If C is a SVNS in X and $A \subseteq B \subseteq C$, then $\mathrm{BL}_{wHyb}(A,C) \le \mathrm{BL}_{wHyb}(A,B)$
and $\mathrm{BL}_{wHyb}(A,C) \le \mathrm{BL}_{wHyb}(B,C)$.

Proof 1.

From the definition of SVNS, we write,
$0 \le T_A(x) + I_A(x) + F_A(x) \le 3$ and
$0 \le T_B(x) + I_B(x) + F_B(x) \le 3$

$$\Rightarrow\ 0 \le \max\begin{pmatrix}|T_A(x_i)-T_B(x_i)|, |I_A(x_i)-I_B(x_i)|,\\ |F_A(x_i)-F_B(x_i)|\end{pmatrix} \le 1$$

$$\Rightarrow \begin{matrix}0 \le |T_A(x_i)-T_B(x_i)| + |I_A(x_i)-I_B(x_i)|\\ +|F_A(x_i)-F_B(x_i)| \le 3\end{matrix};$$

$$\Rightarrow\ 0 \le \mathrm{BL}_{wHyb}(A,B) \le 1.$$

Proof 2.

For any two SVNSs A and B,

for $A = B$, we have,
$\Rightarrow T_A(x) = T_B(x), I_A(x) = I_B(x), F_A(x) = F_B(x)$
$\Rightarrow |T_A(x) - T_B(x)| = 0, |I_A(x) - I_B(x)| = 0,$
$|F_A(x) - F_B(x)| = 0$
$\Rightarrow \mathrm{BL}_{wHyb}(A, B) = 1$.

Conversely,

for $\mathrm{BL}_{wHyb}(A, B) = 1$, we have,
$|T_A(x) - T_B(x)| = 0$, $|I_A(x) - I_B(x)| = 0$,
$|F_A(x) - F_B(x)| = 0$
$\Rightarrow T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$
$\Rightarrow A = B$.

Proof 3.

For any two SVNSs A and B, we have,
$|T_A(x) - T_B(x)| = |T_B(x) - T_A(x)|$,
$|I_A(x) - I_B(x)| = |I_B(x) - I_A(x)|$,
$|F_A(x) - F_B(x)| = |F_B(x) - F_A(x)|$
$\Rightarrow \mathrm{BL}_{wHyb}(A, B) = \mathrm{BL}_{wHyb}(B, A)$.

Proof 4.

For $A \subseteq B \subseteq C$, we have,
$T_A(x) \le T_B(x) \le T_C(x)$, $I_A(x) \ge I_B(x) \ge I_C(x)$,
$F_A(x) \ge F_B(x) \ge F_C(x)$ for all $x \in X$.
$\Rightarrow |T_A(x) - T_B(x)| \le |T_A(x) - T_C(x)|$,
$|T_B(x) - T_C(x)| \le |T_A(x) - T_C(x)|$;
$|I_A(x) - I_B(x)| \le |I_A(x) - I_C(x)|$,
$|I_B(x) - I_C(x)| \le |I_A(x) - I_C(x)|$;
$|F_A(x) - F_B(x)| \le |F_A(x) - F_C(x)|$,
$|F_B(x) - F_C(x)| \le |F_A(x) - F_C(x)|$.
$\Rightarrow \mathrm{BL}_{wHyb}(A, C) \le \mathrm{BL}_{wHyb}(A, B)$ and
$\mathrm{BL}_{wHyb}(A, C) \le \mathrm{BL}_{wHyb}(B, C)$.

4. A new entropy measure for SVNSs

Entropy strategy [52] is an important contribution for determining indeterminate information. Zhang *et al*. [53] introduced the fuzzy entropy. Vlachos and Sergiadis [54] proposed entropy function for intuitionistic fuzzy sets. Majumder and Samanta [55] developed some entropy measures for SVNSs. When attribute weights are completely unknown to decision makers, the entropy measure is used to calculate attribute weights. In this paper, we define an entropy measure for determining unknown attribute weights.

Definition 8. The entropy function of a SVNS $P = \left\langle T_{ij}^{P}(x), I_{ij}^{P}(x), F_{ij}^{P}(x)\right\rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) is defined as follows:

$$E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}^{P}(x) + F_{ij}^{P}(x)\right)\left(1 - 2I_{ij}^{P}(x)\right)^2 \tag{7}$$

$$w_j = \frac{1 - E_j(P)}{n - \sum_{j=1}^{n} E_j(P)} \tag{8}$$

Here, $\sum_{j=1}^{n} w_j = 1$

Theorem 6. The entropy function $E_j(P)$ satisfies the following properties:

P1. $E_j(P) = 0$, if $T_{ij} = 1, F_{ij} = I_{ij} = 0$.
P2. $E_j(P) = 1$, if $\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle = \langle 0.5, 0.5, 0.5\rangle$.
P3. $E_j(P) \ge E_j(Q)$, if $T_{ij}^{P} + F_{ij}^{P} \le T_{ij}^{Q} + F_{ij}^{Q}$; $I_{ij}^{P} \ge I_{ij}^{Q}$.
P4. $E_j(P) = E_j(P^c)$.

Proof 1.

$T_{ij} = 1, F_{ij} = I_{ij} = 0$

$$\Rightarrow E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[(1+0)\right] = 1 - \frac{n}{n} = 0$$

Proof 2.

$\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle = \langle 0.5, 0.5, 0.5\rangle$.

$$\Rightarrow E_j(P) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left[(0.5 + 0.5) \times 0\right] = 1 - 0 = 1$$

Proof 3.

$T_{ij}^{P} + F_{ij}^{P} \le T_{ij}^{Q} + F_{ij}^{Q}$, $I_{ij}^{P} \ge I_{ij}^{Q}$

$$\Rightarrow \sum_{i=1}^{m}\left(T_{ij}^{P} + F_{ij}^{P}\right)\left(1 - 2I_{ij}^{P}\right)^2 \le \sum_{i=1}^{m}\left(T_{ij}^{Q} + F_{ij}^{Q}\right)\left(1 - 2I_{ij}^{Q}\right)^2$$

$$\Rightarrow \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}^{P} + F_{ij}^{P}\right)\left(1 - 2I_{ij}^{P}\right)^2 \le \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}^{Q} + F_{ij}^{Q}\right)\left(1 - 2I_{ij}^{Q}\right)^2$$

$$\Rightarrow 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}^{P} + F_{ij}^{P}\right)\left(1 - 2I_{ij}^{P}\right)^2 \ge 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}^{Q} + F_{ij}^{Q}\right)\left(1 - 2I_{ij}^{Q}\right)^2$$

$\Rightarrow E_j(P) \ge E_j(Q)$.

Proof 4.

Since $\left\langle T_{ij}, I_{ij}, F_{ij}\right\rangle^c = \left\langle F_{ij}, 1 - I_{ij}, T_{ij}\right\rangle$, we have
$E_j(P) = E_j(P^c)$.

5. MAGDM strategy based on weighted hybrid binary logarithm similarity measure for SVNSs

Assume that (P_1, P_2, ..., P_m) be the alternatives, (C_1, C_2, ..., C_n) be the attributes of each alternative, and {D_1, D_2, ..., D_r} be the decision makers. Decision makers provide the rating of alternatives based on the predefined attribute. Each decision maker constructs a neutrosophic decision matrix associated with the alternatives based on each attribute shown in Equation (9). Using the following steps, we present the MAGDM strategy (see figure 1) based on weighted hybrid binary logarithm similarity measure (WHBLSM).

Step 1: Determine the relation between the alternatives and the attributes

At first, each decision maker prepares decision matrix. The relation between alternatives P_i (i = 1, 2, ..., m) and the attribute C_j (j = 1, 2, ..., n) corresponding to each decision maker is presented in the Equation (9).

$$D_r[P\,|\,C]=$$

$$\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array}\begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ \langle T_{11}^{D_r}, I_{11}^{D_r}, F_{11}^{D_r}\rangle & \langle T_{12}^{D_r}, I_{12}^{D_r}, F_{12}^{D_r}\rangle & \cdots & \langle T_{1n}^{D_r}, I_{1n}^{D_r}, F_{1n}^{D_r}\rangle \\ \langle T_{21}^{D_r}, I_{21}^{D_r}, F_{21}^{D_r}\rangle & \langle T_{22}^{D_r}, I_{22}^{D_r}, F_{22}^{D_r}\rangle & \cdots & \langle T_{2n}^{D_r}, I_{2n}^{D_r}, F_{2n}^{D_r}\rangle \\ \cdots & \cdots & \ddots & \cdots \\ \langle T_{m1}^{D_r}, I_{m1}^{D_r}, F_{m1}^{D_r}\rangle & \langle T_{m2}^{D_r}, I_{m2}^{D_r}, F_{m2}^{D_r}\rangle & \cdots & \langle T_{mn}^{D_r}, I_{mn}^{D_r}, F_{mn}^{D_r}\rangle \end{pmatrix} \tag{9}$$

Here, $\langle T_{ij}^{D_r}, I_{ij}^{D_r}, F_{ij}^{D_r}\rangle$ (i = 1, 2, ..., m; j = 1, 2, ..., n) is the single valued neutrosophic rating value of the alternative P_i with respect to the attribute C_j corresponding to the decision maker D_r.

Step 2: Determine the core decision matrix

We form a new decision matrix, called core decision matrix to combine all the decision maker's opinions into a group opinion. Core decision matrix minimizes the biasness which is imposed by different decision makers and hence credibility to the final decision increases. The core decision matrix is presented in Equation (10).

$$D[P\,|\,C]=$$

$$\begin{array}{c} \\ P_1 \\ P_2 \\ \vdots \\ P_m \end{array}\begin{pmatrix} C_1 & C_2 & \cdots & C_n \\ \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \cdots & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} \\ \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \cdots & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} \\ \cdots & \cdots & \ddots & \cdots \\ \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} & \cdots & \frac{\oplus_{t=1}^{r}\langle T_{11}^{D_t}, I_{11}^{D_t}, F_{11}^{D_t}\rangle}{r} \end{pmatrix}$$

(10)

Step 3: Determine the ideal solution

The evaluation of attributes can be categorized into benefit attribute and cost attribute. An ideal alternative can be determined by using a maximum operator for the benefit attributes and a minimum operator for the cost attributes for determining the best value of each attribute among all the alternatives. An ideal alternative [42] is presented as follows:

$P^* = \{C_1^*, C_2^*, \ldots, C_m^*\}$.

where the benefit attribute is

$$C_j^* = \left\langle \max_i T_{C_j}^{(P_i)}, \min_i I_{C_j}^{(P_i)}, \min_i F_{C_j}^{(P_i)} \right\rangle \tag{11}$$

and the cost attribute is

$$C_j^* = \left\langle \min_i T_{C_j}^{(P_i)}, \max_i I_{C_j}^{(P_i)}, \max_i F_{C_j}^{(P_i)} \right\rangle \tag{12}$$

Step 4: Determine the attribute weights

Using Equation (8), determine the weights of the attribute.

Step 5: Determine the WHBLSM values

Using Equation (6), calculate the weighted similarity measures for each alternative.

Step 6: Ranking the priority

All the alternatives are preference ranked based on the decreasing order of calculated measure values. The highest value reflects the best alternative.

Step 7: End.

6. An illustrative example

Suppose that a state government wants to construct an eco-tourism park for the development of state tourism and especially for mental refreshment of children. After initial screening, three potential spots namely, spot-1 (P_1), spot-2 (P_2), and spot-3 (P_3) remain for further selection. A team of three decision makers, namely, D_1, D_2, and D_3 has been constructed for selecting the most suitable spot with respect to the following attributes.

- Ecology (C_1),
- Costs (C_2),
- Technical facility (C_3),
- Transport (C_4),
- Risk factors (C_5)

The steps of decision-making strategy to select the best potential spot to construct an eco-tourism park based on the proposed strategy are stated below:

6.1. Steps of MAGDM strategy

We present MAGDM strategy based on the proposed WHBLSM using the following steps.

Step 1: Determine the relation between alternatives and attributes

The relation between alternatives P_1, P_2 and P_3 and the attribute set {C_1, C_2, C_3, C_4, C_5} corresponding to the set of decision makers {D_1, D_2, D_3} are presented in Equations (13), (14), and (15).

$$D_1[P\,|\,C] = \left(\begin{array}{cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ P_1 & \langle 0.7, 0.4, 0.4\rangle & \langle 0.7, 0.4, 0.3\rangle & \langle 0.8, 0.1, 0.1\rangle & \langle 0.7, 0.2, 0{,}1\rangle & \langle 0.6, 0.5, 0.5\rangle \\ P_2 & \langle 0.4, 0.3, 0.6\rangle & \langle 0.5, 0.2, 0.5\rangle & \langle 0.6, 0.2, 0.2\rangle & \langle 0.7, 0.3, 0.3\rangle & \langle 0.4, 0.3, 0.4\rangle \\ P_3 & \langle 0.4, 0.2, 0.3\rangle & \langle 0.8, 0.1, 0.3\rangle & \langle 0.5, 0.4, 0.4\rangle & \langle 0.5, 0.2, 0.2\rangle & \langle 0.7, 0.3, 0.2\rangle \end{array}\right) \quad (13)$$

$$D_2[P\,|\,C] = \left(\begin{array}{cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ P_1 & \langle 0.5, 0.2, 0.3\rangle & \langle 0.7, 0.4, 0.4\rangle & \langle 0.8, 0.2, 0.2\rangle & \langle 0.5, 0.2, 0{,}2\rangle & \langle 0.5, 0.5, 0.4\rangle \\ P_2 & \langle 0.5, 0.4, 0.4\rangle & \langle 0.5, 0.2, 0.4\rangle & \langle 0.5, 0.3, 0.3\rangle & \langle 0.8, 0.3, 0.3\rangle & \langle 0.4, 0.1, 0.4\rangle \\ P_3 & \langle 0.4, 0.2, 0.5\rangle & \langle 0.8, 0.2, 0.2\rangle & \langle 0.5, 0.3, 0.3\rangle & \langle 0.7, 0.2, 0.2\rangle & \langle 0.7, 0.4, 0.2\rangle \end{array}\right) \quad (14)$$

$$D_3[P\,|\,C] = \left(\begin{array}{cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ P_1 & \langle 0.7, 0.4, 0.3\rangle & \langle 0.8, 0.2, 0.1\rangle & \langle 0.6, 0.3, 0.3\rangle & \langle 0.7, 0.2, 0{,}5\rangle & \langle 0.5, 0.6, 0.5\rangle \\ P_2 & \langle 0.6, 0.2, 0.3\rangle & \langle 0.5, 0.1, 0.3\rangle & \langle 0.7, 0.4, 0.4\rangle & \langle 0.5, 0.3, 0.4\rangle & \langle 0.3, 0.4, 0.4\rangle \\ P_3 & \langle 0.6, 0.2, 0.3\rangle & \langle 0.6, 0.4, 0.2\rangle & \langle 0.5, 0.3, 0.3\rangle & \langle 0.7, 0.4, 0.2\rangle & \langle 0.5, 0.6, 0.4\rangle \end{array}\right) \quad (15)$$

Step 2: Determine the core decision matrix

Using Equation (10), we construct the core decision matrix for all decision makers shown in Equation (16).

$$D[P\,|\,C] \left(\begin{array}{cccccc} & C_1 & C_2 & C_3 & C_4 & C_5 \\ P_1 & \langle 0.984, 0.324, 0.332\rangle & \langle 0.988, 0.324, 0.232\rangle & \langle 0.989, 0.184, 0.184\rangle & \langle 0.956, 0.203, 0.219\rangle & \langle 0.961, 0.452, 0.219\rangle \\ P_2 & \langle 0.938, 0.292, 0.420\rangle & \langle 0.956, 0.162, 0.395\rangle & \langle 0.979, 0.292, 0.292\rangle & \langle 0.989, 0.304, 0.334\rangle & \langle 0.908, 0.232, 0.404\rangle \\ P_3 & \langle 0.949, 0.203, 0.359\rangle & \langle 0.994, 0.203, 0.232\rangle & \langle 0.956, 0.334, 0.334\rangle & \langle 0.984, 0.255, 0.203\rangle & \langle 0.984, 0.420, 0.255\rangle \end{array}\right) \quad (16)$$

Step 3: Determine the ideal solution

Here, C_3 and C_4 denote benefit attributes and C_1, C_2 and C_5 denote cost attributes. Using Equations (11) and (12), we calculate the ideal solutions as follows:

$$P^* = \left\{\begin{array}{l} \langle 0.938, 0.324, 0.420\rangle, \langle 0.956, 0.324, 0.395\rangle, \\ \langle 0.989, 0.184, 0.184\rangle, \langle 0.989, 0.203, 0.203\rangle, \\ \langle 0.908, 0.452, 0.404\rangle \end{array}\right\}.$$

Step 4: Determine the attribute weights

Using Equation (8), we calculate the attribute weights as follows:

$[w_1, w_2, w_3, w_4, w_5]$ =
[0.1680, 0.3300, 0.2285, 0.2485, 0.0250]

Step 5: Determine the weighted hybrid binary logarithm similarity measures

Using Equation (6), we calculate similarity values for alternatives shown in Table 1.

Step 6: Ranking the alternatives

Ranking order of alternatives is prepared as the descending order of similarity values. Highest value indicates the best alternative. Ranking results are shown in Table 1 for different values of λ.

Step 7. End.

7. Sensitivity analysis

In this section, we discuss the variation of ranking results (see Table 1) for different values of λ. From the results shown in Tables 1, we observe that the proposed strategy provides the same ranking order for different values of λ.

8. Comparison analysis

In this section, we solve the problem with different existing strategies [33, 37, 38, 56]. Outcomes are furnished in the Table 2 and figure 2.

9. Contributions of the proposed strategy

- We propose two types of binary logarithm similarity measures and their hybrid similarity measure for SVNS environment. We have proved their basic properties.
- To calculate unknown weights structure of attributes in SVNS environment, we have proposed a new entropy function.
- We develop a decision making strategy based on the proposed weighted hybrid binary logarithm similarity measure (WHBLSM).
- We have solved a illustrative example to show the feasibility, applicability, and effectiveness of the proposed strategy.

10. Conclusion

Conclusions in the paper are concise as follows:

1. We have proposed hybrid binary logarithm similarity measure and weighted hybrid binary logarithm similarity measure for dealing indeterminacy in decision making situation.
2. We have defined a new entropy function to determine unknown attribute weights.
3. We have developed a new MAGDM strategy based on the proposed weighted hybrid binary logarithm similarity measure.
4. We have presented a numerical example to illustrate the proposed strategy.
5. We have conducted a sensitivity analysis
6. We have presented comparative analyses between the obtained results from the proposed strategies and different existing strategies in the literature. The proposed weighted hybrid binary logarithm similarity measure can be applied to solve MAGDM problems in clustering analysis, pattern recognition, personnel selection, etc.
7. Future research can be continued to investigate the proposed similarity measures in neutrosophic hybrid environment for tackling uncertainty, inconsistency and indeterminacy in decision making. The concept of the paper can be applied in practical decision-making, supply chain management, data mining, cluster analysis, teacher selection etc.

Table 1 Ranking order for different values of λ.

Similarity measures	(λ)	Measure values	Ranking order
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.10	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9426; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9233; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9101	$P_1 \succ P_2 \succ P_3$
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.25	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9479; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9296; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9153	$P_1 \succ P_2 \succ P_3$
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.40	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9532; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9357; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9207	$P_1 \succ P_2 \succ P_3$
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.55	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9585; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9419; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9260	$P_1 \succ P_2 \succ P_3$
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.70	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9638; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9482; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9313	$P_1 \succ P_2 \succ P_3$
$\mathrm{BL}_{wHyb}(P^*, P_i)$	0.90	$\mathrm{BL}_{wHyb}(P^*,P_1)$=0.9708; $\mathrm{BL}_{wHyb}(P^*,P_2)$=0.9565; $\mathrm{BL}_{wHyb}(P^*,P_3)$=0.9384	$P_1 \succ P_2 \succ P_3$

Table 2 Ranking order for different existing strategies

Similarity measures	Measure values for P_1, P_2 and P_3	Ranking order
Mondal and Pramanik [37]	0.8901, 0.8679, 0.8093	$P_1 \succ P_2 \succ P_3$
Ye [33]	0.8409, 0.8189, 0.7766	$P_1 \succ P_2 \succ P_3$
Biswas et al. [56] (λ=0.55)	0.9511, 0.9219, 0.9007	$P_1 \succ P_2 \succ P_3$
Ye and Fu [38]	0.9161, 0.8758, 0.7900	$P_1 \succ P_2 \succ P_3$
Proposed strategy (λ=0.55)	0.9585, 0.9419, 0.9260	$P_1 \succ P_2 \succ P_3$

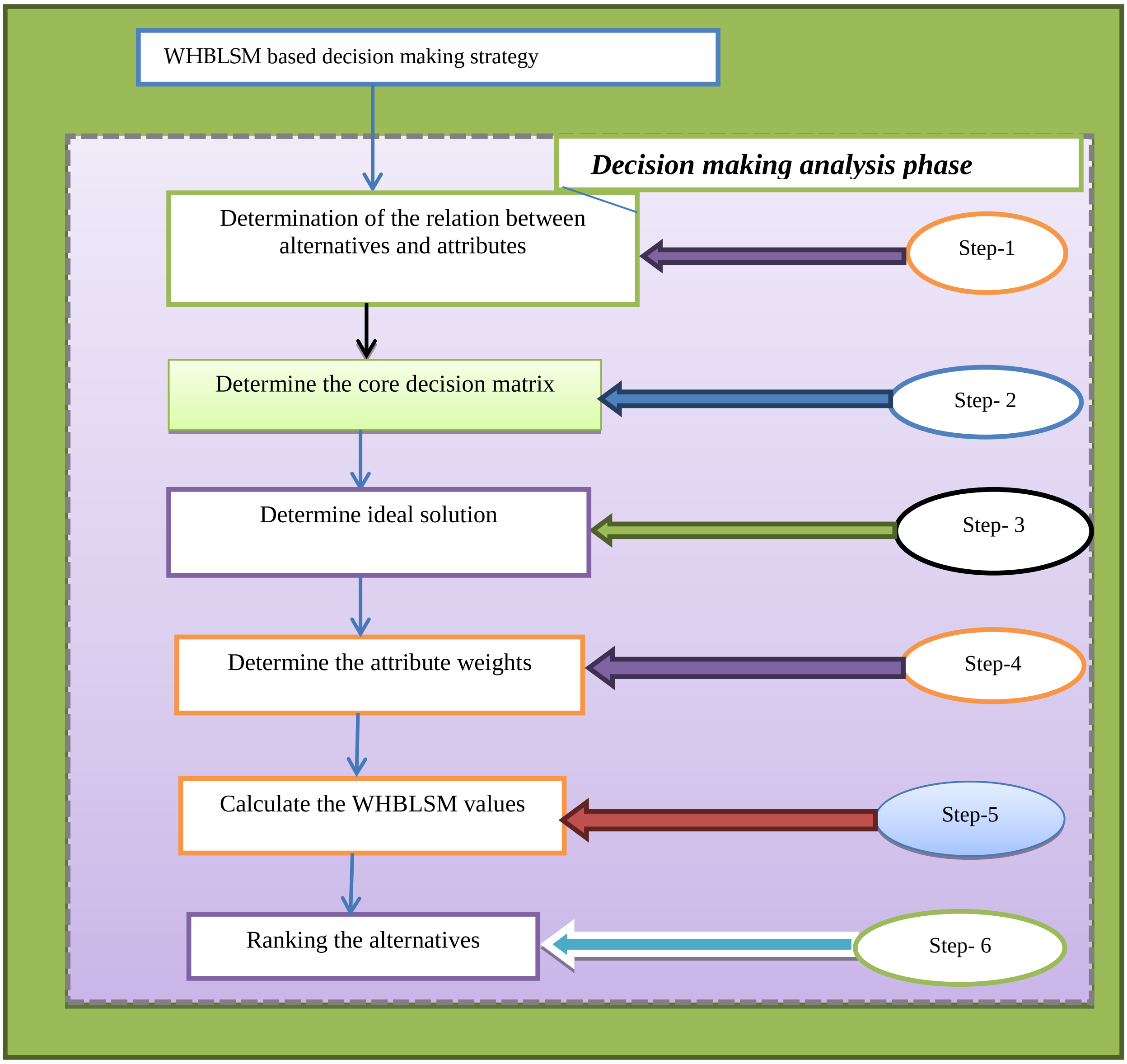

Fig. 1: Decision making phases of the proposed approach

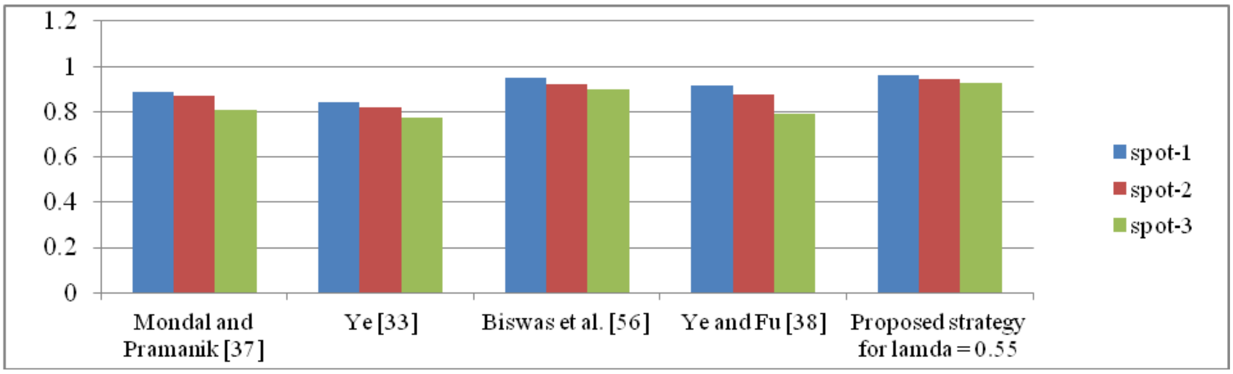

Fig. 2: Ranking order of different strategies

References

[1] F. Smarandache, A unifying field in logics, neutrosophy: neutrosophic probability, set and logic. Rehoboth, American Research Press, 1998.

[2] H. Wang, F. Smarandache, Y. Q. Zhang, and R. Sunderraman. Single valued neutrosophic sets. Multispace and Multistructure, 4(2010), 410–413.

[3] Y. Guo, C. Zhou, H. P. Chan, A. Chughtai, J. Wei, L. M. Hadjiiski, E. A. Kazerooni. Automated iterative neutrosophic lung segmentation for image analysis in thoracic computed tomography. Medical Physics. 40(8) (2013), 081912. doi: 10.1118/1.4812679

[4] K. M. Amin, A. I. Shahin, Y. Guo. A novel breast tumor classification algorithm using neutrosophic score features. Measurement, 81 (2016), 210–220.

[5] Y. X. Ma, J. Q. Wang, J. Wang, X. H. Wu. An interval neutrosophic linguistic multi-criteria group decision-making strategy and its application in selecting medical treatment options. Neural Computing and Applications, 28(9) (2017), 2745–2765.

[6] J. Ye. Improved cross entropy measures of single valued neutrosophic sets and interval neutrosophic sets and their multicriteria decision making strategies. Cybernetics and Information Technologies, 15(4) (2015), 13–26.

[7] J. J. Peng, J. Q. Wang, X. H. Wu, J. Wang, X. H. Chen. Multi-valued neutrosophic sets and power aggregation operators with their applications in multi-criteria group decision-making problems. International Journal of Computational Intelligence Systems, 8(2) (2015), 345–363.

[8] R. Şahin, and P. Liu. Maximizing deviation strategy for neutrosophic multiple attribute decision making with incomplete weight information. Neural Computing and Applications, 27(7) (2017), 2017–2029.

[9] J. Ye. Simplified neutrosophic harmonic averaging projection-based strategy for multiple attribute decision making problems. International Journal of Machine Learning and Cybernetics, 8(3) (2017), 981–987.

[10] P. Chi, P. Liu. An extended TOPSIS method for the multiple attribute decision making problems based on interval neutrosophic set, Neutrosophic Sets and Systems, 1 (2013), 63–70.

[11] P. Biswas, S. Pramanik, and B. C. Giri. Entropy based grey relational analysis method for multi-attribute decision making under single valued neutrosophic assessments. Neutrosophic Sets and Systems 2(2014), 102–110.

[12] P. Biswas, S. Pramanik, and B. C. Giri. A new methodology for neutrosophic multi-attribute decision making with unknown weight information. Neutrosophic Sets and Systems 3 (2014), 42–52.

[13] P. Biswas, S. Pramanik, and B. C. Giri. Aggregation of triangular fuzzy neutrosophic set information and its application to multi-attribute decision making. Neutrosophic Sets and Systems, 12 (2016), 20–40.

[14] P. Biswas, S. Pramanik, and B. C. Giri. Value and ambiguity index based ranking method of single-valued trapezoidal

neutrosophic numbers and its application to multi-attribute decision making. Neutrosophic Sets and Systems 12 (2016), 127–138.

[15] P. Biswas, S. Pramanik, and B. C. Giri. Multi-attribute group decision making based on expected value of neutrosophic trapezoidal numbers. New Trends in Neutrosophic Theory and Applications-Vol-II. Pons Editions, Brussells (2017). In Press.

[16] P. Biswas, S. Pramanik, and B. C. Giri. Non-linear programming approach for single-valued neutrosophic TOPSIS method. New Mathematics and Natural Computation, (2017). In Press.

[17] S. Pramanik, S. Dalapati, and T. K. Roy. Logistics center location selection approach based on neutrosophic multi-criteria decision making. New Trends in Neutrosophic Theories and Applications, Pons-Edtitons, Brussels, 2016, 161–174.

[18] K. Mondal, and S. Pramanik. Neutrosophic decision making model of school choice. Neutrosophic Sets and Systems, 7 (2015), 62–68.

[19] S. Pramanik, T. K. Roy. Neutrosophic game theoretic approach to Indo-Pak conflict over Jammu-Kashmir. Neutrosophic Sets and Systems, 2 (2014), 82–101.

[20] S. Pramanik, and S. N. Chackrabarti. A study on problems of construction workers in West Bengal based on neutrosophic cognitive maps. International Journal of Innovative Research in Science, Engineering and Technology, 2(11) (2013), 6387–6394.

[21] K. Mondal, and S. Pramanik. A study on problems of Hijras in West Bengal based on neutrosophic cognitive maps. Neutrosophic Sets and Systems, 5(2014), 21–26.

[22] J. Ye. Fault diagnoses of steam turbine using the exponential similarity measure of neutrosophic numbers. Journal of Intelligent and Fuzzy Systems, 30(4) (2016), 1927–1934.

[23] K. Hu, J. Ye, E. Fan, S. Shen, L. Huang, J. Pi. A novel object tracking algorithm by fusing color and depth information based on single valued neutrosophic cross-entropy. Journal of Intelligent and Fuzzy Systems, 32(3) (2017), 1775–1786.

[24] Y. Guo, A. Şengür. A novel image segmentation algorithm based on neutrosophic similarity clustering. Applied Soft Computing, 25 (2014), 391–398.

[25] D. Koundal, S. Gupta, S. Singh. Automated delineation of thyroid nodules in ultrasound images using spatial neutrosophic clustering and level set. Applied Soft Computing, 40 (2016), 86–97.

[26] D. Koundal, S. Gupta, S. Singh. Speckle reduction strategy for thyroid ultrasound images in neutrosophic domain. IET Image Processing, 10(2) (2016), 167–75.

[27] J. Ye. Multicriteria decision-making strategy using the correlation coefficient under single-valued neutrosophic environment. International Journal of General Systems, 42(4) (2013), 386–394.

[28] J. Ye. Single valued neutrosophic clustering algorithms based on similarity measures. Journal of Classification, 34(1) (2017), 148–162.

[29] S. Broumi, F. Smarandache. Neutrosophic refined similarity measure based on cosine function. Neutrosophic Sets and Systems, 6 (2014), 41–47.

[30] S. Pramanik, P. Biswas, B. C. Giri. Hybrid vector similarity measures and their applications to multi-attribute decision making under neutrosophic environment. Neural Computing and Applications, 28(5) (2017), 1163–1176.

[31] X. Peng, J. Dai. Approaches to single-valued neutrosophic MADM based on MABAC, TOPSIS and new similarity measure with score function. Neural Computing and Applications, 2016, 1–16. doi: https://doi.org/10.1007/s00521-016-2607-y

[32] J. Ye. Vector similarity measures of simplified neutrosophic sets and their application in multicriteria decision making. International Journal of Fuzzy Systems, 16(2) (2014), 204–211.

[33] J. Ye. Improved cosine similarity measures of simplified neutrosophic sets for medical diagnosis. Artificial intelligence in medicine, 63(3) (2015), 171–179.

[34] P. Biswas, S. Pramanik, and B. C. Giri. Cosine similarity measure based multi-attribute decision-making with trapezoidal fuzzy neutrosophic numbers. Neutrosophic Sets Systems, 2015, 8 (2015), 47–57.

[35] S. Pramanik and K. Mondal. Weighted Fuzzy Similarity Measure Based on Tangent Function and its Application to Medical Diagnosis. International Journal of Innovative Research in Science, Engineering and Technology, 4(2) (2015), 158–164.

[36] K. Mondal and S. Pramanik. Intuitionistic fuzzy similarity measure based on tangent function and its application to multi-attribute decision making. Global Journal of Advanced Research, 2(2) (2015), 464–471.

[37] K. Mondal, S. Pramanik. Neutrosophic tangent similarity measure and its application to multiple attribute decision making. Neutrosophic sets and systems, 9 (2015), 80–87.

[38] J. Ye, J. Fu. Multi-period medical diagnosis strategy using a single valued neutrosophic similarity measure based on tangent function. Computer Methods and Programs in Biomedicine, 123 (2016), 142–149.

[39] M. S. Can, O. F. Ozguven. PID tuning with neutrosophic similarity measure. International Journal of Fuzzy Systems, 19(2) (2016), 489–503.

[40] K. Mondal, S. Pramanik. Multi-criteria group decision making approach for teacher recruitment in higher education

[41] under simplified neutrosophic environment. Neutrosophic Sets and Systems, 6 (2014), 28–34.

[42] P. Liu, F. Teng. An extended TODIM strategy for multiple attribute group decision-making based on 2-dimension uncertain linguistic variable. Complexity, 21(5) (2016), 20–30.

[43] S. Pramanik, S. Dalapati, S. Alam, F. Smarandache, T. K. Roy. NS-Cross Entropy-Based MAGDM under Single-Valued Neutrosophic Set Environment. Information, 9(2) (2018), 37; doi:10.3390/info9020037

[44] J. Ye. Multiple attribute group decision-making strategy with completely unknown weights based on similarity measures under single valued neutrosophic environment. Journal of Intelligent and Fuzzy Systems, 27(6) (2014), 2927–2935.

[45] J. Ye. Clustering methods using distance-based similarity measures of single-valued neutrosophic sets. Journal of Intelligent Systems, 23(4) (2014), 379–389.

[46] K. Mondal, S Pramanik, and B. C. Giri. Single valued neutrosophic hyperbolic sine similarity measure based strategy for MADM problems. Neutrosophic Sets and Systems, 19 (2018) (ACCEPTED).

[47] K. Mondal, S Pramanik, and B. C. Giri. Interval neutrosophic tangent similarity measure and its application to MADM problems. Neutrosophic Sets and Systems, 19 (2018) (ACCEPTED).

[48] Z. Lu, J. Ye. Logarithmic similarity measure between interval-valued fuzzy sets and its fault diagnosis method. Information, 9(2) (2018), 36. doi: 10.3390/info9020036

[49] B. Ashtiani, F. Haghighirad, A. Makui. G. A. Montazer. Extension of fuzzy TOPSIS method based on interval-valued fuzzy sets. Applied Soft Computing, 9(2) (2009), 457–461.

[50] J. Ye. A multi-criteria decision-making method using aggregation operators for simplified neutrosophic sets. Journal of Intelligent & Fuzzy Systems, 26(5) (2014), 2459–2466.

[51] J. N. Mitchell. Computer multiplication and division using binary logarithms. IRE Transactions on Electronic Computers, (4) (1962), 512–517.

[52] K. Mondal, S. Pramanik, B. C. Giri, F. Smarandache, J. Ye. Hybrid logarithm similarity measure based MAGDM strategy under SVNS environment. *Preprints* 2018, 2018030231 (doi:10.20944/preprints201803.0231.v1)

[53] C. E. Shannon Prediction and entropy of printed English. Bell Labs Technical Journal, 30(1) (1951), 50–64.

[54] H. Zhang, W. Zhang, C. Mei. Entropy of interval-valued fuzzy sets based on distance and its relationship with similarity measure. Knowledge-Based Systems, 22(6) (2009), 449–454.

[55] I. K. Vlachos, G. D. Sergiadis. Intuitionistic fuzzy information–applications to pattern recognition. Pattern Recognition Letters, 28(2) (2007), 197–206.

[56] P. Majumdar, S. K. Samanta. On similarity and entropy of neutrosophic sets. Journal of Intelligence and Fuzzy Systems, 26 (2014), 1245–1252.

[57] P. Biswas, S. Pramanik, and B. C. Giri. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. Neural Computing and Applications, 27(3) (2016), 727–737.

[58] Abdel-Basset, M., Mohamed, M., Smarandache, F., & Chang, V. (2018). Neutrosophic Association Rule Mining Algorithm for Big Data Analysis. *Symmetry*, *10*(4), 106.

[59] Abdel-Basset, M., & Mohamed, M. (2018). The Role of Single Valued Neutrosophic Sets and Rough Sets in Smart City: Imperfect and Incomplete Information Systems. Measurement. Volume 124, August 2018, Pages 47-55

[60] Abdel-Basset, M., Gunasekaran, M., Mohamed, M., & Smarandache, F. A novel method for solving the fully neutrosophic linear programming problems. *Neural Computing and Applications*, 1-11.

[61] Abdel-Basset, M., Manogaran, G., Gamal, A., & Smarandache, F. (2018). A hybrid approach of neutrosophic sets and DEMATEL method for developing supplier selection criteria. *Design Automation for Embedded Systems*, 1-22.

[62] Abdel-Basset, M., Mohamed, M., & Chang, V. (2018). NMCDA: A framework for evaluating cloud computing services. *Future Generation Computer Systems*, *86*, 12-29.

[63] Abdel-Basset, M., Mohamed, M., Zhou, Y., & Hezam, I. (2017). Multi-criteria group decision making based on neutrosophic analytic hierarchy process. *Journal of Intelligent & Fuzzy Systems*, *33*(6), 4055-4066.

[64] Abdel-Basset, M.; Mohamed, M.; Smarandache, F. An Extension of Neutrosophic AHP–SWOT Analysis for Strategic Planning and Decision-Making. *Symmetry* 2018, *10*, 116.

Received : March 19, 2018. **Accepted** : April 9, 2018.