Operators on Single Valued Trapezoidal Neutrosophic Numbers and SVTN-Group Decision Making

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Abstract: In this paper, we first introduce single valued trapezoidal neutrosophic (SVTN) numbers with their properties. We then define some operations and distances of the SVTN-numbers. Based on these new operations, we also define some aggregation operators, including SVTN-ordered weighted geometric operator, SVTN-hybrid geometric operator, SVTN-ordered weighted arithmetic operator and SVTN-hybrid arithmetic operator. We then examine the properties of these SVTN-information aggregation operators. By using the SVTN-weighted geometric operator and SVTN-hybrid geometric operator, we also define a multi attribute group decision making method, called SVTN-group decision making method. We finally give an illustrative example and comparative analysis to verify the developed method and to demonstrate its practicality and effectiveness.

Keywords: Single valued neutrosophic sets, neutrosophic numbers, trapezoidal neutrosophic numbers, SVTN-numbers, SVTN-group decision making.

1 Introduction

In real decision making, there usually are many multiple attribute group decision making (MAGDM) problems. Due to the ambiguity of people's thinking and the complexity of objective things, the attribute values of the MAGDM problems cannot always be expressed by exact and crisp values and it may be easier to describe them by neutrosophic information. Zadeh [77] initiated fuzzy set theory. It is one of the most effective tools for processing fuzzy information which has only one membership, and is unable to express non-membership. Therefore, Atanassov [3] presented the intuitionistic fuzzy sets by adding a nonmembership function. Also, Atanassov and Gargov [4] proposed the interval-valued intuitionistic fuzzy set by extending the membership function and nonmembership function to the interval numbers. These sets can only handle incomplete information, not the indeterminate information and inconsistent information. For this reason, Smarandache [53, 54, 55] introduced a new concept that is called neutrosophic set by adding an independent indeterminacy-membership on the basis of intuitionistic fuzzy sets from philosophical point of view, which is a generalization of the concepts of classical sets, probability sets, rough sets [43], fuzzy sets [77, 23], intuitionistic fuzzy sets [3], paraconsistent sets, dialetheist sets, paradoxist sets and tautological sets. In theory of neutrosophic sets, truth-membership, indeterminacy-membership and falsity-membership are represented independently. Also, Wang et al. [62] proposed the interval neutrosophic sets by extending the truth-membership, indeterminacy-membership, and falsity-membership functions to interval numbers. After

Smarandache, Broumi et al. [5, 6, 7], Biswas et al.[8, 9, 10, 11, 12, 13, 14, 15], Kahraman and Otay [32], Mondal et al. [35, 36, 37, 38, 39, 40] and Pramanik et al. [44, 45, 46, 47] studied on some decision making problems based on neutrosophic information. Recently, fuzzy and neutrosophic models have been studied by many authors, such as [1, 2, 19, 20, 28, 29, 30, 48, 49, 50, 52, 57, 58, 62, 63, 80, 81, 82, 83].

Gani et al. [27] presented a method called weighted average rating method for solving group decision making problem by using an intuitionistic trapezoidal fuzzy hybrid aggregation operator. Wan et al. [65] investigated MAGDM problems, in which the ratings of alternatives are expressed with triangular intuitionistic fuzzy numbers. Wei [66, 67], introduced some new group decision making methods by developing aggregation operators with intuitionistic fuzzy information. Xu and Yager [60], presented some new geometric aggregation operators, such as intuitionistic fuzzy weighted geometric operator, intuitionistic fuzzy ordered weighted geometric operator, and intuitionistic fuzzy hybrid geometric operator. Wu and Cao [68] developed some geometric aggregation operators with intuitionistic trapezoidal fuzzy numbers and examined their desired properties. Power average operator of real numbers is extended to four kinds of power average operators of trapezoidal intuitionistic fuzzy numbers by Wan [64]. Farhadinia and Ban [25] initiated a novel method to extend a similarity measure of generalized trapezoidal fuzzy numbers to similarity measures of generalized trapezoidal intuitionistic fuzzy numbers and generalized interval-valued trapezoidal fuzzy numbers. Ye [71] proposed an extended technique for order preference by similarity to ideal solution method for group decision making with interval-valued intuitionistic fuzzy numbers to solve the partner selection problem under incomplete and uncertain information environment. Recently, some intuitionistic models with intuitionistic values have been studied by many authors. For example, on intuitionistic fuzzy sets [26, 59, 76], on interval-valued intuitionistic fuzzy sets [16, 26], interval-valued intuitionistic trapezoidal fuzzy numbers [26, 69], on triangular intuitionistic fuzzy number [17, 24, 26, 33, 34, 61, 78], on trapezoidal intuitionistic fuzzy numbers [18, 26, 31, 34, 41, 42, 51, 72, 75, 79], on generalized trapezoidal fuzzy numbers, on generalized trapezoidal intuitionistic fuzzy numbers and generalized interval-valued trapezoidal fuzzy numbers [25].

A neutrosophic set can handle a incomplete, indeterminate and inconsistent information from philosophical point of view. Ye [74] and ¸Subas¸ [56] introduced single valued neutrosophic numbers, which is a generaliza-tion of fuzzy numbers and intuitionistic fuzzy numbers. The neutrosophic numbers are special single valued neutrosophic sets on the real number sets, which are useful to deal with ill-known quantities in decision data and decision making problems themselves. Then, Ye [73] and Deli and ¸Subas¸ [21, 22] developed new methods on single valued neutrosophic numbers based on multi-criteria decision making problem. But, multi-criteria group decision making problem has not yet been studied.

The paper is organized as follows. In the next section, we give some basic definitions and properties of single valued trapezoidal neutrosophic (SVTN) numbers. In Section 3, some operations for SVTN-numbers and distance between two SVTN-number are presented. In Section 4, we introduce some new geometric aggregation operators, including SVTN-ordered weighted geometric operator, SVTN-hybrid geometric operator, SVTN-ordered weighted arithmetic operator and SVTN-hybrid arithmetic operator. In Section 5, we develope a group decision making method, so called SVTN-group decision making method to solve MAGDM problems based on the SVTN-weighted geometric operator and the SVTN-hybrid geometric operators. We then present an illustrative example to verify the developed method and to demonstrate its practicality. In Section 6 we give a comparative analysis. In Section 7, we conclude the paper and give some remarks.

2 Preliminary

In this section, some basic concepts and definitions on fuzzy sets, intuitionistic fuzzy sets, neutrosophic sets, single valued neutrosophic sets and single valued neutrosophic numbers are given.

Definition 2.1. [77] Let E be a universe. Then, a fuzzy set X over E is defined by

$$X = \{(\mu_X(x)/x) : x \in E\}$$

where $\mu_X : E \to [0.1]$ is called membership function of X. For each $x \in E$, the value $\mu_X(x)$ represents the degree of x belonging to the fuzzy set X.

Definition 2.2. [3] Let E be a universe. Then, an intuitionistic fuzzy set K over E is defined by

$$K = \{< x, \mu_K(x), \gamma_K(x) >: \ x \in E\}$$

where $\mu_K : E \to [0,1]$ and $\gamma_K : E \to [0,1]$ such that $0 \leq \mu_K(x) + \gamma_K(x) \leq 1$ for any $x \in E$. For each $x \in E$, the values $\mu_K(x)$ and $\gamma_K(x)$ are the degree of membership and degree of non-membership of x, respectively.

Definition 2.3. [54] Let E be a universe. Then, a neutrosophic set A over E is defined by

$$A = \{< x, (T_A(x), I_A(x), F_A(x)) >: x \in E\}.$$

where $T_A(x)$, $I_A(x)$ and $F_A(x)$ are called truth-membership function, indeterminacy-membership function and falsity-membership function, respectively. They are respectively defined by $T_A : E \to]^-0, 1^+[, \quad I_A : E \to]^-0, 1^+[, \quad F_A : E \to]^-0, 1^+[$ such that $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$.

Definition 2.4. [63] Let E be a universe. Then, a single valued neutrosophic set over E is a neutrosophic set over E, but the truth-membership function, indeterminacy-membership function and falsity-membership function are respectively defined by

$$T_A : E \to [0,1], \quad I_A : E \to [0,1], \quad F_A : E \to [0,1]$$

such that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$.

Definition 2.5. [22, 56] A single valued trapezoidal neutrosophic number $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$ is a special neutrosophic set on the real number set R, whose truth-membership, indeterminacy-membership, and a falsity-membership are given as follows:

$$\mu_{\tilde{a}}(x) = \begin{cases} (x - a_1)w_{\tilde{a}}/(b_1 - a_1), & (a_1 \leq x < b_1) \\ w_{\tilde{a}}, & (b_1 \leq x \leq c_1) \\ (d_1 - x)w_{\tilde{a}}/(d_1 - c_1), & (c_1 < x \leq d_1) \\ 0, & otherwise, \end{cases}$$

$$\nu_{\tilde{a}}(x) = \begin{cases} (b_1 - x + u_{\tilde{a}}(x - a_1))/(b_1 - a_1), & (a_1 \leq x < b_1) \\ u_{\tilde{a}}, & (b_1 \leq x \leq c_1) \\ (x - c_1 + u_{\tilde{a}}(d_1 - x))/(d_1 - c_1), & (c_1 < x \leq d_1) \\ 1, & otherwise \end{cases}$$

and

$$\lambda_{\tilde{a}}(x) = \begin{cases} (b_1 - x + y_{\tilde{a}}(x - a_1))/(b_1 - a_1), & (a_1 \leq x < b_1) \\ y_{\tilde{a}}, & (b_1 \leq x \leq c_1) \\ (x - c_1 + y_{\tilde{a}}(d_1 - x))/(d_1 - c_1), & (c_1 < x \leq d_1) \\ 1, & otherwise \end{cases}$$

respectively.

Sometimes, we use the $\tilde{a}_i = \langle (a_i, b_i, c_i, d_i); w_i, u_i, y_i \rangle$, instead of $\tilde{a}_i = \langle (a_i, b_i, c_i, c_i); w_{\tilde{a}_i}, u_{\tilde{a}_i}, y_{\tilde{a}_i} \rangle$.

Note that the single valued trapezoidal neutrosophic number is abbreviated as SVTN-number and the set of all SVTN-numbers on R will be denoted by Ω.

3 Operations and Distances of SVTN-Numbers

In this section, we give operations and distances of SVTN-numbers and investigate their related properties.

Definition 3.1. [73] Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_{\tilde{a}}, u_{\tilde{a}}, y_{\tilde{a}} \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_{\tilde{b}}, u_{\tilde{b}}, y_{\tilde{b}} \rangle \in \Omega$ and $\gamma \geq 0$ be any real number. Then,

1. $\tilde{a} \oplus \tilde{b} = \langle (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2); w_{\tilde{a}} + w_{\tilde{b}} - w_{\tilde{a}} w_{\tilde{b}}, u_{\tilde{a}} u_{\tilde{b}}, y_{\tilde{a}} y_{\tilde{b}} \rangle$
2. $\tilde{a} \otimes \tilde{b} = \langle (a_1 a_2, b_1 b_2, c_1 c_2, d_1 d_2); w_{\tilde{a}} w_{\tilde{b}}, u_{\tilde{a}} + u_{\tilde{b}} - u_{\tilde{a}} u_{\tilde{b}}, y_{\tilde{a}} + y_{\tilde{b}} - y_{\tilde{a}} y_{\tilde{b}} \rangle$
3. $\gamma \tilde{a} = \langle (\gamma a_1, \gamma b_1, \gamma c_1, \gamma d_1); 1 - (1 - w_{\tilde{a}})^\gamma, u_{\tilde{a}}^\gamma, y_{\tilde{a}}^\gamma \rangle$
4. $\tilde{a}^\gamma = \langle (a_1^\gamma, b_1^\gamma, c_1^\gamma, d_1^\gamma); w_{\tilde{a}}^\gamma, 1 - (1 - u_{\tilde{a}})^\gamma, 1 - (1 - y_{\tilde{a}})^\gamma \rangle$

Theorem 3.2. *Let* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_1); w_2, u_2, y_2 \rangle \in \Omega$. *Then,* $\tilde{a} \oplus \tilde{b}$, $\tilde{a} \otimes \tilde{b}$, $\gamma \tilde{a}$ *and* $\tilde{a}^\gamma$ *are also SVTN-numbers.*

Proof: It is easy from Definition 3.1.

Theorem 3.3. *Let* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2); w_2, u_2, y_2 \rangle$, $\tilde{c} = \langle (a_3, b_3, c_3, d_3); w_3, u_3, y_3 \rangle \in \Omega$ *and* γ, γ_1, γ_2 *be positif real numbers. Then, the followings are valid.*

1. $\tilde{a} \oplus \tilde{b} = \tilde{b} \oplus \tilde{a}$
2. $\tilde{a} \otimes \tilde{b} = \tilde{b} \otimes \tilde{a}$
3. $(\tilde{a} \otimes \tilde{b}) \otimes \tilde{c} = \tilde{a} \otimes (\tilde{b} \otimes \tilde{c})$
4. $(\tilde{a} \oplus \tilde{b}) \oplus \tilde{c} = \tilde{a} \oplus (\tilde{b} \oplus \tilde{c})$
5. $\tilde{a} \otimes (\tilde{b} \oplus \tilde{c}) = (\tilde{a} \otimes \tilde{b}) \oplus (\tilde{a} \otimes \tilde{c})$
6. $(\tilde{a} \otimes \tilde{b})^\gamma = \tilde{b}^\gamma \otimes \tilde{a}_1^\gamma$
7. $\tilde{a}^{\gamma_1} \otimes \tilde{a}^{\gamma_2} = \tilde{a}^{(\gamma_1 + \gamma_2)}$ *or* $\tilde{b}^{\gamma_2} \otimes \tilde{b}^{\gamma_2} = \tilde{b}^{(\gamma_1 + \gamma_2)}$

Proof: It is easy from Definition 3.1.

Definition 3.4. Let $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2);\, w_2, u_2, y_2 \rangle \in \Omega$. Then, the distance between $\tilde{a}$ and $\tilde{b}$ is defined by

$$\begin{aligned} d_h(\tilde{a}, \tilde{b}) = \; & \tfrac{1}{6}\big(|(1+w_1-u_1-y_1)a_1-(1+w_2-u_2-y_2)a_2|+ \\ & |(1+w_1-u_1-y_1)b_1-(1+w_2-u_2-y_2)b_2|+ \\ & |(1+w_1-u_1-y_1)c_1-(1+w_2-u_2-y_2)c_2|+ \\ & |(1+w_1-u_1-y_1)d_1-(1+w_2-u_2-y_2)d_2| \quad \big) \end{aligned}$$

Example 3.5. Assume that $\tilde{a} = \langle (1, 4, 5, 6); 0.3, 0.4, 0.7 \rangle$, $\tilde{b} = \langle (1, 2, 5, 7); 0.7, 0.5, 0.1 \rangle \in \Omega$. Then, the distance of $\tilde{a}$ and $\tilde{b}$ is computed by

$$\begin{aligned} d_h(\tilde{a}, \tilde{b}) = \; & \tfrac{1}{6}\big(|(1+0.3-0.4-0.7)1-(1+0.7-0.5-0.1)1|+ \\ & |(1+0.3-0.4-0.7)4-(1+0.7-0.5-0.1)2|+ \\ & |(1+0.3-0.4-0.7)5-(1+0.7-0.5-0.1)5|+ \\ & |(1+0.3-0.4-0.7)6-(1+0.7-0.5-0.1)7|\big) \\ & \cong 7.78 \end{aligned}$$

Theorem 3.6. *Let* $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2);\, w_2, u_2, y_2 \rangle \in \Omega$. *Then,* $d_h(\tilde{a}, \tilde{b})$ *meet the nonnegative, symmetric and triangle inequality (or metric).*

Proof: Clearly, the $d_h(\tilde{a}, \tilde{b})$ meet the nonnegative, symmetric properties. For $\tilde{a} = \langle (a_1, b_1, c_1, d_1); w_1, u_1, y_1 \rangle$, $\tilde{b} = \langle (a_2, b_2, c_2, d_2);\, w_2, u_2, y_2 \rangle$, $\tilde{c} = \langle (a_3, b_3, c_3, d_3);\, w_3, u_3, y_3 \rangle \in \Omega$, to prove the triangle inequality, since

$$\begin{aligned} & |(1+w_1-u_1-y_1)a_1-(1+w_2-u_2-y_2)a_2|+ \\ & |(1+w_2-u_2-y_2)a_2-(1+w_3-u_3-y_3)a_3| \\ & \geq |(1+w_1-u_1-y_1)a_1-(1+w_3-u_3-y_3)a_3| \end{aligned}$$

$$\begin{aligned} & |(1+w_1-u_1-y_1)b_1-(1+w_2-u_2-y_2)b_2|+ \\ & |(1+w_2-u_2-y_2)b_2-(1+w_3-u_3-y_3)b_3| \\ & \geq |(1+w_1-u_1-y_1)b_1-(1+w_3-u_3-y_3)b_3| \end{aligned}$$

$$\begin{aligned} & |(1+w_1-u_1-y_1)c_1-(1+w_2-u_2-y_2)c_2|+ \\ & |(1+w_2-u_2-y_2)c_2-(1+w_3-u_3-y_3)c_3| \\ & \geq |(1+w_1-u_1-y_1)c_1-(1+w_3-u_3-y_3)c_3| \end{aligned}$$

$$\begin{aligned} & |(1+w_1-u_1-y_1)d_1-(1+w_2-u_2-y_2)d_2|+ \\ & |(1+w_2-u_2-y_2)d_2-(1+w_3-u_3-y_3)d_3| \\ & \geq |(1+w_1-u_1-y_1)d_1-(1+w_3-u_3-y_3)d_3| \end{aligned}$$

we have

$$\begin{array}{l}|(1+w_1-u_1-y_1)a_1-(1+w_2-u_2-y_2)a_2|+\\|(1+w_2-u_2-y_2)a_2-(1+w_3-u_3-y_3)a_3|\\+|(1+w_1-u_1-y_1)b_1-(1+w_2-u_2-y_2)b_2|+\\|(1+w_2-u_2-y_2)b_2-(1+w_3-u_3-y_3)b_3|\\+|(1+w_1-u_1-y_1)c_1-(1+w_2-u_2-y_2)c_2|+\\|(1+w_2-u_2-y_2)c_2-(1+w_3-u_3-y_3)c_3|\\+|(1+w_1-u_1-y_1)d_1-(1+w_2-u_2-y_2)d_2|+\\|(1+w_2-u_2-y_2)d_2-(1+w_3-u_3-y_3)d_3|\\\geq|(1+w_1-u_1-y_1)a_1-(1+w_3-u_3-y_3)a_3|+\\|(1+w_1-u_1-y_1)b_1-(1+w_3-u_3-y_3)b_3|\\+|(1+w_1-u_1-y_1)c_1-(1+w_3-u_3-y_3)c_3|+\\|(1+w_1-u_1-y_1)d_1-(1+w_3-u_3-y_3)d_3|\end{array}$$

and then,

$$d_h(\tilde{a},\tilde{b})+d_h(\tilde{b},\tilde{c})\geq d_h(\tilde{a},\tilde{c})$$

Definition 3.7. [56] Let $\tilde{a}=\langle(a_1,b_1,c_1,d_1);w_1,u_1,y_1\rangle\in\Omega$. Then, a normalized SVTN-number of $\tilde{a}$ is defined by

$$\langle(\tfrac{a_1}{a_1+b_1+c_1+d_1},\ \tfrac{b_1}{a_1+b_1+c_1+d_1},\tfrac{c_1}{a_1+b_1+c_1+d_1},\tfrac{d_1}{a_1+b_1+c_1+d_1});w_1,u_1,y_1\rangle$$

Example 3.8. Assume that $\tilde{a}=\langle(1,4,5,10);0.3,0.4,0.7\rangle\in\Omega$. Then, a normalized SVTN-number of $\tilde{a}$ is computed as

$$\langle(0.05,0.2,0.25,0.5);0.3,0.4,0.7\rangle$$

Definition 3.9. The SVTN-numbers $\tilde{a}^+=\langle(1,1,1,1);1,0,0\rangle$, $\tilde{a}_s^+=\langle(1,1,1,1);1,1,0\rangle$, $\tilde{a}^-=\langle(0,0,0,0);0,1,1\rangle$ and $\tilde{a}_s^-=\langle(0,0,0,0);0,0,1\rangle$ are called SVTN-positive ideal solution, strongly SVTN-positive ideal solution, SVTN-negative ideal solution and strongly SVTN-negative ideal solution, respectively.

Definition 3.10. Let $\tilde{a}_i=\langle(a_i,b_1,c_i,d_i);w_i,u_i,y_i\rangle\in\Omega$ for all $i=1,2$ and $\tilde{a}^+$, $\tilde{a}_s^+$, $\tilde{a}^-$ and $\tilde{a}_s^-$ be SVTN-positive ideal solution, strongly SVTN-positive ideal solution, SVTN-negative ideal solution and strongly SVTN-negative ideal solution, respectively. Then, the distance between $\tilde{a}_i$ and $\tilde{a}^+$, $\tilde{a}_s^+$, $\tilde{a}^-$, $\tilde{a}_s^-$ are denoted as $d_h(\tilde{a}_i,\tilde{a}^+)$, $d_h(\tilde{a}_i,\tilde{a}_s^+)$, $d_h(\tilde{a}_i,\tilde{a}^-)$, $d_h(\tilde{a}_i,\tilde{a}_s^-)$ for all $i=1,2$, respectively. Then,

1. If $d_h(\tilde{a}_1,\tilde{a}^+)<d_h(\tilde{a}_2,\tilde{a}^+)$, then $\tilde{a}_2$ is smaller than $\tilde{a}_1$, denoted by $\tilde{a}_1>\tilde{a}_2$
2. If $d_h(\tilde{a}_1,\tilde{a}^+)=d_h(\tilde{a}_2,\tilde{a}^+)$;
   (a) If $d_h(\tilde{a}_1,\tilde{a}_s^+)<d_h(\tilde{a}_2,\tilde{a}_s^+)$, then $\tilde{a}_2$ is smaller than $\tilde{a}_1$, denoted by $\tilde{a}_1>\tilde{a}_2$
   (b) If $d_h(\tilde{a}_1,\tilde{a}_s^+)=d_h(\tilde{a}_2,\tilde{a}_s^+)$;
      i. If $d_h(\tilde{a}_1,\tilde{a}^-)<d_h(\tilde{a}_2,\tilde{a}^-)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1<\tilde{a}_2$
      ii. If $d_h(\tilde{a}_1,\tilde{a}^-)=d_h(\tilde{a}_2,\tilde{a}^-)$;
         A. If $d_h(\tilde{a}_1,\tilde{a}_s^-)<d_h(\tilde{a}_2,\tilde{a}_s^-)$, then $\tilde{a}_1$ is smaller than $\tilde{a}_2$, denoted by $\tilde{a}_1<\tilde{a}_2$
         B. If $d_h(\tilde{a}_1,\tilde{a}_s^-)=d_h(\tilde{a}_2,\tilde{a}_s^-)$; $\tilde{a}_1$ and $\tilde{a}_2$ are the same, denoted by $\tilde{a}_1=\tilde{a}_2$

Example 3.11. Assume that $\tilde{a_1} = \langle(2,3,5,6); 0.3, 0.4, 0.7\rangle$, $\tilde{a_2} = \langle(1,3,6,7); 0.7, 0.5, 0.1\rangle$, $\tilde{a}^+ = \langle(1,1,1,1); 1,0,0\rangle \in \Omega$. Then,

$$\begin{aligned} d_h(\tilde{a}_1, \tilde{a}^+) &= \tfrac{1}{6}\big(|(1+0.3-0.4-0.7)2 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.3-0.4-0.7)3 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.3-0.4-0.7)5 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.3-0.4-0.7)6 - |(1+1-0.0-0.0)1|\big) \\ &= \tfrac{7}{60} \end{aligned}$$

and

$$\begin{aligned} d_h(\tilde{a}_2, \tilde{a}^+) &= \tfrac{1}{6}\big(|(1+0.7-0.5-0.1)1 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.7-0.5-0.1)3 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.7-0.5-0.1)6 - |(1+1-0.0-0.0)1|+ \\ &\quad |(1+0.7-0.5-0.1)7 - |(1+1-0.0-0.0)1|\big) \\ &= \tfrac{65}{60} \end{aligned}$$

Since $d_h(\tilde{a}_1, \tilde{a}^+) < d_h(\tilde{a}_2, \tilde{a}^+)$, $\tilde{a}_2$ is smaller than $\tilde{a}_1$ (or $\tilde{a}_1 > \tilde{a}_2$).

From now on we use $I_n = \{1, 2, ..., n\}$ $I_m = \{1, 2, ..., m\}$ and $I_t = \{1, 2, ..., t\}$ as an index set for $n \in N$, $m \in N$ and $t \in N$, respectively.

4 SVTN-Weighted Operators

In this section, we present some arithmetic and geometric operators including SVTN-weighted geometric operator, SVTN-ordered weighted geometric operator, SVTN-hybrid geometric operator, SVTN-weighted arithmetic operator, SVTN-ordered weighted arithmetic operator and SVTN-hybrid arithmetic operator with their properties.

4.1 SVTN-Weighted Geometric Operators

In this subsection, we introduce some SVTN-weighted geometric operators on the SVTN-numbers.

Definition 4.1. [73] Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for all $j \in I_n$. Then, SVTN-weighted geometric operator, denoted by S_{go}, is defined by $S_{go}: \Omega^n \to \Omega$,

$$S_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}_1^{w_1} \otimes \tilde{a}_2^{w_2} \otimes \cdots \otimes \tilde{a}_n^{w_n}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for every $j \in I_n$ such that $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$.

Theorem 4.2. *[73] Let $\tilde{a}_j = \langle(a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for $j \in I_n$ and S_{go} be the SVTN-weighted geometric operator. Then, their aggregated value by using $S_{go}: \Omega^n \to \Omega$, operator is also a SVTN-number and*

$$\begin{aligned} S_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) &= \prod_{j=1}^n \tilde{a}_j^{w_j} \\ &= \langle(\prod_{j=1}^n a_j^{w_j}, \prod_{j=1}^n b_j^{w_j}, \prod_{j=1}^n c_j^{w_j}, \prod_{j=1}^n d_j^{w_j}); \\ &\quad \prod_{j=1}^n w_{\tilde{a}_j}^{w_j}, 1-\prod_{j=1}^n (1-u_{\tilde{a}_j})^{w_j}, 1-\prod_{j=1}^n (1-y_{\tilde{a}_j})^{w_j}\rangle \end{aligned}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for all $j \in I_n$ such that $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$.

Theorem 4.3. *[73] Let* $a\tilde{\ }_j = \langle (a_j, b_j, c_j, d_j); w_{a\tilde{\ }_j}, u_{a\tilde{\ }_j}, y_{a\tilde{\ }_j}\rangle \in \Omega$ *for* $j \in I_n$*. Then,*

1. *If* $\tilde{a}_j = \tilde{a}$*, for all* $j \in I_n$*, then* $S_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$*,*
2. $min_{j\in I}\{\tilde{a}_j\} \leq S_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq max_{j\in I}\{\tilde{a}_j\}$*,*
3. *If* $\tilde{a}^*_j = \langle (a^*_j, b^*_j, c^*_j, d_j)^*; w^*_{\tilde{a}_j}, u^*_{\tilde{a}_j}, y^*_{\tilde{a}_j}\rangle \in \Omega$ *and* $\tilde{a}_j \leq \tilde{a_j}^*$ *for all* $j \in I_n$*,*
then $S_{go}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq S_{go}(\tilde{a}^*_1, \tilde{a}^*_2, ..., \tilde{a}^*_n)$*.*

Definition 4.4. Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for all $j \in I_n$. Then, an SVTN-ordered weighted geometric operator, denoted by S_{ogo}, is defined by $S_{ogo} : \Omega^n \to \Omega$,

$$S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}^{w_1}_{\sigma(1)} \otimes \tilde{a}^{w_2}_{\sigma(2)} \otimes \cdots \otimes \tilde{a}^{w_n}_{\sigma(n)}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for every $j \in I$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Here, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of $(1, 2, ..., n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j \in I_n$.

Theorem 4.5. *Let* $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ *for* $j \in I_n$ *and* S_{ogo} *be an SVTN-ordered weighted geometric operator. Then, their aggregated value by using* S_{ogo} *operator is also a SVTN-number an*

$$\begin{aligned} S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^n \tilde{a}^{w_j}_{\sigma(j)} \quad &= \quad \langle (\prod_{j=1}^n a^{w_j}_{\sigma(j)}, \prod_{j=1}^n b^{w_j}_{\sigma(j)}, \prod_{j=1}^n c^{w_j}_{\sigma(j)}, \prod_{j=1}^n d^{w_j}_{\sigma(j)}); \\ &\prod_{j=1}^n w^{w_j}_{\tilde{a}_{\sigma(j)}}, 1 - \prod_{j=1}^n (1 - u_{\tilde{a}_{\sigma(j)}})^{w_j}, 1 - \prod_{j=1}^n (1 - y_{\tilde{a}_{\sigma(j)}})^{w_j}\rangle \end{aligned}$$

where $w = (w_1, w_2, ..., w_n)^T$ *is a weight vector of* $\tilde{a}_j$ *for all* $j \in I_n$ *such that* $w_j \in [0, 1]$ *and* $\sum_{j=1}^n w_j = 1$*.*

Theorem 4.6. *Let* $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ *for* $j \in I_n$*. Then,*

1. *If* $\tilde{a}_j = \tilde{a}$*, then* $S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$*.*
2. $min_j\{\tilde{a}_j\} \leq S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq max_j\{\tilde{a}_j\}$
3. *If* $\tilde{a}^*_j = \langle (a^*_j, b^*_j, c^*_j, d_j)^*; w^*_{\tilde{a}_j}, u^*_{\tilde{a}_j}, y^*_{\tilde{a}_j}\rangle \in \Omega$ *and* $\tilde{a_j}^{\leq}\tilde{a_j}^*$*, then*
$S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq S_{ogo}(\tilde{a}^*_1, \tilde{a}^*_2, ..., \tilde{a}^*_n)$
4. *If* $\tilde{\tilde{a}}_j \in \Omega$*, then*
$S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = S_{ogo}(\tilde{\tilde{a}}_1, \tilde{\tilde{a}}_2, ..., \tilde{\tilde{a}}_n)$ *where* $(\tilde{\tilde{a}}_1, \tilde{\tilde{a}}_2, ..., \tilde{\tilde{a}}_n)$ *is any permutation of* $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$*.*

Theorem 4.7. *Let* $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ *for* $j \in I_n$ *and* S_{ogo} *be the SVTN-geometric averaging operator. Then, for all* $j \in I_n$*,*

1. *If* $w = (1, 0, ..., 0)^T$*, then* $S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = max_j\{\tilde{a}_j\}$*.*
2. *If* $w = (0, 0, ..., 1)^T$*, then* $S_{ogo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = min_j\{\tilde{a}_j\}$*.*

Definition 4.8. Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$. Then, an SVTN-hybrid geometric operator, denoted by $S^{\tilde{s}}_{hgo}$, is defined by

$$S^{\tilde{s}}_{hgo} : \Omega^n \to \Omega, \quad S^{\tilde{s}}_{hgo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{\tilde{a}}^{\tilde{s}_1}_{\sigma(1)} \otimes \tilde{\tilde{a}}^{\tilde{s}_2}_{\sigma(2)} \otimes \cdots \otimes \tilde{\tilde{a}}^{\tilde{s}_n}_{\sigma(n)}$$

where for $j \in I_n$, $\tilde{\tilde{a}}_{\sigma(j)}$ is the jth largest of the weighted SVTN-numbers $\tilde{\tilde{a}}_j$, $\tilde{\tilde{a}}_j = \tilde{a}_j^{nw_j}$, $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)^T$ is a vector associated with the $S^{\tilde{s}}_{hgo}$ such that $\tilde{s}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{s}_j = 1$.

Theorem 4.9. *Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$ and $S^{\tilde{s}}_{hgo}$ be the SVTN-hybrid geometric operator. Then, their aggregated value by using $S^{\tilde{s}}_{hgo}$ operator is also a SVTN-number and*

$$\begin{aligned} S^{\tilde{s}}_{hgo}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \prod_{j=1}^{n} \tilde{\bar{a}}^{\tilde{s}_j}_{\sigma(j)} \quad &= \langle (\prod_{j=1}^{n} \bar{a}^{\tilde{s}_j}_{\sigma(j)}, \prod_{j=1}^{n} \bar{b}^{\tilde{s}_j}_{\sigma(j)} \prod_{j=1}^{n} \bar{c}^{\tilde{s}_j}_{\sigma(j)}, \prod_{j=1}^{n} \bar{d}^{\tilde{s}_j}_{\sigma(j)}); \\ &\prod_{j=1}^{n} w^{\tilde{s}_j}_{\tilde{\bar{a}}_{\sigma(j)}}, 1 - \prod_{j=1}^{n}(1 - u_{\tilde{\bar{a}}_{\sigma(j)}})^{\tilde{s}_j}, 1 - \prod_{j=1}^{n}(1 - y_{\tilde{\bar{a}}_{\sigma(j)}})^{\tilde{s}_j} \rangle \end{aligned}$$

where for $j \in I_n$, $\tilde{\bar{a}}_{\sigma(j)}$ is the jth largest of the weighted SVTN-numbers $\tilde{\bar{a}}_j$, $\tilde{\bar{a}}_j = \tilde{a}_j^{nw_j}$, $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$, and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)^T$ is a vector associated with the $S^{\tilde{s}}_{hgo}$ such that $\tilde{s}_j \in [0, 1]$ and $\sum_{j=1}^{n} \tilde{s}_j = 1$.

Corollary 4.10. *Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$. Then, SVTN-weighted geometric operator S_{go} and SVTN-ordered weighted geometric operator S_{ogo} operator is a special case of the SVTN-hybrid geometric operator $S^{\tilde{s}}_{hgo}$.*

4.2 SVTN-Weighted arithmetic Operators

In this subsection, we introduce some SVTN-weighted arithmetic operators on the SVTN-numbers.

Definition 4.11. [73] Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Omega$ for all $j \in I_n$. Then, SVTN-weighted arithmetic operator, denoted by $S_{ao} : \Omega^n \to \Omega$, is defined by

$$S_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = w_1\tilde{a}_1 \oplus w_2\tilde{a}_2 \oplus \cdots \oplus w_n\tilde{a}_n$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for every $j \in I_n$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 4.12. *[73] Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Omega$ for $j \in I_n$ and S_{ao} be the SVTN-weighted arithmetic operator. Then, their aggregated value by using S_{ao} operator is also a SVTN-number and*

$$\begin{aligned} S_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) &= \sum_{j=1}^{n} w_j\tilde{a}_j = \langle (\sum_{j=1}^{n} w_j a_j, \sum_{j=1}^{n} w_j b_j, \sum_{j=1}^{n} w_j c_j, \sum_{j=1}^{n} w_j d_j); \\ &= 1 - \prod_{j=1}^{n}(1 - w_{\tilde{a}_j})^{w_j}, \prod_{j=1}^{n} u^{w_j}_{\tilde{a}_j}, \prod_{j=1}^{n} y^{w_j}_{\tilde{a}_j} \rangle \end{aligned}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for all $j \in I_n$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.

Theorem 4.13. *[73] Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Omega$ for $j \in I_n$. Then,*

1. *If $\tilde{a}_j = \tilde{a}$, for all $j \in I_n$, then $S_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$,*
2. *$min_j\{\tilde{a}_j\} \leq S_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq max_j\{\tilde{a}_j\}$,*
3. *If $\tilde{a}^*_j = \langle (a^*_j, b^*_j, c^*_j, d_j)^*; w^*_{\tilde{a}_j}, u^*_{\tilde{a}_j}, y^*_{\tilde{a}_j} \rangle \in \Omega$ and $\tilde{a}_j \leq \tilde{a}_j{}^*$ for all $j \in I_n$, then $S_{ao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq S_{ao}(\tilde{a}^*_1, \tilde{a}^*_2, ..., \tilde{a}^*_n)$.*

Definition 4.14. Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j} \rangle \in \Omega$ for all $j \in I_n$. Then, an SVTN-ordered weighted arithmetic operator, denoted by $S_{oao} : \Omega^n \to \Omega$, is defined by

$$S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = w_1\tilde{a}_{\sigma(1)} \oplus w_2\tilde{a}_{\sigma(2)} \oplus \cdots \oplus w_n\tilde{a}_{\sigma(n)}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for every $j \in I$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^{n} w_j = 1$.
Here, $(\sigma(1), \sigma(2), ..., \sigma(n))$ is a permutation of $(1, 2, ..., n)$ such that $a_{\sigma(j-1)} \geq a_{\sigma(j)}$ for all $j \in I_n$.

Theorem 4.15. *Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for $j \in I_n$ and S_{oao} be an SVTN-ordered weighted arithmetic operator. Then, their aggregated value by using S_{oao} operator is also a SVTN-number and*

$$\begin{array}{rcl} S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) &=& \sum_{j=1}^n w_j \tilde{a}_{\sigma(j)} = \langle (\sum_{j=1}^n w_j a_{\sigma(j)}, \sum_{j=1}^n w_j b_{\sigma(j)}, \sum_{j=1}^n w_j c_{\sigma(j)}, \sum_{j=1}^n w_j d_{\sigma(j)}); \\ &=& 1 - \prod_{j=1}^n (1 - w_{\tilde{a}_{\sigma(j)}})^{w_j}, \prod_{j=1}^n u_{\tilde{a}_{\sigma(j)}}^{w_j}, \prod_{j=1}^n y_{\tilde{a}_{\sigma(j)}}^{w_j}\rangle \end{array}$$

where $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ for all $j \in I_n$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Theorem 4.16. *Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for $j \in I_n$ and S_{oao} be the SVTN-arithmetic averaging operator. Then, for all $j \in I_n$,*

1. *If $\tilde{a}_j = \tilde{a}$, then $S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{a}$.*
2. *$min_j\{\tilde{a}_j\} \leq S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq max_j\{\tilde{a}_j\}$*
3. *If $\tilde{a}_j^* = \langle (a_j^*, b_j^*, c_j^*, d_j)^*; w_{\tilde{a}_j}^*, u_{\tilde{a}_j}^*, y_{\tilde{a}_j}^*\rangle \in \Omega$ and $\tilde{a}_j \overset{\leq}{} \tilde{a}_j{}^*$, then $S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) \leq S_{oao}(\tilde{a}_1^*, \tilde{a}_2^*, ..., \tilde{a}_n^*)$*
4. *If $\tilde{\acute{a}}_j \in \Omega$, then $S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = S_{oao}(\tilde{\acute{a}}_1, \tilde{\acute{a}}_2, ..., \tilde{\acute{a}}_n)$ where $(\tilde{\acute{a}}_1, \tilde{\acute{a}}_2, ..., \tilde{\acute{a}}_n)$ is any permutation of $(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n)$.*

Theorem 4.17. *Let $\tilde{a}_j = \langle (a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}\rangle \in \Omega$ for $j \in I_n$ and S_{oao} be the SVTN-arithmetic averaging operator. Then, for all $j \in I_n$,*

1. *If $w = (1, 0, ..., 0)^T$, then $S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = max_j\{\tilde{a}_j\}$.*
2. *If $w = (0, 0, ..., 1)^T$, then $S_{oao}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = min_j\{\tilde{a}_j\}$.*

Definition 4.18. Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$. Then, an SVTN-hybrid arithmetic operator, denoted by $S_{hao}^{\tilde{s}} : \Omega^n \to \Omega$,, is defined by

$$S_{hao}^{\tilde{s}}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \tilde{s}_1 \bar{\tilde{a}}_{\sigma(1)} \oplus \tilde{s}_2 \bar{\tilde{a}}_{\sigma(2)} \oplus \cdots \oplus \tilde{s}_n \bar{\tilde{a}}_{\sigma(n)}$$

where for $j \in I_n$, $\bar{\tilde{a}}_{\sigma(j)}$ is the jth largest of the weighted SVTN-numbers $\bar{\tilde{a}}_j$, $\bar{\tilde{a}}_j = \tilde{a}_j^{nw_j}$, $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)^T$ is a vector associated with the $S_{hao}^{\tilde{s}}$ such that $\tilde{s}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{s}_j = 1$.

Theorem 4.19. *Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$ and $S_{hao}^{\tilde{s}}$ be the SVTN-hybrid arithmetic operator. Then, their aggregated value by using $S_{hao}^{\tilde{s}}$ operator is also a SVTN-number and*

$$\begin{array}{rcl} S_{hao}^{\tilde{s}}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) &=& \sum_{j=1}^n \bar{\tilde{a}}_{\sigma(j)} \tilde{s}_j = \langle (\sum_{j=1}^n \bar{a}_{\sigma(j)}^{\tilde{s}_j}, \sum_{j=1}^n \bar{b}_{\sigma(j)}^{\tilde{s}_j} \sum_{j=1}^n \bar{c}_{\sigma(j)}^{\tilde{s}_j}, \sum_{j=1}^n \bar{d}_{\sigma(j)}^{\tilde{s}_j}); \\ &=& 1 - \prod_{j=1}^n (1 - w_{\bar{\tilde{a}}_{\sigma(j)}} \tilde{s}_j), \prod_{j=1}^n u_{\bar{\tilde{a}}_{\sigma(j)}} \tilde{s}_j, \prod_{j=1}^n y_{\bar{\tilde{a}}_{\sigma(j)}} \tilde{s}_j\rangle \end{array}$$

where for $j \in I_n$, $\bar{\tilde{a}}_{\sigma(j)}$ is the jth largest of the weighted SVTN-numbers $\bar{\tilde{a}}_j$, $\bar{\tilde{a}}_j = \tilde{a}_j^{nw_j}$, $w = (w_1, w_2, ..., w_n)^T$ is a weight vector of $\tilde{a}_j$ such that $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$, and $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)^T$ is a vector associated with the $S_{hao}^{\tilde{s}}$ such that $\tilde{s}_j \in [0, 1]$ and $\sum_{j=1}^n \tilde{s}_j = 1$.

Corollary 4.20. *Let $\tilde{a}_j = ((a_j, b_j, c_j, d_j); w_{\tilde{a}_j}, u_{\tilde{a}_j}, y_{\tilde{a}_j}) \in \Omega$ for $j \in I_n$. Then, SVTN-weighted arithmetic operator S_{ao} and SVTN-weighted arithmetic operator S_{oao} operator is a special case of the SVTN-hybrid arithmetic operator $S_{hao}^{\tilde{s}}$.*

5 SVTN-Group Decision Making Method

In this section, by using the $S^{\tilde{s}}_{hgo}$ and S_{go} operators we define a multi attribute group decision making method called SVTN-group decision making method.

Definition 5.1. Let $B = \{B_1, B_2, ..., B_m\}$ be a set of alternatives, $U = \{u_1, u_2, ..., u_n\}$ be a set of attributes, $D = \{d_1, d_2, ..., d_t\}$ be a set of decision makers, $\tilde{s} = (\tilde{s}_1, \tilde{s}_2, ..., \tilde{s}_n)^T$ be a weighting vector of the attributes where $\tilde{s}_j \in [0, 1]$ for $j \in I_n$ and $\sum_{j=1}^{n} \tilde{s}_j = 1$, and $w = (w_1, w_2, ..., w_t)^T$ be a weighting vector of the decision makers such that $w_j \in [0, 1]$ for $j \in I_n$ and $\sum_{j=1}^{t} w_j = 1$. If $\tilde{a}^k_{ij} = \langle (a^k_{ij}, b^k_{ij}, c^k_{ij}, d^k_{ij}); w^k_{ij}, u^k_{ij}, y^k_{ij}\rangle \in \Omega$, then

$$[\tilde{a}^k_{ij}]_{m\times n} = \begin{array}{c} \\ B_1 \\ B_2 \\ \vdots \\ B_m \end{array} \begin{array}{c} \begin{array}{cccc} u_1 & u_2 & \cdots & u_n \end{array} \\ \begin{pmatrix} \tilde{a}^k_{11} & \tilde{a}^k_{12} & \cdots & \tilde{a}^k_{1n} \\ \tilde{a}^k_{21} & \tilde{a}^k_{22} & \cdots & \tilde{a}^k_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{a}^k_{m1} & \tilde{a}^k_{m2} & \cdots & \tilde{a}^k_{mn} \end{pmatrix} \end{array}$$

is called an SVTN-group decision making matrix of the decision maker d_k for each $k \in I_t$. The matrix is also written shortly as

$$[\tilde{a}^k_{ij}]_{m\times n} = \langle (a^k_{ij}, b^k_{ij}, c^k_{ij}, d^k_{ij}); w^k_{ij}, u^k_{ij}, y^k_{ij}\rangle$$

Now, we can give an algorithm of the SVTN-group decision making method as follows;

Algorithm:

Step 1. Construct

$[\tilde{a}^k_{ij}]_{m\times n} = \langle (a^k_{ij}, b^k_{ij}, c^k_{ij}, d^k_{ij}); w^k_{ij}, u^k_{ij}, y^k_{ij}\rangle$ of d_k for each $k \in I_t$.

Step 2. Compute $\tilde{a}^k_i = S_{go}(\tilde{a}^k_{i1}, \tilde{a}^k_{i2}, ..., \tilde{a}^k_{in}) = \prod_{j=1}^{n} (\tilde{a}^k_{ij})^{w_j}$ for each $k \in I_t$ and $i \in I_m$ to derive the individual overall preference SVTN-values $\tilde{a}^k_i$ of the alternative B_i.

Step 3. Compute $\tilde{a}_i = S^{\tilde{s}}_{hgo}(\tilde{a}^1_i, \tilde{a}^2_i, ..., \tilde{a}^t_i) =< (a_i, b_i, c_i); w_{\tilde{a}_i}, u_{\tilde{a}_i}, y_{\tilde{a}_i} >$ for each $i \in I_m$ to derive the collective overall preference SVTN-values $\tilde{a}_i$ of the alternative B_i.

Step 4. Compute $d_h(\tilde{a}_i, \tilde{a}^+)$ for each $i \in I_m$.

Step 5. Rank all alternatives B_i according to the $d_h(\tilde{a}_i, \tilde{a}^+)$ for each $i \in I_m$.

Example 5.2. (It's adopted from [70]) Let us suppose there is a risk investment company, which wants to invest a sum of money in the best option. There is a panel with five possible alternatives (engineer construction projects) to invest the money. The risk investment company must take a decision according to four attributes: u_1 = "risk analysis", u_2 = "growth analysis", u_3 = "social-political impact analysis", u_4 = "environmental impact analysis". The five possible alternatives $B_i\,(i = 1, 2, ..., 5)$ are to be evaluated using the SVTN-numbers by the four decision makers (whose weighting vector $w = (0.2, 0.4, 0.1, 0.3)^T$) under the above four attributes (whose weighting vector $\tilde{s} = (0.25, 0.25, 0.25, 0.25)^T$), and construct, respectively,

Step 1. For each $k = 1, 2, 3, 4$, the decision maker d_k construct own decision matrices $[\tilde{a}^k_{ij}]_{5x4}$ as Table 1:

Step 2. For each $k = 1, 2, 3, 4$ and $i = 1, 2, 3, 4, 5$ compute $\tilde{a}_i^k = S_{go}(\tilde{a}_{i1}^k, \tilde{a}_{i2}^k, ..., \tilde{a}_{in}^k)$ as follows:

$$\tilde{a}_1^1 = \big((0.170, 0.411, 0.606, 0.814); 0.442, 0.749, 0.409\big)$$
$$\tilde{a}_2^1 = \big((0.194, 0.342, 0.517, 0.800); 0.534, 0.543, 0.302\big)$$
$$\tilde{a}_3^1 = \big((0.224, 0.259, 0.517, 0.628); 0.237, 0.513, 0.281\big)$$
$$\tilde{a}_4^1 = \big((0.214, 0.332, 0.464, 0.774); 0.460, 0.518, 0.407\big)$$
$$\tilde{a}_5^1 = \big((0.139, 0.209, 0.401, 0.580); 0.186, 0.587, 0.332\big)$$
$$\tilde{a}_1^2 = \big((0.226, 0.278, 0.459, 0.763); 0.540, 0.423, 0.500\big)$$
$$\tilde{a}_2^2 = \big((0.285, 0.388, 0.592, 0.728); 0.379, 0.686, 0.522\big)$$
$$\tilde{a}_3^2 = \big((0.476, 0.581, 0.700, 0.814); 0.394, 0.349, 0.300\big)$$
$$\tilde{a}_4^2 = \big((0.230, 0.332, 0.613, 0.738); 0.564, 0.714, 0.346\big)$$
$$\tilde{a}_5^2 = \big((0.132, 0.147, 0.355, 0.531); 0.293, 0.396, 0.635\big)$$
$$\tilde{a}_1^3 = \big((0.115, 0.155, 0.459, 0.599); 0.275, 0.806, 0.674\big)$$
$$\tilde{a}_2^3 = \big((0.298, 0.375, 0.592, 0.806); 0.309, 0.387, 0.679\big)$$
$$\tilde{a}_3^3 = \big((0.107, 0.112, 0.150, 0.513); 0.491, 0.537, 0.670\big)$$
$$\tilde{a}_4^3 = \big((0.200, 0.310, 0.565, 0.673); 0.500, 0.346, 0.693\big)$$
$$\tilde{a}_5^3 = \big((0.164, 0.176, 0.355, 0.650); 0.426, 0.527, 0.519\big)$$
$$\tilde{a}_1^4 = \big((0.182, 0.302, 0.537, 0.781); 0.275, 0.627, 0.527\big)$$
$$\tilde{a}_2^4 = \big((0.154, 0.305, 0.428, 0.693); 0.225, 0.568, 0.617\big)$$
$$\tilde{a}_3^4 = \big((0.000, 0.232, 0.504, 0.675); 0.354, 0.551, 0.513\big)$$
$$\tilde{a}_4^4 = \big((0.200, 0.300, 0.417, 0.660); 0.509, 0.481, 0.342\big)$$
$$\tilde{a}_5^4 = \big((0.000, 0.182, 0.374, 0.625); 0.282, 0.424, 0.270\big)$$

$$\tilde{a}_1^1 = \big((0.170, 0.411, 0.606, 0.814); 0.442, 0.749, 0.409\big)$$
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$$\tilde{a}_3^2 = \big((0.476, 0.581, 0.700, 0.814); 0.394, 0.349, 0.300\big)$$
$$\tilde{a}_4^2 = \big((0.230, 0.332, 0.613, 0.738); 0.564, 0.714, 0.346\big)$$
$$\tilde{a}_5^2 = \big((0.132, 0.147, 0.355, 0.531); 0.293, 0.396, 0.635\big)$$
$$\tilde{a}_1^3 = \big((0.115, 0.155, 0.459, 0.599); 0.275, 0.806, 0.674\big)$$
$$\tilde{a}_2^3 = \big((0.298, 0.375, 0.592, 0.806); 0.309, 0.387, 0.679\big)$$
$$\tilde{a}_3^3 = \big((0.107, 0.112, 0.150, 0.513); 0.491, 0.537, 0.670\big)$$
$$\tilde{a}_4^3 = \big((0.200, 0.310, 0.565, 0.673); 0.500, 0.346, 0.693\big)$$
$$\tilde{a}_5^3 = \big((0.164, 0.176, 0.355, 0.650); 0.426, 0.527, 0.519\big)$$
$$\tilde{a}_1^4 = \big((0.182, 0.302, 0.537, 0.781); 0.275, 0.627, 0.527\big)$$
$$\tilde{a}_2^4 = \big((0.154, 0.305, 0.428, 0.693); 0.225, 0.568, 0.617\big)$$
$$\tilde{a}_3^4 = \big((0.000, 0.232, 0.504, 0.675); 0.354, 0.551, 0.513\big)$$
$$\tilde{a}_4^4 = \big((0.200, 0.300, 0.417, 0.660); 0.509, 0.481, 0.342\big)$$
$$\tilde{a}_5^4 = \big((0.000, 0.182, 0.374, 0.625); 0.282, 0.424, 0.270\big)$$

Step 3. Assume that $w = (0.2, 0.4, 0.1, 0.3)^T$ and $\tilde{s} = (0.25, 0.25, 0.25)^T$. We can compute

$$\tilde{a}_i = S_{hgo}^{\tilde{s}}(\tilde{a}_i^1, \tilde{a}_i^2, ..., \tilde{a}_i^t) =< (a_i, b_i, c_i); w_{\tilde{a}_i}, u_{\tilde{a}_i}, y_{\tilde{a}_i} >$$

for each $i = 1, 2, 3, 4, 5$ as follows:

$$\begin{aligned}
\tilde{a}_1 &= \big((0.187, 0.291, 0.509, 0.760); 0.396, 0.569, 0.513\big)\\
\tilde{a}_2 &= \big((0.219, 0.351, 0.523, 0.738); 0.340, 0.584, 0.536\big)\\
\tilde{a}_3 &= \big((0.000, 0.298, 0.524, 0.698); 0.352, 0.451, 0.414\big)\\
\tilde{a}_4 &= \big((0.214, 0.320, 0.512, 0.714); 0.519, 0.553, 0.404\big)\\
\tilde{a}_5 &= \big((0.000, 0.171, 0.370, 0.579); 0.274, 0.450, 0.479\big)
\end{aligned}$$

Step 4. Compute $d_h(\tilde{a}_i, \tilde{a}_i^+)$ for each alternative B_i, $i = 1, 2, 3, 4, 5$, as follows:

$$\begin{aligned}
&d_h(\tilde{a}_1, \tilde{a}^+) = 1.242, d_h(\tilde{a}_2, \tilde{a}^+) = 1.266,\\
&d_h(\tilde{a}_3, \tilde{a}^+) = 1.210, d_h(\tilde{a}_4, \tilde{a}^+) = 1.169,\\
&d_h(\tilde{a}_5, \tilde{a}^+) = 1.269
\end{aligned}$$

Then we get the rank,

$$d_h(\tilde{a}_5, \tilde{a}^+) > d_h(\tilde{a}_2, \tilde{a}^+) > d_h(\tilde{a}_1, \tilde{a}^+) > d_h(\tilde{a}_3, \tilde{a}^+) > d_h(\tilde{a}_4, \tilde{a}^+)$$

Step 5. Therefore, we can rank all alternatives B_i according to the $d_h(\tilde{a}_i, \tilde{a}_i^+)$ for each $i = 1, 2, 3, 4, 5$.

$$B_5 < B_2 < B_1 < B_3 < B_4$$

and thus the most desirable alternative is B_4.

6 Comparative Analysis and Discussion

In this section, a comparative study is presented to show the flexibility and feasibility of the introduced SVTN-group decision making method. Different methods used to solve the same SVTN-group decision making problem with SVTN-information is given by Ye [73]. The ranking results obtained by different methods are summarized in Table 2.

From the results presented in Table 2, the best alternative in proposed method and Ye's method [73] with geometric operator is B_4, whilst the worst one is B_5. In contrast, by using the methods in the proposed method and Ye's method [73] with arithmetic operator, the best is B_3, whilst the worst is B_5. There are a number of reasons why differences exist between the final rankings of the methods. First, the author uses a score and accurate function in Ye's method [73] with arithmetic operator and Ye's method [73] with geometric opera-tor. Moreover, different aggregation operators, which is arithmetic and geometric operator, lead to different rankings because the operators emphasize the decision makers judgments differently. The proposed method is different in that it contains two major phrases. First, the proposed method uses both SVTN-weighted geo-metric operator and the SVTN-hybrid geometric operator to aggregate the SVTN-numbers. Second, based on distance measure, the method uses SVTN-positive ideal solution and SVTN-negative ideal solution to rank the SVTN-information. Finally, the ranking of the proposed method is similar to other methods. Therefore, the proposed method is flexible and feasible.

7 Conclusion

Due to the ambiguity of people's thinking and the complexity of objective things, the attribute values of the MAGDM problems cannot always be expressed by exact and crisp values and it may be easier to escribe them by neutrosophic information. This paper introduced an MAGDM in which the attribute values are expressed with the SVTN-numbers, which are solved by developing a new decision method based on geometric aggregation operators of SVTN-numbers. The proposed method with SVTN-numbers is more suitable for real scientific and engineering applications, because the proposed decision-making method includes much more information and can deal with indeterminate and inconsistent decision-making problems. In the future, we shall further develop more aggregation operators for SVTN-numbers and apply them to solve practical applications in areas such as group decision making, expert system, information fusion system, fault diagnoses, medical diagnoses and so on.

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Table 1. The decision matrices of decision maker d_k.

$$[\tilde{a}^1_{ij}]_{5x4} = \begin{pmatrix} ((0.5,0.7,0.8,0.9);0.5,0.6,0.7) & ((0.1,0.3,0.4,0.7);0.4,0.9,0.3) & ((0.1,0.1,0.8,0.9);0.7,0.5,0.3) & ((0.2,0.7,0.8,0.9);0.4,0.5,0.3) \\ ((0.4,0.5,0.7,0.8);0.3,0.2,0.3) & ((0.2,0.3,0.5,0.8);0.7,0.4,0.2) & ((0.3,0.4,0.7,0.8);0.7,0.4,0.6) & ((0.1,0.3,0.4,0.8);0.5,0.8,0.3) \\ ((0.5,0.6,0.7,0.8);0.2,0.7,0.2) & ((0.1,0.1,0.4,0.5);0.1,0.4,0.3) & ((0.2,0.3,0.5,0.7);0.4,0.7,0.3) & ((0.4,0.5,0.6,0.7);0.7,0.4,0.3) \\ ((0.4,0.5,0.6,0.7);0.2,0.5,0.4) & ((0.2,0.3,0.4,0.9);0.5,0.5,0.6) & ((0.1,0.3,0.4,0.7);0.5,0.8,0.2) & ((0.2,0.3,0.5,0.7);0.7,0.4,0.1) \\ ((0.3,0.5,0.6,0.7);0.2,0.5,0.4) & ((0.1,0.2,0.5,0.8);0.1,0.8,0.3) & ((0.3,0.4,0.4,0.8);0.1,0.5,0.4) & ((0.1,0.1,0.2,0.3);0.5,0.1,0.3) \end{pmatrix}$$

$$[\tilde{a}^2_{ij}]_{5x4} = \begin{pmatrix} ((0.1,0.1,0.2,0.5);0.8,0.4,0.7) & ((0.2,0.3,0.4,0.8);0.8,0.4,0.3) & ((0.1,0.1,0.8,0.9);0.3,0.3,0.3) & ((0.6,0.7,0.8,0.9);0.3,0.5,0.6) \\ ((0.3,0.4,0.7,0.8);0.2,0.4,0.8) & ((0.3,0.4,0.7,0.9);0.7,0.9,0.3) & ((0.6,0.7,0.7,0.8);0.8,0.5,0.7) & ((0.2,0.3,0.4,0.5);0.2,0.2,0.4) \\ ((0.6,0.7,0.8,0.9);0.5,0.1,0.3) & ((0.6,0.7,0.8,0.9);0.3,0.4,0.3) & ((0.2,0.3,0.5,0.7);0.7,0.4,0.3) & ((0.4,0.5,0.6,0.7);0.4,0.4,0.3) \\ ((0.4,0.5,0.6,0.7);0.6,0.4,0.5) & ((0.2,0.3,0.7,0.8);0.7,0.8,0.3) & ((0.2,0.3,0.4,0.7);0.3,0.9,0.3) & ((0.2,0.3,0.6,0.7);0.5,0.6,0.3) \\ ((0.1,0.1,0.4,0.8);0.3,0.4,0.2) & ((0.2,0.2,0.5,0.6);0.2,0.3,0.1) & ((0.10.3,0.4,0.8);0.5,0.4,0.9) & ((0.1,0.1,0.2,0.3);0.4,0.5,0.9) \end{pmatrix}$$

$$[\tilde{a}^3_{ij}]_{5x4} = \begin{pmatrix} ((0.1,0.1,0.2,0.5);0.5,0.9,0.9) & ((0.1,0.2,0.4,0.8);0.1,0.6,0.3) & ((0.4,0.5,0.8,0.9);0.8,0.7,0.3) & ((0.1,0.1,0.8,0.4);0.5,0.9,0.8) \\ ((0.3,0.4,0.7,0.8);0.5,0.1,0.3) & ((0.4,0.4,0.7,0.9);0.5,0.3,0.9) & ((0.3,0.5,0.7,0.8);0.5,0.4,0.3) & ((0.2,0.3,0.4,0.7);0.1,0.6,0.3) \\ ((0.1,0.1,0.1,0.4);0.4,0.7,0.8) & ((0.1,0.1,0.1,0.4);0.8,0.6,0.8) & ((0.2,0.3,0.7,0.9);0.1,0.1,0.3) & ((0.1,0.1,0.2,0.7);0.5,0.4,0.3) \\ ((0.4,0.5,0.8,0.9);0.5,0.4,0.3) & ((0.2,0.3,0.7,0.8);0.5,0.4,0.9) & ((0.4,0.5,0.8,0.9);0.5,0.4,0.3) & ((0.1,0.2,0.3,0.4);0.5,0.2,0.4) \\ ((0.3,0.3,0.4,0.8);0.5,0.2,0.3) & ((0.2,0.2,0.5,0.8);0.5,0.7,0.3) & ((0.1,0.2,0.4,0.8);0.1,0.5,0.3) & ((0.1,0.1,0.2,0.4);0.5,0.4,0.8) \end{pmatrix}$$

$$[\tilde{a}^4_{ij}]_{5x4} = \begin{pmatrix} ((0.5,0.7,0.8,0.9);0.1,0.9,0.3) & ((0.2,0.3,0.5,0.8);0.5,0.6,0.6) & ((0.1,0.2,0.4,0.5);0.5,0.4,0.3) & ((0.1,0.2,0.5,0.8);0.2,0.3,0.6) \\ ((0.2,0.4,0.5,0.6);0.5,0.4,0.3) & ((0.2,0.3,0.4,0.8);0.1,0.4,0.8) & ((0.1,0.2,0.5,0.8);0.5,0.4,0.3) & ((0.1,0.3,0.4,0.6);0.3,0.8,0.5) \\ ((0.1,0.2,0.3,0.4);0.1,0.2,0.3) & ((0.0,0.1,0.6,0.7);0.5,0.7,0.3) & ((0.0,0.1,0.4,0.7);0.4,0.7,0.2) & ((0.4,0.5,0.8,0.9);0.5,0.4,0.8) \\ ((0.2,0.3,0.4,0.7);0.5,0.6,0.3) & ((0.2,0.3,0.4,0.5);0.5,0.2,0.4) & ((0.2,0.3,0.6,0.7);0.6,0.9,0.3) & ((0.2,0.3,0.4,0.9);0.5,0.4,0.3) \\ ((0.0,0.1,0.2,0.8);0.5,0.4,0.3) & ((0.1,0.3,0.7,0.9);0.5,0.4,0.3) & ((0.4,0.5,0.7,0.8);0.2,0.6,0.5) & ((0.0,0.1,0.2,0.3);0.1,0.4,0.1) \end{pmatrix}$$

Table 2. The ranking results of different methods.

Methods	Ranking results
The proposed method with arithmetic operator	$B_5 < B_2 < B_1 < B_4 < B_3$
The proposed method with geometric operator	$B_5 < B_2 < B_1 < B_3 < B_4$
Ye's method [73] *with geometric operator*	$B_5 < B_2 < B_3 < B_1 < B_4$
Ye's method [73] *with arithmetic operator*	$B_5 < B_2 < B_1 < B_4 < B_3$

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