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Secondary k-column symmetric Neutrosophic Fuzzy Matrices

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Abstract: **Objective:** The objective of this study is to establish the results of secondary k- column symmetric (CS) Neutrosophic fuzzy matrices. **Methods and Findings:** We have applied CS condition in neutrosophic environment to find the relation between s-k CS, s- CS, k- CS and CS. **Novelty**: We establish the necessary and sufficient criteria for s-k CS Neutrosophic fuzzy matrices and various g-inverses of an s – k CS Neutrosophic fuzzy matrices to be an s – k CS. The generalized inverses of an s – k CS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

Keywords: Neutrosophic fuzzy matrices (NFM), s-column symmetric, k-column symmetric, column symmetric.

1. Introduction

Zadeh [1] has studied fuzzy set (FS). Atanassov [2] introduced intuitionistic FSs. Smarandache [3] has discussed the concept of neutrosophic sets. Khan, Shyamal, and Pal [4] have studied intuitionistic fuzzy matrices (IFMs) for the first time. Atanassov [5,6] has discussed IFS and Operations over IV IFS. Hashimoto [7] has studied Canonical form of a transitive matrix. Kim and Roush [8] have studied generalized fuzzy matrices. Lee [9] has studied Secondary Skew Symmetric, Secondary Orthogonal Matrices. Hill and Waters [10] have analyzed On k-Real and k-Hermitian matrices. Meenakshi [11] has studied Fuzzy Matrix: Theory and Applications.

Anandhkumar [12,13] has studied Pseudo Similarity of NFM and On various Inverse of NFM. Punithavalli and Anandhkumar [14] have studied Reverse Sharp and Left-T And Right- T Partial Ordering on IFM. Pal and Susanta Kha [15] have studied IV Intuitionistic Fuzzy Matrices. Vidhya and Irene Hepzibah [16] have discussed on Interval Valued NFM. Anandhkumar et.al [17,18] has focused on Reverse Sharp and Left-T Right-T Partial Ordering on NFM and IFM. Anandhkumar,et.al have studied [19] Partial orderings, Characterizations and Generalization of k-idempotent NFM. Here, we introduce the Secondary k-CS NFM and introduce some basic operators on NFMs.

1.1 Literature Review

Meenakshi and Jaya Shree [20] have studied On k-kernel symmetric matrices. Meenakshi and Krishanmoorthy [21] have characterized On Secondary k-Hermitian matrices. Meenakshi and Jaya Shree [22] have studied On k -range symmetric matrices. Jaya shree [23] has studied Secondary κ-Kernel Symmetric Fuzzy Matrices. Shyamal and Pal [24] Interval valued Fuzzy matrices. Meenakshi and Kalliraja [25] have studied Regular Interval valued Fuzzy matrices. Anandhkumar [26] has studied Kernal and k-kernal Intuitionistic Fuzzy matrices. Jaya Shree [27] has discussed Secondary κ-range symmetric fuzzy matrices. Anandhkumar et.al.,[28] have studied Generalized Symmetric NFM. Kaliraja and Bhavani [29] have studied Interval Valued Secondary κ-Range Symmetric Fuzzy Matrices,

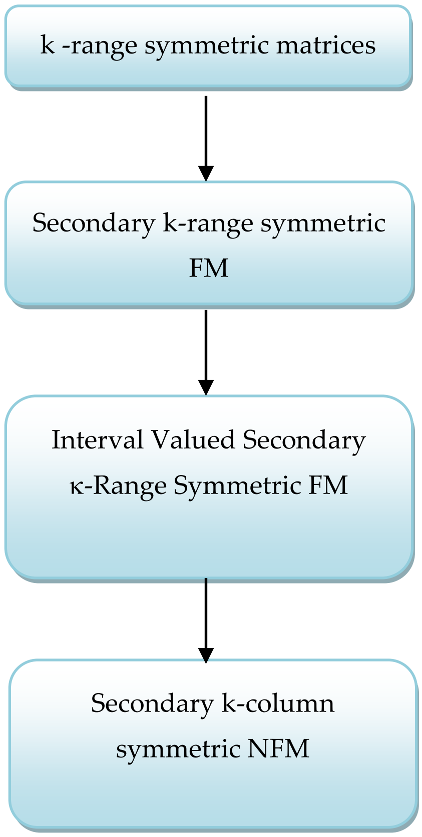
Let P be any fuzzy matrix, $P^{\dagger}$ occurs then this will coincides with the transpose of the matrix (P^T). The fuzzy matrix P belongs to Fn is known to be kernel symmetric matrix, then this shows that $N(P) = N(P^T)$ which does not implies $R[P] = R[P^T]$. But the converse is true. Symmetric matrices are established in the field of complex entries for the theory of k - hermitian matrices. This idea make use of the development of k - EP matrices in the generalization of k - hermitian matrices and also EP matrices. Hill and Waters [30] have initiated the study on κ - real and κ - Hermitian matrices. The concept of Theorems on products of EPr matrices introduced by Baskett and Katz [30]. It is commonly known that for complex matrices, the concepts of range and kernel symmetric are equivalent. But this is fails for Interval valued fuzzy matrices.

The concept of interval valued s - k Hermitian and interval valued kernel symmetric matrices for fuzzy matrices. We also expanded many basic conclusions on these two types of matrices. An Interval valued secondary s - k kernel symmetric fuzzy matrix can be described. Suitable standards for determining g - inverses of an Interval valued secondary s - k - kernel symmetric fuzzy matrices are interval valued secondary s - k - kernel symmetric are found. We establish the necessary and sufficient canditions for an interval valued s - k kernel symmetric fuzzy matrices. Meenakshi, Krishnamoorthy and Ramesh [31] have studied on s - k - EP matrices. Meenakshi and Krishnamoorthy [32] have introduced the idea of s - k hermitian matrices.

Shyamal and Pal [33] have studied Interval valued Fuzzy matrices. The definition of k-symmetric matrices was introduced by the following authors Ann Lec [34] has studied Secondary symmetric and skew symmetric secondary orthogonal matrices. . Anandhkumar et.al [35] have discussed Interval Valued Secondary k-Range Symmetric NFM.

Table:1 Extension of Neutrosophic Fuzzy Matrices based on previous works

References	**Extension of Neutrosophic Fuzzy Matrices from Fuzzy Matrices**	**Year**
[20]	On k-kernel symmetric matrices	2009
[22]	On k -range symmetric matrices	2009
[23]	Secondary k-Kernel Symmetric Fuzzy Matrices	2014
[27]	Secondary k-range symmetric FM	2018
[29]	Interval Valued Secondary κ-Range Symmetric Fuzzy Matrices	2022
Proposed	Secondary k-column symmetric Neutrosophic Fuzzy Matrices	2023



From Table 1 and process flow, it is observed that the previous studies are on k-Kernel, K-range, Secondary k-Kernel and Secondary k- range using fuzzy matrices. It is evident that there is a research gap of these studies in Neutrosophic environment. So, based on the above observation, we have established the results of K-column and Secondary k- column in neutrosophic fuzzy matrices.

Notations:

P^T = Transpose of the matrix P

P^+ = Moore-penrose inverse of P

CS = Column symmetric

C(P) = Column space of P

2. Generalized Symmetric NFM

Definition: 2.1 Let P be a NFM, if C[P] = C[P^T] then P is said to be CS.

Example:2.1 Let us consider $P = \begin{bmatrix} <0.3,0.5,0.4> & <0,0,1> & <0.7,0.2,0.5> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.7,0.2,0.5> & <0,0,1> & <0.3,0.2,0.4> \end{bmatrix}$,

The following NFM are not CS

$$P=\begin{bmatrix} <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \\ <0,0,1> & <0,0,1> & <1,1,0> \end{bmatrix},\ P^T=\begin{bmatrix} <1,1,0> & <0,0,1> & <0,0,1> \\ <1,1,0> & <1,1,0> & <0,0,1> \\ <0,0,1> & <1,1,0> & <1,1,0> \end{bmatrix},$$

$$\left(<1,1,0> \quad <0,0,1> \quad <0,0,1>\right)^T \in C(\mathrm{P}), \qquad \left(<1,1,0> \quad <0,0,1> \quad <0,0,1>\right)^T \notin C(\mathrm{P}^T)$$

$$\left(<1,1,0> \quad <1,1,0> \quad <0,0,1>\right)^T \in C(\mathrm{P}), \qquad \left(<1,1,0> \quad <1,1,0> \quad <0,0,1>\right)^T \in C(\mathrm{P}^T)$$

$$\left(<0,0,1> \quad <1,1,0> \quad <1,1,0>\right)^T \in C(\mathrm{P}), \qquad \left(<0,0,1> \quad <1,1,0> \quad <1,1,0>\right)^T \in C(\mathrm{P}^T)$$

$$C(\mathrm{P}) \notin C(\mathrm{P}^T)$$

Definition 2.2: A NFM P $\in$ F_n is s-symmetric NFM $\Leftrightarrow$ $P = VP^TV$.

Example:2.2 Let us consider $P=\begin{bmatrix} <0.4,0.3,0.2> & <0,0,1> & <0.5,0.4,0.3> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.5,0.4,0.3> & <0,0,1> & <0.3,0.2,0.4> \end{bmatrix},$

$$V=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

Definition 2.3: A NFM P $\in$ F_n is s-CS NFM $\Leftrightarrow$ $C(P) = C(VP^TV)$.

Example:2.3 Let us consider $P=\begin{bmatrix} <0.7,0.4,0.5> & <0,0,1> & <0.8,0.2,0.1> \\ <0,0,1> & <0,0,1> & <0,0,1> \\ <0.8,0.2,0.1> & <0,0,1> & <0.5,0.7,0.3> \end{bmatrix},$

$$V=\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

Definition 2.4: A NFM P $\in$ F_n is s-k-CS NFM $\Leftrightarrow$ $C(P) = C(KVP^TVK)$.

Example:2.4 Let us consider $P=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix},$

$$\mathrm{K}=\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix},\ V=\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix},$$

Preliminary: 2.1 Let V is a permutation NFM its satisfies the conditions

(i) $VV^T = V^TV = I_n$

(ii) $V^T = V$

(iii) C(P) = C(VP)

(iv) C(P) = C(KP).

Remark 2.1: We notice that $P = KVP^TVK$ implies that $C(P) = C(KVP^TVK)$

This is illustrating the following example

Example 2.5. Consider a NFM , $V=\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}$,

$P=\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}$, $K=\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$,

$$KVP^TVK = \begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}\begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix}$$
$$\begin{bmatrix} <0,0,0> & <1,1,0> \\ <1,1,0> & <0,0,0> \end{bmatrix}\begin{bmatrix} <1,1,0> & <0,0,0> \\ <0,0,0> & <1,1,0> \end{bmatrix}$$

$$KVP^TVK = \begin{bmatrix} <0.7,0.3,0.4> & <0.5,0.3,0.4> \\ <0.5,0.3,0.4> & <0.7,0.3,0.5> \end{bmatrix} = P$$

Therefore, $C(P) = C(KVP^TVK)$

Example 2.6. Consider a NFM

$$K = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}, V = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$P = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0.5,0.3,0.4> & <1,1,0> & <0,0,0> \\ <0.4,0.2,0.6> & <0.5,0.3,0.4> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}\begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix}$$

$$KV = \begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,0,0> & <0,0,0> & <1,1,0> \\ <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \end{bmatrix} \begin{bmatrix} <0,0,0> & <1,1,0> & <0,0,0> \\ <1,1,0> & <0,0,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,1,0> \end{bmatrix}$$

$$VK = \begin{bmatrix} <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <1,1,0> & <0,1,0> \end{bmatrix}$$

$$P^T VK = \begin{bmatrix} <0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4> \\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \end{bmatrix}$$

$$KVP^T VK = \begin{bmatrix} <0,1,0> & <0,1,0> & <0,1,0> \\ <0,1,0> & <0,1,0> & <1,1,0> \\ <1,1,0> & <0,1,0> & <0,1,0> \end{bmatrix} \begin{bmatrix} <0.5,0.8,0.4> & <0.4,0.8,0.6> & <0,0,0.4> \\ <0,0.7,0> & <0.5,0.7,0> & <0,0.7,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \end{bmatrix}$$

$$KVP^T VK = \begin{bmatrix} <0,0,0> & <0,0.2,0> & <0,0,0> \\ <0,0,0> & <0,0,0> & <1,0,0> \\ <0.5,0,0> & <0.4,0,0> & <0,0,0> \end{bmatrix} \neq P$$

$P \neq KVP^T VK$ is not s- κ –symmetric iff not s- κ-CS.

Theorem 2.1: For NFM P $\in F_n$, the subsequent are equivalent :

(i) C(P) = C (P^T).

(ii) P^T = PH= KP for several IFM H, K and ρ (P) = r.

Lemma 2.1: For NFM P $\in$ F_n and a PM K, C(P) = C(Q) iff C(KPK^T) = C(KQK^T)

Theorem 2.2 .For NFM P $\in F_n$ the subsequent are equivalent

(i) C (P) = C(KVP^TVK)

(ii) C (KVP) = C($(KVP)^T$)

(iii) C(PKV) = C($(PKV)^T$)

(iv) C(VP) = C($K(VP)^TK$)

(v) C(PK)= C($V(PK)^TV$)

(vi) $C(P^T) = C(KV(P)VK)$

(vii) $C(P) = C(P^T VK)$

(viii) $C(P^T) = C(PKV)$

(ix) $P = VKP^T VKH_1$ for $H_1 \in F_n$

(x) $P = H_1 KVP^T VK$ for $H_1 \in F_n$

(xi) $P^T = KVPVKH$ for $H \in F_n$

(xii) $P^T = HKVPKV$ for $H \in F_n$

Proof: (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iv)

$\Leftrightarrow$ P is s- κ- Cs

$\Leftrightarrow C(P) = C(KVP^T VK)$

$\Leftrightarrow C(KVP) = C((KVP)^T$ [Preliminary 2.1]

$\Leftrightarrow$ KVP is Column symmetric

$\Leftrightarrow$ VP is κ- Column symmetric

So , (i) $\Leftrightarrow$ (ii) $\Leftrightarrow$ (iv) hence.

(i) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (v)

P is s- κ – CS

$\Leftrightarrow C(P) = C(KVP^T VK)$ [By Definition 2.4]

$\Leftrightarrow C(KVP) = C((KVP)^T)$ [Preliminary 2.1]

$\Leftrightarrow C(PKV) = C((PKV)^T)$

$\Leftrightarrow$ PKV is Column symmetric

$\Leftrightarrow$ PK is s- Column symmetric

So , (i) $\Leftrightarrow$ (iii) $\Leftrightarrow$ (v) hence.

(ii) $\Leftrightarrow$ (vii)

KVP is Column symmetric $\Leftrightarrow$ $C(KVP) = C((KVP)^T)$

$\Leftrightarrow C(P) = C((KVP)^T)$ [Preliminary 2.1]

$\Leftrightarrow C(P) = C(P^T VK)$

So , (ii) $\Leftrightarrow$ (vii) hence.

(iii) $\Leftrightarrow$ (viii):

PVK is Column symmetric $\Leftrightarrow$ $C(PVK) = C((PVK)^T)$

$\Leftrightarrow C(PVK) = C(P^T)$ [Preliminary 2.1]

So ,(iii) $\Leftrightarrow$ (viii) hence.

(i) $\Leftrightarrow$ (vi)

P is s- κ- Column symmetric $\Leftrightarrow C(P) = C(KVP^TVK)$

$\Leftrightarrow C(KVP) = C((KVP)^T)$ [Preliminary 2.1]

$\Leftrightarrow (KVP)^T$ is Column symmetric

$\Leftrightarrow$ P^TVK is Column symmetric

$\Leftrightarrow$ P^T is s- κ - Column symmetric

So , (i) $\Leftrightarrow$ (vi) hence.

(i) $\Leftrightarrow$ (xi) $\Leftrightarrow$ (x)

P is s- κ- Column symmetric $\Leftrightarrow$ $C(P) = C(KVP^TVK)$

$\Leftrightarrow C(P^T) = C(KVPVK)$

$\Leftrightarrow$ $P^T = KVPVKH$ [By Theorem 2.1]

$\Leftrightarrow P = H_1KV P^TVK$ for $H_1 \in F_n$

So , (i) $\Leftrightarrow$ (xi) $\Leftrightarrow$ (x) hence.

(ii) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (ix)

KVP is Column symmetric $\Leftrightarrow$ VP is κ- Column symmetric

$\Leftrightarrow C(VP) = C(K(VP)^TK)$

$\Leftrightarrow$ $C(P) = C(P^TVK)$ [Preliminary 2.1]

$\Leftrightarrow C(P^T) = C(KVP)$

$\Leftrightarrow$ $P^T = HKVP$ for $H \in F_n$ [By Theorem 2.1]

$\Leftrightarrow$ $P^T = HKVPKV$

$\Leftrightarrow$ $P = VKP^TVKH_1$ for $H_1 \in F_n$

So , (ii) $\Leftrightarrow$ (xii) $\Leftrightarrow$ (ix) hence.

Corollary 2.1: For NFM $P \in F_n$ the subsequent are equivalent:

(i) $C(P) = C(VP^TV)$

(ii) $C(VP) = C(VP)^T$

(iii) $C(PV) = C(PV)^T$

(iv) P is s-CS

(v) $C(P^T) = C(VPV)$

(vi) $C(P) = C(P^TV)$

(vii) $C(P^T) = C(PV)$

(viii) $C(KVP) = C((VP)^T)$

(ix) $P = VP^TVH_1$ for $H_1 \in F_n$

(x) $P = H_1VP^TV$ for $H_1 \in F_n$

(xi) $P^T = VPVH$ for $H \in F_n$

(xii) $P^T = HVPV$ for $H \in F$

Theorem 2.3: For NFM $P \in F_n$. Then any two of the subsequent imply the other one:

(i) $C(P) = C(KP^TK)$

(ii) $C(P) = C(VKP^TKV)$

(iii) $C(P^T) = C((VKP)^T)$

Proof: (i) & (ii) $\Leftrightarrow$ (iii)

P is s- κ – Cs

$\Rightarrow C(P) = C(P^TVK)$

$\Rightarrow C(KPK) = C(KP^TK)$ [By Lemma 2.1]

Hence (i) & (ii) $\Rightarrow$ $C(P^T) = C((VPK)^T)$

So,(iii) hence.

(i) & (iii) $\Leftrightarrow$ (ii)

P is κ- Column symmetric $\Rightarrow C(P) = C(KP^TK)$

$\Rightarrow C(KPK) = C(P^T)$ [By Lemma 2.1]

Hence (i) & (iii)

$\Rightarrow$ $C(KPK)= C((VPK)^T)$

$\Rightarrow C(P) = C(P^TVK)$

$\Rightarrow$ $C(P) = C((KVP)^T)$

$\Rightarrow$ P is s- κ CS [By Theorem 2.2]

So, (ii) hence.

(iii) & (ii) implies (i)

P is s- κ – Cs

$\Rightarrow$ $C(P)= C(P^TVK)$

$\Rightarrow C(KPK) = C(KP^TV)$ [Preliminary 2.1]

Hence (ii) & (iii) $\Rightarrow$ $C(KPK) = C(P^T)$

$\Rightarrow C(P) = C(KP^TK)$ [By Lemma 2.1]

$\Rightarrow$ P is κ – Column symmetric

Therefore,(i) hold. Hence the Theorem

3.s- κ-Column Symmetric Regular NFM

In this section, it was discovered that there are various generalized inverses of matrices in NFM. The comparable standards for different g-inverses of s-k CS NFM to be s-k CS are also established. The generalized inverses of an s – κ CS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.

Theorem 3.1: Let $\in$ F_n, Z $\in$ P {1,2} and PZ, ZP, are s- κ-CS NFM. Then P is s- κ - CS NFM ⇔ Z is s- κ – CS NFM.

Proof: $C(KVP) = C(KVPZP) \subseteq C(ZP)$ [since P = PZP]

$= C(ZVVP) = N(ZVKKVP) \subseteq C(KVP)$

Hence, $C(KVP) = C(ZP)$

$= C(KV(ZP)^T VK)$

$= C(P^T Z^T VK)$

$= C(Z^T VK)$

$= C((KVZ)^T)$

$C((KVP)^T) = C(P^T VK)$

$= C(Z^T P^T VK)$

$= C((KVPZ)^T)$

$= C(KVPZ)$

$= C(KVZ)$

KVZ is column symmetric $\Leftrightarrow C(KVP) = N((KVP)^T)$

$\Leftrightarrow C((KVZ)^T) = N(KVZ)$

$\Leftrightarrow$ KVZ is CS

$\Leftrightarrow$ Z is s- κ- CS

Theorem 3.2: Let $P \in F_n$, $Z \in P\{1,2,3\}$, $C(KVP) = C((KVZ)^T)$.Then P is s-κ-CS NFM $\Leftrightarrow$ Z is s- κ - CS NFM.

Proof: Given $Z \in P\{1,2,3\}$, we have $PZP = P, ZPZ = Z, (PZ)^T = PZ$

$C((KVP)^T) = C(Z^T P^T VK)$ [By using PZP = P]

$= C(KV(PZ)^T)$

$= C((PZ)^T)$ [Preliminary 2.1]

$= C(PZ)$ [$(PZ)^T = PZ$]

$= C(Z)$ [By using Z = ZPZ]

$= C(KVZ)$ [Preliminary 2.1]

KVP is column symmetric NFM $\Leftrightarrow C(KVP) = C((KVP)^T)$

$\Leftrightarrow C((KVZ)^T) = C(KVZ)$

$\Leftrightarrow$ KVZ is column symmetric

$\Leftrightarrow$ Z is s- κ - column symmetric.

Theorem 3.3: Let $P \in F_n$, $Z \in P\{1,2,4\}$, $C((KVP)^T) = C(KVZ)$. Then P is s- κ- CS NFM $\Leftrightarrow$ Z is s- κ- CS NFM.

Proof: Given $Z \in P\{1, 2, 4\}$,

$PZP = P, ZPZ = Z, (ZP)^T = ZP$

$C(KVP) = C(P)$ [Preliminary 2.1]

$= C(ZP)$ [ZPZ = Z, PZP = P] = $N((ZP)^T)$ [$(ZP)^T = ZP$]

$= C(P^T Z^T)$

$= C(Z^T)$

$= C((KVZ)^T)$. [Preliminary 2.1]

KVP is column symmetric NFM $\Leftrightarrow C(KVP) = C((KVP)^T$

$\Leftrightarrow C((KVZ)^T) = C(KVZ)$

$\Leftrightarrow$ KVZ is CS NFM

$\Leftrightarrow$ Z is s- κ – CS NFM.

4.Conclusion:

Firstly, we present equivalent characterizations of an k- CS, CS, s- CS, s-k CS NFM. Also,we give the example of s-k-symmetric NFM is s-k- CS Neutrosophic fuzzy matrix the opposite isn't always true. We discussed various generalized inverses of NFM and generalized inverses of an s – k CS P corresponding to the sets P{1, 2}, P{1, 2, 3} and P{1, 2, 4} are characterized.Finally, to conclude we have introduced the concept of secondary k-CS neutrosophic fuzzy matrices. In future we will work on interval valued secondary k-CS neutrosophic fuzzy matrices.

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