NSS

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On Neutrosophic Soft Prime Ideal

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Abstract The motivation of the present paper is to extend the concept of neutrosophic soft prime ideal over a ring. In this paper the concept of neutrosophic soft completely prime ideals, neutrosophic soft completely semi-prime ideals and neutrosophic soft prime k - ideals have been introduced. These are illustrated with suitable examples also. Several related properties, theorems and structural characteristics of each are studied here.

Keywords Neutrosophic soft completely prime ideals; Neutrosophic soft completely semi-prime ideals; Neutrosophic soft prime k - ideals.

1 Introduction

Because of the insufficiency in the available information situation, evaluation of membership values and nonmembership values are not always possible to handle the uncertainties appearing in daily life situations. So there exists an indeterministic part upon which hesitation survives. The neutrosophic set theory by Smarandache [1,2] which is a generalisation of fuzzy set and intuitionistic fuzzy set theory, makes description of the objective world more realistic, practical and very promising in nature. The neutrosophic logic includes the information about the percentage of truth, indeterminacy and falsity grade in several real world problems in law, medicine, engineering, management, industrial, IT sector etc which are not available in intuitionistic fuzzy set theory. But each of the theories suffers from inherent difficulties because of the inadequacy of parametrization tools. Molodtsov [3] introduced a nice concept of soft set theory which is free from the parametrization inadequacy syndrome of different theories dealing with uncertainty. The parametrization tool of soft set theory makes it very convenient and easy to apply in practice. The classical algebraic structures were extended over fuzzy set, intuitionistic fuzzy set, soft set, fuzzy soft set and intuitionistic fuzzy soft set by so many authors, for instance, Rosenfeld [4], Malik and

Mordeson [5,6], Lavanya and Kumar [8], Bakhadach et al. [9], Dutta et al. [10-12], Maji et al. [13], Aktas and Cagman [14], Augunoglu and Aygun [15], Zhang [16], Maheswari and Meera [17] and others.

The notion of neutrosophic soft set theory (NSS) has been innovated by Maji [18]. Later, it has been modified by Deli and Broumi [19]. Cetkin et al. [20,21], Bera and Mahapatra [22-26] and others have produced their research works on fundamental algebraic structures on the NSS theory context.

This paper presents the notion of neutrosophic soft completely prime ideals, neutrosophic soft completely semi-prime ideals and neutrosophic soft prime k-ideals along with investigation of some related properties and theorems. The content of the present paper is designed as following :

Section 2 gives some preliminary useful definitions related to it. In Section 3, neutrosophic soft completely prime ideals is defined and illustrated by suitable examples along with investigation of its structural characteristics. Section 4 deals with the notion of neutrosophic soft completely semi-prime ideals with development of related theorems. The concept of neutrosophic soft prime k-ideals along with some properties has been introduced in Section 6. Finally, the conclusion of our work has been stated in Section 7.

2 Preliminaries

We recall some basic definitions related to fuzzy set, soft set, neutrosophic soft set for the sake of completeness.

2.1 Definition [24]

1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t - norm if $*$ satisfies the following conditions :
(i) $*$ is commutative and associative.
(ii) $*$ is continuous.
(iii) $a * 1 = 1 * a = a,\ \forall a \in [0,1]$.
(iv) $a * b \le c * d$ if $a \le c,\ b \le d$ with $a, b, c, d \in [0,1]$.

A few examples of continuous t-norm are $a * b = ab, a * b = \min\{a, b\}, a * b = \max\{a + b - 1, 0\}$.

2. A binary operation $\diamond : [0,1] \times [0,1] \to [0,1]$ is said to be continuous t - conorm (s - norm) if $\diamond$ satisfies the following conditions :
(i) $\diamond$ is commutative and associative.
(ii) $\diamond$ is continuous.
(iii) $a \diamond 0 = 0 \diamond a = a,\ \forall a \in [0,1]$.
(iv) $a \diamond b \le c \diamond d$ if $a \le c,\ b \le d$ with $a, b, c, d \in [0,1]$.

A few examples of continuous s-norm are $a \diamond b = a + b - ab, a \diamond b = \max\{a, b\}, a \diamond b = \min\{a + b, 1\}$.

2.2 Definition [1]

Let X be a space of points (objects), with a generic element in X denoted by x. A neutrosophic set A in X is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]^-0, 1^+[$. That is $T_A, I_A, F_A : X \to]^-0, 1^+[$. There is no restriction on the sum of $T_A(x), I_A(x), F_A(x)$ and so, $^-0 \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$.

2.3 Definition [3]

Let U be an initial universe set and E be a set of parameters. Let $P(U)$ denote the power set of U. Then for $A \subseteq E$, a pair (F, A) is called a soft set over U, where $F : A \to P(U)$ is a mapping.

2.4 Definition [18]

Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then for $A \subseteq E$, a pair (F, A) is called an NSS over U, where $F : A \to NS(U)$ is a mapping.

This concept has been redefined by Deli and Broumi [19] as given below.

2.5 Definition [19]

1. Let U be an initial universe set and E be a set of parameters. Let $NS(U)$ denote the set of all NSs of U. Then, a neutrosophic soft set N over U is a set defined by a set valued function f_N representing a mapping $f_N : E \to NS(U)$ where f_N is called approximate function of the neutrosophic soft set N. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $NS(U)$ and therefore it can be written as a set of ordered pairs,

$$\begin{aligned} N &= \{(e, f_N(e)) : e \in E\} \\ &= \{(e, \{< x, T_{f_{N(e)}}(x), I_{f_{N(e)}}(x), F_{f_{N(e)}}(x) >: x \in U\}) : e \in E\} \end{aligned}$$

where $T_{f_{N(e)}}(x), I_{f_{N(e)}}(x), F_{f_{N(e)}}(x) \in [0, 1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $f_N(e)$. Since supremum of each T, I, F is 1 so the inequality $0 \leq T_{f_{N(e)}}(x) + I_{f_{N(e)}}(x) + F_{f_{N(e)}}(x) \leq 3$ is obvious.

2. Let N_1 and N_2 be two NSSs over the common universe (U, E). Then N_1 is said to be the neutrosophic soft subset of N_2 if $T_{f_{N_1}(e)}(x) \leq T_{f_{N_2}(e)}(x)$, $I_{f_{N_1}(e)}(x) \geq I_{f_{N_2}(e)}(x)$, $F_{f_{N_1}(e)}(x) \geq F_{f_{N_2}(e)}(x)$, $\forall e \in E$ and $\forall x \in U$.

We write $N_1 \subseteq N_2$ and then N_2 is the neutrosophic soft superset of N_1.

2.6 Proposition [22]

An NSS N over the group (G, o) is called a neutrosophic soft group iff followings hold on the assumption that $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

$$\begin{aligned} T_{f_N(e)}(xoy^{-1}) &\geq T_{f_N(e)}(x) * T_{f_N(e)}(y), \\ I_{f_N(e)}(xoy^{-1}) &\leq I_{f_N(e)}(x) \diamond I_{f_N(e)}(y), \\ F_{f_N(e)}(xoy^{-1}) &\leq F_{f_N(e)}(x) \diamond F_{f_N(e)}(y));\ \forall x, y \in G, \forall e \in E. \end{aligned}$$

2.7 Definition [24]

1. A neutrosophic soft ring N over the ring $(R, +, \cdot)$ is called a neutrosophic soft left ideal over R if $f_N(e)$ is a neutrosophic left ideal of R for each $e \in E$ i.e.,
(i) $f_N(e)$ is a neutrosophic subgroup of $(R, +)$ for each $e \in E$ and

$$(ii) \begin{cases} T_{f_N(e)}(x \cdot y) \geq T_{f_N(e)}(y) \\ I_{f_N(e)}(x \cdot y) \leq I_{f_N(e)}(y) \\ F_{f_N(e)}(x \cdot y) \leq F_{f_N(e)}(y);\ \text{for } x, y \in R. \end{cases}$$

2. A neutrosophic soft ring N over the ring $(R, +, \cdot)$ is called a neutrosophic soft right ideal over R if $f_N(e)$ is a neutrosophic right ideal of R for each $e \in E$ i.e.,
(i) $f_N(e)$ is a neutrosophic subgroup of $(R, +)$ for each $e \in E$ and

$$(ii) \begin{cases} T_{f_N(e)}(x \cdot y) \geq T_{f_N(e)}(x) \\ I_{f_N(e)}(x \cdot y) \leq I_{f_N(e)}(x) \\ F_{f_N(e)}(x \cdot y) \leq F_{f_N(e)}(x);\ \text{for } x, y \in R. \end{cases}$$

3. A neutrosophic soft ring N over the ring $(R, +, \cdot)$ is called a neutrosophic soft ideal over R if $f_N(e)$ is a both neutrosophic left and right ideal of R for each $e \in E$.

2.8 Definition [25]

1. Let $\varphi : U \to V$ and $\psi : E \to E$ be two functions where E is the parameter set for each of the crisp sets U and V. Then the pair (φ, ψ) is called an NSS function from (U, E) to (V, E). We write, $(\varphi, \psi) : (U, E) \to (V, E)$. If M is an NSS over U via parametric set E, we shall write (M, E) an NSS over U.
2. Let (M, E), (N, E) be two NSSs defined over U, V respectively and (φ, ψ) be an NSS function from (U, E) to (V, E). Then,
(i) The image of (M, E) under (φ, ψ), denoted by $(\varphi, \psi)(M, E)$, is an NSS over V and is defined by :
$(\varphi, \psi)(M, E) = (\varphi(M), \psi(E)) = \{< \psi(a), f_{\varphi(M)} >: a \in E\}$ where $\forall b \in \psi(E), \forall y \in V$,

$$T_{f_{\varphi(M)}(b)}(y) = \begin{cases} \max_{\varphi(x)=y} \max_{\psi(a)=b} [T_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 0 \quad , & \text{otherwise.} \end{cases}$$

$$I_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [I_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \quad , & \text{otherwise.} \end{cases}$$

$$F_{f_{\varphi(M)}(b)}(y) = \begin{cases} \min_{\varphi(x)=y} \min_{\psi(a)=b} [F_{f_M(a)}(x)], & \text{if } x \in \varphi^{-1}(y) \\ 1 \quad , & \text{otherwise.} \end{cases}$$

(ii) The pre-image of (N, E) under (φ, ψ), denoted by $(\varphi, \psi)^{-1}(N, E)$, is an NSS over U and is defined by :
$(\varphi, \psi)^{-1}(N, E) = (\varphi^{-1}(N), \psi^{-1}(E))$ where $\forall a \in \psi^{-1}(E), \forall x \in U$,

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x) &= T_{f_N[\psi(a)]}(\varphi(x)) \\ I_{f_{\varphi^{-1}(N)}(a)}(x) &= I_{f_N[\psi(a)]}(\varphi(x)) \\ F_{f_{\varphi^{-1}(N)}(a)}(x) &= F_{f_N[\psi(a)]}(\varphi(x)) \end{aligned}$$

If ψ and φ is injective (surjective), then (φ, ψ) is injective (surjective).

2.9 Definition [26]

1. An NSS M over (R, E) is said to be constant if each $f_M(e)$ is constant for $e \in E$ i.e., $(T_{f_{M(e)}}(x), I_{f_{M(e)}}(x), F_{f_{M(e)}}(x))$ is same $\forall e \in E$, $\forall x \in R$.

For M to be nonconstant, if for each $e \in E$ the triplet $(T_{f_{M(e)}}(x), I_{f_{M(e)}}(x), F_{f_{M(e)}}(x))$ is atleast of two different kinds $\forall x \in R$.

2. Let R be a ring and M, N be two NSSs over (R, E). Then $MoN = L$ (say) is also an NSS over (R, E) and is defined as following, for $e \in E$ and $x \in R$,

$$T_{f_{L(e)}}(x) = \begin{cases} \max_{x=yz}[T_{f_{M(e)}}(y) * T_{f_{N(e)}}(z)] \\ 0 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

$$I_{f_{L(e)}}(x) = \begin{cases} \min_{x=yz}[I_{f_{M(e)}}(y) \diamond I_{f_{N(e)}}(z)] \\ 1 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

$$F_{f_{L(e)}}(x) = \begin{cases} \min_{x=yz}[F_{f_{M(e)}}(y) \diamond F_{f_{N(e)}}(z)] \\ 1 \quad \text{if } x \text{ is not expressible as } x = yz. \end{cases}$$

3. A neutrosophic soft ideal P over (R, E) is said to be a neutrosophic soft prime ideal if (i) P is not constant neutrosophic soft ideal, (ii) for any two neutrosophic soft ideals M, N over (R, E), $MoN \subseteq P \Rightarrow$ either $M \subseteq P$ or $N \subseteq P$.

2.10 Theorem [26]

1. Let P be an NSS over (R, E) such that cardinality of $f_P(e)$ is 2 i.e., $|f_P(e)| = 2$ and $[f_P(e)](0_r) = (1, 0, 0)$ for each $e \in E$. If $P_0 = \{x \in R : [f_P(e)](x) = [f_P(e)](0_r)\}$ is a prime ideal over R, then P is a neutrosophic soft prime ideal over (R, E).

2. Let P be an NSS over (R, E). Then P is a neutrosophic soft left (right) ideal over (R, E) iff $\widehat{P} = \{x \in R : [f_P(e)](x) = (1, 0, 0)\}$ with $0_r \in \widehat{P}$ is a left (right) ideal of R.

3. $S(\neq \phi) \subset R$ is an ideal of R iff there exists a neutrosophic soft ideal M over (R, E) where $f_M : E \longrightarrow NS(R)$ is defined as, $\forall e \in E$,

$$[f_M(e)](x) = \begin{cases} (r_1, r_2, r_3) & \text{if } x \in S \\ (t_1, t_2, t_3) & \text{if } x \notin S. \end{cases}$$

with $r_1 > t_1$, $r_2 < t_2$, $r_3 < t_3$ and $r_1, r_2, r_3, t_1, t_2, t_3 \in [0, 1]$.
In particular, $S(\neq \phi) \subset R$ is an ideal of R iff the characteristic function χ_S is a

neutrosophic soft ideal over (R, E) where $\chi_S : E \longrightarrow NS(R)$ is defined as, $\forall e \in E$,

$$[\chi_S(e)](x) = \begin{cases} (1,0,0) & \text{if } x \in S \\ (0,1,1) & \text{if } x \notin S. \end{cases}$$

4. An NSS M over (R, E) is a neutrosophic soft left (right) ideal iff each nonempty level set $[f_M(e)]_{(\alpha,\beta,\gamma)}$ of the neutrosophic set $f_M(e)$ is a left (right) ideal of R where $\alpha \in Im\, T_{f_M(e)}$, $\beta \in Im\, I_{f_M(e)}$, $\gamma \in Im\, F_{f_M(e)}$.

5. Let P be a neutrosophic soft left (right) ideal over (R, E). Then $P_0 = \{x \in R : [f_P(e)](x) = [f_P(e)](0_r)\}$ is a left (right) ideal of R.

6. Let P be a neutrosophic soft prime ideal over (R, E). Then $P_0 = \{x \in R : [f_P(e)](x) = [f_P(e)](0_r)\}$ is a prime ideal of R.

2.11 Definition [7]

A left k-ideal I of a semiring S is a left ideal such that if $a \in I$ and $x \in S$ and if either $a + x \in I$ or $x + a \in I$, then $x \in I$.
Right k-ideal of a semiring is defined dually. A non-empty subset I of a semiring S is called a k-ideal if it is both a left k-ideal and a right k-ideal.

3 Neutrosophic soft completely prime ideal

Here first we have defined a completely prime ideal of a ring and then defined a neutrosophic soft completely prime ideal. These are illustrated with suitable examples. Along with several related properties and theorems have been developed.

Through out this paper, unless otherwise stated, E is treated as the parametric set and $e \in E$, an arbitrary parameter. Moreover the standard t-norm and s-norm are taken into consideration wherever needed through out this paper i.e., $a * b = \min\{a, b\}$ and $a \diamond b = \max\{a, b\}$.

3.1 Definition

An ideal S of a ring R is called a completely prime ideal of R if for $x, y \in R$, $xy \in S \Rightarrow$ either $x \in S$ or $y \in S$.

3.1.1 *Example*

1. For the ring $(\mathbf{Z}, +, \cdot)$ ($\mathbf{Z}$ being the set of integers), an ideal $(2\mathbf{Z}, +, \cdot)$ is a completely prime ideal.

2. We assume a ring $R = \{0, x, y, z\}$. The two binary operations addition and multiplication on R are given by the following tables :

Table 1

$+$	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

Table 2

$\cdot$	0	x	y	z
0	0	0	0	0
x	0	0	0	0
y	0	0	y	y
z	0	0	y	y

It is an abelian ring. With respect to these two tables, $\{0, x\}$ and $\{0, y\}$ are two ideals of R. From 2nd table, it is evident that $\{0, x\}$ is a completely prime ideal of R but $\{0, y\}$ is not so because $z \cdot z = y$ though $z \notin \{0, y\}$.

3. Consider the another ring $R = \{0, x, y, z\}$ with two binary operations addition and multiplication on R are given by the following tables :

Table 3

$+$	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

Table 4

$\cdot$	0	x	y	z
0	0	0	0	0
x	0	0	0	0
y	0	0	0	0
z	0	x	y	x

It is not an abelian ring. With respect to these two tables, $\{0, x\}$ is an ideal of R but not completely prime ideal. Because $y \cdot z = 0, z \cdot z = x, y \cdot y = 0$ but $y, z \notin \{0, x\}$.

3.2 Proposition

If S is a completely prime ideal of a ring R then S is a prime ideal of R.

Proof. Let S be a completely prime ideal of a ring R and A, B be two ideals of R such that $AB \subseteq S$. Suppose $A \not\subseteq S$ and $B \not\subseteq S$. Then there exists $x \in A$ and $y \in B$ such that $x, y \notin S$. But $xy \in S$ as $AB \subseteq S$. Since S is a completely prime ideal of R, so either $x \in S$ or $y \in S$ and this leads a contradiction to the fact $x, y \notin S$. Hence S is a prime ideal of R.

3.3 Definition

A neutrosophic soft ideal N over (R, E) is called a neutrosophic soft completely prime ideal if $\forall x, y \in R$ and $\forall e \in E$,

$$\begin{cases} T_{f_N(e)}(x \cdot y) \leq \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\} \\ I_{f_N(e)}(x \cdot y) \geq \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\} \\ F_{f_N(e)}(x \cdot y) \geq \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\}. \end{cases}$$

3.3.1 *Example*

Consider the Example [3.1.1](2). We define an NSS M over (R, E) as following, $\forall r \in R$ and $\forall e \in E$,

$$[f_M(e)](r) = \begin{cases} (1, 0.3, 0.1) & \text{if } r \in \{0, x\} \\ (0.8, 0.6, 0.4) & \text{if } r \notin \{0, x\}. \end{cases}$$

Then M is a neutrosophic soft completely prime ideal over (R, E).

3.4 Theorem

An NSS N is a neutrosophic soft completely prime ideal over (R, E) iff for $e \in E$, $|f_N(e)| = 2$, $[f_N(e)](0_r) = (1, 0, 0)$ and $\widehat{N} = \{x \in R : [f_N(e)](x) = (1, 0, 0)\}$ is a completely prime ideal of R.

Proof. Let N be a neutrosophic soft completely prime ideal over (R, E). Then N is a neutrosophic soft ideal over (R, E) and so $\widehat{N}$ is an ideal over R by Theorem [2.11](2). To prove $\widehat{N}$ is a complete prime ideal, let $xy \in \widehat{N}$ for $x, y \in R$. Then $[f_N(e)](xy) = (1, 0, 0)$ for $e \in E$. But,

$$1 = T_{f_N(e)}(xy) \leq \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\},$$
$$0 = I_{f_N(e)}(xy) \geq \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\},$$
$$0 = F_{f_N(e)}(xy) \geq \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\};$$

This implies that

$$T_{f_N(e)}(0_r) = 1 \leq \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\},$$
$$I_{f_N(e)}(0_r) = 0 \geq \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\},$$
$$F_{f_N(e)}(0_r) = 0 \geq \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\};$$

This shows that,

$$\text{either } T_{f_N(e)}(0_r) \leq T_{f_N(e)}(x) \text{ or } T_{f_N(e)}(0_r) \leq T_{f_N(e)}(y),$$
$$\text{either } I_{f_N(e)}(0_r) \geq I_{f_N(e)}(x) \text{ or } I_{f_N(e)}(0_r) \geq I_{f_N(e)}(y),$$
$$\text{either } F_{f_N(e)}(0_r) \geq F_{f_N(e)}(x) \text{ or } F_{f_N(e)}(0_r) \geq F_{f_N(e)}(y);$$

But $T_{f_N(e)}(0_r) \geq T_{f_N(e)}(x)$, $I_{f_N(e)}(0_r) \leq I_{f_N(e)}(x)$, $F_{f_N(e)}(0_r) \leq F_{f_N(e)}(x)$, $\forall x \in R$. Hence $T_{f_N(e)}(x) = T_{f_N(e)}(0_r)$, $I_{f_N(e)}(x) = I_{f_N(e)}(0_r)$, $F_{f_N(e)}(x) = F_{f_N(e)}(0_r)$, $\forall x \in R$ i.e., $x, y \in \widehat{N}$. Thus $\widehat{N}$ is a complete prime ideal.

Conversely suppose $\widehat{N}$ is a completely prime ideal with the given conditions. As $\widehat{N}$ is an ideal of R, so N is a neutrosophic soft ideal over (R, E) by Theorem [2.11](2). For contrary, suppose N is not neutrosophic soft completely prime ideal. Then,

$$T_{f_N(e)}(xy) > \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\},$$
$$I_{f_N(e)}(xy) < \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\},$$
$$F_{f_N(e)}(xy) < \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\};$$

Since $|f_N(e)| = 2$ and $[f_N(e)](0_r) = (1, 0, 0)$ then there exists $x, y \in R$ so that $[f_N(e)](x) = [f_N(e)](y) = (r_1, r_2, r_3) \neq (1, 0, 0)$ (say) for $0 \leq r_1 < 1$ and $0 < r_2, r_3 \leq 1$. Then,

$$T_{f_N(e)}(xy) > r_1,\ I_{f_N(e)}(xy) < r_2,\ F_{f_N(e)}(xy) < r_3$$
$$\Rightarrow\ T_{f_N(e)}(xy) = 1,\ I_{f_N(e)}(xy) = F_{f_N(e)}(xy) = 0$$
$$\Rightarrow\ [f_N(e)](xy) = (1, 0, 0)$$
$$\Rightarrow\ xy \in \widehat{N}$$

Since $\widehat{N}$ is completely prime ideal, so either $x \in \widehat{N}$ or $y \in \widehat{N}$ i.e., $[f_N(e)](x) = [f_N(e)](y) = (1, 0, 0)$. A contradiction arises to the fact that $[f_N(e)](x) = [f_N(e)](y) = (r_1, r_2, r_3) \neq (1, 0, 0)$. Thus,

$$T_{f_N(e)}(xy) \leq \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\},$$
$$I_{f_N(e)}(xy) \geq \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\},$$
$$F_{f_N(e)}(xy) \geq \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\};$$

and so N is a neutrosophic soft completely prime ideal over (R, E).

3.5 Theorem

Let N be a neutrosophic soft completely prime ideal over (R, E) with $|f_N(e)| = 2$, $[f_N(e)](0_r) = (1, 0, 0)$ for each $e \in E$. Then N is a neutrosophic soft prime ideal over (R, E).

Proof. Let the condition hold. By Theorem [3.4], $\widehat{N} = \{x \in R : [f_N(e)](x) = (1, 0, 0)\}$ is a completely prime ideal of R. Then by Proposition [3.2], $\widehat{N}$ is a prime ideal of R. Hence N is a neutrosophic soft prime ideal over (R, E) by Theorem [2.11](1).

3.6 Theorem

Let R be a ring. Then $S(\neq \phi) \subset R$ be a completely prime ideal of R iff an NSS N over (R, E) is a neutrosophic soft completely prime ideal where $f_N : E \longrightarrow NS(R)$ is defined as :

$$[f_N(e)](x) = \begin{cases} (r_1, r_2, r_3) & \text{if } x \in S \\ (t_1, t_2, t_3) & \text{if } x \notin S. \end{cases}$$

with $r_1 > t_1$, $r_2 < t_2$, $r_3 < t_3$ and $r_1, r_2, r_3, t_1, t_2, t_3 \in [0, 1]$.

Proof. First let $S(\neq \phi) \subset R$ be a completely prime ideal of R. Then S is an ideal of R and so by Theorem [2.11](3), N is a neutrosophic soft ideal over (R, E). To end the theorem, we shall just show that N is completely prime. For contrary, suppose

$$\begin{aligned} &T_{f_N(e)}(xy) > \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\}, \\ &I_{f_N(e)}(xy) < \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\}, \\ &F_{f_N(e)}(xy) < \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\}; \end{aligned}$$

Then by definition of $f_N(e)$, we have $[f_N(e)](xy) = (r_1, r_2, r_3)$ and $[f_N(e)](x) = [f_N(e)](y) = (t_1, t_2, t_3)$. This implies $xy \in S$ but $x, y \notin S$ which is a contradiction to the fact that S is a completely prime ideal of R. Hence N is a neutrosophic soft completely prime ideal over (R, E).

Conversely, let N in given form be a neutrosophic soft completely prime ideal over (R, E). Then N is a neutrosophic soft ideal over (R, E) and so by Theorem [2.11](3), S is an ideal of R. To show S is a completely prime ideal of R, let $xy \in S$. Then,

$$\begin{aligned} & [f_N(e)](xy) = (r_1, r_2, r_3) \\ \Rightarrow\ & T_{f_N(e)}(xy) = r_1,\ I_{f_N(e)}(xy) = r_2,\ F_{f_N(e)}(xy) = r_3 \\ \Rightarrow\ & \max\{T_{f_N(e)}(x), T_{f_N(e)}(y)\} \geq r_1,\ \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\} \leq r_2, \\ & \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\} \leq r_3 \\ \Rightarrow\ & \text{either } T_{f_N(e)}(x) \geq r_1,\ I_{f_N(e)}(x) \leq r_2,\ F_{f_N(e)}(x) \leq r_3 \\ & \text{or } T_{f_N(e)}(y) \geq r_1,\ I_{f_N(e)}(y) \leq r_2,\ F_{f_N(e)}(y) \leq r_3 \\ \Rightarrow\ & \text{either } x \in S \text{ or } y \in S \end{aligned}$$

Thus S is a completely prime ideal of R.

3.6.1 Corollary

A non empty subset S of a ring R is a completely prime ideal iff the characteristic function χ_S is a neutrosophic soft completely prime ideal over (R, E) where $\chi_S : E \longrightarrow NS(R)$ is defined by :

$$[\chi_S(e)](x) = \begin{cases} (1,0,0) & \text{if } x \in S \\ (0,1,1) & \text{if } x \notin S. \end{cases}$$

Proof. It is the particular case of Theorem [3.6].

3.7 Theorem

An NSS M over (R, E) is a neutrosophic soft completely prime ideal means each nonempty level set $[f_M(e)]_{(\alpha,\beta,\gamma)}$ of the neutrosophic set $f_M(e)$, $e \in E$ is a completely prime ideal of R where $\alpha \in Im\, T_{f_M(e)}$, $\beta \in Im\, I_{f_M(e)}$, $\gamma \in Im\, F_{f_M(e)}$.

Proof. Here M is a neutrosophic soft completely prime ideal over (R, E). Then M is a neutrosophic soft ideal over (R, E) and so by Theorem [2.11](4), $[f_M(e)]_{(\alpha,\beta,\gamma)}$ is an ideal of R. To complete the theorem, let $xy \in [f_M(e)]_{(\alpha,\beta,\gamma)}$. Then,

$$\begin{aligned} & T_{f_M(e)}(xy) \geq \alpha,\ I_{f_M(e)}(xy) \leq \beta,\ F_{f_M(e)}(xy) \leq \gamma \\ \Rightarrow\ & \max\{T_{f_M(e)}(x), T_{f_M(e)}(y)\} \geq \alpha,\ \min\{I_{f_M(e)}(x), I_{f_M(e)}(y)\} \leq \beta, \\ & \min\{F_{f_M(e)}(x), F_{f_M(e)}(y)\} \leq \gamma \\ \Rightarrow\ & \text{either } T_{f_M(e)}(x) \geq \alpha,\ I_{f_M(e)}(x) \leq \beta,\ F_{f_M(e)}(x) \leq \gamma \\ & \text{or } T_{f_M(e)}(y) \geq \alpha,\ I_{f_M(e)}(y) \leq \beta,\ F_{f_M(e)}(y) \leq \gamma \\ \Rightarrow\ & \text{either } x \in [f_M(e)]_{(\alpha,\beta,\gamma)} \text{ or } y \in [f_M(e)]_{(\alpha,\beta,\gamma)} \end{aligned}$$

Thus $[f_M(e)]_{(\alpha,\beta,\gamma)}$ is a completely prime ideal of R.

3.8 Proposition

Let S be a completely prime ideal of a ring R. Then there exists a neutrosophic soft completely prime ideal M over (R, E) such that $[f_M(e)]_{(\alpha,\beta,\gamma)} = S$ for $e \in E$ and $\alpha, \beta, \gamma \in (0, 1)$.

Proof. As S is a completely prime ideal of a ring R, so S is an ideal of R. For $\alpha, \beta, \gamma \in (0, 1)$ define an NSS M over (R, E) as following :

$$[f_M(e)](x) = \begin{cases} (\alpha,\beta,\gamma) & \text{if } x \in S \\ (0,1,1) & \text{if } x \notin S. \end{cases}$$

Then by Theorem [2.11](3), M is a neutrosophic soft ideal over (R, E). If possible let M is not a neutrosophic soft completely prime ideal over (R, E). Then,

$$\begin{aligned} & T_{f_M(e)}(xy) > \max\{T_{f_M(e)}(x), T_{f_M(e)}(y)\}, \\ & I_{f_M(e)}(xy) < \min\{I_{f_M(e)}(x), I_{f_M(e)}(y)\}, \\ & F_{f_M(e)}(xy) < \min\{F_{f_M(e)}(x), F_{f_M(e)}(y)\}; \end{aligned}$$

Then by definition of $f_M(e)$, we have $[f_M(e)](xy) = (\alpha, \beta, \gamma)$ and $[f_M(e)](x) = [f_M(e)](y) = (0, 1, 1)$. This implies $xy \in S$ but $x, y \notin S$ which is a contradiction to the fact that S is a completely prime ideal of R. Hence M is a neutrosophic soft completely prime ideal over (R, E). Obviously $[f_M(e)]_{(\alpha,\beta,\gamma)} = S$ for each $e \in E$.

3.9 Theorem

Let $(\varphi, \psi) : (R_1, E) \longrightarrow (R_2, E)$ be a neutrosophic soft homomorphism where R_1, R_2 be two rings. Suppose (M, E) and (N, E) be two neutrosophic soft left (right) ideals over R_1 and R_2, respectively. Then,
1. $(\varphi, \psi)(M, E)$ is a neutrosophic soft left (right) ideal over R_2 if (φ, ψ) is epimorphism.
2. $(\varphi, \psi)^{-1}(N, E)$ is a neutrosophic soft left (right) ideal over R_1.

Proof. **1.** Let $b \in \psi(E)$ and $y_1, y_2, s \in R_2$. For $\varphi^{-1}(y_1) = \phi$ or $\varphi^{-1}(y_2) = \phi$, the proof is straight forward.
So, we assume that there exists $x_1, x_2, r \in R_1$ such that $\varphi(x_1) = y_1, \varphi(x_2) = y_2, \varphi(r) = s$. Then,

$$\begin{aligned} T_{f_{\varphi(M)}(b)}(y_1 - y_2) &= \max_{\varphi(x)=y_1-y_2} \max_{\psi(a)=b} [T_{f_M(a)}(x)] \\ &\geq \max_{\psi(a)=b} [T_{f_M(a)}(x_1 - x_2)] \\ &\geq \max_{\psi(a)=b} [T_{f_M(a)}(x_1) * T_{f_M(a)}(x_2)] \\ &= \max_{\psi(a)=b} [T_{f_M(a)}(x_1)] * \max_{\psi(a)=b} [T_{f_M(a)}(x_2)] \\ T_{f_{\varphi(M)}(b)}(sy_1) &= \max_{\varphi(x)=sy_1} \max_{\psi(a)=b} [T_{f_M(a)}(x)] \\ &\geq \max_{\psi(a)=b} [T_{f_M(a)}(rx_1)] \\ &\geq \max_{\psi(a)=b} [T_{f_M(a)}(x_1)] \end{aligned}$$

Since, this inequality is satisfied for each $x_1, x_2 \in R_1$ satisfying $\varphi(x_1) = y_1, \varphi(x_2) = y_2$ so we have,

$$\begin{aligned} & T_{f_{\varphi(M)}(b)}(y_1 - y_2) \\ \geq\ & (\max_{\varphi(x_1)=y_1} \max_{\psi(a)=b} [T_{f_M(a)}(x_1)]) * (\max_{\varphi(x_2)=y_2} \max_{\psi(a)=b} [T_{f_M(a)}(x_2)]) \\ =\ & T_{f_{\varphi(M)}(b)}(y_1) * T_{f_{\varphi(M)}(b)}(y_2) \end{aligned}$$

Also, $T_{f_{\varphi(M)}(b)}(sy_1) \geq \max_{\varphi(x_1)=y_1} \max_{\psi(a)=b} [T_{f_M(a)}(x_1)] = T_{f_{\varphi(M)}(b)}(y_1)$

Next,

$$\begin{aligned} I_{f_{\varphi(M)}(b)}(y_1 - y_2) &= \min_{\varphi(x)=y_1-y_2} \min_{\psi(a)=b} [I_{f_M(a)}(x)] \\ &\leq \min_{\psi(a)=b} [I_{f_M(a)}(x_1 - x_2)] \\ &\leq \min_{\psi(a)=b} [I_{f_M(a)}(x_1) \diamond I_{f_M(a)}(x_2)] \\ &= \min_{\psi(a)=b} [I_{f_M(a)}(x_1)] \diamond \min_{\psi(a)=b} [I_{f_M(a)}(x_2)] \end{aligned}$$

$$\begin{aligned} I_{f_{\varphi(M)}(b)}(sy_1) &= \min_{\varphi(x)=sy_1} \min_{\psi(a)=b} [I_{f_{M(a)}}(x)] \\ &\leq \min_{\psi(a)=b} [I_{f_{M(a)}}(rx_1)] \\ &\leq \min_{\psi(a)=b} [I_{f_{M(a)}}(x_1)] \end{aligned}$$

Since, this inequality is satisfied for each $x_1, x_2 \in R_1$ satisfying $\varphi(x_1) = y_1, \varphi(x_2) = y_2$ so we have,

$$\begin{aligned} & I_{f_{\varphi(M)}(b)}(y_1 - y_2) \\ \leq & \ (\min_{\varphi(x_1)=y_1} \min_{\psi(a)=b} [I_{f_{M(a)}}(x_1)]) \diamond (\min_{\varphi(x_2)=y_2} \min_{\psi(a)=b} [I_{f_{M(a)}}(x_2)]) \\ = & \ I_{f_{\varphi(M)}(b)}(y_1) \diamond I_{f_{\varphi(M)}(b)}(y_2) \end{aligned}$$

Also, $I_{f_{\varphi(M)}(b)}(sy_1) \leq \min_{\varphi(x_1)=y_1} \min_{\psi(a)=b} [I_{f_{M(a)}}(x_1)] = I_{f_{\varphi(M)}(b)}(y_1)$.
Similarly, we can show that
$F_{f_{\varphi(M)}(b)}(y_1 - y_2) \leq F_{f_{\varphi(M)}(b)}(y_1) \diamond F_{f_{\varphi(M)}(b)}(y_2)$, $F_{f_{\varphi(M)}(b)}(sy_1) \geq F_{f_{\varphi(M)}(b)}(y_1)$;
This completes the proof.

2. For $a \in \psi^{-1}(E)$ and $x_1, x_2 \in R_1$, we have,

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x_1 - x_2) &= T_{f_N[\psi(a)]}(\varphi(x_1 - x_2)) \\ &= T_{f_N[\psi(a)]}(\varphi(x_1) - \varphi(x_2)) \\ &\geq T_{f_N[\psi(a)]}(\varphi(x_1)) * T_{f_N[\psi(a)]}(\varphi(x_2)) \\ &= T_{f_{\varphi^{-1}(N)}(a)}(x_1) * T_{f_{\varphi^{-1}(N)}(a)}(x_2) \\ T_{f_{\varphi^{-1}(N)}(a)}(rx_1) &= T_{f_N[\psi(a)]}(\varphi(rx_1)) \\ &= T_{f_N[\psi(a)]}(\varphi(r)\varphi(x_1)) \\ &\geq T_{f_N[\psi(a)]}(s\varphi(x_1)) \\ &\geq T_{f_N[\psi(a)]}(\varphi(x_1)) \\ &= T_{f_{\varphi^{-1}(N)}(a)}(x_1) \end{aligned}$$

Next,

$$\begin{aligned} I_{f_{\varphi^{-1}(N)}(a)}(x_1 - x_2) &= I_{f_N[\psi(a)]}(\varphi(x_1 - x_2)) \\ &= I_{f_N[\psi(a)]}(\varphi(x_1) - \varphi(x_2)) \\ &\leq I_{f_N[\psi(a)]}(\varphi(x_1)) \diamond I_{f_N[\psi(a)]}(\varphi(x_2)) \\ &= I_{f_{\varphi^{-1}(N)}(a)}(x_1) \diamond I_{f_{\varphi^{-1}(N)}(a)}(x_2) \\ I_{f_{\varphi^{-1}(N)}(a)}(rx_1) &= I_{f_N[\psi(a)]}(\varphi(rx_1)) \\ &= I_{f_N[\psi(a)]}(\varphi(r)\varphi(x_1)) \\ &\leq I_{f_N[\psi(a)]}(s\varphi(x_1)) \\ &\leq I_{f_N[\psi(a)]}(\varphi(x_1)) \\ &= I_{f_{\varphi^{-1}(N)}(a)}(x_1) \end{aligned}$$

Similarly, $F_{f_{\varphi^{-1}(N)}(a)}(x_1 - x_2) \leq F_{f_{\varphi^{-1}(N)}(a)}(x_1) \diamond F_{f_{\varphi^{-1}(N)}(a)}(x_2)$ and
$F_{f_{\varphi^{-1}(N)}(a)}(rx_1) \leq F_{f_{\varphi^{-1}(N)}(a)}(x_1)$;
This proves the 2nd part.

3.10 Theorem

Let (φ, ψ) be a neutrosophic soft homomorphism from a ring R_1 to a ring R_2. Suppose (M, E) and (N, E) are neutrosophic soft completely prime ideals over R_1 and R_2, respectively. Then,
1. $(\varphi, \psi)(M, E)$ is a neutrosophic soft completely prime ideal over R_2.
2. $(\varphi, \psi)^{-1}(N, E)$ is a neutrosophic soft completely prime ideal over R_1.

Proof. **1.** If possible, let (M, E) be a neutrosophic soft completely prime ideal over R_1 but $(\varphi, \psi)(M, E)$ is not so over R_2. Then for $b \in \psi(E)$ and $y_1, y_2 \in R_2$,

$$\begin{aligned}
& T_{f_{\varphi(M)}(b)}(y_1 y_2) > \max\{T_{f_{\varphi(M)}(b)}(y_1), T_{f_{\varphi(M)}(b)}(y_2)\} \\
\Rightarrow\ & \max_{\varphi(x)=y_1 y_2} \max_{\psi(a)=b} [T_{f_M(a)}(x)] > \max\{(\max_{\varphi(x)=y_1} \max_{\psi(a)=b} [T_{f_M(a)}(x)]), \\
& \quad (\max_{\varphi(x)=y_2} \max_{\psi(a)=b} [T_{f_M(a)}(x)])\} \\
\Rightarrow\ & \max_{\varphi(x)=y_1 y_2} [T_{f_M(a)}(x)] > \max\{(\max_{\varphi(x)=y_1} [T_{f_M(a)}(x)]), (\max_{\varphi(x)=y_2} [T_{f_M(a)}(x)])\} \\
\Rightarrow\ & \max_{\varphi(x)=y_1 y_2} [T_{f_M(a)}(x)] \geq \max\{T_{f_M(a)}(x_1), T_{f_M(a)}(x_2)\}
\end{aligned}$$

Since the inequality holds for each $x_1, x_2 \in R_1$ satisfying $\varphi(x_1) = y_1, \varphi(x_2) = y_2$ so we have $T_{f_M(a)}(x_1 x_2) > \max\{T_{f_M(a)}(x_1), T_{f_M(a)}(x_2)\}$ which is a contradiction to the truth that (M, E) is a neutrosophic soft completely prime ideal over R_1. We can reach to the same conclusion taking the indeterminacy membership function (I) and falsity membership function (F) also. Hence we get the first result.
2. For $a \in \psi^{-1}(E)$ and $x_1, x_2 \in R_1$, we have,

$$\begin{aligned}
T_{f_{\varphi^{-1}(N)}(a)}(x_1 x_2) &= T_{f_N[\psi(a)]}(\varphi(x_1 x_2)) \\
&= T_{f_N[\psi(a)]}(\varphi(x_1)\varphi(x_2)) \\
&\leq \max\{T_{f_N[\psi(a)]}(\varphi(x_1)), T_{f_N[\psi(a)]}(\varphi(x_2))\} \\
&= \max\{T_{f_{\varphi^{-1}(N)}(a)}(x_1), T_{f_{\varphi^{-1}(N)}(a)}(x_2)\} \\
I_{f_{\varphi^{-1}(N)}(a)}(x_1 x_2) &= I_{f_N[\psi(a)]}(\varphi(x_1 x_2)) \\
&= I_{f_N[\psi(a)]}(\varphi(x_1)\varphi(x_2)) \\
&\geq \min\{I_{f_N[\psi(a)]}(\varphi(x_1)), I_{f_N[\psi(a)]}(\varphi(x_2))\} \\
&= \min\{I_{f_{\varphi^{-1}(N)}(a)}(x_1), I_{f_{\varphi^{-1}(N)}(a)}(x_2)\} \\
F_{f_{\varphi^{-1}(N)}(a)}(x_1 x_2) &= F_{f_N[\psi(a)]}(\varphi(x_1 x_2)) \\
&= F_{f_N[\psi(a)]}(\varphi(x_1)\varphi(x_2)) \\
&\geq \min\{F_{f_N[\psi(a)]}(\varphi(x_1)), F_{f_N[\psi(a)]}(\varphi(x_2))\} \\
&= \min\{F_{f_{\varphi^{-1}(N)}(a)}(x_1), F_{f_{\varphi^{-1}(N)}(a)}(x_2)\}
\end{aligned}$$

This shows the 2nd result.

4 Neutrosophic Soft Completely Semi-Prime Ideal

In this section the concept of semi-prime ideal, completely semi-prime ideal of a ring R and neutrosophic soft completely semi-prime ideal are focussed.

4.1 Definition

1. An ideal I of a ring R is called a semi-prime ideal if there is another ideal J of R such that $JJ \subseteq I \Rightarrow J \subseteq I$.
2. An ideal J of a ring R is called a completely semi-prime ideal if for $x \in R$, $xx \in J \Rightarrow x \in J$. xx is denoted by x^2.

4.1.1 *Example*

1. Let $R = \{0, x, y, z\}$ be a ring. The two binary operations addition and multiplication on R are given by the following tables :

Table 5

$+$	0	x	y	z
0	0	x	y	z
x	x	0	z	y
y	y	z	0	x
z	z	y	x	0

Table 6

$\cdot$	0	x	y	z
0	0	0	0	0
x	0	x	x	0
y	0	x	y	z
z	0	0	z	z

Then $\{0, x\}$ is a completely semi-prime ideal of R as $0{\cdot}0 = 0, x{\cdot}x = x, y{\cdot}y = y, z{\cdot}z = z$.
2. Consider the Example [3.1.1](3). Then $\{0, x\}$ is not a completely semi-prime ideal, because $z \cdot z = x, y \cdot y = 0$ but $y, z \notin \{0, x\}$.

4.2 Proposition

Every completely prime ideal of a ring R is a completely semi-prime ideal of R.

Proof. By taking $y = x$, the proof follows directly from Definition [3.1].

4.3 Definition

Let R be a ring and E be a parametric set. A neutrosophic soft ideal N over (R, E) is called a neutrosophic soft completely semi-prime ideal if $\forall x, y \in R$ and $\forall e \in E$,
$T_{f_N(e)}(x^2) \leq T_{f_N(e)}(x)$, $I_{f_N(e)}(x^2) \geq I_{f_N(e)}(x)$, $F_{f_N(e)}(x^2) \geq F_{f_N(e)}(x)$.

4.3.1 *Example*

Consider the Example [4.1.1](1). We define an NSS M over (R, E) as following, $\forall r \in R$ and $\forall e \in E$,

$$[f_M(e)](r) = \begin{cases} (0.4, 0.1, 0.5) & \text{if } r \in \{0, x\} \\ (0.2, 0.5, 0.8) & \text{if } r \notin \{0, x\}. \end{cases}$$

Then M is a neutrosophic soft completely semi-prime ideal over (R, E).

4.4 *Lemma*

A neutrosophic soft ideal N over (R, E) is a neutrosophic soft completely semi-prime ideal iff $[f_N(e)](x^2) = [f_N(e)](x)$, for every $e \in E, x \in R$.

Proof. Let N be a neutrosophic soft ideal over (R, E) with $[f_N(e)](x^2) = [f_N(e)](x)$, $\forall e \in E$ and $\forall x \in R$. Then by Definition [4.3], N is a neutrosophic soft completely semi-prime ideal over (R, E).

Conversely, if N is a neutrosophic soft completely semi-prime ideal by Definition [4.3], $T_{f_N(e)}(x^2) \leq T_{f_N(e)}(x)$, $I_{f_N(e)}(x^2) \geq I_{f_N(e)}(x)$, $F_{f_N(e)}(x^2) \geq F_{f_N(e)}(x)$ and as N is a neutrosophic soft ideal over (R, E), then $T_{f_N(e)}(x^2) \geq T_{f_N(e)}(x)$, $I_{f_N(e)}(x^2) \leq I_{f_N(e)}(x)$, $F_{f_N(e)}(x^2) \leq F_{f_N(e)}(x)$. Hence $[f_N(e)](x^2) = [f_N(e)](x)$ for every $e \in E, x \in R$.

4.5 Theorem

An NSS N over (R, E) is a neutrosophic soft completely semi-prime ideal iff for $e \in E$, $S = \{x \in R : [f_N(e)](x) = [f_N(e)](0_r)\}, 0_r$ being the additive identity of ring R, is a completely semi-prime ideal of R.

Proof. Let N be a neutrosophic soft completely semi-prime ideal over (R, E). Then $[f_N(e)](x^2) = [f_N(e)](x)$ for every $e \in E, x \in R$. Now let $x^2 \in S$. Then $[f_N(e)](x^2) = [f_N(e)](0_r) \Rightarrow [f_N(e)](x) = [f_N(e)](0_r) \Rightarrow x \in S$. Hence S is a completely semi-prime ideal of R.

Conversely, if S is a completely semi-prime ideal of R. Then $x^2 \in S \Rightarrow x \in S$. Since $x^2 \in S$, then $[f_N(e)](x^2) = [f_N(e)](0_r)$ and $[f_N(e)](x) = [f_N(e)](0_r) \Rightarrow [f_N(e)](x^2) = [f_N(e)](x)$. Hence by Lemma [4.4], N is a neutrosophic soft completely semi-prime ideal over (R, E).

4.6 Theorem

An NSS N is a neutrosophic soft completely semi-prime ideal over (R, E) iff $[f_N(e)]_{(\alpha,\beta,\gamma)}$ is a completely semi-prime ideal of R where $\alpha \in Im\, T_{f_N(e)}$, $\beta \in Im\, I_{f_N(e)}$, $\gamma \in Im\, F_{f_N(e)}$.

Proof. Let N be a neutrosophic soft completely semi-prime ideal over (R, E). Then $[f_N(e)](x^2) = [f_N(e)](x)$. Now,

$$\begin{aligned} & x^2 \in [f_N(e)]_{(\alpha,\beta,\gamma)} \\ \Rightarrow\ & T_{f_N(e)}(x^2) \geq \alpha, I_{f_N(e)}(x^2) \leq \beta, F_{f_N(e)}(x^2) \leq \gamma \\ \Rightarrow\ & T_{f_N(e)}(x) \geq \alpha, I_{f_N(e)}(x) \leq \beta, F_{f_N(e)}(x) \leq \gamma \\ \Rightarrow\ & x \in [f_N(e)]_{(\alpha,\beta,\gamma)} \end{aligned}$$

Hence, $[f_N(e)]_{(\alpha,\beta,\gamma)}$ is a completely semi-prime ideal of R.

Conversely, let $[f_N(e)]_{(\alpha,\beta,\gamma)}$ be a completely semi-prime ideal of R. Then $x^2 \in [f_N(e)]_{(\alpha,\beta,\gamma)} \Rightarrow x \in ([f_N(e)]_{(\alpha,\beta,\gamma)}$ i.e.,

$$\begin{aligned} & T_{f_N(e)}(x^2) \geq \alpha, I_{f_N(e)}(x^2) \leq \beta, F_{f_N(e)}(x^2) \leq \gamma \\ \Rightarrow\ & T_{f_N(e)}(x) \geq \alpha, I_{f_N(e)}(x) \leq \beta, F_{f_N(e)}(x) \leq \gamma \end{aligned}$$

Now, suppose $[f_N(e)](x^2) \neq [f_N(e)](x)$. Let $[f_N(e)](x) = (t_1, t_2, t_3)$. Then $x^2 \notin [f_N(e)]_{(t_1,t_2,t_3)}$ but $x \in [f_N(e)]_{(t_1,t_2,t_3)}$ which is a contradiction as $[f_N(e)]_{(\alpha,\beta,\gamma)}$ is a completely semi-prime ideal of R. Hence $[f_N(e)](x^2) = [f_N(e)](x)$ and so N is a neutrosophic soft completely semi-prime ideal over (R, E) by Lemma [4.4].

4.7 Theorem

Let (φ, ψ) be a neutrosophic soft homomorphism from a ring R_1 to a ring R_2. Suppose (M, E) and (N, E) are neutrosophic soft completely semi-prime ideals over R_1 and R_2, respectively. Then,
1. $(\varphi, \psi)(M, E)$ is a neutrosophic soft completely semi-prime ideal over R_2.
2. $(\varphi, \psi)^{-1}(N, E)$ is a neutrosophic soft completely semi-prime ideal over R_1.

Proof. **1.** If possible, let (M, E) be a neutrosophic soft completely semi-prime ideal over R_1 but $(\varphi, \psi)(M, E)$ is not so over R_2. Then for $b \in \psi(E)$ and $y \in R_2$,

$$\begin{aligned} & T_{f_{\varphi(M)}(b)}(y^2) > T_{f_{\varphi(M)}(b)}(y) \\ \Rightarrow\ & \max_{\varphi(x)=y^2} \max_{\psi(a)=b} [T_{f_M(a)}(x)] > \max_{\varphi(x)=y} \max_{\psi(a)=b} [T_{f_M(a)}(x)] \\ \Rightarrow\ & \max_{\varphi(x)=y^2} [T_{f_M(a)}(x)] > \max_{\varphi(x)=y} [T_{f_M(a)}(x)] \\ \Rightarrow\ & \max_{\varphi(x)=y^2} [T_{f_M(a)}(x)] \geq T_{f_M(a)}(x) \end{aligned}$$

Since the inequality holds for each $x \in R_1$ satisfying $\varphi(x) = y$, so we have $T_{f_M(a)}(x^2) > T_{f_M(a)}(x)$ which is a contradiction to the fact that (M, E) is a neutrosophic soft completely semi-prime ideal over R_1. We can reach to the same conclusion taking the indeterminacy membership function (I) and falsity membership function (F) also. Hence we get the first result.
2. For $a \in \psi^{-1}(E)$ and $x \in R_1$, we have,

$$\begin{aligned} T_{f_{\varphi^{-1}(N)}(a)}(x^2) &= T_{f_N[\psi(a)]}(\varphi(x^2)) = T_{f_N[\psi(a)]}(\varphi(x))^2 \leq T_{f_N[\psi(a)]}(\varphi(x)) = T_{f_{\varphi^{-1}(N)}(a)}(x), \\ I_{f_{\varphi^{-1}(N)}(a)}(x^2) &= I_{f_N[\psi(a)]}(\varphi(x^2)) = I_{f_N[\psi(a)]}(\varphi(x))^2 \geq I_{f_N[\psi(a)]}(\varphi(x)) = I_{f_{\varphi^{-1}(N)}(a)}(x), \\ F_{f_{\varphi^{-1}(N)}(a)}(x^2) &= F_{f_N[\psi(a)]}(\varphi(x^2)) = F_{f_N[\psi(a)]}(\varphi(x))^2 \geq F_{f_N[\psi(a)]}(\varphi(x)) = F_{f_{\varphi^{-1}(N)}(a)}(x); \end{aligned}$$

This proves the 2nd result.

5 Neutrosophic soft prime k-ideal

5.1 Definition

A neutrosophic soft ideal N over (R, E) is said to be a neutrosophic soft k-ideal over (R, E) if $\forall x, y \in R$ and $\forall e \in E$,

$$\begin{cases} T_{f_N(e)}(x) \geq \min\{T_{f_N(e)}(x+y), T_{f_N(e)}(y)\} \\ I_{f_N(e)}(x) \leq \max\{I_{f_N(e)}(x+y), I_{f_N(e)}(y)\} \\ F_{f_N(e)}(x) \leq \max\{F_{f_N(e)}(x+y), F_{f_N(e)}(y)\}. \end{cases}$$

5.1.1 *Example*

1. Let $\mathbf{Z}$ be the set of all integers and $E = \{e_1, e_2, e_3\}$ be a parametric set. We consider an NSS N over $(\mathbf{Z}, E)$ given by the following table :

Table 7

	$f_N(e_1)$	$f_N(e_2)$	$f_N(e_3)$
$\mathbf{Z}_1$	$(0.3, 0.8, 0.5)$	$(0.4, 0.5, 0.7)$	$(0.7, 0.6, 0.4)$
$\mathbf{Z}_2$	$(0.4, 0.6, 0.3)$	$(0.6, 0.2, 0.4)$	$(0.7, 0.4, 0.2)$
$\mathbf{Z}_3$	$(0.6, 0.2, 0.1)$	$(1, 0, 0)$	$(0.9, 0.1, 0.1)$

where $\mathbf{Z}_1 = \{\pm 1, \pm 3, \pm 5, \cdots\}, \mathbf{Z}_2 = \{\pm 2, \pm 4, \pm 6, \cdots\}, \mathbf{Z}_3 = \{0\}$. Then N is a neutrosophic soft k-ideal over $(\mathbf{Z}, E)$. To verify it, we shall show
(i) $f_N(e)$ is neutrosophic subgroup of $(\mathbf{Z}, +)$ for each $e \in E$.
(ii) $f_N(e)$ is both neutrosophic left and right ideal of $\mathbf{Z}$ for each $e \in E$.
(iii) $f_N(e)$ is neutrosophic k-ideal of $\mathbf{Z}$ for each $e \in E$.

If $x \in \mathbf{Z}_1, y \in \mathbf{Z}_2$ then $x - y \in \mathbf{Z}_1$. We then write $\mathbf{Z}_1 - \mathbf{Z}_2 = \mathbf{Z}_1$ and so on.
Here $\mathbf{Z}_1 - \mathbf{Z}_1 = \mathbf{Z}_2$ or $\mathbf{Z}_3$, $\mathbf{Z}_1 - \mathbf{Z}_2 = \mathbf{Z}_1$, $\mathbf{Z}_1 - \mathbf{Z}_3 = \mathbf{Z}_3$, $\mathbf{Z}_2 - \mathbf{Z}_2 = \mathbf{Z}_2$ or $\mathbf{Z}_3$, $\mathbf{Z}_2 - \mathbf{Z}_3 = \mathbf{Z}_2$, $\mathbf{Z}_3 - \mathbf{Z}_3 = \mathbf{Z}_3$. Then Table 7 shows the result (i) obviously.

Next $\mathbf{Z}_1.\mathbf{Z}_1 = \mathbf{Z}_1$, $\mathbf{Z}_2.\mathbf{Z}_2 = \mathbf{Z}_2$, $\mathbf{Z}_3.\mathbf{Z}_3 = \mathbf{Z}_3$, $\mathbf{Z}_2.\mathbf{Z}_1 = \mathbf{Z}_1.\mathbf{Z}_2 = \mathbf{Z}_2$, $\mathbf{Z}_1.\mathbf{Z}_3 = \mathbf{Z}_3.\mathbf{Z}_1 = \mathbf{Z}_3$, $\mathbf{Z}_2.\mathbf{Z}_3 = \mathbf{Z}_3.\mathbf{Z}_2 = \mathbf{Z}_3$. Then the result (ii) also holds by Table 7.

Finally $\mathbf{Z}_1 + \mathbf{Z}_1 = \mathbf{Z}_2$ or $\mathbf{Z}_3$, $\mathbf{Z}_1 + \mathbf{Z}_2 = \mathbf{Z}_1$, $\mathbf{Z}_1 + \mathbf{Z}_3 = \mathbf{Z}_3$, $\mathbf{Z}_2 + \mathbf{Z}_2 = \mathbf{Z}_2$ or $\mathbf{Z}_3$, $\mathbf{Z}_2 + \mathbf{Z}_3 = \mathbf{Z}_2$, $\mathbf{Z}_3 + \mathbf{Z}_3 = \mathbf{Z}_3$. The Table 7 then meets the result (iii) clearly.

2. Let $\mathbf{R}$ be the set of real numbers and $E = \{e_1, e_2, e_3\}$ be a parametric set. Consider an NSS M over $(\mathbf{R}, E)$ given by the following table :

Table 8

	$f_M(e_1)$	$f_M(e_2)$	$f_M(e_3)$
$\mathbf{Q}$	$(0.6, 0.1, 0.3)$	$(0.8, 0.2, 0.4)$	$(0.5, 0.6, 0.7)$
$\mathbf{Q}^c$	$(0.5, 0.4, 0.7)$	$(0.4, 0.5, 0.6)$	$(0.3, 0.7, 1)$

where $\mathbf{Q}$ and $\mathbf{Q}^c$ are the set of rational and irrational numbers, respectively. If $x \in \mathbf{Q}, y \in \mathbf{Q}^c$ then $x - y \in \mathbf{Q}^c$. We write $\mathbf{Q} - \mathbf{Q}^c = \mathbf{Q}^c$ and so on.
Then $\mathbf{Q} - \mathbf{Q} = \mathbf{Q}$, $\mathbf{Q} - \mathbf{Q}^c = \mathbf{Q}^c$, $\mathbf{Q}^c - \mathbf{Q}^c = \mathbf{Q}$ or $\mathbf{Q}^c$. Clearly $f_M(e)$ is neutrosophic subgroup of $(\mathbf{R}, +)$ for each $e \in E$ by Table 8.
Next, $\mathbf{Q}.\mathbf{Q} = \mathbf{Q}$, $\mathbf{Q}.\mathbf{Q}^c = \mathbf{Q}^c$, $\mathbf{Q}^c.\mathbf{Q}^c = \mathbf{Q}$ or $\mathbf{Q}^c$. Then Table 8 shows that $f_M(e)$ is neutrosophic ideal of $\mathbf{R}$ for each $e \in E$.
Finally $\mathbf{Q} + \mathbf{Q} = \mathbf{Q}$, $\mathbf{Q} + \mathbf{Q}^c = \mathbf{Q}^c$, $\mathbf{Q}^c + \mathbf{Q}^c = \mathbf{Q}$ or $\mathbf{Q}^c$. Then $f_M(e)$ is neutrosophic k-ideal of $\mathbf{R}$ for each $e \in E$ by Table 8.
Hence M is a neutrosophic soft k-ideal over $(\mathbf{R}, E)$.

5.2 Definition

A neutrosophic soft k-ideal P over (R, E) is said to be a neutrosophic soft prime k-ideal if (i) P is not constant over (R, E), (ii) for any two neutrosophic soft ideals M, N over (R, E), $MoN \subseteq P \Rightarrow$ either $M \subseteq P$ or $N \subseteq P$.

5.3 Theorem

Let P be a neutrosophic soft prime k-ideal over (R, E). Then $P_0 = \{x \in R : [f_P(e)](x) = [f_P(e)](0_r), \forall e \in E\}$ is a prime k-ideal of R.

Proof. Let $x, x+y \in P_0$ for $x, y \in R$. Then $[f_P(e)](x) = [f_P(e)](x+y) = [f_P(e)](0_r)$. Since P is a neutrosophic soft k-ideal over (R, E), so $\forall e \in E$,

$$T_{f_P(e)}(y) \geq \min\{T_{f_P(e)}(x+y), T_{f_P(e)}(x)\} = T_{f_P(e)}(0_r),$$
$$I_{f_P(e)}(y) \leq \max\{I_{f_P(e)}(x+y), I_{f_P(e)}(x)\} = I_{f_P(e)}(0_r),$$
$$F_{f_P(e)}(y) \leq \max\{F_{f_P(e)}(x+y), F_{f_P(e)}(x)\} = F_{f_P(e)}(0_r);$$

But $T_{f_P(e)}(0_r) \geq T_{f_P(e)}(y)$, $I_{f_P(e)}(0_r) \leq I_{f_P(e)}(y)$, $F_{f_P(e)}(0_r) \leq F_{f_P(e)}(y)$, $\forall e \in E$. Thus $T_{f_P(e)}(y) = T_{f_P(e)}(0_r)$, $I_{f_P(e)}(y) = I_{f_P(e)}(0_r)$, $F_{f_P(e)}(y) \leq F_{f_P(e)}(0_r)$, $\forall e \in E$ i.e., $[f_P(e)](y) = [f_P(e)](0_r)$ and so $y \in P_0$. Hence P_0 is a k-ideal of R. Also by Theorem [2.11](6), P_0 is a prime ideal of R. This completes the proof.

5.4 Theorem

Let P be a neutrosophic soft prime k-ideal over $(\mathbf{Z}, E), \mathbf{Z}$ being the set of integers with $P_0 = \{x \in R : [f_P(e)](x) = [f_P(e)](0), \forall e \in E\} = n\mathbf{Z}, n$ being a natural number. Then $|f_P(e)| \leq r$, where r is the number of distinct positive divisor of n.

Proof. Let $a(\neq 0)$ be an integer and $d = gcd(a, n)$. Then there exists $r, s \in \mathbf{Z} - \{0\}$ such that $ns = ar + d$ or $ar = ns + d$. We shall now estimate following two cases :
Case 1 : When $ns = ar + d$, then $\forall e \in E$ and as $n \in P_0 = n\mathbf{Z}$,

$$T_{f_P(e)}(ar+d) = T_{f_P(e)}(ns) \geq T_{f_P(e)}(n) = T_{f_P(e)}(0) \geq T_{f_P(e)}(ar),$$
$$I_{f_P(e)}(ar+d) = I_{f_P(e)}(ns) \leq I_{f_P(e)}(n) = I_{f_P(e)}(0) \leq I_{f_P(e)}(ar),$$
$$F_{f_P(e)}(ar+d) = F_{f_P(e)}(ns) \leq F_{f_P(e)}(n) = F_{f_P(e)}(0) \leq F_{f_P(e)}(ar);$$

Again P is a neutrosophic soft k-ideal over $(\mathbf{Z}, E)$. So,

$$T_{f_P(e)}(d) \geq \min\{T_{f_P(e)}(ar+d), T_{f_P(e)}(ar)\} = T_{f_P(e)}(ar) \geq T_{f_P(e)}(a),$$
$$I_{f_P(e)}(d) \leq \max\{I_{f_P(e)}(ar+d), I_{f_P(e)}(ar)\} = I_{f_P(e)}(ar) \leq I_{f_P(e)}(a),$$
$$F_{f_P(e)}(d) \leq \max\{F_{f_P(e)}(ar+d), F_{f_P(e)}(ar)\} = F_{f_P(e)}(ar) \leq F_{f_P(e)}(a);$$

Case 2 : When $ar = ns + d$, then $\forall e \in E$ and as $n \in P_0 = n\mathbf{Z}$,

$$T_{f_P(e)}(ns+d) = T_{f_P(e)}(ar) \geq T_{f_P(e)}(a),$$
$$I_{f_P(e)}(ns+d) = I_{f_P(e)}(ar) \leq I_{f_P(e)}(a),$$
$$F_{f_P(e)}(ns+d) = F_{f_P(e)}(ar) \leq F_{f_P(e)}(a);$$

Again,

$$T_{f_P(e)}(ns) \geq T_{f_P(e)}(n) = T_{f_P(e)}(0) \geq T_{f_P(e)}(a),$$
$$I_{f_P(e)}(ns) \leq I_{f_P(e)}(n) = I_{f_P(e)}(0) \leq I_{f_P(e)}(a),$$
$$F_{f_P(e)}(ns) \leq F_{f_P(e)}(n) = F_{f_P(e)}(0) \leq F_{f_P(e)}(a);$$

Now as P is a neutrosophic soft k-ideal over $(\mathbf{Z}, E)$ so,

$$T_{f_P(e)}(d) \geq \min\{T_{f_P(e)}(ns+d), T_{f_P(e)}(ns)\} \geq T_{f_P(e)}(a),$$
$$I_{f_P(e)}(d) \leq \max\{I_{f_P(e)}(ns+d), I_{f_P(e)}(ns)\} \leq I_{f_P(e)}(a),$$
$$F_{f_P(e)}(d) \leq \max\{F_{f_P(e)}(ns+d), F_{f_P(e)}(ns)\} \leq F_{f_P(e)}(a);$$

Thus in either case $\forall e \in E$,
$T_{f_P(e)}(d) \geq T_{f_P(e)}(a)$, $I_{f_P(e)}(d) \leq I_{f_P(e)}(a)$, $F_{f_P(e)}(d) \leq F_{f_P(e)}(a)$;
Further since d is a divisor of a, there exists $t \in \mathbf{Z} - \{0\}$ such that $a = dt$. So $\forall e \in E$,
$T_{f_P(e)}(a) = T_{f_P(e)}(dt) \geq T_{f_P(e)}(d)$, $I_{f_P(e)}(a) = I_{f_P(e)}(dt) \leq I_{f_P(e)}(d)$,
$F_{f_P(e)}(a) = F_{f_P(e)}(dt) \leq F_{f_P(e)}(d)$;
Hence $T_{f_P(e)}(d) = T_{f_P(e)}(a)$, $I_{f_P(e)}(d) = I_{f_P(e)}(a)$, $F_{f_P(e)}(d) = F_{f_P(e)}(a)$, $\forall e \in E$. Thus for any integer $a(\neq 0)$ there exists a divisor d of n such that $[f_P(e)](d) = [f_P(e)](a)$, $\forall e \in E$.
If $a = 0$ then $T_{f_P(e)}(a) = T_{f_P(e)}(0) = T_{f_P(e)}(n)$, $I_{f_P(e)}(a) = I_{f_P(e)}(0) = I_{f_P(e)}(n)$,
$F_{f_P(e)}(a) = F_{f_P(e)}(0) = F_{f_P(e)}(n)$, $\forall e \in E$.
This follows the theorem.

5.5 *Lemma*

For a neutrosophic soft prime k-ideal N over $(\mathbf{Z}, E)$($\mathbf{Z}$ being the set of integers), $N_0 = p\mathbf{Z}$ is a prime k-ideal of $\mathbf{Z}$ iff p is either zero or prime.

This result is similar to the matter incase of prime ideal in the ring of integers in classical sense. So the proof is omitted.

5.6 Theorem

Let N be a neutrosophic soft prime k-ideal over $(\mathbf{Z}, E)$, $\mathbf{Z}$ being the set of integers. Then $|f_N(e)| = 2$ for each $e \in E$.
Conversely, if N is an NSS over $(\mathbf{Z}, E)$ such that for each $e \in E$, $[f_N(e)](x) = (1, 0, 0)$ when $p|x$ and $[f_N(e)](x) = (\alpha, \beta, \gamma)$ when $p \not| x$, p being a fixed prime and $\beta > 0, \gamma > 0, \alpha < 1$, then N be a neutrosophic soft prime k-ideal over $(\mathbf{Z}, E)$.

Proof. Let N be a neutrosophic soft prime k-ideal over $(\mathbf{Z}, E)$ with $N_0 = p\mathbf{Z}$. By Theorem [5.3], N_0 is a prime k-ideal of $\mathbf{Z}$. Hence by Lemma [5.5], p is prime i.e., p has only two distinct divisors namely $1, p$. So by Theorem [5.4], $|f_N(e)| \leq 2$. But N being a neutrosophic soft prime k-ideal can not be constant, so $|f_N(e)| = 2$, $\forall e \in E$.
Conversely, let N be an NSS over $(\mathbf{Z}, E)$ satisfying the given conditions. Let $x, y \in \mathbf{Z}$.
If $T_{f_N(e)}(x) = \alpha$ or $T_{f_N(e)}(y) = \alpha$ then $T_{f_N(e)}(x+y) = 1$ or α and so
$T_{f_N(e)}(x+y) \geq \min\{T_{f_N(e)}(x), T_{f_N(e)}(y)\}$.
If $T_{f_N(e)}(x) = 1$ and $T_{f_N(e)}(y) = 1$ then $p|x$ and $p|y$. It implies $p|(x+y)$ and
$T_{f_N(e)}(x+y) = 1 = \min\{T_{f_N(e)}(x), T_{f_N(e)}(y)\}$.
Thus in either case $T_{f_N(e)}(x+y) \geq \min\{T_{f_N(e)}(x), T_{f_N(e)}(y)\}$, $\forall x, y \in \mathbf{Z}$, $\forall e \in E$.
Next, if $I_{f_N(e)}(x) = \beta$ or $I_{f_N(e)}(y) = \beta$ then $I_{f_N(e)}(x+y) = 0$ or β and so,
$I_{f_N(e)}(x+y) \leq \max\{I_{f_N(e)}(x), I_{f_N(e)}(y)\}$.
If $I_{f_N(e)}(x) = 0$ and $T_{f_N(e)}(y) = 0$ then $p|x$ and $p|y$. It implies $p|(x+y)$ and
$I_{f_N(e)}(x+y) = 0 = \min\{I_{f_N(e)}(x), I_{f_N(e)}(y)\}$.

Thus in either case $I_{f_N(e)}(x+y) \leq \max\{I_{f_N(e)}(x), I_{f_N(e)}(y)\}$, $\forall x, y \in \mathbf{Z}$, $\forall e \in E$.
Finally, if $F_{f_N(e)}(x) = \beta$ or $F_{f_N(e)}(y) = \beta$ then $F_{f_N(e)}(x+y) = 0$ or β and so $F_{f_N(e)}(x+y) \leq \max\{F_{f_N(e)}(x), F_{f_N(e)}(y)\}$.
If $F_{f_N(e)}(x) = 0$ and $F_{f_N(e)}(y) = 0$ then $p|x$ and $p|y$. It implies $p|(x+y)$ and $F_{f_N(e)}(x+y) = 0 = \min\{F_{f_N(e)}(x), F_{f_N(e)}(y)\}$.
Thus in either case $F_{f_N(e)}(x+y) \leq \max\{F_{f_N(e)}(x), F_{f_N(e)}(y)\}$, $\forall x, y \in \mathbf{Z}$, $\forall e \in E$.
Further if $[f_N(e)](x) = (\alpha, \beta, \gamma)$ then either $[f_N(e)](xy) = (\alpha, \beta, \gamma)$ or $[f_N(e)](xy) = (1, 0, 0)$ i.e., $T_{f_N(e)}(xy) \geq T_{f_N(e)}(x)$, $I_{f_N(e)}(xy) \leq I_{f_N(e)}(x)$, $F_{f_N(e)}(xy) \leq F_{f_N(e)}(x)$.
If $[f_N(e)](x) = (1, 0, 0)$ then $p|x$ and so $p|xy$. Then $[f_N(e)](x) = [f_N(e)](xy) = (1, 0, 0)$. Thus in either case we have $\forall x, y \in \mathbf{Z}$ and $\forall e \in E$,
$T_{f_N(e)}(xy) \geq T_{f_N(e)}(x)$, $I_{f_N(e)}(xy) \leq I_{f_N(e)}(x)$, $F_{f_N(e)}(xy) \leq F_{f_N(e)}(x)$.
So N is a neutrosophic soft ideal over $(\mathbf{Z}, E)$.
We shall now prove that N is a neutrosophic soft k-ideal over $(\mathbf{Z}, E)$.
If $[f_N(e)](x+y) = (\alpha, \beta, \gamma)$ or $[f_N(e)](y) = (\alpha, \beta, \gamma)$, then the inequalities in Definition [5.1] are obvious.
If $[f_N(e)](x+y) = (1, 0, 0)$ or $[f_N(e)](y) = (1, 0, 0)$, then $p|(x+y)$ and $p|y$. It implies $p|x$ and so $[f_N(e)](x) = (1, 0, 0)$. Thus the inequalities in Definition [5.1] hold clearly.
Therefore N is a neutrosophic soft k-ideal over $(\mathbf{Z}, E)$ and so N_0 is a k-ideal over $\mathbf{Z}$.
Finally, we shall prove that N is a neutrosophic soft prime k-ideal over $(\mathbf{Z}, E)$.
To prove it, we shall first show that $N_0 = p\mathbf{Z}$ is a prime k-ideal of $\mathbf{Z}$. Now,
$x \in N_0 \Leftrightarrow [f_N(e)](x) = [f_N(e)](0) = (1, 0, 0) \Leftrightarrow p|x \Leftrightarrow x = pm, m \in \mathbf{Z} \Leftrightarrow x \in p\mathbf{Z}$.
Thus $N_0 = p\mathbf{Z}, p$ being a prime and so N_0 is a prime k-ideal of $\mathbf{Z}$ by Lemma [5.5].
Further, $|f_N(e)| = 2, \forall e \in E$ namely $(1, 0, 0)$ and (α, β, γ). So N is not constant over $(\mathbf{Z}, E)$. Now assume two neutrosophic soft ideals S, Q over $(\mathbf{Z}, E)$ such that $SoQ \subseteq N$ and $S \not\subseteq N, Q \not\subseteq N$. Then there exists $x, y \in \mathbf{Z}$ such that $T_{f_S(e)}(x) > T_{f_N(e)}(x)$, $I_{f_S(e)}(x) < I_{f_N(e)}(x)$, $F_{f_S(e)}(x) < F_{f_N(e)}(x)$ and $T_{f_Q(e)}(y) > T_{f_N(e)}(y)$, $I_{f_Q(e)}(y) < I_{f_N(e)}(y)$, $F_{f_Q(e)}(y) < F_{f_N(e)}(y)$, $\forall e \in E$. Then $[f_N(e)](x) = [f_N(e)](y) = (\alpha, \beta, \gamma)$ obviously and so $x, y \notin N_0$. It implies $xy \notin N_0$ as it is a prime k-ideal of an abelian ring $\mathbf{Z}$. So $[f_N(e)](xy) = (\alpha, \beta, \gamma)$. Thus $T_{f_{SoQ}(e)}(xy) \leq T_{f_N(e)}(xy) = \alpha$, $I_{f_{SoQ}(e)}(xy) \geq I_{f_N(e)}(xy) = \beta$, $F_{f_{SoQ}(e)}(xy) \geq F_{f_N(e)}(xy) = \gamma$. But,

$$T_{f_{SoQ}(e)}(xy) \geq T_{f_S(e)}(x) * T_{f_Q(e)}(y) > \alpha,$$
$$I_{f_{SoQ}(e)}(xy) \leq I_{f_S(e)}(x) \diamond I_{f_Q(e)}(y) < \beta,$$
$$F_{f_{SoQ}(e)}(xy) \leq F_{f_S(e)}(x) \diamond F_{f_Q(e)}(y) < \gamma;$$

It opposes the fact. This ends the theorem.

6 Conclusion

The aim of this paper is to put forward the study of the concept neutrosophic soft prime ideal introduced in [26]. Here we have studied about neutrosophic soft completely prime ideal, neutrosophic soft completely semi-prime ideal and neutrosophic soft prime k-ideal. They are defined and illustrated by suitable examples. Their related properties and structural characteristics have been investigated also. Moreover a number of theorems have been developed in virtue of these notions. The concepts will bring a new opportunity in research and development of algebraic structures over NSS theory context, we expect.

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