



NC-VIKOR Based MAGDM Strategy under Neutrosophic Cubic Set Environment

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Abstract. Neutrosophic cubic set consists of interval neutrosophic set and single valued neutrosophic set simultaneously. Due to its unique structure, neutrosophic cubic set can express hybrid information consisting of single valued neutrosophic information and interval neutrosophic information simultaneously. VIKOR (VIsekriterijumska optimizacija i KOmpromisno Resenje) strategy is an important decision making strategy which selects the optimal alternative by utilizing maximum group utility and minimum of an individual regret. In this paper, we propose VIKOR strategy in neutrosophic cubic set environment, namely NC-VIKOR. We first define NC-VIKOR strategy in neutrosophic

cubic set environment to handle multi-attribute group decision making (MAGDM) problems, which means we combine the VIKOR with neutrosophic cubic number to deal with multi-attribute group decision making problems. We have proposed a new strategy for solving MAGDM problems. Finally, we solve MAGDM problem using our newly proposed NC-VIKOR strategy to show the feasibility, applicability and effectiveness of the proposed strategy. Further, we present sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives.

Keywords: MAGDM, NCS, NC-VIKOR strategy.

1. Introduction

Smarandache [1] introduced neutrosophic set (NS) by defining the truth membership function, indeterminacy function and falsity membership function as independent components by extending fuzzy set [2] and intuitionistic fuzzy set [3]. Each of three independent component of NS belongs to $[0, 1^+]$. Wang et al. [4] introduced single valued neutrosophic set (SVNS) where each of truth, indeterminacy and falsity membership degree belongs to $[0, 1]$. Many researchers developed and applied the NS and SVNS in various areas of research such as conflict resolution [5], clustering analysis [6-9], decision making [10-39], educational problem [40, 41], image processing [42-45], medical diagnosis [46, 47], social problem [48, 49]. Wang et al. [50] proposed interval neutrosophic set (INS). Ye [51] defined similarity measure of two interval neutrosophic sets and applied it to solve multi criteria decision making (MCDM) problem. By combining SVNS and INS Jun et al. [52], and Ali et al. [53] proposed neutrosophic cubic set (NCS). Thereafter, Zhan et al. [54] presented

two weighted average operators on NCSs and applied the operators for MADM problem. Banerjee et al. [55] introduced the grey relational analysis based MADM strategy in NCS environment. Lu and Ye [56] proposed three cosine measures between NCSs and presented MADM strategy in NCS environment. Pramanik et al. [57] defined similarity measure for NCSs and proved its basic properties and presented a new multi criteria group decision making strategy with linguistic variables in NCS environment. Pramanik et al. [58] proposed the score and accuracy functions for NCSs and prove their basic properties. In the same study, Pramanik et al. [59] developed a strategy for ranking of neutrosophic cubic numbers (NCNs) based on the score and accuracy functions. In the same study, Pramanik et al. [58] first developed a TODIM (Tomada de decisao interativa e multicritério), called the NC-TODIM and presented new NC-TODIM [58] strategy for solving (MAGDM) in NCS environment. Shi and Ye [59] introduced Dombi aggregation operators of NCSs and applied them for MADM problem. Pramanik et al. [60] proposed an ex-

tended technique for order preference by similarity to ideal solution (TOPSIS) strategy in NCS environment for solving MADM problem. Ye [61] present operations and aggregation method of neutrosophic cubic numbers for MADM. Pramanik et al. [62] presented some operations and properties of neutrosophic cubic soft set.

Opricovic [63] proposed the VIKOR strategy for a MAGDM problem with conflicting attributes [64-65]. In 2015, Bausys and Zavadskas [66] extended the VIKOR strategy to INS environment and applied it to solve MCDM problem. Further, Hung et al. [67] proposed VIKOR method for interval neutrosophic MAGDM. Poursmaeil et al. [68] proposed an MAGDM strategy based on TOPSIS and VIKOR in SVNS environment. Liu and Zhang [69] extended VIKOR method in neutrosophic hesitant fuzzy set environment. Hu et al. [70] proposed interval neutrosophic projection based VIKOR method and applied it for doctor selection. Selvakumari et al. [70] proposed VIKOR Method for decision making problem using octagonal neutrosophic soft matrix.

VIKOR strategy in NCS environment is yet to appear in the literature.

Research gap:

MAGDM strategy based on NC-VIKOR. This study answers the following research questions:

- i. Is it possible to extend VIKOR strategy in NCS environment?
- ii. Is it possible to develop a new MAGDM strategy based on the proposed NC-VIKOR method in NCS environment?

Motivation:

The above-mentioned analysis [64-69] describes the motivation behind proposing a novel NC-VIKOR method based MAGDM strategy under the NCS environment. This study develops a novel NC-VIKOR - based MAGDM strategy that can deal with multiple decision-makers.

The objectives of the paper are:

- i. To extend VIKOR strategy in NCS environment.
- ii. To define aggregation operator.
- iii. To develop a new MAGDM strategy based on proposed NC-VIKOR in NCS environment.

To fill the research gap, we propose NC-VIKOR strategy, which is capable of dealing with MAGDM problem in NCS environment.

The main contributions of this paper are summarized below:

- i. We developed a new NC-VIKOR strategy to deal with MAGDM problems in NCS environment.
- ii. We introduce a neutrosophic cubic number aggregation operator and prove its basic properties.

iii. In this paper, we develop a new MAGDM strategy based on proposed NC-VIKOR method under NCS environment to solve MAGDM problems.

iv. In this paper, we solve a MAGDM problem based on proposed NC-VIKOR method.

The remainder of this paper is organized as follows: In the section 2, we review some basic concepts and operations related to NS, SVNS, NCS. In Section 3, we develop a novel MAGDM strategy based on NC-VIKOR to solve the MAGDM problems with NCS environment. In Section 4, we solve an illustrative numerical example using the proposed NC-VIKOR in NCS environment. Then in Section 5, we present the sensitivity analysis. The conclusions of the whole paper and further direction of research are given in Section 6.

2. Preliminaries

Definition 1. Neutrosophic set

Let X be a space of points (objects) with a generic element in X denoted by x , i.e. $x \in X$. A neutrosophic set [1] A in X is characterized by truth-membership function $t_A(x)$, indeterminacy-membership function $i_A(x)$ and falsity-membership function $f_A(x)$, where $t_A(x), i_A(x), f_A(x)$ are the functions from X to $]0, 1^+[$ i.e. $t_A, i_A, f_A: X \rightarrow]0, 1^+[$ that means $t_A(x), i_A(x), f_A(x)$ are the real standard or non-standard subset of $]0, 1^+[$. Neutrosophic set can be expressed as $A = \{ \langle x, (t_A(x), i_A(x), f_A(x)) \rangle : \forall x \in X \}$ and $0 \leq t_A(x) + i_A(x) + f_A(x) \leq 3^+$.

Example 1. Suppose that $X = \{ x_1, x_2, x_3, \dots, x_n \}$ be the universal set of n points. Let A_1 be any neutrosophic set in X . Then A_1 expressed as $A_1 = \{ \langle x_1, (0.7, 0.4, 0.3) \rangle : x_1 \in X \}$.

Definition 2. Single valued neutrosophic set

Let X be a space of points (objects) with a generic element in X denoted by x . A single valued neutrosophic set [4] B in X is expressed as:

$B = \{ \langle x: (t_B(x), i_B(x), f_B(x)) \rangle : x \in X \}$, where $t_B(x), i_B(x), f_B(x) \in [0, 1]$.

For each $x \in X$, $t_B(x), i_B(x), f_B(x) \in [0, 1]$ and $0 \leq t_B(x) + i_B(x) + f_B(x) \leq 3$.

Definition 3. Interval neutrosophic set

An interval neutrosophic set [50] \tilde{A} of a non empty set H is expressed by truth-membership function $t_{\tilde{A}}(h)$ the indeterminacy membership function $i_{\tilde{A}}(h)$ and falsity membership function $f_{\tilde{A}}(h)$. For each $h \in H$, $t_{\tilde{A}}(h), i_{\tilde{A}}(h), f_{\tilde{A}}(h) \subseteq [0, 1]$ and \tilde{A} defined as follows:

$$\tilde{A} = \{ \langle h, [t_{\tilde{A}}^-(h), t_{\tilde{A}}^+(h)], [i_{\tilde{A}}^-(h), i_{\tilde{A}}^+(h)], [f_{\tilde{A}}^-(h), f_{\tilde{A}}^+(h)] : \forall h \in H \}. \text{ Here, } t_{\tilde{A}}^-(h), t_{\tilde{A}}^+(h), i_{\tilde{A}}^-(h), i_{\tilde{A}}^+(h), f_{\tilde{A}}^-(h), f_{\tilde{A}}^+(h) : H \rightarrow]^{-}, 1^{+} [\text{ and } 0 \leq \sup t_{\tilde{A}}^+(h) + \sup i_{\tilde{A}}^+(h) + \sup f_{\tilde{A}}^+(h) \leq 3^+.$$

Here, we consider $t_{\tilde{A}}^-(h), t_{\tilde{A}}^+(h), i_{\tilde{A}}^-(h), i_{\tilde{A}}^+(h), f_{\tilde{A}}^-(h), f_{\tilde{A}}^+(h) : H \rightarrow [0, 1]$ for real applications.

Example 2.

Assume that $H = \{ h_1, h_2, h_3, \dots, h_n \}$ be a non-empty set. Let \tilde{A}_1 be any interval neutrosophic set. Then \tilde{A}_1 expressed as $\tilde{A}_1 = \{ \langle h_1 : [0.30, 0.70], [0.20, 0.45], [0.18, 0.39] : h \in H \}$.

Definition 4. Neutrosophic cubic set

A neutrosophic cubic set [52, 53] in a non-empty set H is defined as $N = \{ \langle h, \tilde{A}(h), A(h) \rangle : \forall h \in H \}$, where \tilde{A} and A are the interval neutrosophic set and neutrosophic set in H respectively. Neutrosophic cubic set can be presented as an order pair $N = \langle \tilde{A}, A \rangle$, then we call it as neutrosophic cubic (NC) number.

Example 3.

Suppose that $H = \{ h_1, h_2, h_3, \dots, h_n \}$ be a non-empty set. Let N_1 be any NC-number. Then N_1 can be expressed as $N_1 = \{ \langle h_1 : [0.35, 0.47], [0.20, 0.43], [0.18, 0.42], (0.7, 0.3, 0.5) \rangle : h_1 \in H \}$.

Some operations of NC-numbers: [52, 53]

i. Union of any two NC-numbers

Let $N_1 = \langle \tilde{A}_1, A_1 \rangle$ and $N_2 = \langle \tilde{A}_2, A_2 \rangle$ be any two NC-numbers in a non-empty set H . Then the union of N_1 and N_2 denoted by $N_1 \cup N_2$ is defined as follows:

$$N_1 \cup N_2 = \langle \tilde{A}_1(h) \cup \tilde{A}_2(h), A_1(h) \cup A_2(h) \rangle \forall h \in H, \text{ where}$$

$$\tilde{A}_1(h) \cup \tilde{A}_2(h) = \{ \langle h, [\max\{ t_{\tilde{A}_1}^-(h), t_{\tilde{A}_2}^-(h) \}, \max\{ t_{\tilde{A}_1}^+(h), t_{\tilde{A}_2}^+(h) \}], [\min\{ i_{\tilde{A}_1}^-(h), i_{\tilde{A}_2}^-(h) \}, \min\{ i_{\tilde{A}_1}^+(h), i_{\tilde{A}_2}^+(h) \}], [\min\{ f_{\tilde{A}_1}^-(h), f_{\tilde{A}_2}^-(h) \}, \min\{ f_{\tilde{A}_1}^+(h), f_{\tilde{A}_2}^+(h) \}] \rangle : h \in H \} \text{ and } A_1(h) \cup A_2(h) = \{ \langle h, \max\{ t_{A_1}(h), t_{A_2}(h) \}, \min\{ i_{A_1}(h), i_{A_2}(h) \}, \min\{ f_{A_1}(h), f_{A_2}(h) \} \rangle : \forall h \in H \}.$$

Example 4.

Assume that

$$N_1 = \langle [0.39, 0.47], [0.17, 0.43], [0.18, 0.36], (0.6, 0.3, 0.4) \rangle \text{ and } N_2 = \langle [0.56, 0.70], [0.27, 0.42], [0.15, 0.26], (0.7, 0.3, 0.6) \rangle \text{ be two NC-numbers. Then } N_1 \cup N_2 = \langle [0.56, 0.7], [0.17, 0.42], [0.15, 0.26], (0.7, 0.3, 0.4) \rangle.$$

ii. Intersection of any two NC-numbers

Intersection of N_1 and N_2 denoted by $N_1 \cap N_2$ is defined as follows:

$$N_1 \cap N_2 = \langle \tilde{A}_1(h) \cap \tilde{A}_2(h), A_1(h) \cap A_2(h) \rangle \forall h \in H, \text{ where } \tilde{A}_1(h) \cap \tilde{A}_2(h) = \{ \langle h, [\min\{ t_{\tilde{A}_1}^-(h), t_{\tilde{A}_2}^-(h) \}, \min\{ t_{\tilde{A}_1}^+(h), t_{\tilde{A}_2}^+(h) \}], [\max\{ i_{\tilde{A}_1}^-(h), i_{\tilde{A}_2}^-(h) \}, \max\{ i_{\tilde{A}_1}^+(h), i_{\tilde{A}_2}^+(h) \}], [\max\{ f_{\tilde{A}_1}^-(h), f_{\tilde{A}_2}^-(h) \}, \max\{ f_{\tilde{A}_1}^+(h), f_{\tilde{A}_2}^+(h) \}] \rangle : h \in H \} \text{ and } A_1(h) \cap A_2(h) = \{ \langle h, \min\{ t_{A_1}(h), t_{A_2}(h) \}, \max\{ i_{A_1}(h), i_{A_2}(h) \}, \max\{ f_{A_1}(h), f_{A_2}(h) \} \rangle : \forall h \in H \}.$$

Example 5.

Assume that

$$N_1 = \langle [0.45, 0.57], [0.27, 0.33], [0.18, 0.46], (0.7, 0.3, 0.5) \rangle \text{ and } N_2 = \langle [0.67, 0.75], [0.22, 0.44], [0.17, 0.21], (0.8, 0.4, 0.4) \rangle \text{ be two NC numbers. Then } N_1 \cap N_2 = \langle [0.45, 0.57], [0.22, 0.33], [0.18, 0.46], (0.7, 0.3, 0.4) \rangle.$$

iii. Compliment of a NC-number

Let $N_1 = \langle \tilde{A}_1, A_1 \rangle$ be a NCS in H . Then compliment of $N_1 = \langle \tilde{A}_1, A_1 \rangle$ is denoted by $N_1^c = \langle h, \tilde{A}_1^c(h), A_1^c(h) \rangle : \forall h \in H \}$.

$$\text{Here, } \tilde{A}_1^c = \{ \langle h, [t_{\tilde{A}_1^c}^+(h), t_{\tilde{A}_1^c}^-(h)], [i_{\tilde{A}_1^c}^+(h), i_{\tilde{A}_1^c}^-(h)], [f_{\tilde{A}_1^c}^+(h), f_{\tilde{A}_1^c}^-(h)] \rangle : \forall h \in H \}, \text{ where, } t_{\tilde{A}_1^c}^-(h) = \{1\} - t_{\tilde{A}_1}^-(h), t_{\tilde{A}_1^c}^+(h) = \{1\} - t_{\tilde{A}_1}^+(h), i_{\tilde{A}_1^c}^-(h) = \{1\} - i_{\tilde{A}_1}^-(h), i_{\tilde{A}_1^c}^+(h) = \{1\} - i_{\tilde{A}_1}^+(h), f_{\tilde{A}_1^c}^-(h) = \{1\} - f_{\tilde{A}_1}^-(h), f_{\tilde{A}_1^c}^+(h) = \{1\} - f_{\tilde{A}_1}^+(h)$$

$$= \{1\} - f_{\tilde{A}_1}^+(h), \text{ and } t_{\tilde{A}_1^c}(h) = \{1\} - t_{A_1}(h), i_{\tilde{A}_1^c}(g) = \{1\} - i_{A_1}(h), f_{\tilde{A}_1^c}(h) = \{1\} - f_{A_1}(h).$$

Example 6.

Assume that N_1 be any NC-number in H in the form:
 $N_1 = \langle [0.45, .57], [0.27, .33], [0.18, .46], (.7, .3, .5) \rangle$.
 Then compliment of N_1 is obtained as $N_1^c = \langle [0.18, 0.46], [0.67, 0.73], [0.45, 0.57], (0.5, 0.7, 0.7) \rangle$.

iv. Containment

Let $N_1 = \langle \tilde{A}_1, A_1 \rangle = \{ \langle h, [t_{\tilde{A}_1}^-(h), t_{\tilde{A}_1}^+(h)], [i_{\tilde{A}_1}^-(h), i_{\tilde{A}_1}^+(h)], [f_{\tilde{A}_1}^-(h), f_{\tilde{A}_1}^+(h)], (t_{A_1}(h), i_{A_1}(h), f_{A_1}(h)) \rangle : h \in H \}$ and $N_2 = \langle \tilde{A}_2, A_2 \rangle = \{ \langle h, [t_{\tilde{A}_2}^-(h), t_{\tilde{A}_2}^+(h)], [i_{\tilde{A}_2}^-(h), i_{\tilde{A}_2}^+(h)], [f_{\tilde{A}_2}^-(h), f_{\tilde{A}_2}^+(h)], (t_{A_2}(h), i_{A_2}(h), f_{A_2}(h)) \rangle : h \in H \}$

be any two NC-numbers in a non-empty set H, then, (i) $N_1 \subseteq N_2$ if and only if

$$t_{\tilde{A}_1}^-(h) \leq t_{\tilde{A}_2}^-(h), t_{\tilde{A}_1}^+(h) \leq t_{\tilde{A}_2}^+(h),$$

$$i_{\tilde{A}_1}^-(h) \geq i_{\tilde{A}_2}^-(h), i_{\tilde{A}_1}^+(h) \geq i_{\tilde{A}_2}^+(h),$$

$$f_{\tilde{A}_1}^-(h) \geq f_{\tilde{A}_2}^-(h), f_{\tilde{A}_1}^+(h) \geq f_{\tilde{A}_2}^+(h)$$

and $t_{A_1}(h) \leq t_{A_2}(h)$,

$i_{A_1}(h) \geq i_{A_2}(h), f_{A_1}(h) \geq f_{A_2}(h)$ for all $h \in H$.

Definition 7.

Let $N_1 = \langle [a_1, a_2], [b_1, b_2], [c_1, c_2], (a, b, c) \rangle$ and $N_2 = \langle [d_1, d_2], [e_1, e_2], [f_1, f_2], (d, e, f) \rangle$ be any two NC-numbers, then distance [58] between them is defined by $D(N_1, N_2) =$

$$\frac{1}{9} [|a_1 - d_1| + |a_2 - d_2| + |b_1 - e_1| + |b_2 - e_2| + |c_1 - f_1| + |c_2 - f_2| + |a - d| + |b - e| + |c - f|] \quad (1)$$

Definition 2.14: Procedure of normalization

In general, benefit type attributes and cost type attributes can exist simultaneously in MAGDM problem. Therefore the decision matrix must be normalized. Let a_{ij} be a NC-numbers to express the rating value of i-th alternative with respect to j-th attribute (Ψ_j). When attribute $\Psi_j \in C$ or $\Psi_j \in G$ (where C and G be the set of cost type attribute and set of benefit type attributes respectively) The normalized values for cost type attribute and benefit type attribute are calculated by using the following expression (2).

$$a_{ij}^* = \begin{cases} a_{ij} & \text{if } \Psi_j \in G \\ 1 - a_{ij} & \text{if } \Psi_j \in C \end{cases} \quad (2)$$

Where, a_{ij} is the performance rating of i th alternative for attribute Ψ_j and $\max a_j$ is the maximum performance rating among alternatives for attribute Ψ_j .

VIKOR strategy

The VIKOR strategy is an MCDM or multi-criteria decision analysis strategy to deal with multi-criteria optimization problem. This strategy focuses on ranking and selecting the best alternatives from a set of feasible alternatives in the presence of conflicting criteria for a decision problem. The compromise solution [63, 64] reflects a feasible solution that is the closest to the ideal, and a compromise means an agreement established by mutual concessions. The L_p -metric is used to develop the strategy [65]. The VIKOR strategy is developed using the following form of L_p -metric

$$L_{pi} = \left\{ \sum_{j=1}^n \left[\frac{(\Omega_j^+ - \Omega_{ij}^-)}{(\Omega_j^+ - \Omega_j^-)} \right]^p \right\}^{\frac{1}{p}}$$

$1 \leq p \leq \infty; i=1, 2, 3, \dots, m$.

In the VIKOR strategy, L_{i1} (as S_i) and $L_{\infty i}$, i (as R_i) are utilized to formulate ranking measure. The solution obtained by $\min S_i$ reflects the maximum group utility (“majority” rule), and the solution obtained by $\min R_i$ indicates the minimum individual regret of the “opponent”.

Suppose that each alternative is evaluated by each criterion function, the compromise ranking is prepared by comparing the measure of closeness to the ideal alternative. The m alternatives are denoted as $A_1, A_2, A_3, \dots, A_m$. For the alternative A_i , the rating of the j th aspect is denoted by Ω_{ij} , i.e. Ω_{ij} is the value of j th criterion function for the alternative A_i ; n is the number of criteria.

The compromise ranking algorithm of the VIKOR strategy is presented using the following steps:

Step 1: Determine the best Ω_j^+ and the worst Ω_j^- values of all criterion functions $j = 1, 2, \dots, n$. If the

j-th function represents a benefit then:

$$\Omega_j^+ = \max_i \Omega_{ij}, \quad \Omega_j^- = \min_i \Omega_{ij}$$

Step 2: Compute the values S_i and R_i ; $i = 1, 2, \dots, m$, by these relations:

$$S_i = \sum_{j=1}^n w_j (\Omega_j^+ - \Omega_{ij}) / (\Omega_j^+ - \Omega_j^-),$$

$$R_i = \max_j w_j (\Omega_j^+ - \Omega_{ij}) / (\Omega_j^+ - \Omega_j^-),$$

Here, w_j is the weight of the criterion that expresses its relative importance.

Step 3: Compute the values Q_i ; $i = 1, 2, \dots, m$, using the following relation:

$$Q_i = v(S_i - S^-) / (S^+ - S^-) + (1-v)(R_i - R^-) / (R^+ - R^-)$$

Here, $S^+ = \max_i S_i$, $S^- = \min_i S_i$

$$R^+ = \max_i R_i, \quad R^- = \min_i R_i$$

Here, v represents ‘‘the decision making mechanism coefficient’’ (or ‘‘the maximum group utility’’). Here we consider $v = 0.5$.

Step 4: Preference ranking order of the alternatives is done by sorting the values of S , R and Q in decreasing order.

3. VIKOR strategy for solving MAGDM problem in NCS environment

In this section, we propose a MAGDM strategy in NCS environment. Assume that $\Phi = \{\Phi_1, \Phi_2, \Phi_3, \dots, \Phi_r\}$ be a set of r alternatives and $\Psi = \{\Psi_1, \Psi_2, \Psi_3, \dots, \Psi_s\}$ be a set of s attributes. Assume that $W = \{w_1, w_2, w_3, \dots, w_s\}$ be the weight vector of the attributes, where $w_k \geq 0$ and $\sum_{k=1}^s w_k = 1$. Assume that $E = \{E_1, E_2, E_3, \dots, E_M\}$ be the set of M decision makers and $\zeta = \{\zeta_1, \zeta_2, \zeta_3, \dots, \zeta_M\}$ be the set of weight vector of decision makers, where $\zeta_p \geq 0$ and $\sum_{p=1}^M \zeta_p = 1$.

The proposed MAGDM strategy consists of the following steps:

Step: 1. Construction of the decision matrix

Let $DM^p = (a_{ij}^p)_{r \times s}$ ($p = 1, 2, 3, \dots, t$) be the p -th decision matrix, where information about the alternative Φ_i provided by the decision maker or expert E_p with respect to attribute Ψ_j ($j = 1, 2, 3, \dots, s$). The p -th decision matrix denoted by DM^p (See Equation (3)) is constructed as follows:

$$DM^p = \begin{pmatrix} & \Psi_1 & \Psi_2 & \dots & \Psi_s \\ \Phi_1 & a_{11}^p & a_{12}^p & \dots & a_{1s}^p \\ \Phi_2 & a_{21}^p & a_{22}^p & & \Psi_2^p \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \Phi_r & a_{r1}^p & a_{r2}^p & \dots & a_{rs}^p \end{pmatrix} \quad (3)$$

Here $p = 1, 2, 3, \dots, M$; $i = 1, 2, 3, \dots, r$; $j = 1, 2, 3, \dots, s$.

Step: 2. Normalization of the decision matrix

In decision making situation, cost type attributes and benefit type attributes play an important role to select the best alternative. Cost type attributes and benefit type attributes may exist simultaneously, so the decision matrices need to be normalized. We use Equation (2) for normalizing the cost type attributes and benefit type attributes. After normalization, the normalized decision matrix (Equation (3)) is represented as follows (see Equation 4):

$$DM^p = \begin{pmatrix} & \Psi_1 & \Psi_2 & \dots & \Psi_s \\ \Phi_1 & a_{11}^{*p} & a_{12}^{*p} & \dots & a_{1s}^{*p} \\ \Phi_2 & a_{21}^{*p} & a_{22}^{*p} & & a_{2s}^{*p} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ \Phi_r & a_{r1}^{*p} & a_{r2}^{*p} & \dots & a_{rs}^{*p} \end{pmatrix} \quad (4)$$

Here, $p = 1, 2, 3, \dots, M$; $i = 1, 2, 3, \dots, r$; $j = 1, 2, 3, \dots, s$.

Step: 3. Aggregated decision matrix

For obtaining group decision, we aggregate all the individual decision matrices (DM^p , $p=1, 2, \dots, M$) to an aggregated decision matrix (DM) using the neutrosophic cubic numbers weighted aggregation (NCNWA) operator as follows:

$$a_{ij} = NCNWA_{\zeta} (a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) = (\zeta_1 a_{ij}^1 \oplus \zeta_2 a_{ij}^2 \oplus \zeta_3 a_{ij}^3 \oplus \dots \oplus \zeta_M a_{ij}^M) = < \left(\left[\sum_{p=1}^M \zeta_p t_{ij}^{-(p)}, \sum_{p=1}^M \zeta_p t_{ij}^{+(p)} \right], \left[\sum_{p=1}^M \zeta_p i_{ij}^{-(p)}, \sum_{p=1}^M \zeta_p i_{ij}^{+(p)} \right], \left[\sum_{p=1}^M \zeta_p f_{ij}^{-(p)}, \sum_{p=1}^M \zeta_p f_{ij}^{+(p)} \right], \left(\sum_{p=1}^M \zeta_p t_{ij}^{(p)}, \sum_{p=1}^M \zeta_p i_{ij}^{(p)}, \sum_{p=1}^M \zeta_p f_{ij}^{(p)} \right) \right) > \quad (5)$$

The NCNWA operator satisfies the following properties:

1. Idempotency
2. Monotonicity
3. Boundedness

Property: 1. Idempotency

If all $a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M = a$ are equal, then

$$a_{ij} = NCNWA_{\zeta} (a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) = a$$

Proof:

Since $a_{ij}^1 = a_{ij}^2 = \dots = a_{ij}^M = a$, based on the Equation

(5), we get

$$\begin{aligned}
 a_{ij} &= \text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) = \\
 &(\zeta_1 a_{ij}^1 \oplus \zeta_2 a_{ij}^2 \oplus \zeta_3 a_{ij}^3 \oplus \dots \oplus \zeta_M a_{ij}^M) = \\
 &(\zeta_1 a \oplus \zeta_2 a \oplus \zeta_3 a \oplus \dots \oplus \zeta_M a) = \\
 &\langle \left([t^- \sum_{p=1}^M \zeta_p, t^+ \sum_{p=1}^M \zeta_p], [i^- \sum_{p=1}^M \zeta_p, i^+ \sum_{p=1}^M \zeta_p], \right. \\
 & \left. [f^- \sum_{p=1}^M \zeta_p, f^+ \sum_{p=1}^M \zeta_p], (t \sum_{p=1}^M \zeta_p, i \sum_{p=1}^M \zeta_p, f \sum_{p=1}^M \zeta_p) \right\rangle > \\
 &= \langle ([t^-, t^+], [i^-, i^+], [f^-, f^+], (t, i, f)) \rangle = a.
 \end{aligned}$$

Property: 3. Monotonicity

Assume that $\{a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M\}$ and $\{a_{ij}^{*1}, a_{ij}^{*2}, \dots, a_{ij}^{*M}\}$ be any two set of collections of M NC-numbers with the condition $a_{ij}^p \leq a_{ij}^{*p}$ ($p = 1, 2, \dots, M$), then

$$\text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \leq \text{NCNWA}_\zeta(a_{ij}^{*1}, a_{ij}^{*2}, \dots, a_{ij}^{*M}).$$

Proof:

From the given condition $t_{ij}^{-(p)} \leq t_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p t_{ij}^{-(p)} &\leq \zeta_p t_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p t_{ij}^{-(p)} &\leq \sum_{p=1}^M \zeta_p t_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $t_{ij}^{+(p)} \leq t_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p t_{ij}^{+(p)} &\leq \zeta_p t_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p t_{ij}^{+(p)} &\leq \sum_{p=1}^M \zeta_p t_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $i_{ij}^{-(p)} \geq i_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p i_{ij}^{-(p)} &\geq \zeta_p i_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p i_{ij}^{-(p)} &\geq \sum_{p=1}^M \zeta_p i_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $i_{ij}^{+(p)} \geq i_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p i_{ij}^{+(p)} &\geq \zeta_p i_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p i_{ij}^{+(p)} &\geq \sum_{p=1}^M \zeta_p i_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $f_{ij}^{-(p)} \geq f_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p f_{ij}^{-(p)} &\geq \zeta_p f_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p f_{ij}^{-(p)} &\geq \sum_{p=1}^M \zeta_p f_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $f_{ij}^{+(p)} \geq f_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p f_{ij}^{+(p)} &\geq \zeta_p f_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p f_{ij}^{+(p)} &\geq \sum_{p=1}^M \zeta_p f_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $t_{ij}^{(p)} \leq t_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p t_{ij}^{(p)} &\leq \zeta_p t_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p t_{ij}^{(p)} &\leq \sum_{p=1}^M \zeta_p t_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $i_{ij}^{(p)} \geq i_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p i_{ij}^{(p)} &\geq \zeta_p i_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p i_{ij}^{(p)} &\geq \sum_{p=1}^M \zeta_p i_{ij}^{*(p)}.
 \end{aligned}$$

From the given condition $t_{ij}^{(p)} \leq t_{ij}^{*(p)}$, we have

$$\begin{aligned}
 \zeta_p f_{ij}^{(p)} &\geq \zeta_p f_{ij}^{*(p)} \\
 \Rightarrow \sum_{p=1}^M \zeta_p f_{ij}^{(p)} &\geq \sum_{p=1}^M \zeta_p f_{ij}^{*(p)}.
 \end{aligned}$$

From the above relations, we obtain

$$\text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \leq \text{NCNWA}_\zeta(a_{ij}^{*1}, a_{ij}^{*2}, \dots, a_{ij}^{*M}).$$

Property: 2. Boundedness

Let $\{a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M\}$ be any collection of M NC-numbers.

If

$$\begin{aligned}
 a^+ &= \langle [\max_p \{t_{ij}^{-(p)}\}, [\max_p \{t_{ij}^{+(p)}\}], [\min_p \{i_{ij}^{-(p)}\}, \min_p \{i_{ij}^{+(p)}\}], \\
 &[\min_p \{f_{ij}^{-(p)}\}, \min_p \{f_{ij}^{+(p)}\}], (\max_p \{t_{ij}^p\}, \min_p \{i_{ij}^p\}, \min \{f_{ij}^p\}) \rangle > \\
 &\quad i \\
 a^- &= \langle [\min_p \{t_{ij}^{-(p)}\}, [\min_p \{t_{ij}^{+(p)}\}], [\max_p \{i_{ij}^{-(p)}\}, \max_p \{i_{ij}^{+(p)}\}], \\
 &[\max_p \{f_{ij}^{-(p)}\}, \max_p \{f_{ij}^{+(p)}\}], (\min_p \{t_{ij}^p\}, \max_p \{i_{ij}^p\}, \max \{f_{ij}^p\}) \rangle.
 \end{aligned}$$

Then, $a^- \leq \text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \leq a^+$.

Proof:

From Property 1 and Property 2, we obtain

$$\text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \geq \text{NCNWA}_\zeta(a^-, a^-, \dots, a^-) = a^-$$

and

$$\text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \leq \text{NCNWA}_\zeta(a^+, a^+, \dots, a^+) = a^+.$$

So, we have

$$a^- \leq \text{NCNWA}_\zeta(a_{ij}^1, a_{ij}^2, \dots, a_{ij}^M) \leq a^+.$$

Therefore, the aggregated decision matrix is defined as follows:

$$DM = \begin{pmatrix} & \Psi_1 & \Psi_2 & \dots & \Psi_s \\ \Phi_1 & a_{11} & a_{12} & \dots & a_{1s} \\ \Phi_2 & a_{21} & a_{22} & & a_{2s} \\ \dots & \dots & \dots & & \dots \\ \Phi_r & a_{r1} & a_{r2} & \dots & a_{rs} \end{pmatrix} \quad (6)$$

Here, $i = 1, 2, 3, \dots, r; j = 1, 2, 3, \dots, s; p = 1, 2, \dots, M$.

Step: 4. Define the positive ideal solution and negative ideal solution

$$a_{ij}^+ = \langle [\max_i t_{ij}^-, \max_i t_{ij}^+], [\min_i i_{ij}^-, \min_i i_{ij}^+], \quad (7)$$

$$[\min_i f_{ij}^-, \min_i i_{ij}^+], [\max_i t_{ij}^-, \min_i f_{ij}^-, \min_i f_{ij}^+] \rangle$$

$$a_{ij}^- = \langle [\min_i t_{ij}^-, \min_i t_{ij}^+], [\max_i i_{ij}^-, \max_i i_{ij}^+], \quad (8)$$

$$[\max_i f_{ij}^-, \max_i i_{ij}^+], [\min_i t_{ij}^-, \max_i f_{ij}^-, \max_i f_{ij}^+] \rangle$$

Step: 5. Compute Γ_i and Z_i
 and Γ_i represent the average and worst group scores for the alternative A_i respectively with the relations

$$\Gamma_i = \sum_{j=1}^s \frac{w_j \times D(a_{ij}^+, a_{ij}^*)}{D(a_{ij}^+, a_{ij}^-)} \quad (9)$$

$$Z_i = \max_j \left\{ \frac{w_j \times D(a_{ij}^+, a_{ij}^*)}{D(a_{ij}^+, a_{ij}^-)} \right\} \quad (10)$$

Here, w_j is the weight of Ψ_j .
 The smaller values of Γ_i and Z_i correspond to the better average and worse group scores for alternative A_i , respectively.

Step: 6. Calculate the values of ϕ_i ($i = 1, 2, 3, \dots, r$)

$$\phi_i = \gamma \frac{(\Gamma_i - \Gamma^-)}{(\Gamma^+ - \Gamma^-)} + (1-\gamma) \frac{(Z_i - Z^-)}{(Z^+ - Z^-)} \quad (11)$$

Here, $\Gamma_i^- = \min_i \Gamma_i, \Gamma_i^+ = \max_i \Gamma_i,$
 $Z_i^- = \min_i Z_i, Z_i^+ = \max_i Z_i$ (12)

and γ depicts the decision making mechanism coefficient. If $\gamma > 0.5$, it is for “the maximum group utility”; If $\gamma < 0.5$, it is “the minimum regret”; and it is both if $\gamma = 0.5$.

Step: 7. Rank the priority of alternatives

Rank the alternatives by $\phi_i,$ and Γ_i according to the rule of traditional VIKOR strategy. The smaller value reflects the better alternative.

4. Illustrative example

To demonstrate the feasibility, applicability and effectiveness of the proposed strategy, we solve a MAGDM problem adapted from [51]. We assume that an investment company wants to invest a sum of money in the best option. The investment company forms a decision making board involving of three members (E_1, E_2, E_3) who evaluate the four alternatives to invest money. The alternatives are Car company (Φ_1), Food company (Φ_2), Computer company (Φ_3) and Arms company (Φ_4). Decision makers take decision to evaluate alternatives based on the attributes namely, risk factor (Ψ_1), growth factor (Ψ_2), environment impact (Ψ_3). We consider three criteria as benefit type based on Pramanik et al. [58]. Assume that the weight vector of attributes is $W = (0.36, 0.37, 0.27)^T$ and weight vector of decision makers or experts is $\zeta = (0.26, 0.40, 0.34)^T$. Now, we apply the proposed MAGDM strategy using the following steps.

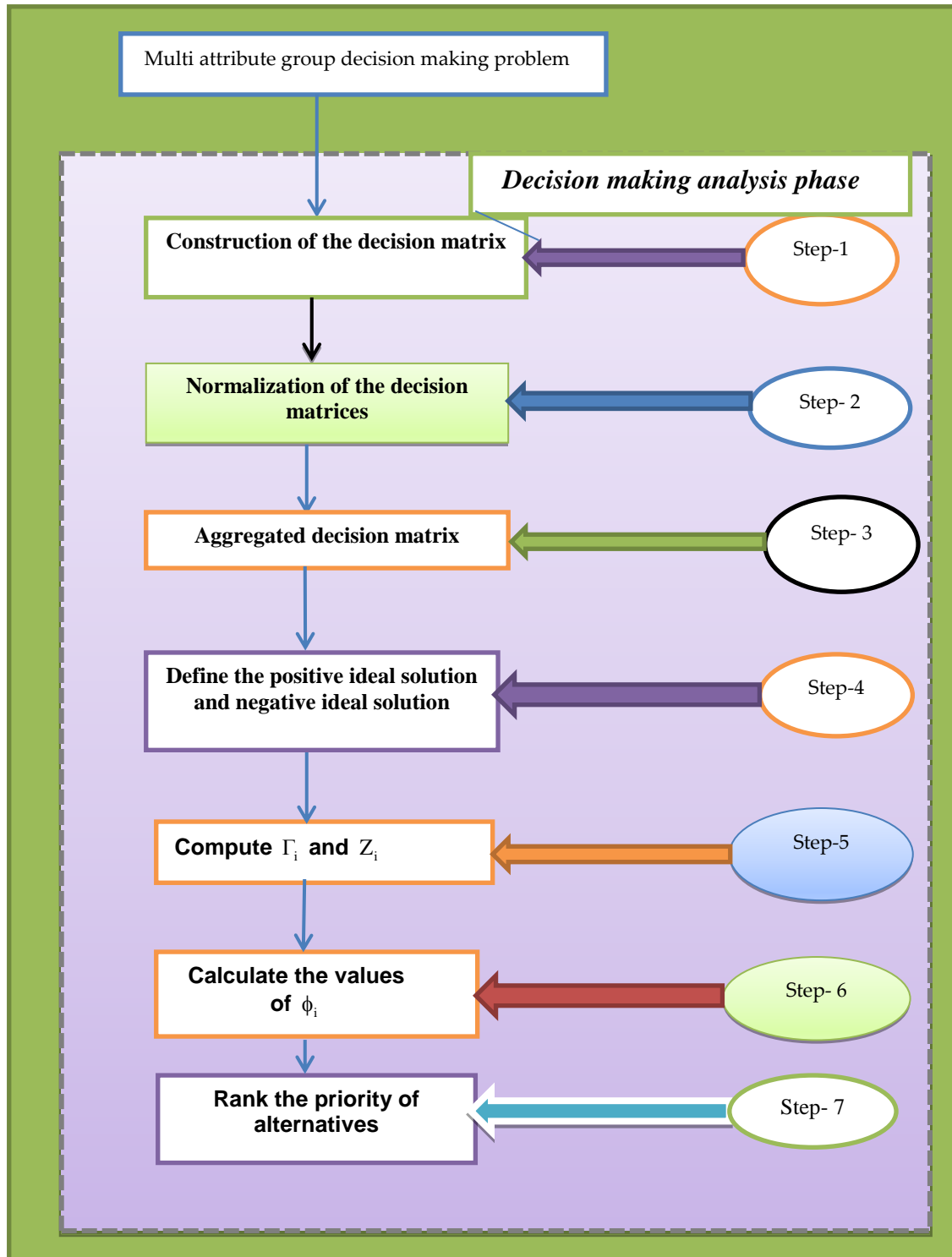


Figure.1 Decision making procedure of proposed MAGDM method

Step: 1. Construction of the decision matrix

We construct the decision matrices as follows:

Decision matrix for DM¹ in NCN form

$$\begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ \Phi_1 < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > \\ \Phi_2 < [6, .8], [2, .3], [2, .4], (.8, .3, .4) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \\ \Phi_3 < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [6, .8], [2, .3], [2, .4], (.8, .3, .4) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > \\ \Phi_4 < [3, .4], [5, .6], [5, .7], (.4, .6, .7) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \end{pmatrix} \tag{13}$$

Decision matrix for DM² in NCN form

$$\begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ \Phi_1 < [3, .4], [5, .6], [5, .7], (.4, .6, .7) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \\ \Phi_2 < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \\ \Phi_3 < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > \\ \Phi_4 < [6, .8], [2, .3], [2, .4], (.8, .3, .4) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \end{pmatrix} \tag{14}$$

Decision matrix for DM³ in NC-number form

$$\begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ \Phi_1 < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > \\ \Phi_2 < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > \\ \Phi_3 < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [6, .8], [2, .3], [2, .4], (.8, .3, .4) > < [6, .8], [2, .3], [2, .4], (.8, .3, .4) > \\ \Phi_4 < [7, .9], [1, .2], [1, .2], (.9, .2, .2) > < [4, .5], [4, .5], [4, .5], (.5, .5, .5) > < [3, .4], [5, .6], [5, .7], (.4, .6, .7) > \end{pmatrix} \tag{15}$$

Step: 2. Normalization of the decision matrix

Since all the criteria are considered as benefit type, we do not need to normalize the decision matrices (DM¹, DM², DM³).

Step: 3. Aggregated decision matrix

Using equation eq. (5), the aggregated decision matrix of (13, 14, 15) is presented below:

$$\begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ \Phi_1 < [44, .56], [36, .46], [36, .51], (.56, .46, .50) > < [48, .60], [32, .42], [32, .42], (.60, .42, .42) > < [62, .80], [18, .28], [18, .28], (.80, .28, .28) > \\ \Phi_2 < [45, .58], [35, .45], [35, .47], (.58, .45, .47) > < [50, .64], [30, .40], [30, .40], (.64, .40, .40) > < [60, .76], [20, .30], [20, .30], (.76, .30, .30) > \\ \Phi_3 < [62, .80], [18, .28], [18, .28], (.80, .28, .28) > < [64, .84], [16, .26], [16, .32], (.84, .26, .32) > < [47, .60], [33, .43], [33, .47], (.60, .43, .47) > \\ \Phi_4 < [56, .73], [24, .34], [24, .41], (.73, .34, .41) > < [40, .50], [40, .50], [40, .50], (.50, .50, .50) > < [56, .73], [24, .34], [24, .37], (.73, .34, .37) > \end{pmatrix} \tag{16}$$

Step: 4. Define the positive ideal solution and negative ideal solution

The positive ideal solution a_{ij}⁺ =

$$\begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ < [62, .80], [18, .28], [18, .28], (.80, .28, .28) > < [64, .84], [16, .26], [16, .32], (.84, .26, .32) > < [62, .80], [18, .28], [18, .28], (.80, .28, .28) > \end{pmatrix}$$

and the negative ideal solution

$$a_{ij}^- = \begin{pmatrix} \Psi_1 & \Psi_2 & \Psi_3 \\ < [44, .56], [36, .46], [36, .51], (.56, .46, .50) > < [40, .50], [40, .50], [40, .50], (.50, .50, .50) > < [47, .60], [33, .43], [33, .43], (.60, .43, .47) > \end{pmatrix}$$

Step: 5. Compute and Γ_i Z_i

Using Equation (9) and Equation (10), we obtain

$$\begin{aligned} \Gamma_1 &= \left(\frac{0.36 \times 0.2}{0.37}\right) + \left(\frac{0.37 \times 0.16}{0.25}\right) + \left(\frac{0.27 \times 0}{0.16}\right) = 0.43, \\ \Gamma_2 &= \left(\frac{0.36 \times 0.18}{0.37}\right) + \left(\frac{0.37 \times 0.14}{0.25}\right) + \left(\frac{0.27 \times 0.02}{0.16}\right) = 0.42, \\ \Gamma_3 &= \left(\frac{0.36 \times 0}{0.37}\right) + \left(\frac{0.37 \times 0}{0.25}\right) + \left(\frac{0.27 \times 0.19}{0.16}\right) = 0.32, \\ \Gamma_4 &= \left(\frac{0.36 \times 0.08}{0.37}\right) + \left(\frac{0.37 \times 0.25}{0.25}\right) + \left(\frac{0.27 \times 0.07}{0.16}\right) = 0.57. \end{aligned}$$

And

$$\begin{aligned} Z_1 &= \max \left\{ \left(\frac{0.36 \times 0.2}{0.37}\right), \left(\frac{0.37 \times 0.16}{0.25}\right), \left(\frac{0.27 \times 0}{0.16}\right) \right\} = 0.24, \\ Z_2 &= \max \left\{ \left(\frac{0.36 \times 0.18}{0.37}\right), \left(\frac{0.37 \times 0.14}{0.25}\right), \left(\frac{0.27 \times 0.02}{0.16}\right) \right\} = 0.21, \\ Z_3 &= \max \left\{ \left(\frac{0.36 \times 0}{0.37}\right), \left(\frac{0.37 \times 0}{0.25}\right), \left(\frac{0.27 \times 0.19}{0.16}\right) \right\} = 0.32, \\ Z_4 &= \max \left\{ \left(\frac{0.36 \times 0.08}{0.37}\right), \left(\frac{0.37 \times 0.25}{0.25}\right), \left(\frac{0.27 \times 0.07}{0.16}\right) \right\} = 0.37. \end{aligned}$$

Step: 6. Calculate the values of ϕ_i

Using Equations (11), (12) and $\gamma=0.5$, we obtain

$$\phi_1 = 0.5 \times \frac{(0.43 - 0.32)}{0.25} + 0.5 \times \frac{(0.24 - 0.21)}{0.16} = 0.31,$$

$$\phi_2 = 0.5 \times \frac{(0.42 - 0.32)}{0.25} + 0.5 \times \frac{(0.21 - 0.21)}{0.16} = 0.2,$$

$$\phi_3 = 0.5 \times \frac{(0.32 - 0.32)}{0.25} + 0.5 \times \frac{(0.32 - 0.21)}{0.16} = 0.34,$$

$$\phi_4 = 0.5 \times \frac{(0.57 - 0.32)}{0.25} + 0.5 \times \frac{(0.37 - 0.21)}{0.16} = 1.$$

Step: 7. Rank the priority of alternatives

The preference order of the alternatives based on the traditional rules of the VIKOR strategy is $\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$.

5. The influence of parameter γ

Table 1 shows how the ranking order of alternatives (Φ_i) changes with the change of the value of γ

Values of γ	Values of ϕ_i	Preference order of alternatives
$\gamma = 0.1$	$\phi_1 = 0.22, \phi_2 = \mathbf{0.04}, \phi_3 = 0.62, \phi_4 = 1$	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$
$\gamma = 0.2$	$\phi_1 = 0.24, \phi_2 = \mathbf{0.08}, \phi_3 = 0.55, \phi_4 = 1$	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$
$\gamma = 0.3$	$\phi_1 = 0.26, \phi_2 = \mathbf{0.12}, \phi_3 = 0.48, \phi_4 = 1$	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$
$\gamma = 0.4$	$\phi_1 = 0.29, \phi_2 = \mathbf{0.16}, \phi_3 = 0.41, \phi_4 = 1$	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$
$\gamma = 0.5$	$\phi_1 = 0.31, \phi_2 = \mathbf{0.2}, \phi_3 = 0.34, \phi_4 = 1$	$\Phi_2 \succ \Phi_1 \succ \Phi_3 \succ \Phi_4$
$\gamma = 0.6$	$\phi_1 = 0.34, \phi_2 = \mathbf{0.24}, \phi_3 = 0.28, \phi_4 = 1$	$\Phi_2 \succ \Phi_3 \succ \Phi_1 \succ \Phi_4$
$\gamma = 0.7$	$\phi_1 = 0.36, \phi_2 = 0.28, \phi_3 = \mathbf{0.21}, \phi_4 = 1$	$\Phi_3 \succ \Phi_2 \succ \Phi_1 \succ \Phi_4$
$\gamma = 0.8$	$\phi_1 = 0.39, \phi_2 = 0.32, \phi_3 = \mathbf{0.14}, \phi_4 = 1$	$\Phi_3 \succ \Phi_2 \succ \Phi_1 \succ \Phi_4$
$\gamma = 0.9$	$\phi_1 = 0.42, \phi_2 = 0.36, \phi_3 = \mathbf{0.07}, \phi_4 = 1$	$\Phi_3 \succ \Phi_2 \succ \Phi_1 \succ \Phi_4$

Table1. Values of ϕ_i ($i = 1, 2, 3, 4$) and ranking of alternatives for different values of γ .

Figure 2 represents the graphical representation of alternatives (A_i) versus ϕ_i ($i = 1, 2, 3, 4$) for different values of γ .

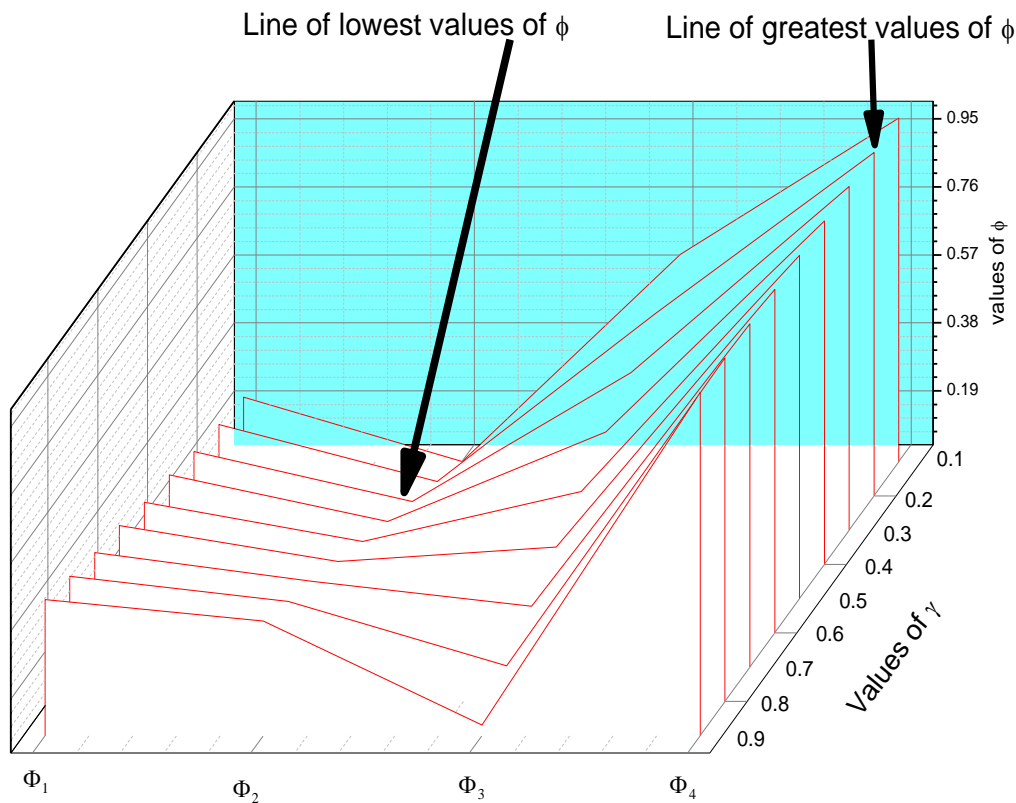


Fig 2. Graphical representation of ranking of alternatives for different values of γ .

6. Conclusions

In this paper, we have extended the traditional VIKOR strategy to NC-VIKOR. We introduced neutrosophic cubic numbers weighted aggregation (NCNWA) operator and applied it to aggregate the individual opinion to group opinion prove its three properties. We developed a novel NC-VIKOR based MAGDM strategy in neutrosophic cubic set environment. Finally, we solve a MAGDM problem to show the feasibility, applicability and efficiency of the proposed MAGDM strategy. We present a sensitivity analysis to show the impact of different values of the decision making mechanism coefficient on ranking order of the alternatives. The proposed NC-VIKOR based MAGDM strategy can be employed to solve a variety of problems such as logistics center selection [28, 74], teacher selection [75], renewable energy selection[70], fault diagnosis[71], brick selection [76, 77], weaver selection [78], etc.

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