



# Law of Included Infinitely-Many-Middles within the frame of Neutrosophy

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**Abstract:** In this paper, we extend, for the first time, the Law of Included Multiple-Middles to the Law of Infinitely-Many-Middles. And we present several practical applications. Also, we discuss Aristotle's Syllogism, Principle of Identity, and Principle of NonContradiction.

**Keywords:** Excluded Middle; Included Middle; Included Multiple-Middles; Included Infinitely-Many-Middles; Syllogism; Many-Valued Syllogism; Identity; NonIdentity; NonContradiction, Anti-NonContradiction.

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## A. Short History

We present below the evolution from the Law of Excluded Middle to the Law of Included Infinitely-Many-Multiples.

### 1. Law of Excluded Middle

The **Law of Excluded Middle** was enounced by the Ancient Greek philosopher Aristotle (384 - 322 BC) on his opus on logic and reasoning [1, 2] that was based on analysis and dichotomy:

"There cannot be an indeterminate between contraries, but of one subject we must either affirm or deny anyone predicate".

Therefore, a proportion is either 100% true or 100% false, as in Boolean logic. Or, an element either belong 100% to a set, or does not belong 100% to the set (as in the classical set theory).

### 2. Law of Included Middle

The **Law of Included Middle** is the denial of previous, and it supports the idea that between contraries there may be a middle. It is based on trichotomy. Several philosophers and logicians developed it, such as Stephane Lupasco's logic of contradiction, using the non-standard logic, followed by Basarab Nicolescu's levels of reality, and J.-J. Wunenburger. Gonthier pleads for a low necessity in using the logic of contradiction. [5]

With the introduction of modern sets and logics, such as fuzzy set/logic (Zadeh, 1965), intuitionistic fuzzy set/logic (Atanassov, 1983), neutrosophic set/logic/probability (Smarandache, 1995), the Law of Included Middle became evident and useful in our everyday life where we deal with approximate partial membership/non-membership/truth/falsehood while in neutrosophic probability besides the chance of occurrence of an event, there has been added the middle term: indeterminate-chance of occurrence or not.

Neutrosophic set and logic explicitly presented the *middle term I* (indeterminacy or neutrality) in between the opposite terms (membership/truth), and *F* (non-membership/falsehood).

### 3. Law of Included Multiple-Middles

The **Law of Included Multiple-Middles** is an extension of the previous, and it was enounced by Smarandache [4] in 2014.

Neutrosophy [6] is a branch of philosophy that studies the dynamics of the opposites  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  together with their neutrality  $\langle \text{neut}A \rangle$ , where  $\langle A \rangle$  is an item (idea, proposition, theory, etc.),  $\langle \text{anti}A \rangle$  is its opposite, while  $\langle \text{neut}A \rangle$  is the neutrality in between them (i.e. neither  $\langle A \rangle$  nor  $\langle \text{anti}A \rangle$ ).

Of course, we are referring to the neutrosophic triads  $\langle A \rangle$ ,  $\langle \text{neut}A \rangle$ ,  $\langle \text{anti}A \rangle$  that make sense in our real world.

Neutrosophy, together with Neutrosophic Set/logic/probability, have been refined [7] in 2013, by refining splitting/multiplicating  $\langle A \rangle$ ,  $\langle \text{neut}A \rangle$ ,  $\langle \text{anti}A \rangle$  as follows:

$\langle A \rangle$  as  $\langle A_1 \rangle$ ,  $\langle A_2 \rangle$ , ...,  $\langle A_p \rangle$ ;

$\langle \text{neut}A \rangle$  as  $\langle \text{neut}A_1 \rangle$ ,  $\langle \text{neut}A_2 \rangle$ , ...,  $\langle \text{neut}A_r \rangle$ ;

and  $\langle \text{anti}A \rangle$  as  $\langle \text{anti}A_1 \rangle$ ,  $\langle \text{anti}A_2 \rangle$ , ...,  $\langle \text{anti}A_s \rangle$ ;

where  $p, r, s \geq 0$  are integers,

and at least one of  $p, r, s \geq 1$  in order to ensure that at least one neutrosophic component amongst  $\langle A \rangle$ ,  $\langle \text{neut}A \rangle$ ,  $\langle \text{anti}A \rangle$  is refined/split/multiplied.

This definition also permits the refinement of fuzzy set/logic (for  $p \geq 2$  and  $r = s = 0$ ), and of intuitionistic fuzzy set/logic (for  $p \geq 1, r = 0, s \geq 1$  and at least one of  $p$  or  $s \geq 2$ ).

By taking  $p = 1, r \geq 2$ , and  $s = 1$ , we defined the Law of Included Multiple-Middles:

Between the opposites  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  there are included multiple-middles:  $\langle \text{neut}A_1 \rangle$ ,  $\langle \text{neut}A_2 \rangle$ , ...,  $\langle \text{neut}A_r \rangle$ .

It is of course based on a multichotomical analysis.

i) Between the opposite colors White and Black there are many colors such as: yellow, rose, red, blue, etc.

ii) Pentagonal Neutrosophic logic, where each proposition is characterized by five degrees of truth, such as  $(T, C, V, U, F)$

where the opposites are:

$T$ =degree of truth and  $F$ =degree of falsehood,

and the three included-middles are:

$C$ =degree of contradiction

$V$ =degree of vagueness

$U$ =degree of unknowingness

For example, the logical proposition:

$P$ =Artificial intelligence will take over the world, evaluated by experts, this proposition may be 40% true ( $T$ ), 20% contradictory ( $C$ ), 30% vague ( $V$ ), 60% unknown ( $U$ ) and 50% false ( $F$ ).

$P(0.4, 0.2, 0.3, 0.6, 0.5)$ .

### 4. Law of Included Infinitely-Many-Middles

In between the opposites  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  there are infinitely many middles, denoted by

$\langle \text{neut}A_i \rangle, i=1, 2, \dots, \infty$ .

(i) Practical Example

Between the White and Black colors there are infinitely-many nuances of colors.

(ii) Between 100% True and 100% False, there are included infinitely many middles, which are truth-values of the form:  $d\%$  True and  $(1-d)\%$  False, thus a logical proposition may be, for example:

1% True and 99% False, 2% True and 98% False, etc.

(iii) Similarly, between 100% membership and 100% non-membership, there are included infinitely many middles of the form:  $d\%$  membership and  $(1-d)\%$  non-membership.

### 5. Syllogism

Aristotle studied it:

if " $A \rightarrow B$ " and " $B \rightarrow C$ " are totally true, then " $A \rightarrow C$ " is also totally true.

This is in classical logic.

## 6. Many-Valued Syllogism

In many-valued logics, where " $A \rightarrow B$ " and " $B \rightarrow C$ " are partially true, then " $A \rightarrow C$ " is partially true as well.

## 7. Principle of NonContradiction

It was enounced by Aristotle [1, 2], that  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  cannot be true at the same time:

"the same attribute cannot at the same time belong and not belong to the same subject and in the same respect",

and

"it is impossible for anyone to believe the same thing to be and not to be, as something Heraclitus says".

## 8. Principle of Anti NonContradiction

We name this principle as "Anti Contradiction" that occur in the many-valued logics in order to distinguish it from the Principle of NonContradiction (also called Contradiction).

The above principle, related to the Law of Excluded Middle, does not work any longer in the modern theories. Again, with the introduction of modern set theories, it is possible to have both, degree of belonging and degree of not-belonging simultaneously of an element to a set, for example John (0.6, 0.4), meaning that John belongs (works) only 60% for his company and 40% does not. And similarly with respect to the modern logics, where a logical proposition may be partially true and partially false.

In fuzzy and fuzzy extension theories (except neutrosophic theories),  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  may be partially (not totally) true at the same time.

In fuzzy logic, if a proposition  $P$  is 50% true, then its negation  $\neg P$  is also  $100\% - 50\% = 50\%$  true.

In neutrosophic logic, if a proposition has the truth-value  $P$  is  $(a, 0.5, a)$ , where  $0 \leq a \leq 1$ , then its negation  $\neg P$  is also  $(a, 1 - 0.5, a) = (a, 0.5, a)$ .

In neutrosophic theories,  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  may be partially or totally true at the same time. For example, a paradox is proposition that is 100% true and 100% false at the same time, therefore  $\langle A \rangle$  and  $\langle \text{anti}A \rangle$  are totally true at the same time.

## 9. Principle of Identity

As enounced by Aristotle:

$A=A$  (an item is equal to itself).

This is true if one considers the item  $\langle A \rangle$  under the same parameters that characterize it, and having the same corresponding values:

$$A(P_1 = v_1, P_2 = v_2, \dots, P_n = v_n) = A(P_1 = v_1, P_2 = v_2, \dots, P_n = v_n).$$

## 10. Principle of NonIdentity

The Principle of Identity, by Aristotle, that  $A = A$ , works when the entity  $A$  is compared to itself with respect to the same parameters that characterize  $A$ , with each parameter measured at the same scale and on the same time.

But, if the parameters that characterize  $A$  are different, or their corresponding values are different, then one has non-equality.

For example, if  $A = \text{Andrew}$ , then  $\text{Andrew}(\text{at age } 5) \neq \text{Andrew}(\text{at age } 70)$  physically, intellectually, and psychically.

As such, one may also define a Principle of NonIdentity, when A is different from A in at least one circumstance. And, in general, an item (person, animal, object, etc.) is not equal to itself at different times:

$$\text{item (at time } t1) \neq \text{item (at time } t2).$$

## B. Conclusion

We have presented the Law of Excluded Middle by Aristotle, then the Law of Included Middle, Law of Included Multiple-Middles, and we introduced for the first time the Law of Included Infinitely-Many-Middles.

Afterwards, several comments we made on Aristotle's Syllogism, Principle of NonContradiction, and Principle of Identity, that, in the many-valued logics, may have degrees of partial truth and partial falsehood even partial indeterminacy - depending on each application.

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