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Convert between neutrosophic complex numbers forms

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Abstract: Complex numbers have been studied in previous papers, but the papers did not deal with the conversion from algebraic form of a neutrosophic complex number form to exponential or trigonometric form and vice versa, and this prompted us to search for a method that facilitates this conversion process, as this paper dealt with how to move from algebraic form of a neutrosophic complex number to exponential or trigonometric form and vice versa, also, we discussed the roots from order n of a neutrosophic complex number, whether this number is given in algebraic or trigonometric form.

Keywords: neutrosophic complex numbers, algebraic form, roots of a neutrosophic complex number, exponential form.

1. Introduction

As Smarandache proposed the Neutrosophic Logic as an alternative to the existing logics to represent a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, and contradiction. Smarandache introduced the concept of neutrosophy as a new school of philosophy [4][8]. He presented the definition of the standard form of neutrosophic real number and conditions for the division of two neutrosophic real numbers to exist [3][5], studying the concept of the Neutrosophic probability [4][6], the Neutrosophic statistics [5][7], and professor Smarandache entered the concept of preliminary calculus of the differential and integral calculus, where he introduced for the first time the notions of neutrosophic mereo-limit, mereo-continuity, mereoderivative, and mereo-integral [1][8]. Y.Alhasan presented the definition of the concept of neutrosophic complex numbers and its properties including the conjugate of neutrosophic complex number, division of neutrosophic complex numbers, the inverted neutrosophic complex number and the absolute value of a neutrosophic complex number and Theories related to the conjugate of neutrosophic complex numbers, the product of a neutrosophic complex number by its conjugate equals the absolute value of number and he studied the general exponential form of a neutrosophic complex number [2-11]. Madeleine Al- Taha presented results on single valued neutrosophic (weak) polygroups [10]. An algebraic approach to neutrosophic euclidean geometry is presented [12].

Complex numbers play a significant role in daily life because they make it much easier to perform mathematical operations and give us a way to solve equations for which there are no real-number-group solutions. The electrical engineering field makes extensive use of complex numbers to calculate electric voltage and measure alternating current.

Paper is divided into four pieces. provides an introduction in the first portion, which includes a review of neutrosophic science. A few definitions and hypotheses of a neutrosophic complex number are covered in the second section. The third section describes the transformation of a neutrosophic complex number from its exponential or trigonometric form to its algebraic form, and vice versa. The paper's conclusion is provided in the fourth section.

2. Preliminaries

2.1. The general exponential form of a neutrosophic complex number [11]

Theorem 1

The general exponential form of the neutrosophic complex number is given by the formula:

$$z = |z|e^{i(\theta+\vartheta I)} = re^{i(\theta+\vartheta I)}$$

Whereas $r = |z|$ is the absolute value of a neutrosophic complex number.

2.2 The general Trigonometric form of a neutrosophic complex number [11]

Definition 1

The following formula:

$$z = \mathrm{r} \ \ (\cos(\theta + \vartheta I) + sin \ \ (\theta + \vartheta I) \ \ i)$$

is called the general trigonometric form of a neutrosophic complex number

Definition 2 [12]

Let f: $R(I) \ \rightarrow \ R(I)$; $f = f(X)$ and $X = x + yI \in R(I)$ the f is called a neutrosophic real function with one neutrosophic variable. a neutrosophic real function $f(X)$ written as follows:

$$f(X) = f(x + yI) = f(x) + I[f(x + y) - f(x)]$$

3. Conversion from exponential or trigonometric form of a neutrosophic complex number to algebraic form

$$z = \acute{r}e^{i(\acute{\theta}+\acute{\vartheta} I)}$$

$$= \acute{r}\left(\ cos\left(\acute{\theta} + \acute{\vartheta} I\right) + i \ \ sin(\acute{\theta} + \acute{\vartheta} I)\right)$$

$$= \acute{r}\left(\ cos\left(\acute{\theta}\right) + I\left[cos\left(\acute{\theta} + \acute{\vartheta}\right) - \ cos\left(\acute{\theta}\right)\right] + i \ \ sin\left(\acute{\theta}\right) + i \ \ I\left[sin\left(\acute{\theta} + \acute{\vartheta}\right) - \ sin\left(\acute{\theta}\right)\right]\right)$$

Whereas $\left(\acute{\theta} + \acute{\vartheta} I\right)$ is the indeterminate angle between two indeterminate parts of the coordinate axes ($x - axis \ \ and \ \ y - axis$).

Example 1

$$z = 4e^{i\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)}$$

$$= 4\left(cos\left(\frac{\pi}{6}+\frac{\pi}{3}I\right) + i \quad sin\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)\right)$$

$$= 4\left(cos\frac{\pi}{6} + I\left[cos\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - cos\frac{\pi}{6}\right] + i\,sin\frac{\pi}{6} + i \quad I\left[sin\left(\frac{\pi}{6}+\frac{\pi}{3}\right) - sin\frac{\pi}{6}\right]\right)$$

$$= 4\left(\frac{\sqrt{3}}{2} + I\left[0 - \frac{\sqrt{3}}{2}\right] + i\frac{1}{2} + i \quad I\left[1 - \frac{1}{2}\right]\right)$$

$$= 2\sqrt{3} - 2\sqrt{3} \quad I + (2 + 2 \quad I)i$$

Example 2

$$z = \frac{1}{6}e^{i(\pi - 2\pi I)}$$

$$= \frac{1}{6}\left(cos(\pi - 2\pi I) + i \quad sin(\pi - 2\pi I)\right)$$

$$= \frac{1}{6}(cos\pi + I[cos(\pi - 2\pi) - cos\,\pi] + i\,sin\,\pi + i \quad I[sin(\pi - 2\pi) - sin\pi])$$

$$= \frac{1}{6}(-1 + I[-1 + 1] + i(0) + i \quad I[0 - 0]) = -\frac{1}{6}$$

3.1 Conversion from algebraic form of a neutrosophic complex number to exponential or trigonometric form

Let $z = \ddot{a} + \ddot{c}I + \ddot{b}i + \ddot{d}iI$, then:

$$\acute{r} = \sqrt{(\ddot{a} + \ddot{c}I)^2 + \left(\ddot{b} + \ddot{d}I\right)^2}$$

$$cos\left(\acute{\theta} + \acute{\vartheta}I\right) = \frac{\ddot{a} + \ddot{c}I}{\acute{r}}$$

$$sin\left(\acute{\theta} + \acute{\vartheta}I\right) = \frac{\ddot{b} + \ddot{d}I}{\acute{r}}$$

We can find angle whereas: $\acute{\theta} + \acute{\vartheta}I = \acute{\mathfrak{J}} + \acute{\varphi}I + 2\pi k \quad ; k \in Z$
hence:

$$z = \acute{r}e^{i(\acute{\mathfrak{J}} + \acute{\varphi}I)}$$

Example 3

$$z = \left(\frac{-1}{\sqrt{2}} + \frac{2}{\sqrt{2}}I\right) + \left(\frac{1}{\sqrt{2}}\right)i$$

$$\acute{r} = \sqrt{(\ddot{a} + \ddot{c}I)^2 + \left(\ddot{b} + \ddot{d}I\right)^2}$$

$$= \sqrt{\left(\frac{-1}{\sqrt{2}} + \frac{2}{\sqrt{2}}I\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$$

$$= \sqrt{\left(\frac{1}{2} - 2\frac{1}{\sqrt{2}}\frac{2}{\sqrt{2}}I + 2I\right) + \frac{1}{2}} = 1$$

then:

$$cos(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{-1}{\sqrt{2}}+\frac{2}{\sqrt{2}}I$$

$$sin(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad \acute{\theta}+\acute{\vartheta}I=\frac{3\pi}{4}+\frac{3\pi}{2}I$$

hence:

$$z=e^{i\left(\frac{3\pi}{4}+\frac{3\pi}{2}I\right)}$$

to check the solution: if we take some values of I, we get on the following

#	$z=e^{i\left(\frac{3\pi}{4}+\frac{3\pi}{2}I\right)}$	$cos\left(\frac{3\pi}{4}+\frac{3\pi}{2}I\right)=\frac{-1}{\sqrt{2}}+\frac{2}{\sqrt{2}}I$	$sin\left(\frac{3\pi}{4}+\frac{3\pi}{2}I\right)=\frac{1}{\sqrt{2}}$	**Result**
$I=0$	$z=e^{i\left(\frac{3\pi}{4}\right)}$	$cos\left(\frac{3\pi}{4}\right)=\frac{-1}{\sqrt{2}}$	$sin\left(\frac{3\pi}{4}\right)=\frac{1}{\sqrt{2}}$	✓
$I=1$	$z=e^{i\left(\frac{9\pi}{4}\right)}$	$cos\left(\frac{9\pi}{4}\right)=\frac{1}{\sqrt{2}}$	$sin\left(\frac{9\pi}{4}\right)=\frac{1}{\sqrt{2}}$	✓

and so on…

Example 4

Let:

$$z_1=\frac{\sqrt{3}-\sqrt{3}\ I}{2}+\frac{1+I}{2}i \quad and \quad z_2=\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I\right)\ i$$

Write of each z_1, z_2 by exponential form.

Solution:

$$z_1=\frac{\sqrt{3}-\sqrt{3}\ I}{2}+\frac{1+I}{2}i$$

$$\acute{r}_1=\sqrt{(\ddot{a}+\ddot{c}I)^2+\left(\ddot{b}+\ddot{d}I\right)^2}$$

$$=\sqrt{\left(\frac{\sqrt{3}-\sqrt{3}\ I}{2}\right)^2+\left(\frac{1+I}{2}\right)^2}$$

$$=\sqrt{\left(\frac{3-6I+3I}{4}\right)+\frac{1+2I+I}{4}}=1$$

then:

$$cos(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{\sqrt{3}-\sqrt{3}\ I}{2}=\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}I$$

$$sin(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{1+I}{2}=\frac{1}{2}+\frac{1}{2}I$$

$$\Rightarrow \qquad \acute{\theta}+\acute{\vartheta}I=\frac{\pi}{6}+\frac{\pi}{3}I$$

hence:

$$z_1=e^{i\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)}$$

to check the solution: if we take some values of I, we get on the following

#	$z=e^{i\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)}$	$cos\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)=\frac{\sqrt{3}}{2}-\frac{\sqrt{3}}{2}I$	$sin\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)=\frac{1}{2}+\frac{1}{2}I$	**Result**
$I=0$	$z=e^{i\left(\frac{\pi}{6}\right)}$	$cos\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}$	$sin\left(\frac{\pi}{6}\right)=\frac{1}{2}$	✓
$I=1$	$z=e^{i\left(\frac{\pi}{2}\right)}$	$cos\left(\frac{\pi}{2}\right)=0$	$sin\left(\frac{\pi}{2}\right)=1$	✓

and so on…

now:

$$z_2=\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I\right)\ i$$

$$\acute{r}_2=\sqrt{(\ddot{a}+\ddot{c}I)^2+(\ddot{b}+\ddot{d}I)^2}$$

$$=\sqrt{\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I\right)^2+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I\right)^2}$$

$$=\sqrt{\left(\frac{6+4\sqrt{3}\ \ +2+2I-4\sqrt{3}\ \ I+6I}{16}-\frac{8I}{16}\right)+\left(\frac{6-4\sqrt{3}\ \ +2+18I-12\sqrt{3}\ \ I+6I}{16}+\frac{16\sqrt{3}\ \ I-24I}{16}\right)}$$
$$=1$$

then:

$$cos(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I$$

$$sin(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I$$

$$\Rightarrow \qquad \acute{\theta}+\acute{\vartheta}I=\frac{\pi}{12}+\frac{\pi}{6}I$$

hence:

$$z_2=e^{i\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)}$$

to check the solution: if we take some values of I, we get on the following

#	$\boldsymbol{z=e^{i\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)}}$	$\boldsymbol{\cos\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)=\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I}$	$\boldsymbol{\sin\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)=\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I}$	**Result**
$\boldsymbol{I=0}$	$z=e^{i\left(\frac{\pi}{12}\right)}$	$\cos\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}+\sqrt{2}}{4}$	$\sin\left(\frac{\pi}{12}\right)=\frac{\sqrt{6}-\sqrt{2}}{4}$	✓
$\boldsymbol{I=1}$	$z=e^{i\left(\frac{\pi}{4}\right)}$	$\cos\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	$\sin\left(\frac{\pi}{4}\right)=\frac{\sqrt{2}}{2}$	✓

and so on…

Example 5

Let:

$$z_1=\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I+\left(\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I\right)i \qquad and \qquad z_2=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I\right)i$$

a) Write of each $z_1, z_2, \frac{z_1}{z_2}$ by exponential and trigonometric form.

b) Write $\frac{z_1}{z_2}$ by algebraic form.

c) conclude that:

$$\cos\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)=\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{2\sqrt{3}-\sqrt{2}-\sqrt{6}}{4}I$$

$$\sin\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)=\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{2+\sqrt{2}-\sqrt{6}}{4}I$$

Solution:

a)

$$z_1=\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I+\left(\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I\right)i$$

$$\acute{r}_1=\sqrt{(\ddot{a}+\ddot{c}I)^2+\left(\ddot{b}+\ddot{d}I\right)^2}$$

$$=\sqrt{\left(\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I\right)^2+\left(\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I\right)^2}=1$$

then:

$$\cos\left(\acute{\theta}+\acute{\vartheta}I\right)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I$$

$$\sin\left(\acute{\theta}+\acute{\vartheta}I\right)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I$$

$$\Longrightarrow \qquad \acute{\theta}+\acute{\vartheta}I=-\frac{\pi}{6}-\frac{\pi}{6}I$$

hence:

$$z_1=e^{-i\left(\frac{\pi}{6}+\frac{\pi}{6}I\right)}$$

now:

$$z_2=\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I\right)i$$

$$\acute{r}_2=\sqrt{(\ddot{a}+\ddot{c}I)^2+\left(\ddot{b}+\ddot{d}I\right)^2}$$

$$=\sqrt{\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)^2+\left(\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I\right)^2}=1$$

then:

$$cos\left(\acute{\theta}+\acute{\vartheta}I\right)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I$$

$$sin\left(\acute{\theta}+\acute{\vartheta}I\right)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I$$

$$\Longrightarrow\qquad \acute{\theta}+\acute{\vartheta}I=-\frac{\pi}{4}-\frac{\pi}{4}I$$

hence:

$$z_2=e^{-i\left(\frac{\pi}{4}+\frac{\pi}{4}I\right)}$$

$$\Longrightarrow\qquad \frac{z_1}{z_2}=\frac{e^{-i\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)}}{e^{-i\left(\frac{\pi}{4}+\frac{\pi}{4}I\right)}}=e^{-i\left(\frac{\pi}{6}+\frac{\pi}{3}I\right)+i\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)}=e^{i\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)}$$

b)

$$\frac{z_1}{z_2}=\frac{\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I+\left(\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I\right)i}{\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I\right)i}$$

$$=\frac{\left(\frac{\sqrt{3}}{2}+\frac{1+\sqrt{3}}{2}I+\left(\frac{-1}{2}+\frac{1-\sqrt{3}}{2}I\right)i\qquad\right)\left(\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{\sqrt{2}}{2}+\frac{2-\sqrt{2}}{2}I\right)i\right)}{\left(\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{-\sqrt{2}}{2}+\frac{-2+\sqrt{2}}{2}I\right)i\right)\left(\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}I\right)+\left(\frac{\sqrt{2}}{2}+\frac{2-\sqrt{2}}{2}I\right)i\right)}$$

$$\frac{z_1}{z_2}=\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{2\sqrt{3}-\sqrt{2}-\sqrt{6}}{4}I\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{2}+2-\sqrt{6}}{4}I\right)i \qquad (*)$$

c)

$$\frac{z_1}{z_2}=e^{i\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)}=cos\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)+i\ \ sin\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)$$

compared with $(*)$, we find:

$$cos\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)+i\ \ sin\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)$$
$$=\left(\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{2\sqrt{3}-\sqrt{2}-\sqrt{6}}{4}I\right)+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{2}+2-\sqrt{6}}{4}I\right)i$$

hence:

$$cos\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)=\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{2\sqrt{3}-\sqrt{2}-\sqrt{6}}{4}I$$

$$sin\left(\frac{\pi}{12}+\frac{\pi}{12}I\right)=\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{2}+2-\sqrt{6}}{4}I$$

3.2 The roots from order n of a neutrosophic complex number

The roots from order n of a complex number y are the set of complex numbers that satisfy:

$$z^n = z_0$$

let z be one of these roots whear:

$$z = \acute{r}e^{i(\acute{\theta}+\acute{\vartheta}I)}$$

then:

$$\acute{r}^n e^{in(\acute{\theta}+\acute{\vartheta}I)} = \acute{r}_0 e^{i(\acute{\theta}_0+\acute{\vartheta}_0 I)}$$

$$\acute{r}^n = \acute{r}_0 \qquad\qquad \Longrightarrow \qquad\qquad \acute{r} = \sqrt[n]{\acute{r}_0}$$

there is $k \in \mathbb{Z}$, where:

$$n\left(\acute{\theta}+\acute{\vartheta}I\right) = \acute{\theta}_0+\acute{\vartheta}_0 I + 2\pi k$$

$$\acute{\theta}+\acute{\vartheta}I = \frac{\acute{\theta}_0}{n}+\frac{\acute{\vartheta}_0 I}{n}+\frac{2\pi k}{n}$$

so the general formula for the roots is:

$$z_k = \sqrt[n]{\acute{r}_0}\ \ e^{i\left(\frac{\acute{\theta}_0}{n}+\frac{\acute{\vartheta}_0 I}{n}+\frac{2\pi k}{n}\right)} \qquad\qquad ;k=0\ \ ,1\ \ ,2\ \ ,\dots\ \ ,n-1$$

Example 6

Find the cube roots of:

$$z=\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{2}}I+\frac{1}{\sqrt{2}}i$$

Solution:

$$\acute{r} = \sqrt{(\ddot{a}+\ddot{c}I)^2+\left(\ddot{b}+\ddot{d}I\right)^2}$$

$$=\sqrt{\left(\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{2}}I\right)^2+\left(\frac{1}{\sqrt{2}}\right)^2}$$

$$=\sqrt{\left(\frac{1}{2}-2\frac{1}{\sqrt{2}}\frac{2}{\sqrt{2}}I+2I\right)+\frac{1}{2}}=1$$

then:

$$cos\left(\acute{\theta}+\acute{\vartheta}I\right)=\frac{\ddot{a}+\ddot{c}I}{\acute{r}}=\frac{1}{\sqrt{2}}-\frac{2}{\sqrt{2}}I$$

$$\sin(\acute{\theta}+\acute{\vartheta}I)=\frac{\ddot{b}+\ddot{d}I}{\acute{r}}=\frac{1}{\sqrt{2}}$$

$$\Longrightarrow \qquad \acute{\theta}+\acute{\vartheta}I=\frac{\pi}{4}+\frac{\pi}{2}I$$

hence:

$$z=e^{i\left(\frac{\pi}{4}+\frac{\pi}{2}I\right)}$$

$$z_k=\sqrt[n]{\acute{r}_0}\;\;e^{i\left(\frac{\acute{\theta}_0}{n}+\frac{\acute{\vartheta}_0 I}{n}+\frac{2\pi k}{n}\right)} \qquad ;k=0\;,1\;\;,2\;\;,\dots\;\;,n-1$$

$$z_k=\sqrt[3]{1}\;\;e^{i\left(\frac{\pi}{12}+\frac{\pi}{6}I+\frac{2\pi k}{3}\right)} \quad ; \quad k=0\;,1\;\;,2$$

$$k=0 \qquad \Longrightarrow \qquad z_0=\;e^{i\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)}=\cos\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)+i\;\;\sin\left(\frac{\pi}{12}+\frac{\pi}{6}I\right)$$

$$z_0$$

$$=\frac{\sqrt{6}+\sqrt{2}}{4}+\frac{\sqrt{2}-\sqrt{6}}{4}I+\left(\frac{\sqrt{6}-\sqrt{2}}{4}+\frac{3\sqrt{2}-\sqrt{6}}{4}I\right)i$$

$$k=1 \qquad \Longrightarrow \qquad z_1=\;e^{i\left(\frac{3\pi}{4}+\frac{\pi}{6}I\right)}=\cos\left(\frac{3\pi}{4}+\frac{\pi}{6}I\right)+i\;\;\sin\left(\frac{3\pi}{4}+\frac{\pi}{6}I\right)$$

$$=\cos\frac{3\pi}{4}+I\left[\cos\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)-\cos\frac{3\pi}{4}\right]+i\;\;\left(\sin\frac{3\pi}{4}+I\left[\sin\left(\frac{3\pi}{4}+\frac{\pi}{6}\right)-\sin\frac{3\pi}{4}\right]\right)$$

$$=\cos\frac{3\pi}{4}+I\left[\cos\frac{11\pi}{12}-\cos\frac{3\pi}{4}\right]+i\;\;\left(\sin\frac{3\pi}{4}+I\left[\sin\frac{11\pi}{12}-\sin\frac{3\pi}{4}\right]\right)$$

$$=\frac{-\sqrt{2}}{2}+I\left[\frac{-\sqrt{6}-\sqrt{2}}{4}+\frac{\sqrt{2}}{2}\right]+\left(\frac{\sqrt{2}}{2}+I\left[\frac{\sqrt{6}-\sqrt{2}}{4}-\frac{\sqrt{2}}{2}\right]\right)i$$

$$z_1=\frac{-\sqrt{2}}{2}+\frac{-\sqrt{6}+\sqrt{2}}{4}I+\left(\frac{\sqrt{2}}{2}+\frac{\sqrt{6}-3\sqrt{2}}{4}I\right)i$$

$$k=2 \qquad \Longrightarrow \qquad z_2=\;e^{i\left(\frac{17\pi}{12}+\frac{\pi}{6}I\right)}=\cos\left(\frac{17\pi}{12}+\frac{\pi}{6}I\right)+i\;\;\sin\left(\frac{17\pi}{12}+\frac{\pi}{6}I\right)$$

$$=\cos\frac{17\pi}{12}+I\left[\cos\left(\frac{17\pi}{12}+\frac{\pi}{6}\right)-\cos\frac{17\pi}{12}\right]+i\;\;\left(\sin\frac{17\pi}{12}+I\left[\sin\left(\frac{17\pi}{12}+\frac{\pi}{6}\right)-\sin\frac{17\pi}{12}\right]\right)$$

$$= cos\frac{17\pi}{12} + I\left[\cos\frac{19\pi}{12} - cos\frac{17\pi}{12}\right] + i\ \left(sin\frac{17\pi}{12} + I\left[sin\frac{19\pi}{12} - sin\frac{17\pi}{12}\right]\right)$$

$$= \frac{-\sqrt{6}+\sqrt{2}}{4} + I\left[\frac{\sqrt{6}-\sqrt{2}}{4} - \left(\frac{-\sqrt{6}+\sqrt{2}}{4}\right)\right] + \left(\frac{-\sqrt{6}-\sqrt{2}}{4} + I\left[\frac{-\sqrt{6}-\sqrt{2}}{4} - \left(\frac{-\sqrt{6}-\sqrt{2}}{4}\right)\right]\right)i$$

$$z_2 = \frac{-\sqrt{6}+\sqrt{2}}{4} + \frac{\sqrt{6}-\sqrt{2}}{2}I + \frac{-\sqrt{6}-\sqrt{2}}{4}i$$

4. Conclusions

The importance of this paper comes from the fact that it presented a scientific method for converting from the algebraic form of a neutrosophic complex number to the exponential or trigonometric form and vice versa. Where this method was harnessed to find the roots from order n of a neutrosophic complex number and write its by algebraic form of a neutrosophic complex number. This article is regarded as one of the key studies on neutrosophic complex numbers.

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