Neutrosophic Totally Semi-Continuous Functions

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Abstract: The principal objective of this article is to develop the perception of neutrosophic totally continuous and neutrosophic totally-semi-continuous functions in neutrosophic topological space using neutrosophic semi-open and neutrosophic semi-closed sets. Neutrosophic strongly semi-continuous and slightly semi-continuous functions have been presented and investigated. Some important properties of these functions are also given in neutrosophic topological spaces. Relations between these newly defined functions and other classes of neutrosophic functions are established.

Keywords: Neutrosophic semi-open set, Neutrosophic semi-closed set, Neutrosophic clopen, Neutrosophic semi-clopen

1. Introduction

The contribution of mathematics to present-day Technology in researching fast trends must be addressed. The theories presented differently from classical methods in studies by Zadeh[19]; a fuzzy set was investigated as a mathematical tool for handling uncertainties, with each element having a degree of membership, truth(t), an intuitionistic fuzzy introduced by Atanassov [2] utilizing falsehood (f), the degree of non-membership, Neutrosophic Set in which Smarandache [14,15,16] presented Neutrality (i), the degree of indeterminacy, as an independent concept have great importance in this contribution of mathematics in recent years.

The neutrosophic set is concerned with the origin, nature, and scope of neutrality. Research on the neutrosophic set is crucial since it provides access to a variety of scientific and technical applications. The universe is full of uncertainty, therefore the neutrosophic set can locate a suitable location for investigation. The neutrosophic set on the real line is distinguished by single-valued neutrosophic numbers (SVN) and single-valued bipolar neutrosophic numbers (SVbN). When a human decision is based on positive and negative ideas, the SVbN number is crucial in decision-making problems. All SVbN number membership functions (truth, indeterminacy, and falsity) are contained in the positive and negative sections between -1 and 0 and 0 and 1, respectively. Using these concepts, the authors of [6, 7, 8, 9, 10] have created applications in the health field, such as multi-criteria decision-making in COVID-19 vaccines and the possibility of multiattribute decision-making on water resource management challenges, in recent years. The neutrosophic crisp set was converted by Salama et al. [12, 13] into neutrosophic topological spaces. In [13], Salama et al. presented continuous functions and neutrosophic closed sets. Generalized neutrosophic closed sets were provided in [4]. [1] Presents a set of neutrosophic semi-open, pre-open, and semipre-open sets. The authors explored the concept of neutrosophic almost -contra-continuous functions in [5].. With its potential application in practical settings, neutrosophic topology has now become a fertile field for study.

The review of the literature reveals that there is still work to be done on the qualities of neutrosophic totally and totally semi-continuous functions. By observing these, we are directed to work in continuous and semi-continuous functions based on the neutrosophic set. In addition, neutrosophic strongly semi-continuous and slightly semi-continuous functions are introduced, and these continuous functions are characterized.

For the sake of presentation clarity, abbreviations are used throughout this article.

Abbreviations are listed here.

Array of Words	Shortenings
Neutrosophic Topology	NT
Neutrosophic Topological Space	NTS
Neutrosophic Set	NS
Neutrosophic Sets	NSs
Neutrosophic Open Set	NOS
Neutrosophic Closed Set	NCS
Neutrosophic Point	NP
Neutrosophic Semi-Open Set	NSOS
Neutrosophic Semi-Closed Set	NSCS
Neutrosophic Semi-Open Sets	NSOSs

2. Methodologies

Definition: 2.1[14,15]: A NS Ψ over a finite non-empty set S_1 has the form $\Psi = \{t, \; T_\Psi(t),$ $I_\Psi \; (t), \; F_\Psi(t) \colon t \epsilon S_1\}$, wherein $T, I, F\colon \; S_1 \longrightarrow]0^-, 1^+[$ are the truth, indeterminacy, besides false membership functions, in that order.

Definition: 2.2[14,15]: Let $S_1 \neq \emptyset$ and the NSs Ψ and Ω be defined as

$\Psi = \{\langle d, \mu_\Psi(\mathrm{d}), \sigma_\Psi(\mathrm{d}), \Omega_\Psi(d)\rangle : \mathrm{d} \in S_1\}$, $\Omega = \{\langle d, \mu_\Omega(\mathrm{d}), \sigma_\Omega(\mathrm{d}), \Omega_\Omega(\mathrm{d})\rangle : x \in S_1\}$. Then

I. $\Psi \subseteq \Omega$ *iff* $\mu_\Psi(d) \leq \mu_\Omega(d)$, $\sigma_\Psi(d) \leq \sigma_\Omega(d)$ *and* $\Omega_\Psi(d) \geq \Omega_\Omega(d)$ *for all* $d \in S_1$;

II. $\Psi = \Gamma$ *iff* $\Psi \subseteq \Omega$ *and* $\Omega \subseteq \Psi$;

III. $\overline{\Psi} = \{\langle x, \Gamma_\Psi(d), \sigma_\Psi(d), \mu_\Psi(d)\rangle : d \in S_1\}$; *[Complement of* Ψ*]*

IV. $\Psi \cap \Omega = \{\langle d, \mu_\Psi(d) \wedge \mu_\Gamma(d), \sigma_\Psi(d) \wedge \sigma_\Omega(d), \Gamma_\Psi(d) \vee \Omega_\Omega(d)\rangle : d \in S_1\}$;

V. $\Psi \cup \Omega = \{\langle d, \mu_\Psi(d) \vee \mu_\Omega(d), \sigma_\Psi(d) \vee \sigma_\Omega(d), \Gamma_\Psi(d) \wedge \gamma_\Omega(d)\rangle : d \in S_1\}$;

VI. $[\,]\ \Psi = \{\langle d, \mu_\Psi(d), \sigma_\Psi(d), 1 - \mu_\Psi(d)\rangle : d \in S_1\}$;

VII. $\langle\rangle\ \Psi = \{\langle d, 1 - \Gamma_\Psi(d), \sigma_\Psi(d), \Gamma_\Psi(d)\rangle : d \in S_1\}$.

The primary goal is to create the resources for NTS development, so we make the NSs $0_\aleph$ along with $1_\aleph$ in S_1 as follows:

Definition: 2.3[15,16]: $0_\aleph = \{\langle \omega, 0,0,1\rangle : \omega \in S_1\}$ along with $1_\aleph = \{\langle \omega, 1,1,0\rangle : \omega \in S_1\}$.

Definition: 2.4[12]: A NT $S_1 \neq \emptyset$ is a family ξ_1 of NSs in S_1 observing the rules listed below:

I. $0_\aleph, 1_\aleph \in \xi_1$,

II. $W_1 \cap W_2 \in T$ *being* $W_1, W_2 \in \xi_1$,

III. $\cup W_i \in \xi_1$ *for random family* $\{W_i | i \in \Psi\} \subseteq \xi_1$.

Here (S_1, ξ_1) or just S_1 is labeled as NTS, with each NS in ξ_1 being noted as NOS. The complement $\overline{\Omega}$ of a NOS Ω in S_1 is noted as NCS in S_1.

Definition: 2.5[12, 13]: Consider an NS Ω in NTS S_1. Accordingly,
$\aleph int(\Omega) = \cup \{E | E$ is NOS in S_1 with $E \subseteq \Omega\}$ is titled as neutrosophic interior ($\aleph int$ in short) of Ω;
$\aleph cl(\Omega) = \cap \{\mathfrak{B} | \mathfrak{B}$ is a NCS in S_1 with $\mathfrak{B} \supseteq \Omega\}$ is entitled as neutrosophic closure (in short $\aleph cl$) of Ω.

Definition: 2.6[1]: For r, t, s the real standard else non-standard members of $]0^-, 1^+[$. A NS $\mho_{r,t,s}$ is termed a neutrosophic point (NP) over S_1 defined by $\mho_{r,t,s}(\mathrm{q}) = \begin{cases} (r,t,s), & if \quad \mho = \mathrm{q} \\ (0,0,1) & if \quad \mho \neq \mathrm{q} \end{cases}$ for q $\in S_1$, where r, t, s are the truth, indeterminacy, and falsity membership values of $\mho$.

Definition 2.7[1]: Consider a $\aleph$S Ψ in an NTS (S_1, Γ), neutrosophic semi-open set (NSOS) whenever $\Psi \subseteq \aleph cl(\aleph int(\Psi))$. The complement of NSOS is known as NSCS.

Definition 2.8[11]: A space S_1 is mentioned as neutrosophic semi-connected if S_1 cannot be expressed as the union of two non-empty disjoint NSOSs.

Definition 2.9[3]: Conder a mapping $\eta: S_1 \to S_2$ is neutrosophic semi-continuous whenever $\eta^{-1}(\Omega)$ is NSOS of S_1 for each $\Omega \, \epsilon \, S_2$

3. Results and Discussion.

This section introduces neutrosophic totally continuous, neutrosophic totally semi-continuous, and neutrosophic strongly semi-continuous functions, and their characterizations and interrelationships are discussed.

Definition 3.1: A neutrosophic function $\eta: S_1 \to S_2$ is termed neutrosophic totally continuous (in short (Nt-c.) (resp. neutrosophic totally-semi-continuous in short (Nts-c.) whenever the inverse image of every NOS (resp. NSOS) of S_2 is a neutrosophic clopen (NOS and NCS) (resp. neutrosophic semi-clopen) subset of S_1

Remark 3.2: Every Nt-c function is also a Nts-c. But the opposite doesn't have to be true.

Example 3.3: Let $S_1 = S_2 = \{a, b\}$ we define a mapping $\eta: (S_1, \tau) \to (S_2, \sigma)$ by $\eta^{-1}(C) = C, \eta^{-1}(D) = D, \eta^{-1}(E) = E, \eta^{-1}(0_\aleph) = 0_\aleph, \eta^{-1}(1_\aleph) = 1_\aleph$ where $\tau = \{A, B, C, D, 0_\aleph, 1_\aleph\}$ and $\sigma = \{C, D, E, 0_\aleph, 1_\aleph\}$. Here $A = \{\langle 0.7,0.3,0.8\rangle\langle 0.5,0.8,0.9\rangle\}, B = \{\langle 0.8,0.7,0.7\rangle\langle 0.9,0.2,0.5\rangle\}, C = \{\langle 0.8,0.3,0.7\rangle\langle 0.9,0.2,0.5\rangle\}$, $D = \{\langle 0.7,0.3,0.8\rangle\langle 0.5,0.2,0.9\rangle\}$, $E = \{\langle 0.7,0.7,0.8\rangle\langle 0.5,0.8,0.9\rangle\}$. Here $C, D, and\ E$ are neutrosophic semi-clopen in S_2, whereas E is not neutrosophic semi-clopen in S_1. Hence each Nts-c need not be Nt-c.

Definition 3.4: A neutrosophic function $\eta: S_1 \to S_2$ is known as the neutrosophic strongly semi-continuous (in brief, Nss-c) iff inverse image of each neutrosophic subset of S_2 is a neutrosophic semi-clopen subset of S_1

Remark 3.5: Neutrosophic strongly semi-continuity → Neutrosophic totally-semi-continuity → Neutrosophic semi-continuity

Example 3.6: The example that follows is not Nss-c, but rather Nts-c functoin.

Let $S_1 = S_2 = \{a, b\}$ define a map $\eta: (S_1, \tau) \to (S_2, \sigma)$ by $\eta^{-1}(C) = C$, $\eta^{-1}(D) = D$, $\eta^{-1}(0_\aleph) = 0_\aleph$, $\eta^{-1}(1_\aleph) = 1_\aleph$ where $\tau = \{A, B, C, D, 0_\aleph, 1_\aleph\}$ and $\sigma = \{C, D, 0_\aleph, 1_\aleph\}$, where $A = \{\langle 0.4,0.6,0.8\rangle\langle 0.3,0.5,0.7\rangle\}$, $B = \{\langle 0.8,0.4,0.4\rangle\langle 0.7,0.5,0.3\rangle\}, C = \{\langle 0.4,0.6,0.8\rangle\langle 0.3,0.5,0.8\rangle\}$,

$D = \{\langle 0.7,0.3,0.8\rangle\langle 0.5,0.8,0.9\rangle\}$ and $E = \{\langle 0.6,0.5,0.4\rangle\langle 0.7,0.5,0.4\rangle\}$. Here $C, D, and\ E$ are neutrosophic clopen in S_1, but E is not neutrosophic semi-clopen in S_2. Hence every Nss-c function need not be Nss-c .

Proposition 3.7: Each and every Nss-c function into NT_1-space is a Nts-c function.

Prof: In an NT_1-space, singletons are neutrosophic closed sets. Henceforth $\eta^{-1}(D_1)$ is neutrosophic semi-clopen in S_1 for every subset D_1 of S_2.

Remark 3.8: It is very evident that the classes of Nts-c functions along with Nss-c functions coincide when a range is NT_1-space.

Definition 3.9: A neutrosophic function $\eta: S_1 \rightarrow S_2$ is termed as neutrosophic slightly semi-continuous(in short Nsls-c) if for each $p \in S_1$ and every neutrosophic clopen subset Q of S_2 containing $\eta(Q)$, there exists an NSOS, R of S_1 such that $\eta(R) \subseteq Q$.

Proposition 3.10: Every Nsls-c function that enters a discrete space is considered to be a Nss-c function.

Proof: Let $\eta: S_1 \rightarrow S_2$ be a Nsls-c function from a space S_1 into a discrete space S_2. Consider D_1 be any neutrosophic subset of S_2. Then D_1 is a neutrosophic-clopen subset of S_2. Hence $\eta^{-1}(D_1)$ is the neutrosophic semi-clopen of S_1. Thus, η is Nss-c.

Proposition 3.11: If η is a Nts-c function from a neutrosophic semi-connected space S_1 onto any space S_2, then S_2 is an indiscrete space.

Proof: If possible, suppose that S_2 is not indiscrete. Let D_1 be a proper non-empty neutrosophic open subset of S_2. Then $\eta^{-1}(D_1)$ is a proper non-empty neutrosophic semi-clopen subset of S_1, which is a discrepancy to the fact that S_1 is neutrosophic semi-connected.

Proposition 3.12: If $\eta: S_1 \rightarrow S_2$ be a Nsls-c and $\mu: S_2 \rightarrow S_3$ is Nt-c, then $(\mu \ o \ \eta)$ is Nts-c function.

Proof: Let D_1 be a NOS of S_3. Then $\mu^{-1}(D_1)$ is an NSCS of S_2. Since η Nsls-c, therefore $\eta^{-1}(\mu^{-1}(D_1)) = (\eta o \mu)^{-1}(D_1)$ is a neutrosophic semi-clopen subset of S_1. Hence $(\mu \ o \ \eta)$ is Nts-c function.

Definition 3.13: A NTS S_1 is said to be neutrosophic semi-T_2 if, for any pair of distinct points q_1, q_2 of S_1, there exist disjoint NSOSs Q_1 and Q_2 such that $q_1 \in Q_1$ and $q_2 \in Q_2$.

Remark 3.14: A NTS S_1 is neutrosophic semi-T_2 if and only if for any pair of different points q_1, q_2 of S_1 such that $q_1 \in Q_1$, $q_2 \in Q_2$, and $Nscl(Q_1) \cap Nscl(Q_2) = \emptyset$.

Proposition 3.15: Let $\eta: S_1 \rightarrow S_2$ be a Nts-c injection. If S_2 is NT_0, then S_1 is neutrosophic semi-T_2.

Proof: Consider q_1 and q_2 to be any pair of different points of S_1. Then $\eta(q_1) \neq \eta(q_2)$. As S_2 is NT_0, there exists a NOS, Q containing, say, $\eta(q_1)$ but not $\eta(q_2)$. Then $q_1 \in \eta^{-1}(Q)$ and $q_2 \notin \eta^{-1}(Q)$. Since η is Nts-c, $\eta^{-1}(Q)$ is a neutrosophic semi-clopen subset of S_1. Also, $q_1 \in \eta^{-1}(Q)$ and $q_2 \in S_1 - \eta^{-1}(Q)$. By Remark 3.14, it follows that S_1 is neutrosophic semi-T_2.

4. Comparison Analysis

To highlight the primary contributions of the current study, comparisons with pertinent earlier studies are shown and discussed in this section. Currently, a NS is being created to express incomplete, inconsistent, partial, and ambiguous facts. A NS is expressed to deal with uncertainty using the membership function of Truth, Indeterminacy, and Falsehood. In many practical difficulties, the neutrosophic set leads to more logical conclusions. The neutrosophic set reveals data inconsistencies and can address practical issues. Real or complex continuous functions fail to deal with data containing uncertainty, but Neutrosophic totally semi-continuous functions deal with this. Continuous and neutrosophic continuous functions cannot be compared as both sets are different in these concepts. One is a crisp set, and another is a NS. Even Fuzzy totally continuous functions cannot handle uncertainty. The idea of neutrosophic totally a semi-continuity function is crucial to both functional analysis as well as fixed-point theory. It has numerous applications in information sciences, artificial intelligence, economic theory, decision theory, etc. The key difference is that if h is real, it can only approach zero from the left and right direction in a real line. If h is complex, it can approach zero from infinite directions and any spiral path, etc., in a complex plane. If h is neutrosophic, it can approach zero not only from an endless number of directions in a neutrosophic plane.

5. Conclusion

The Neutrosophic method makes use of all three membership functions (truth, indeterminacy, and falsity). Prepared to face adversity. The perception of the classic set, fuzzy set, and intuitionistic fuzzy set are all generalized by the neutrosophic set. Neutrosophic totally continuous and semi-continuous functions are defined and characterized using neutrosophic semi-open and semi-closed sets. Its properties are analyzed, and some implications are given. Counterexamples also show that the converse statements of these implications are only sometimes true. In addition, neutrosophic strongly semi-continuous along with neutrosophic slightly semi-continuous functions are presented. In the future, we would like to use the neutrosophic semi and nearly continuous mapping to investigate various characteristics in the neutrosophic semi and almost topological group. These ideas will open up new avenues for future research and development of neutrosophic soft topological spaces.

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Received: March 16, 2023. Accepted: July 18, 2023