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Unification of some generalized aggregation operators in neutrosophic cubic environment and its applications in multi expert decision-making analysis

Majid Khan [1], Muhammad Gulistan [1,2]

[1]Department of Mathematics and Statistic Hazara University, Mansehra Pakistan; majid_swati@yahoo.com, gulistanmath@hu.edu.pk,

[2]Department of Electrical and Computer Engineering, University of Alberta Canada; mgulista@ualberta.ca

***** Correspondence: gulistanmath@hu.edu.pk,

Abstract: This manuscript aims to provide a platform that merges different aggregation operators into a concise and computationally trackable single aggregation operator. This generalization can produce numerous aggregation operators while the complex framework can be streamlined and analyzed more efficiently. Generalized aggregation operators have become increasingly important in decision-making (DM) theory due to the growing complexity and diversity of DM problems.

These unified aggregation operators are furnished upon numerical examples from MEMADM and MADM problems as applications. Finally, a comparative analysis with some existing methods is conducted to examine the properties and behavior of the proposed aggregation operator.

Keywords: Neutrosophic Cubic Set (NCS); Decision Making (DM); Multi Attribute Decision making (MADM); Multi Expert Multi Attribute Decision making (MEMADM).

1. Introduction

Complex phenomena such as uncertainty occur in daily life problems. Due to the vagueness that is inevitably involved in many areas of life problems, the common notions of sets still need to deal with the situation. Many successful attempts have been made to handle uncertainty in the system description. Zadeh initiated his idea of a Fuzzy set (FS) [22]. In recent years, FS attracted researchers, and it has been applied in the many fields like medical sciences, social sciences and engineering. Over time, it is further extended onto the interval-valued fuzzy set (IVFS)[23], intuitionistic fuzzy set(IFS)[2], interval-valued intuitionistic fuzzy set(IVIFS)[3], and cubic set(CS)[7]. Smrandache proposed a neutrosophic set (NS) [15], which is a generalization of IFS[16]. NS is characterized by three independent constituents, the truth, indeterminacy, and falsehood. NS was classified into the single-valued neutrosophic set (SVNS)[20]. Later the neutrosophic interval set (INS) [19] was proposed. Jun characterized the NS and INS to form a neutrosophic cubic set (NCS)[8]. NCS is a generalization of NS, and INS [9], enables us to express more information. This makes NCS a persuasive tool to deal with uncertain and imprecise data more efficiently.

The role of aggregation operators in DM is essential. The DM is a challenging job in an inconsistent and vague environment. NCS can minimize the uncertain situation. NCS provides a platform to handle a complex frame of the environment to its structure of both interval and crisp value simultaneously. This complexion of NCS attracted many researchers to apply it in different fields of DM. Khan *et al.*, [9] proposed Einstein geometric operators in NCS. Zhan *et al.*, [24] used NCS on MCDM. Banerjee *et al.* [4] worked on the grey rational analysis (GRA) method in NCS. Lu and Ye [12] proposed a cosine measure of NCS. Pramanik *et al.* [13] defined similarity measures on NCS. Shi and Ji [14] suggested Dombi aggregation operators on NCS.

Khan et al. generalized some aggregation operators on NCS in their work as generalized aggregation operators[11]. In addition, khan et al. generalized Shapley and Choquet integral aggregation operators in NCS[10]. These two generalizations are a precious platform for deducing aggregation operators.

The aggregation operators have a vast scope in data analysis, machine learning, artificial intelligence, and decision-making theory. The aggregation operators enable one to arrive at a single decision, considering various degrees of uncertainty and imprecision in complex work frames. The development of a new generalized aggregation operator with improved performance and flexibility in the active research area can further expand their scope of application.

1.1 Motivation

The Choquet integral can handle overall interaction, and shapley takes the interaction among the criteria and can measure the weights [17]. Motivated by this ([10,11]) work, the author(s) attempted to further generalization of these generalizations so that a single platform is provided to choose an appropriate aggregation operator. This will allow one to not only measure the weight but also choose an aggregation operator of choice according to the situation.

1.2 Contribution

The aggregation operators play a vital role in DM theory to allow a MADM to make a single decision. The choice of an aggregation operator significantly impacts the final decision. Overall aggregation operators are crucial to handling imprecise and complex DM problems. To address such challenges, generalized aggregation operators are needed so that these challenges can be dealt with under one platform. Recently, the author(s) have generalized some aggregation operators like neutrosophic cubic generalized aggregation operators, induced generalized Shapley Choquet integral operator.

The methodologies to measure the unified aggregation in NCS.

- The generalization-induced generalized neutrosophic cubic unified aggregation operator is provided.
- Shapley measures are used to determine the interactive and overall weights.
- These aggregation operators are applied and compared with existing methods to test their validity.

Organization The remaining paper is patterned as follows. The preliminaries are given in section 2. Section 3 deals with the IGNCUSCI aggregation operator proposed, and other generalizations are deduced. In section 4, a numerical example is furnished upon proposed aggregation operators as an application. The data is both tabulated and graphically interpreted. Finally, a comparative analysis is conducted to validate the results obtained by the current study.

Table1.1: Table of abbreviation and notations

Abbreviation/notation	**Description**
DM	Decision Making

MADM	Multi Attribute Decision Making
MEMADM	Multi Expert Multi Attribute Decision making
FS	Fuzzy set
IVFS	Interval value fuzzy set
IFS	Intuitionistic fuzzy set
IVIFS	Interval valued Intuitionistic fuzzy set
CS	Cubic set
NS	Neutrosophic set
INS	Interval Neutrosophic set
NCS	Neutrosophic cubic set
NCWA	Neutrosophic cubic weighted arithmetic
NCWG	Neutrosophic cubic weighted geometric
IGNCSCIA	Induced generalized neutrosophic cubic Shapley choquet integral arithmetic
IGNCSCIG	Induced generalized neutrosophic cubic Shapley choquet integral geometric
GNCU	Generalized neutrosophic cubic unified
IGNCUSCI	induced generalized neutrosophic cubic unified Shapley choquet integral
Scr	Score
Acu	accuracy

2. Preliminaries

In this section some preliminaries are considered.

Definition 2.1[22] A set $\Gamma = \{(\ell, \mu(\ell)) \mid \ell \in \Xi\}$, where the mapping $\mu : \Xi \to [0,1]$ called a FS and μ is called a membership degree.

Definition 2.2[2] A pair $\{(\ell, (\mu(\ell), \nu(\ell))) \mid \ell \in \Xi\}$ of mappings where $\mu : \Xi \to [0,1]$ and $\nu : \Xi \to [0,1]$ is called an IFS. Where μ and ν are a membership and non-membership function, simply denoted by (μ, ν).

Definition 2.3[15] A set $\wp=\left\{\left(\ell,\left(\wp_T(\ell),\wp_I(\ell),\wp_F(\ell)\right)\right)\mid \ell\in\Xi\right\}$ is called an NS. Where

$\wp_T:\Xi\to\left]0^-,1^+\right[$, $\wp_I:\Xi\to\left]0^-,1^+\right[$, $\wp_F:\Xi\to\left]0^-,1^+\right[$ such that

$0^-\le\wp_T(\ell)+\wp_I(\ell)+\wp_F(\ell)\le 3^+$ and $\wp_T,\wp_I,\wp_F$ are called truth, indeterminacy, and falsity.

Definition 2.4[8] A set $\wp=\left\{\left(\ell,\left(\tilde{\wp}_T(\ell),\tilde{\wp}_I(\ell),\tilde{\wp}_F(\ell)\right),\left(\wp_T(\ell),\wp_I(\ell),\wp_F(\ell)\right)\right)\mid \ell\in\Xi\right\}$ is NCS. Where

$\left(\tilde{\wp}_T,\tilde{\wp}_I,\tilde{\wp}_F\right)$ is an INS and $\left(\wp_T,\wp_I,\wp_F\right)$ is a NS such that $\tilde{0}\le\tilde{\wp}_T(\ell)+\tilde{\wp}_I(\ell)+\tilde{\wp}_F(\ell)\le\tilde{3}$ and

$0\le\wp_T(\ell)+\wp_I(\ell)+\wp_F(\ell)\le 3$. Denoted by $\wp=\left(\tilde{\wp}_T,\tilde{\wp}_I,\tilde{\wp}_F,\wp_T,\wp_I,\wp_F\right)=\left(\left[\wp_T^-,\wp_T^+\right],\left[\wp_I^-,\wp_I^+\right],\left[\wp_F^-,\wp_F^+\right],\wp_T,\wp_I,\wp_F\right)$.

Definition 2.5[9] The sum, product, scalar multiplication and $\sigma-th$ power on two NCS $\wp=\left(\left[\wp_T^-,\wp_T^+\right],\left[\wp_I^-,\wp_I^+\right],\left[\wp_F^-,\wp_F^+\right],\wp_T,\wp_I,\wp_F\right)$ and $N=\left(\left[N_T^-,N_T^+\right],\left[N_I^-,N_I^+\right],\left[N_F^-,N_F^+\right],N_T,N_I,N_F\right)$ is defined by

$$\wp\oplus N=\begin{pmatrix}\left[\wp_T^-+N_T^--\wp_T^-N_T^-,\wp_T^++N_T^+-\wp_T^+N_T^+\right],\left[\wp_I^-+N_I^--\wp_I^-N_I^-,\wp_I^++N_I^+-\wp_I^+N_I^+\right],\\ \left[\wp_F^-N_F^-,\wp_F^+N_F^+\right],\wp_TN_T,\wp_IN_I,\wp_F+N_F-\wp_FN_F\end{pmatrix}$$

$$\wp\otimes N=\begin{pmatrix}\left[\wp_T^-N_T^-,\wp_T^+N_T^+\right],\left[\wp_I^-N_I^-,\wp_I^+N_I^+\right],\left[\wp_F^-+N_F^--\wp_F^-N_F^-,\wp_F^++N_F^+-\wp_F^+N_F^+\right],\\ \wp_T+N_T-\wp_TN_T,\wp_I+N_I-\wp_IN_I,\wp_FN_F\end{pmatrix}$$

$$\sigma\wp=\begin{pmatrix}\left[1-(1-\wp_T^-)^\sigma,1-(1-\wp_T^+)^\sigma\right],\left[1-(1-\wp_I^-)^\sigma,1-(1-\wp_I^+)^\sigma\right],\\ \left[\left(\wp_F^-\right)^\sigma,\left(\wp_F^+\right)^\sigma\right],\left(\wp_T\right)^\sigma,\left(\wp_I\right)^\sigma,1-\left(1-\wp_F\right)^\sigma\end{pmatrix}$$

$$\wp^\sigma=\begin{pmatrix}\left[(\wp_T^-)^\sigma,(\wp_T^+)^\sigma\right],\left[(\wp_I^-)^\sigma,(\wp_I^+)^\sigma\right],\left[1-\left(1-\wp_F^-\right)^\sigma,1-\left(1-\wp_F^+\right)^\sigma\right],\\ 1-\left(1-\wp_T\right)^\sigma,1-\left(1-\wp_I\right)^\sigma,\left(\wp_F\right)^\sigma\end{pmatrix}$$

where σ is scalar.

Definition 2.8[11] The GNCU aggregation operator is generalization of some aggregation operators defined by, $GNCU\left(\wp_1,\wp_2,\ldots,\wp_n\right)=\left(\left(\sum_{j=1}^{m}C_j\left(\sum_{i=1}^{n}w_i^j\wp_i^{\lambda_j}\right)^{1/\lambda_j}\right)^q\right)^{1/q}$

C_j is relevance of sub-aggregation operator with $0\le C_j\le 1$ and $\sum C_j=1$, w_p^q is the pth weight of qth weight vector with $0\le w_p^q\le 1$ and $\sum_{i=1}^{m}w_i^j=1$, $\lambda_j\in R$ is a parameter and A_i is NCS. The parameter q ranges, $q\in(0,\infty)$. Usually, λ_j remain unchanged however in complex types of aggregations one can assigned different values to λ_j.

Definition 2.9 [5,17] The Shapley index is defined as

$$\varpi_S^{Sh}(\theta, N) = \sum_{T \subseteq N \setminus S} \frac{(\wp - u - v)!t!}{(\wp - u + 1)!t!}\left(\theta(K \cup L) - \theta(L)\right), \qquad \forall K \subseteq N$$

Where u, $\wp$ and v respectively denotes the cardinalities of N K and L. θ denotes fuzzy function of fuzzy measure λ on N.

Definition 2.10[5,17] Meng proposed generalized Shapley index by $\lambda-$ fuzzy measure θ_λ on N by

$$\varpi_S^{Sh}(\theta_\lambda, N) = \sum_{T \subseteq N \setminus S} \frac{(\wp - u - v)!v!}{(\wp - u + 1)!v!}\left(\theta_\lambda(K \cup L) - \theta_\lambda(L)\right), \forall K \subseteq N$$

where $\theta_\lambda -$ fuzzy measure expressed as

$$\theta_\lambda(A) = \begin{cases} \frac{1}{\lambda}\left(\prod_{k \in \mathcal{T}}\left((\lambda\theta_\lambda(\Lambda) + 1) - 1\right)\right) & if\, \lambda \neq 0 \\ \sum_{k \in \mathcal{T}} \lambda\theta_\lambda(k) & if\, \lambda = 0 \end{cases}$$

where $\prod_{c \in N}(1 + \theta_\lambda\{c_k\}) = 1 + \theta_\lambda$ measure of λ. Furthermore, if $S = \{k\}$, then

$$\varpi_k^{Sh}(\theta_\lambda, N) = \sum_{T \subseteq N \setminus S} \frac{t!(\wp - t - 1)!}{t!(\wp)!}\left(\theta_\lambda(\Lambda)\left(\prod_{j \in T}\left(1 + \lambda\theta_\lambda(\Lambda)\right)\right)\right), \ \forall K \subseteq N$$

Arithmetic λ –Shapley Choquet integral operator as

$$C_{\varpi_S^{Sh}(\theta_\lambda, N)}\left(k\left(m_{\rho(\Lambda)}\right)\right) = \sum_{b=1}^{n} k\left(m_{\rho(\Lambda)}\right) \Theta\left(\widehat{\Upsilon}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}\right)$$
$$where\left(\widehat{\Upsilon}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}\right) = \left(\varpi^{Sh}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}(\theta_\lambda, N) - \varpi^{Sh}_{(\widehat{\mathcal{T}})_{\rho(\Lambda+1)}}(\theta_\lambda, N)\right)$$

for $\rho(\Lambda)$ as $(\rho(1), \ldots, \rho(\wp))$ present the permutation of $(1, \ldots, \wp)$ such that $k(m_{(1)}) \leq \ldots \leq k(m_{(\wp)})$ and $\widehat{\mathcal{T}} = \{m_1, \ldots, m_\wp\}$ with $\widehat{\mathcal{T}}_{\rho(\wp+1)} = \varphi$.

Geometric λ –Shapley Choquet integral operator as

$$C_{\varpi_S^{Sh}(\eta_\lambda, N)}\left(k\left(m_{\rho(\Lambda)}\right)\right) = \prod_{\Lambda=1}^{\wp}\left(k\left(m_{\rho(\Lambda)}\right)\right)^{\Theta\left(\widehat{\Upsilon}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}\right)}$$
$$where\left(\widehat{\Upsilon}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}\right) = \left(\varpi^{Sh}_{(\widehat{\mathcal{T}})_{\rho(\Lambda)}}(\theta_\lambda, N) - \varpi^{Sh}_{(\widehat{\mathcal{T}})_{\rho(\Lambda+1)}}(\theta_\lambda, N)\right)$$

for $\rho(\Lambda)$ as $(\rho(1), \ldots, \rho(\wp))$ present the permutations of $(1, \ldots, \wp)$ such that $k(m_{(1)}) \leq \ldots \leq k(m_{(\wp)})$ and $\widehat{\mathcal{T}} = \{m_1, \ldots, m_\wp\}$ with $\widehat{\mathcal{T}}_{\rho(\wp+1)} = \varphi$.

Definition 2.11[10] Let $\Xi_{\Lambda} = \left(\left[\left(\Xi_{\Lambda}{}^{L}\right)_{T}, \left(\Xi_{\Lambda}{}^{U}\right)_{T}\right], \left[\left(\Xi_{\Lambda}{}^{L}\right)_{I}, \left(\Xi_{\Lambda}{}^{U}\right)_{I}\right], \left[\left(\Xi_{\Lambda}{}^{L}\right)_{F}, \left(\Xi_{\Lambda}{}^{U}\right)_{F}\right], (\Xi_{\Lambda})_{T}, (\Xi_{\Lambda})_{I}, (\Xi_{\Lambda})_{F}\right)$, where $(\Lambda = 1,...,\hbar)$ be the collection of NCS, and θ be a fuzzy measure on $\hat{T} = \{\Xi_1,...,\Xi_\hbar\}$ such that $\theta(\Xi_\Lambda) = \hat{u}_\Lambda$, then the IGNCSCIA operator is defined as

$$IGNCSCIA_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle,...,\langle \hat{u}_\hbar, \Xi_\hbar\rangle\right) = \left(\bigoplus_{\Lambda=1}^{\hbar} k\left(\Xi_\Lambda\right)^q \Theta\left(\hat{\Upsilon}_{\rho(\hat{T})_{(\Lambda)}}\right)\right)^{1/q}$$

, where $q \in (0,\infty)$ and $\rho(\Lambda)$ as $\left(\rho(1),...,\rho(\Lambda),...,\rho(\hbar)\right)$ being the permutation of $\left(1,..,\Lambda,..,\hbar\right)$ such that, $k(\Xi_{(1)}) \leq ... \leq k(\Xi_{(\hbar)})$ and $\hat{T} = \{\Xi_1,...,\Xi_\hbar\}$ with $\hat{T}_{\rho(\hbar+1)} = \varphi$.

Definition 2.12 [10] Let $\Xi_{\Lambda} = \left(\left[\left(\Xi_{\Lambda}{}^{L}\right)_{T}, \left(\Xi_{\Lambda}{}^{U}\right)_{T}\right], \left[\left(\Xi_{\Lambda}{}^{L}\right)_{I}, \left(\Xi_{\Lambda}{}^{U}\right)_{I}\right], \left[\left(\Xi_{\Lambda}{}^{L}\right)_{F}, \left(\Xi_{\Lambda}{}^{U}\right)_{F}\right], (\Xi_{\Lambda})_{T}, (\Xi_{\Lambda})_{I}, (\Xi_{\Lambda})_{F}\right)$, where $(\Lambda = 1,...,\hbar)$ be a collection of NC values, and θ be a fuzzy measure on $\hat{T} = \{\Xi_1,...,\Xi_\hbar\}$ such that $\theta(\Xi_\Lambda) = \hat{u}_\Lambda$, then the IGNCSCIG, operator is defined as operator is defined as

$$IGNCSCIG_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle,...,\langle \hat{u}_\hbar, \Xi_\hbar\rangle\right) = \frac{1}{q}\left(\bigotimes_{\Lambda=1}^{\hbar}\left(q\left(k\left(\Xi_{\rho(\Lambda)}\right)\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{T})_{\rho(\Lambda)}}\right)}\right)$$

, where $q \in (0,\infty)$ and $\rho(\Lambda)$ as $\left(\rho(1),...,\rho(\Lambda),...,\rho(\hbar)\right)$ extant the permutation of $\left(1,..,\Lambda,..,\hbar\right)$ such that, $k(\Xi_{(1)}) \leq ... \leq k(\Xi_{(\hbar)})$ and $\hat{T} = \{\Xi_1,...,\Xi_\hbar\}$ with $\hat{T}_{\rho(\hbar+1)} = \varphi$.

Definition 2.13 [9] For an NCS $\wp = \left(\left[\wp_T^-, \wp_T^+\right], \left[\wp_I^-, \wp_I^+\right], \left[\wp_F^-, \wp_F^+\right], \wp_T, \wp_I, \wp_F\right)$, the score is defined as

$$Scr\left(\wp\right) = \left[\wp_T^- - \wp_F^- + \wp_T^+ - \wp_F^+ + \wp_T - \wp_F\right]$$

Definition 2.14 [9] For an NCS $\wp = \left(\left[\wp_T^-, \wp_T^+\right], \left[\wp_I^-, \wp_I^+\right], \left[\wp_F^-, \wp_F^+\right], \wp_T, \wp_I, \wp_F\right)$, the accuracy is defined as

$$Acu(\wp) = \frac{1}{9}\left\{\wp_T^- + \wp_I^- + \wp_F^- + \wp_T^+ + \wp_I^+ + \wp_F^+ + \wp_T + \wp_I + \wp_F\right\}.$$

Definition 2.15 [9] Let $\wp_1$, $\wp_2$ be two NCS. Then

1). $Scr\left(\wp_1\right) > Scr\left(\wp_2\right) \Rightarrow \wp_1 > \wp_2$

2). If $Scr\left(\wp_1\right) = Scr\left(\wp_2\right)$

i). $Acu\left(\wp_1\right) > Acu\left(\wp_2\right) \Rightarrow \wp_1 > \wp_2$

ii). $Acu\left(\wp_1\right) = Acu\left(\wp_2\right) \Rightarrow \wp_1 = \wp_2$

3 Induced Generalized Neutrosophic Cubic Unified Shapley Choquet Integral Aggregation Operator

A complex frame of work in decision-making is a scenario that typically requires consideration of multiple and conflicting objectives, uncertainty, and ambiguity. The goal is to find a solution that balances these factors and satisfies the decision-maker's preferences as much as possible. To handle such complexity, the generalization provides a better platform. Hence, aggregation operators are a crucial part of DM theory. Many aggregation operators like arithmetic, geometric, hybrid, Shapley Choquet integral operators, and many more are defined so for. The central theme of this section is to unify the generalized aggregation operator [11] and induced generalized Shapley-Choquet (arithmetic and geometric) aggregation operator [10] to a single aggregation operator that balances inconsistent and uncertain information in the data and satisfies the decision maker's preferences as much as possible. This generalization is named an induced generalized neutrosophic cubic unified Shapley Choquet integral (IGNCUSCI) aggregation operator.

The IGNCUSCI operator is defined as;

$$IGNCUSCI_{\theta_\lambda}\left(\langle \widehat{\cancel{u}}_1, \Xi_1\rangle, \ldots, \langle \widehat{\cancel{u}}_n, \Xi_n\rangle\right) = \left(\left(\sum_{j=1}^{m} C_j \left(\sum_{i=1}^{n}\left(\Theta\left(\widehat{\Upsilon}_{\rho(\widehat{\cancel{T}})_{(i)}}\right)\right)^{j} \left(\cancel{k}(\Xi_i)\right)^{\lambda_j}\right)^{1/\lambda_j}\right)^{q}\right)^{1/q} \quad (3.1)$$

C_j is relevance of sub-aggregation operator with $0 \le C_j \le 1$ and $\sum_{j=1}^{m} C_j = 1$, w_i^j is the ith weight of jth weighing vector with $w_i^j \in [0,1]$ and $\sum_{i=1}^{m} w_i^{\,j} = 1$, $\lambda_j \in R$ is a parameter and A_i is NCS. The parameter q ranges, $q \in (0,\infty)$. The parameter q ranges, $q \in (0,\infty)$. Usually, λ_j remains same but different values will be assigned in complex types of aggregations.

3.1 The Generalized Neutrosophic Cubic Unified Aggregation Operator

For $\widehat{\cancel{u}}_i = \Xi_i = \wp_i$ for all i, shapley measures and choquet integral are considered, the equation (3) reduces into GNCU aggregation operator. The GNCU aggregation operator is generalization of some aggregation operators defined by,

$$GNCU(\wp_1, \wp_2, \ldots, \wp_n) = \left(\left(\sum_{j=1}^{m} C_j \left(\sum_{i=1}^{n} w_i^j \wp_i^{\lambda_j}\right)^{1/\lambda_j}\right)^{q}\right)^{1/q}$$

(3.1..1)

C_j is relevance of sub-aggregation operator with $0 \le C_j \le 1$ and $\sum C_j = 1$, w_p^q is the pth weight of qth vector with $0 \le w_p^q \le 1$ and $\sum_p w_p^q = 1$, $\lambda_j \in R$ is a parameter and A_i is NCS. The parameter q ranges, $q \in (0,\infty)$. Usually, λ_j remain unchanged however in complex types of aggregations different values can be assigned.

Considering the types of problem under discussion different families of aggregation operators are analyzed by values are assign to λ_j and q. Some aggregation operators are deduced by assigning the values.

- If $\lambda = q = 1$ and $C_1 = 1, C_2 = C_3 = \cdots = C_n = 0$, the GNCU aggregation operator reduces to neutrosophic cubic weighted arithmetic NCWA operator.

$$NCWA(\wp_1, \wp_2, \ldots, \wp_n) = \sum_{i=1}^{n} w_i \wp_i$$

- If $\lambda = q = 1$ and $C_2 = 1, C_1 = C_3 = \cdots = C_n = 0$, the GNCU aggregation operator reduces to neutrosophic cubic weighted arithmetic NCOWA operator.

  $NCWA(\wp_1, \wp_2, ..., \wp_n) = \sum_{i=1}^{n} w_{\sigma(i)} \wp_i$, where $\sigma(i)$ represents ordering position.

- If $\lambda \to 1, q = 1$ and $C_1 = 1, C_2 = C_3 = \cdots = C_n = 0$, the GNCU aggregation operator reduces to neutrosophic cubic weighted arithmetic NCWA operator.

$$NCWG(\wp_1, \wp_2, ..., \wp_n) = \prod_{i=1}^{n} (\wp_i)^{w_i}.$$

- If $\lambda \to 1, q = 1$ and $C_2 = 1, C_1 = C_3 = \cdots = C_n = 0$, the GNCU aggregation operator reduces to neutrosophic cubic weighted arithmetic NCWA operator.

$$NCWG(\wp_1, \wp_2, ..., \wp_n) = \prod_{i=1}^{n} (\wp_i)^{w_{\sigma(i)}}$$, where $\sigma(i)$ represents ordering position.

- If $\lambda = 2, q = 1$ and $C_1 = 1, C_2 = C_3 = \cdots = C_n = 0$, the GNCU aggregation operator reduces into NCQA.

$$NCQA(\wp_1, \wp_2, ..., \wp_n) = \left(\sum_{i=1}^{n} w_i^j \wp_i^2 \right)^{1/2}$$

Assigning different values to λ_j and δ, some other family of aggregation operators can be reduced. These values depend on the type of problem under discussion.

The averaging aggregation operators have a most practical operator among their competitors, but in some situations, other operators like geometric, quadratic, cubic operators are in a much better position to evaluate the values.

- If $\lambda = \delta = 1$ and for all *j*, the aggregation operator is deduced to NCUA.

$$NCUA(\wp_1, \wp_2, ..., \wp_n) = \sum_{j=1}^{m} C_j \sum_{i=1}^{n} w_i^j \wp_i$$

- If $\lambda \to 1, \delta = 1$ and for all *j*, the aggregation operator is deduced to NCUG.

$$NCUG(\wp_1, \wp_2, ..., \wp_n) = \sum_{j=1}^{m} C_j \prod_{i=1}^{n} \wp_i^{w_j}$$

- If $\lambda = 2, \delta = 1$ and for all *j*, the aggregation operator is deduced to NCUQA.

$$NCUQA(\wp_1, \wp_2, ..., \wp_n) = \sum_{j=1}^{m} C_j \left(\sum_{i=1}^{n} w_i^j \wp_i^2 \right)^{1/2}$$

- If $\lambda \to 0, \delta \to 0$ and for all *j*, the aggregation operator is deduced to NCGUG.

$$NCGUG(\wp_1, \wp_2, ..., \wp_n) = \prod_{j=1}^{m} C_j \prod_{i=1}^{n} \wp_i^{w_j}$$

- If $\lambda = 2, \delta = 2$ and for all *j*, the aggregation operator is deduced to NCQUQA.

$$NCQUQA(\wp_1, \wp_2, ..., \wp_n) = \left(\sum_{j=1}^{m} C_j^2 \left(\sum_{i=1}^{n} w_i^j \wp_i^2 \right)^{1/2} \right)^{1/2}$$

Note that the complex and simple aggregation operators can be studied by assigning different values not only to λ and δ but to the weight as well.

3.2 Induced generalized neutrosophic cubic unified choquet integral aggregation operator.

In this section some complex aggregation operators are obtained from IGNCUSCI. Observe that different scenario may be constructed by assigning different values to λ and δ. The families of aggregation operators can be deduced by assigning different values to λ and q in IGNCUSCI and $C_1 = 1, C_j = 0$ for $j > 1$ .the following family of aggregations operators can be obtained.

3.2.1 Induced generalized neutrosophic cubic Shapley choquet arithmetic aggregation operators.

In this subsection the family of IGNCSCA operator is reduced from IGNCUSCI.

- If $\lambda = 1$, the IGNCUSCI (3.1.1)is reduced to IGNCSCIA operator.

$$IGNCSCIA_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle, \ldots, \langle \hat{u}_n, \Xi_n\rangle\right) = \left(\overset{\wp}{\underset{i=1}{\oplus}} k\left(\Xi_i\right)^q \Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{(i)}}\right)\right)^{1/q} \tag{3.2.1}$$

- If $q = 1$, IGNCSCIA(3.2.1) reduced into INCSCIA.

$$INCSCIA_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle, \ldots, \langle \hat{u}_n, \Xi_n\rangle\right) = \left(\overset{\wp}{\underset{i=1}{\oplus}} k\left(\Xi_i\right) \Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{(i)}}\right)\right)$$

- If $\hat{u}_i = \Xi_i$ for each i, IGNCSCIA(3.2.1) reduced into GNCSCIA.

$$IGNCSCIA_{\theta_\lambda}\left(A_1, \ldots, A_n\right) = \left(\overset{\wp}{\underset{i=1}{\oplus}} k\left(\Xi_i\right)^q \Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{(i)}}\right)\right)^{1/q}$$

- If $\hat{u}_i = \Xi_i$ and $q = 1$ for each i, IGNCSCIA (3.2.1) reduced into NCSCIA.

$$INCSCIA_{\theta_\lambda}\left(A_1, \ldots, A_n\right) = \left(\overset{\wp}{\underset{i=1}{\oplus}} k\left(\Xi_i\right) \Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{(i)}}\right)\right)$$

3.2.1 Induced generalized neutrosophic cubic Shapley choquet geometric aggregation operators.

In this subsection the family of IGNCSCG operator is reduced from IGNCUSCI.

- If $\lambda \to 1$, the IGNCUSCI reduced into IGNCSCIG.

$$IGNCSCIG_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle, \ldots, \langle \hat{u}_\wp, \Xi_\wp\rangle\right) = \frac{1}{q}\left(\overset{\wp}{\underset{i=1}{\otimes}}\left(q\left(k\left(\Xi_{\rho(i)}\right)\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{\rho(i)}}\right)}\right) \tag{3.2.2}$$

- $q = 1$, IGNCSCIG (3.1.2) reduce into INCSCIG.

$$IGNCSCIG_{\theta_\lambda}\left(\langle \hat{u}_1, \Xi_1\rangle, \ldots, \langle \hat{u}_\wp, \Xi_\wp\rangle\right) = \left(\overset{\wp}{\underset{i=1}{\otimes}}\left(k\left(\Xi_{\rho(i)}\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{\rho(i)}}\right)}\right)$$

- $\hat{u}_i = \Xi_i$ for each i, IGNCSCIG (3.1.2) reduce into GNCSCIG.

$$IGNCSCIG_{\theta_\lambda}\left(A_1, \ldots, A_n\right) = \frac{1}{q}\left(\overset{\wp}{\underset{i=1}{\otimes}}\left(q\left(k\left(\Xi_{\rho(i)}\right)\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{\rho(i)}}\right)}\right)$$

- $\hat{u}_i = \Xi_i$ and $q = 1$ for each i, IGNCSCIG (3.1.2) reduce into NCSCIG.

$$IGNCSCIG_{\theta_\lambda}\left(A_1, \ldots, A_n\right) = \left(\overset{\wp}{\underset{i=1}{\otimes}}\left(k\left(\Xi_{\rho(i)}\right)\right)^{\Theta\left(\hat{\Upsilon}_{\rho(\hat{\mp})_{\rho(i)}}\right)}\right)$$

4 Application 1

In this section a numerical example is furnished upon proposed IGNCUSCI aggregation operator as an application. A company is interested in expanding its foreign investment. There is a list of five possible alternatives (countries) $N = \{₪_1, ₪_2, ₪_3, ₪_4, ₪_5\}$, and to choose the best country(alternative) from given list. The attribute set is listed as $\Lambda = \{\sigma_1, \sigma_2, \sigma_3, \sigma_4\}$ and attributes are resources, policies, economy, and infrastructure. Khan et al., applied different techniques. The proposed operators are applied to the data. The comparative analysis is performed at the end for validation of results.

First the data is transformed into NC form the NC values are.

$$\left(\begin{array}{cc} \sigma_1 & \begin{bmatrix} b_{11} = ([0.70, 0.80], [0.50, 0.70], [0.10, 0.20], 0.90, 0.70, 0.20) \\ b_{12} = ([0.60, 0.80], [0.40, 0.60], [0.40, 0.60], 0.20, 0.30, 0.70) \\ b_{13} = ([0.40, 0.50], [0.50, 0.60], [0.40, 0.60], 0.30, 0.30, 0.30) \\ b_{14} = ([0.60, 0.80], [0.50, 0.60], [0.40, 0.50], 0.50, 0.40, 0.50) \\ b_{15} = ([0.60, 0.70], [0.40, 0.50], [0.40, 0.50], 0.50, 0.40, 0.50) \end{bmatrix} \end{array}\right)$$

$$\left(\begin{array}{cc} \sigma_2 & \begin{bmatrix} b_{21} = ([0.60, 0.80], [0.40, 0.50], [0.30, 0.30], 0.90, 0.80, 0.20) \\ b_{22} = ([0.50, 0.70], [0.40, 0.60], [0.10, 0.30], 0.60, 0.40, 0.20) \\ b_{23} = ([0.60, 0.70], [0.40, 0.60], [0.30, 0.40], 0.80, 0.70, 0.50) \\ b_{24} = ([0.5, 0.60], [0.30, 0.40], [0.40, 0.50], 0.40, 0.40, 0.30) \\ b_{25} = ([0.80, 0.90], [0.30, 0.40], [0.10, 0.20], 0.70, 0.40, 0.30) \end{bmatrix} \end{array}\right)$$

$$\left(\begin{array}{cc} \sigma_3 & \begin{bmatrix} b_{31} = ([0.80, 0.80], [0.40, 0.60], [0.10, 0.20], 0.80, 0.90, 0.10) \\ b_{32} = ([0.60, 0.60], [0.20, 0.30], [0.40, 0.50], 0.50, 0.10, 0.30) \\ b_{33} = ([0.70, 0.80], [0.60, 0.70], [0.10, 0.20], 0.50, 0.40, 0.40) \\ b_{34} = ([0.60, 0.80], [0.50, 0.60], [0.10, 0.20], 0.50, 0.70, 0.30) \\ b_{35} = ([0.70, 0.80], [0.50, 0.60], [0.10, 0.20], 0.50, 0.30, 0.40) \end{bmatrix} \end{array}\right)$$

$$\left(\begin{array}{cc} \sigma_4 & \begin{bmatrix} b_{41} = ([0.70, 0.70], [0.30, 0.40], [0.20, 0.20], 0.80, 0.60, 0.20) \\ b_{42} = ([0.60, 0.80], [0.40, 0.40], [0.20, 0.40], 0.70, 0.30, 0.50) \\ b_{43} = ([0.50, 0.60], [0.50, 0.60], [0.20, 0.30], 0.60, 0.60, 0.40) \\ b_{44} = ([0.80, 0.90], [0.30, 0.40], [0.10, 0.20], 0.70, 0.50, 0.40) \\ b_{45} = ([0.50, 0.70], [0.50, 0.50], [0.20, 0.30], 0.60, 0.50, 0.30) \end{bmatrix} \end{array}\right)$$

The Shapley measure is used to evaluate the interactive dependence between attribute, so their weight are measured by Shapley measure presented in the table.

Table 4.1: *The Criteria along with Fuzzy Shapley Measures*

Λ	$\mu_\lambda(\Lambda)$	Λ	$\mu_\lambda(\Lambda)$

ϕ	0	$\{\sigma_1\}$	0.664
$\{\sigma_2\}$	0.077	$\{\sigma_3\}$	0.593
$\{\sigma_4\}$	0.260	$\{\sigma_1, \sigma\}$	0.144
$\{\sigma_1, \sigma_3\}$	0.659	$\{\sigma_1, \sigma_4\}$	0.327
$\{\sigma_2, \sigma_3\}$	0.671	$\{\sigma_2, \sigma_4\}$	0.3384
$\{\sigma_3, \sigma_4\}$	0.858	$\{\sigma_1, \sigma_2, \sigma_3\}$	0.739
$\{\sigma_1, \sigma, \sigma_4\}$	0.405	$\{\sigma, \sigma_3, \sigma\}$	0.922
$\{\sigma_2, \sigma_3, \sigma_4\}$	0.993	Λ	1

These interactive weights are used to measure the weight for the attributes (2.10).

$$\varpi_1^{Sh} = 0.099, \varpi_2^{Sh} = 0.010, \varpi_3^{Sh} = 0.599, \varpi_4^{Sh} = 0.292 .$$

In order to apply IGNCUSCI to this numerical problem first $\lambda = 1$, this reduce IGNCUSCI(3.1) into IGNCSCIA(3.2.1) operator. Since INCSCIA has different values for q so different values are assigned to q and the aggregated values is obtained. Here the values obtained for □ = 1 are written and other values obtained from different values of q just tabulated in table so that unnecessary length is being avoided.

Then the aggregated values of IGNCUSCI calculated for $\lambda = 1$, □ = 1.

$$= \begin{bmatrix} \tilde{u}_1 = \left(\left[0.735, 0.862\right], \left[0.412, 0.557\right], \left[0.158, 0.204\right], 0.172, 0.267, 0.191\right) \\ \tilde{u}_2 = \left(\left[0.594, 0.746\right], \left[0.383, 0.468\right], \left[0.137, 0.329\right], 0.531, 0.207, 0.375\right) \\ \tilde{u}_3 = \left(\left[0.648, 0.750\right], \left[0.528, 0.670\right], \left[0.165, 0.270\right], 0.588, 0.516, 0.480\right) \\ \tilde{u}_4 = \left(\left[0.662, 0.801\right], \left[0.406, 0.511\right], \left[0.156, 0.267\right], 0.444, 0.449, 0.358\right) \\ \tilde{u}_5 = \left(\left[0.761, 0.872\right], \left[0.451, 0.528\right], \left[0.093, 0.188\right], 0.573, 0.348, 0.369\right) \end{bmatrix}$$

The alternatives are ranked in IGNCUSCI $(\lambda = 1)$ for different values of □ and rankings are tabulated in Table 4.2.

Table 4.2: The Ranking for different values of ɍ

Tabulated view of IGNCUSCI $(\lambda = 1)$ for different values of □	
□	Ranks
□ = 0.1	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
□ = 0.5	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
□ = 1	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
□ = 2	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
□ = 3	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
□ = 5	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$

The graphical presentation is given below for IGNCUSCI $(\lambda = 1)$ and different values of □ operators.

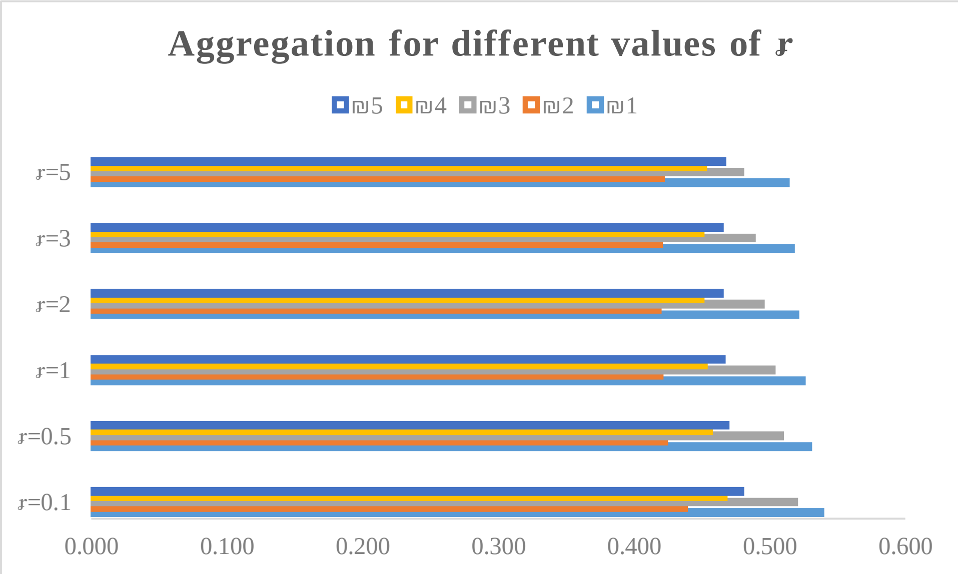

Figure 4.1 The Graphical Presentation of IGNCSCIA

To apply IGNCUSCI to this numerical problem first $\lambda \to 1$ this reduce IGNCUSCI operator into IGNCSCIG operator.

Then the aggregated values of IGNCUSCI $(\lambda \to 1)$ calculated. (ɼ = 1)

$$= \begin{bmatrix} \tilde{u}_{31} = ([0.644, 0.803], [0.339, 0.468], [0.251, 0.271], 0.888, 0.843, 0.869) \\ \tilde{u}_{32} = ([0.518, 0.663], [0.286, 0.372], [0.268, 0.421], 0.634, 0.343, 0.757) \\ \tilde{u}_{33} = ([0.560, 0.664], [0.436, 0.597], [0.261, 0.370], 0.731, 0.646, 0.598) \\ \tilde{u}_{34} = ([0.540, 0.664], [0.311, 0.414], [0.317, 0.415], 0.560, 0.573, 0.714) \\ \tilde{u}_{35} = ([0.645, 0.786], [0.350, 0.436], [0.158, 0.260], 0.665, 0.432, 0.708) \end{bmatrix}$$

The alternatives are ranked in IGNCUSCI $(\lambda \to 1)$ for different values of ɼ and rankings are tabulated in Table 4.3.

Table 4.3: The Ranking Based on values of ɼ

Tabulated view of IGNCUSCI $(\lambda \to 1)$ for different values of ɼ	
ɼ	Ranks
ɼ = 0.1	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
ɼ = 0.5	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
ɼ = 1	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
ɼ = 2	$₪_1 > ₪_3 > ₪_4 > ₪_5 > ₪_2$
ɼ = 3	$₪_3 > ₪_1 > ₪_5 > ₪_4 > ₪_2$
ɼ = 5	$₪_3 > ₪_1 > ₪_5 > ₪_4 > ₪_2$

₪The graphical presentation is given below for IGNCUSCI $(\lambda \to 1)$ and different values of ɼ.

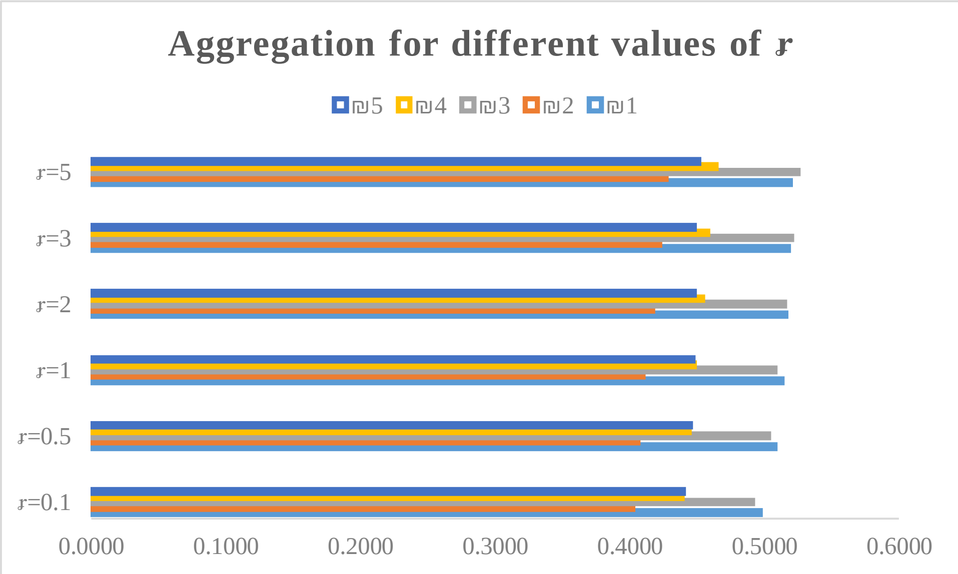

Figure 4.2: The Graphical Presentation of IGNCUSCI $(\lambda \to 1)$

Sensitivity Analysis

The table and figure show that by IGNCSCIG, the aggregated values have different rankings for an alternative. Which makes it a sensitive choice to use it for different values of q. Assigning different values to q and comparing them with other MADM methods, it is observed that the values near 0 in the aggregation operators give results that better match existing methods. This can be overcome by applying a distance-based method like the NCCODAS (see [10]). Otherwise, IGNCSCIA has a ranking that agrees with most existing methods.

Comparative Analysis

To determine the validity of results obtained by current study different existing techniques are applied on above problem. It is worth mentioning that the weight used in these methods were considered from fuzzy Shapley measures. The rankings are tabulated in a table.

Table 4.4: Comparative Analysis with Existence Methods

Method	**Ranking**
NC Einstein geometric aggregation operator in MCDM	$₪_1 > ₪_3 > ₪_2 > ₪_5 > ₪_4$
NC averaging aggregation operators with application MADM	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
GRA MADM in NCS [14]	$₪_1 > ₪_5 > ₪_3 > ₪_2 > ₪_4$
Cosine measure of NCS for MCDM [15]	$₪_1 > ₪_2 > ₪_5 > ₪_3 > ₪_4$
NCCODAS [10]	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
Proposed IGNCSCIA	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
Proposed IGNCSCIG $(q \to 0)$	$₪_1 > ₪_3 > ₪_5 > ₪_4 > ₪_2$
Proposed IGNCSCIG $(q > 2)$	$₪_3 > ₪_1 > ₪_5 > ₪_4 > ₪_2$

From the table it can be observed that the ranking of proposed aggregation operators matches with MADM methods in table with best alternative. Whereas in case of worse alternative at agree with NCCODAS and NC averaging. The second-best alternative is also agreed to NC averaging, NC geometric and NCCODAD method. It is also observed that IGNCSCIG agree to IGNCSCIA for value $(q \leq 2)$ and for $(q > 2)$ the IGNCSCIG ranks different than other method in table. By applying IGNCSCIG one must be aware of sensitive due to different values of q.

Application 2

Cellular companies play a important role in country's stock market. The capital marcket is affected by the performance of these celluar compnies. A company is interested to invest his capital levy in listed companies $\{A_1, A_2, A_3, A_4\}$. Two types of experts are aquired. Attorney D_a look legal matters and market maker D_m expertise in market. Data are collected on the basis of stock market analysis and growth in different areas. The three alternatives are (Ω_1) trends of stock market, (Ω_2) policy directions and (Ω_3) annual performance. The two experts proposed their DM matrices consist of NCS.

The equation 3.1.1 for $\lambda \to 1, q = 1$ is applied throughout this application.

Step 1. We construct the decision maker matrices.

DM matrix for the D_a is

$$\left(\begin{array}{cccc} & \Omega_1 & \Omega_2 & \Omega_3 \\ A_1 & \begin{pmatrix}[0.20,0.60],[0.40,0.60],\\ [0.50,0.80],0.70,0.40,0.30\end{pmatrix} & \begin{pmatrix}[0.10,0.40],[0.50,0.80],\\ [0.40,0.80],0.60,0.70,0.50\end{pmatrix} & \begin{pmatrix}[0.40,0.60],[0.20,0.70],\\ [0.50,0.90],0.40,0.50,0.30\end{pmatrix} \\ A_2 & \begin{pmatrix}[0.30,0.50],[0.60,0.90],\\ [0.30,0.60],0.30,0.60,0.70\end{pmatrix} & \begin{pmatrix}[0.50,0.90],[0.10,0.30],\\ [0.40,0.80],0.80,0.30,0.60\end{pmatrix} & \begin{pmatrix}[0.20,0.70],[0.10,0.60],\\ [0.40,0.70],0.50,0.40,0.70\end{pmatrix} \\ A_3 & \begin{pmatrix}[0.60,0.90],[0.20,0.70],\\ [0.40,0.90],0.50,0.50,0.60\end{pmatrix} & \begin{pmatrix}[0.20,0.60],[0.30,0.70],\\ [0.30,0.80],0.40,0.60,0.50\end{pmatrix} & \begin{pmatrix}[0.50,0.90],[0.70,0.90],\\ [0.10,0.50],0.50,0.60,0.40\end{pmatrix} \\ A_4 & \begin{pmatrix}[0.40,0.80],[0.50,0.90],\\ [0.30,0.80],0.50,0.80,0.50\end{pmatrix} & \begin{pmatrix}[0.20,0.70],[0.40,0.90],\\ [0.50,0.70],0.60,0.40,0.50\end{pmatrix} & \begin{pmatrix}[0.30,0.50],[0.50,0.90],\\ [0.30,0.70],0.30,0.30,0.80\end{pmatrix} \end{array}\right)$$

DM matrix for D_m is

$$\left(\begin{array}{cccc} & \Omega_1 & \Omega_2 & \Omega_3 \\ A_1 & \begin{pmatrix}[0.30,0.60],[0.20,0.60],\\ [0.20,0.60],0.80,0.70,0.20\end{pmatrix} & \begin{pmatrix}[0.30,0.80],[0.40,0.80],\\ [0.30,0.80],0.60,0.70,0.40\end{pmatrix} & \begin{pmatrix}[0.20,0.70],[0.20,0.60],\\ [0.30,0.80],0.50,0.30,0.50\end{pmatrix} \\ A_2 & \begin{pmatrix}[0.20,0.50],[0.60,0.90],\\ [0.30,0.70],0.40,0.80,0.70\end{pmatrix} & \begin{pmatrix}[0.40,0.90],[0.10,0.40],\\ [0.50,0.80],0.60,0.50,0.70\end{pmatrix} & \begin{pmatrix}[0.40,0.90],[0.10,0.40],\\ [0.50,0.80],0.60,0.50,0.70\end{pmatrix} \\ A_3 & \begin{pmatrix}[0.50,0.90],[0.20,0.60],\\ [0.30,0.80],0.70,0.70,0.80\end{pmatrix} & \begin{pmatrix}[0.20,0.50],[0.20,0.70],\\ [0.50,0.80],0.60,0.70,0.20\end{pmatrix} & \begin{pmatrix}[0.30,0.50],[0.30,0.90],\\ [0.20,0.50],0.60,0.50,0.40\end{pmatrix} \\ A_4 & \begin{pmatrix}[0.30,0.50],[0.30,0.90],\\ [0.20,0.50],0.60,0.50,0.40\end{pmatrix} & \begin{pmatrix}[0.40,0.70],[0.20,0.80],\\ [0.30,0.70],0.60,0.70,0.70\end{pmatrix} & \begin{pmatrix}[0.20,0.60],[0.50,0.90],\\ [0.20,0.80],0.40,0.40,0.80\end{pmatrix} \end{array}\right)$$

Step2. Let $W = (0.4, 0.6)^T$, then the single matrix corresponding to weight W is.

$$\begin{pmatrix} & \Omega_1 & \Omega_2 & \Omega_3 \\ A_1 & \begin{pmatrix} [0.255, 0.600], \\ [0.288, 0.673], \\ [0.337, 0.699], \\ 0.765, 0.604, 0.235 \end{pmatrix} & \begin{pmatrix} [0.193, 0.606], \\ [0.443, 0.800], \\ [0.349, 0.868], \\ 0.600, 0.700, 0.477 \end{pmatrix} & \begin{pmatrix} [0.269, 0.658], \\ [0.120, 0.638], \\ [0.388, 0.848], \\ 0.462, 0.388, 0.222 \end{pmatrix} \\ A_2 & \begin{pmatrix} [0.235, 0.558], \\ [0.600, 0.900], \\ [0.300, 0.663], \\ 0.362, 0.736, 0.700 \end{pmatrix} & \begin{pmatrix} [0.235, 0.558], \\ [0.600, 0.900], \\ [0.300, 0.663], \\ 0.362, 0.736, 0.700 \end{pmatrix} & \begin{pmatrix} [0.525, 0.867], \\ [0.152, 0.600], \\ [0.462, 0.845], \\ 0.337, 0.462, 0.330 \end{pmatrix} \\ A_3 & \begin{pmatrix} [0.538, 0.900], \\ [0.356, 0.838], \\ [0.342, 0.848], \\ 0.632, 0.632, 0.713 \end{pmatrix} & \begin{pmatrix} [0.200, 0.538], \\ [0.235, 0.700], \\ [0.428, 0.800], \\ 0.530, 0.663, 0.288 \end{pmatrix} & \begin{pmatrix} [0.368, 0.632], \\ [0.421, 0.900], \\ [0.161, 0.500], \\ 0.563, 0.543, 0.400 \end{pmatrix} \\ A_4 & \begin{pmatrix} [0.510, 0.800], \\ [0.346, 0.632], \\ [0.242, 0.745], \\ 0.500, 0.613, 0.381 \end{pmatrix} & \begin{pmatrix} [0.303, 0.700], \\ [0.264, 0.838], \\ [0.388, 0.700], \\ 0.600, 0.604, 0.612 \end{pmatrix} & \begin{pmatrix} [0.235, 0.558], \\ [0.500, 0.900], \\ [0.242, 0.765], \\ 0.362, 0.362, 0.800 \end{pmatrix} \end{pmatrix}$$

Step3. Let the weights of attributes are $W = \{0.350, 0.300, 0.350\}$, the aggregated value is.

$$\begin{pmatrix} A_1 & \begin{pmatrix} [0.238, 0.620], \\ [0.288, 0.792], \\ [0.357, 0.815], \\ 0.632, 0.576, 0.285 \end{pmatrix} \\ A_2 & \begin{pmatrix} [0.443, 0.766], \\ [0.216, 0.592], \\ [0.538, 0.780], \\ 0.483, 0.573, 0.528 \end{pmatrix} \\ A_3 & \begin{pmatrix} [0.350, 0.662], \\ [0.334, 0.814], \\ [0.313, 0.750], \\ 0.579, 0.613, 0.444 \end{pmatrix} \\ A_4 & \begin{pmatrix} [0.333, 0.677], \\ [0.363, 0.779], \\ [0.289, 0.740], \\ 0.491, 0.536, 0.569 \end{pmatrix} \end{pmatrix}$$

Step4. The alternatives are ranked by score function.

$$S(A_1) = 0.032, S(A_2) = 0.055, S(A_3) = 0.084 \text{ and } S(A_4) = -0.097,$$

$A_3 > A_2 > A_1 > A_4$. The desirable alternative is A_3.

Conclusion

The generalization of aggregation operators plays a critical role in DM and make DM theory more manageable. In this work a unified generalized aggregation operator IGNCUSCI is proposed to counter such situation. The IGNCUSCI can generate a bunch of aggregation operators by assigning value to constraints. Application 1 is evaluated by (3.2.1, 3.2.2) and tested with existing method so that their validity and applicability be tested. It is found that in the case of (3.2.2) a sensitivity arises which needs to be kept in mind.

Application 2 is evaluated using MEMADM applied on (3.1.1). Both applications provide plate form to apply aggregation operators on MADM and MEMADM.

Remark

This idea can be extend to some other aggregation operators like Bonferroni Shapley choquet integral aggregation operators.

Compliance with ethical standards

Conflicts of Interest: The authors declare no conflict of interest.

Ethical approval: This article does not contain any sudies with humain participants or animals performed by any of the authors.

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