On Dominating Energy in Bipolar Single-Valued Neutrosophic Graph

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Abstract: One of the most important concepts in graph theory for dealing with unpredictable phenomena is the concept of domination and it has gained attention from many scholars. Recently, dominating energy of graph plays a vital role in the field of graph energies. If the fuzzy graph (FG) fails to give outstanding results, the neutrosophic set (NS) and neutrosophic graphs (NG) can handle the uncertainty correlated with indeterminate and inconsistent information in any real-world scenario. Recent studies related to domination energy in fuzzy environment only deal with the single membership function. It is more flexible and applicable to use bipolar neutrosophic models because they include both positive and negative influencers. Therefore, this paper is based on some developments of neutrosophic graph theory to deal with situations where imprecision is characterized by positive and negative types of membership functions. A novel concept of the dominating energy graph is proposed based on recently introduced concept of bipolar single-valued neutrosophic graphs (BSVNG). Moreover, this study analyses the concepts of dominating energy graph in BSVNG environment. More precisely, the adjacency matrix of a dominating BSVNG as well as the spectrum of the adjacency matrix and their related theory is developed with the help of illustrative examples. Further, the domination energy of BSVNG is computed. Aside from it, various operations relating to this dominating have been depicted. The complement, union, and join of dominating energy in BSVNG have been investigated by using appropriate examples and some properties of the dominating energy in BSVNG are established.

Keywords: Dominating energy; neutrosophic graph; bipolar single-valued neutrosophic graph

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1. Introduction

The energy of a graph is defined as the sum of the absolute values of its eigenvalues. This concept was proposed by Gutman, motivated by chemical applications. In chemistry, the energy of a given molecular graph is interesting because of its relation to the total π-electron energy of the molecule represented by that graph. A graph with all isolated vertices K_n^c has zero energy, while the complete graph K_n with n vertices have energy $2(n-1)$.

The domination concept in graphs may be utilised to a myriad of issues, including facility location, social network analysis, matching theory, coding theory, communication networks, security systems, clutters, and block cutters. For example, to address challenges in facility location problems in which the number of facilities such as police stations, fire stations, hospitals, and supermarkets is fixed, and one wants to shorten the route that persons must travel to reach the closest facility. Provided that the maximum distance to a facility is defined and efforts are made to lower the number of facilities required to accommodate everyone, a similar situation develops. Furthermore, the domination concept comes up in situations such as monitoring communication or electrical networks, finding sets of representatives, and surveying land.

When considering real life conditions, things are not often precise, and they cannot be described by crisp or deterministic models. Thus, the capacity of making precise statements is quite challenging. In order to handle these vague and imprecise events, Zadeh [1] introduced fuzzy sets together with degrees of membership of elements to these sets . Since that time, ordinary fuzzy sets have been extended to intuitionistic, hesitant, orthopair fuzzy sets, and neutrosophic sets.

Despite all these extensions, fuzzy sets could not handle all types of uncertainties such as indeterminate and inconsistent information. In order to tackle this inadequacy, Smarandache [2] introduced neutrosophic logic and neutrosophic sets . A neutrosophic set is composed of three subsets which are degree of truthiness (T), degree of indeterminacy (I), and degree of falsity (F). These subsets are between]−0, 1+[non-standard unit interval. Thus, a membership function of a neutrosophic number is represented by truth sub-set; non-membership function is represented by falsity sub-set; and hesitancy is represented by indeterminacy sub-set. These features constitute the superiority of neutrosophic sets over the other extensions of fuzzy sets. We utilize neutrosophic sets since their main advantage is the capability in distinguishing relativity and absoluteness of decision makers' preferences.

Bipolarity refers to the tendency of the human mind to analyze and take responsibility based on positive and negative outcomes. The positive analysis is all about reasonable, permitted, appropriate, or considered acceptable, while impossible, rejected or forbidden represents negative analyses. Furthermore, positive thoughts correspond to the preferences as they interpret which objects are preferable to others without rejecting those that do not meet the preferences. Still, negative thoughts correspond to the constraints as they interpret which values or objects must be declined. Based on these consequences, Deli et al. [3] proposed bipolar fuzzy sets and neutrosophic sets to bipolar neutrosophic sets in which positive membership degree, negative membership degree, and operations were studied. Bipolar fuzzy sets have a great value in dealing with uncertainty in real-life problems and useful in dealing with the positive and the negative membership values.

2. Literature review

In 1978, I. Gutman proposed the idea of "graph energy," which is defined as the sum of the absolute values of the eigenvalues of the graph's adjacency matrix. By linking the edge of a graph to the electron energy of a type of molecule, the energy of a graph is employed in quantum theory and many other applications in the context of energy. Later, Gutman and Zhou [4] defined the Laplacian energy of a graph as the sum of the absolute values of the differences of average vertex degree of G to the Laplacian eigenvalues of G. Details on the properties of graph energy and Laplacian energy can be found in [5]–[11].

Meanwhile, after the expansion of fuzzy sets by Zadeh [12], the concept of fuzzy graph was initially proposed by Kauffman [13] and Rosenfeld [14] to deal with the fuzziness in graphs. Fuzzy graphs are effective for representing the structures of object relationship, where the presence of a real object and the link between two objects is ambiguous or uncertain. Some application related to fuzzy graph can be found in [15]-[19]. Apart from that, Anjali and Mathew [20] first proposed the energy of a graph within the fuzzy set environment. In [21], Praba et al. extend the concept of energy in fuzzy graph to the energy of intuitionistic fuzzy graph. Later on, Naz et al. [22] proposed the novel concept of energy graph considering bipolar fuzzy environment and examined some of their properties. Akram and Naz [23] studied on energy of Pythagorean fuzzy graphs and fuzzy digraphs. Akram et al. [24] proposed the concept of energy in bipolar fuzzy graph and demonstrate multi-criteria decision-making approaches in commercial partnerships and smooth communication challenges based on the energy of bipolar fuzzy graphs. Recently, Patra et al. [25] proposed novel techniques of graph energy in interval-valued fuzzy graphs and computed eigenvalues using max–min operators.

The concept of domination is one of the most significant problems in graph theory. In 1998, the novel concepts of domination in fuzzy graphs was first introduced by Somasundaram and Somasundaram [26]. After that, Somasundaram [27] studied the domination in products of fuzzy graphs and discussed various operations on fuzzy graphs such as join, union, composition, Cartesian product and domination parameters. Ghobadi et al. [28] presented an idea of inverse dominating set in fuzzy graph whereas Natarajan and Ayyaswamy [29] initiated the concept of strong (weak) domination in fuzzy graphs. Afterwards, the concepts of cardinality, dominating set, independent set, total dominating number and independent dominating number of bipolar fuzzy graphs was investigated by Karunambigai et al. [30]. Umamageswari et al. [31] introduced the concept of multiple dominating set in bipolar fuzzy graph where the k-dominating set and its domination number in bipolar fuzzy graph were defined.

Later on, Muthuraj et al. [32] defined the non-split total strong (weak) domination in bipolar fuzzy graph and its various parameters. Muneera et al. [33] studied the domination in regular and irregular bipolar fuzzy graphs whereas Akram et al. [34] discussed the different concepts of dominating, total dominating, equitable dominating, total equitable dominating, and independent and equitable independent sets in bipolar fuzzy graphs. Equitable domination in bipolar fuzzy graph, equitable total domination in bipolar fuzzy graph and its various classifications was proposed by Muthuraj and Kanimozhi [35]. Recently, Gong et al. [36] presented the concept of domination in the fuzzy graph to the bipolar frameworks and determined the related expanded concepts of a variety of bipolar fuzzy graphs.

The study of domination has gained attention and it has now been extended to the neutrosophic environment. Hussain et al. [37] introduced the domination number of neutrosophic soft graphs and elaborate them with suitable examples by using strength of path and strength of connectedness. Later on, Banitalebi and Borzooei [38] extend the study of dominating set in the concepts of neutrosophic special dominating set, and define the neutrosophic special domination numbers, neutrosophic special cobondage set and neutrosophic special cobondage numbers in neutrosophic graphs. Ramya et al. [39] presented the concept of complementary domination in single valued neutrosophic graphs (SVNG) and studied the bounds and characteristic of an inverse domination number (IDN) in various SVNG.

Apart from that, the concept of minimum dominating energy was extended to fuzzy graph by Kartheek and Basha [40]. This study defined the properties including various upper and lower bounds for this energy on fuzzy graphs. Praba et al. [41] analyzed the spreading rate of virus on energy of dominating intuitionistic fuzzy graph whereas Kalimulla et al. [42] studied the concept energy of an IFG to dominating energy in operations on IFG. This research employed the various operations such as complement, union, join, Cartesian product and composition to obtain the value of dominating energy. Vijayaragavan et al. [43] obtained the value of dominating Laplacian energy in two products such as Cartesian product and tensor product hence studied the relation between the dominating Laplacian energy in the products in two IFG. Moreover, Sarwar et al. [44] put forward some new concepts of dominating and double dominating energy of m-Polar fuzzy graphs and demonstrate a decision model based on m-polar fuzzy preference relations to solve multi-criteria decision-making problems. Additionally, Akram et al. [45] proposed novel concepts of energy of dominating bipolar fuzzy graph and the energy of double dominating bipolar fuzzy graph.

Since the emergence of the neutrosophic set, many scholars are intended to integrate the study of energy graph, dominating set as well as neutrosophic set. Recently, study from Mullai and Broumi [46] proposed the dominating energy in single-valued neutrosophic graph (SVNG). This study employed the dominating energy in numerous operations such as complement, union and join of neutrosophic graph and provide some theorems related to dominating energy in neutrosophic. Table 1 illustrates the contributions related to domination and energy in fuzzy and neutrosophic graph. Additionally, Figure 1 shows the development of energy graph, domination concept as well as fuzzy and neutrosophic graphs.

Motivated by [45] and [46], we extend this idea and introduce the novel concepts of dominating energy of bipolar single valued neutrosophic graph (BSVNG). Using the notion of eigenvalues of bipolar relations, we study interesting properties and bounds for dominating energy. Before looking deeply into these ideas, the dominating energy of a BSVNG and the dominating energy of different operations on a BSVNG are defined with examples, and certain theorems in dominating energy of BSVNG are developed, as well as other conclusions.

The following is a breakdown of the study's structure: The initial part introduces the historical backgrounds of domination and energy graphs, as well as the concepts of the fuzzy and neutrosophic set. In Part 2, some review of the literature regarding the domination and energy graph in the fuzzy and neutrosophic environment. Part 3 offers a brief introduction to graphs and the neutrosophic set, which will be employed shortly. In Part 4, we establish the dominating energy concept in BSVNG and delve into its aspects. Part 5 demonstrates the dominating energy in various operations in BSVNG and presented new theorems. Finally, Part 6 concludes with a summary of the research's findings as well as its restrictions and a suggestion for further research.

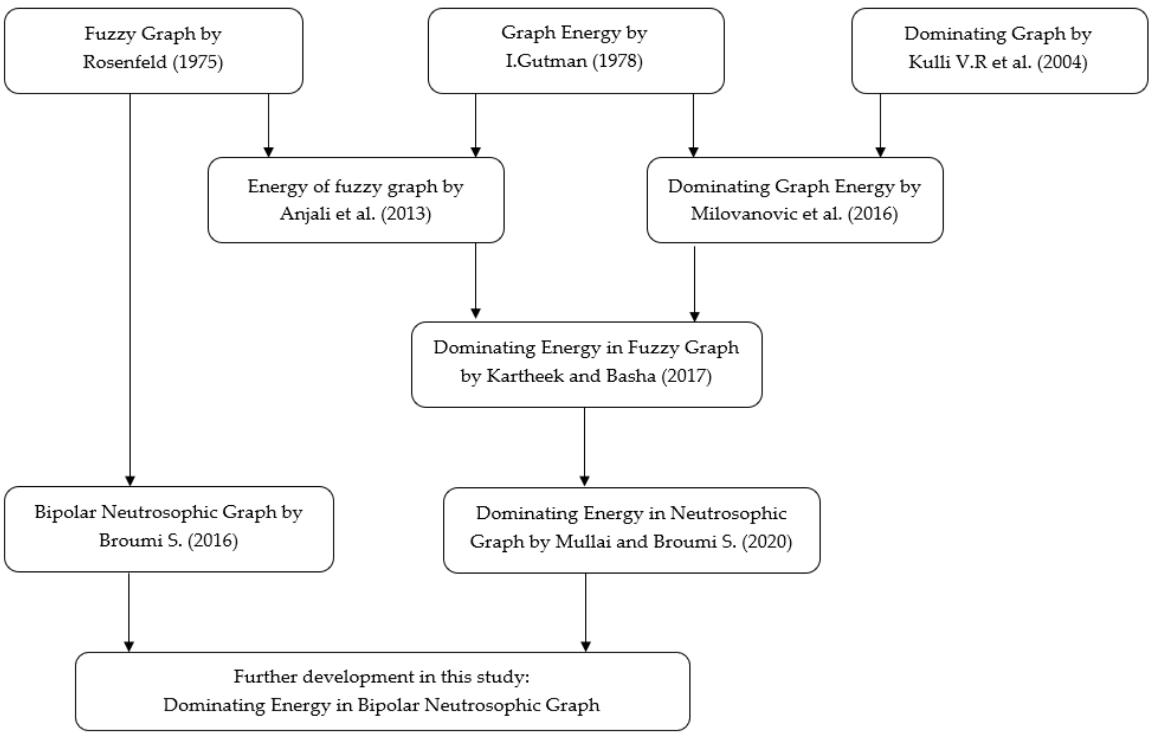

Figure 1: Development of energy, domination, and neutrosophic graphs

Table 1: Contributions related to domination and energy in fuzzy and neutrosophic graphs.

Studied by	Findings	Gaps
Somasundaram & Somasundaram (1998) [26]	Proposed domination concept in FG	Concentrate only uncertainty and ambiguity problem
Somasundaram (2005) [27]	Studied various operation of domination in FG	Concentrate only a few operations
Ghobadi et al., (2008) [28]	Proposed inverse dominating in FG	Did not consider the bipolarity concepts
Natarajan & Ayyaswamy (2010) [29]	Proposed strong (weak) domination in FG	Concentrate only uncertainty and ambiguity problem
Karunambigai et al., (2013) [30]	Studied dominating set in BFG	Did not consider the energy concepts
Umamageswari et al., (2017) [31]	Proposed multiple dominating set of BFG	Did not consider the energy concepts
Muthuraj & Kanimozhi (2019) [32]	Proposed split total strong domination of BFG	Did not investigate the various operation
Muneera et al., (2020) [33]	Studied domination concept in regular and irregular of BFG	Cannot capture the indeterminacy and inconsistent information
Akram et al., (2021) [34]	Proposed various concept of domination in BFG	Did not consider the energy concepts
Muthuraj & Kanimozhi (2020) [35]	Proposed equitable domination in BFG	Did not investigate the various operation
Gong et al., (2021) [36]	Studied domination FG to BFG	Did not consider the energy concepts
Hussain et al., (2019) [37]	Proposed domination number in neutrosophic soft graphs	Did not consider the bipolarity concepts
Banitalebi & Borzooei (2021) [38]	Proposed neutrosophic special domination number	Did not consider the bipolarity concepts
Ramya et al., (2021) [39]	Proposed complementary domination in SVNG	Did not consider the energy concepts
Kartheek & Basha (2017) [40]	Proposed minimum dominating energy in FG	Did not consider the indeterminacy and inconsistent information
Praba et al., (2017) [41]	Proposed dominating energy in IFG	Did not consider the bipolarity concepts
Kalimulla et al., (2018) [42]	Proposed dominating energy of IFG in various operation	Concentrate only a few operations
Vijayaragavan et al., (2019) [43]	Proposed dominating Laplacian energy in product of two IFG	Did not consider the indeterminacy and inconsistent information
Sarwar et al., (2020) [44]	Proposed dominating and double dominating energy of m-polar FG	Did not consider the bipolarity concepts
Akram et al., (2019) [45]	Proposed energy of dominating BFG and doble dominating BFG	Did not consider the indeterminacy and inconsistent information
Mullai & Broumi (2020) [46]	Proposed the dominating energy in SVNG	Did not consider the bipolarity concepts

3. Preliminaries

This section recalls some fundamental definitions with respect to bipolar fuzzy graph (BFG), bipolar single-valued neutrosophic graph (BSVNG) and domination that are necessary for this study.

Definition 3.1 [47]
A bipolar fuzzy graph with a finite set X as the underlying set is a pair of $G=(V,E)$, where $V=(\mu_V^+,\mu_V^-)$ is a bipolar fuzzy set on X and $E=(\mu_E^+,\mu_E^-)$ is a bipolar fuzzy relation on X such that $\mu_E^+(v_m v_n)\le \mu_V^+(v_m)\wedge \mu_V^+(v_n)$ and $\mu_E^-(v_m v_n)\ge \mu_V^+(v_m)\vee \mu_V^+(v_n)$ where $v_m v_n \in E$. We call $V=(\mu_V^+,\mu_V^-)$ the bipolar fuzzy vertex set of X and $E=(\mu_E^+,\mu_E^-)$ the bipolar fuzzy edge set of X.

Definition 3.2 [3]
A bipolar neutrosophic set A on a non-empty set X is an object of the form

$$A=\left\{\left\langle x,T_A^+(x),I_A^+(x),F_A^+(x),T_A^-(x),I_A^-(x),F_A^-(x)\right\rangle : x\in X\right\},$$

where $T_A^+,I_A^+,F_A^+ : X\to[0,1]$ and $T_A^-,I_A^-,F_A^- : X\to[-1,0]$. The positive values $T_A^+(x),I_A^+(x),F_A^+(x)$ denote the truth, indeterminacy and false-memberships degrees of an element $x\in X$, respectively, whereas, $T_A^-(x),I_A^-(x),F_A^-(x)$ denote the implicit counter property of the truth, indeterminacy and false-memberships degrees of the element $x\in X$, respectively, corresponding to the bipolar neutrosophic set A.

Definition 3.3 [48]
Let $A=(T_A^+,I_A^+,F_A^+,T_A^-,I_A^-,F_A^-)$ and $B=(T_B^+,I_B^+,F_B^+,T_B^-,I_B^-,F_B^-)$ be bipolar neutrosophic graph on a set X. If $B=(T_B^+,I_B^+,F_B^+,T_B^-,I_B^-,F_B^-)$ is a bipolar neutrosophic relation on $A=(T_A^+,I_A^+,F_A^+,T_A^-,I_A^-,F_A^-)$, then

$$T_B^+(xy)\le \min\left(T_A^+(x),T_A^+(y)\right), I_B^+(xy)\ge \max\left(I_A^+(x),I_A^+(y)\right), F_B^+(xy)\ge \max\left(F_A^+(x),F_A^+(y)\right),$$
$$T_B^-(xy)\ge \max\left(T_A^-(x),T_A^-(y)\right), I_B^-(xy)\le \min\left(I_A^-(x),I_A^-(y)\right), F_B^-(xy)\le \min\left(F_A^-(x),F_A^-(y)\right),$$
$$\forall x,y\in X.$$

Definition 3.4 [49]
Let $G=(V,E)$ be a bipolar single-valued neutrosophic graph (BSVNG) and $x,y\in V$ in G, then we say that x dominates y if

$$T_E^+(xy)=T_V^+(x)\wedge T_V^+(y), I_E^+(xy)=I_V^+(x)\vee I_V^+(y), F_E^+(xy)=F_V^+(x)\vee F_V^+(y),$$
$$T_E^-(xy)=T_V^-(x)\vee T_V^-(y), I_E^-(xy)=I_V^-(x)\wedge I_V^-(y), F_E^-(xy)=F_V^-(x)\wedge F_V^-(y).$$

4 Dominating energy in bipolar single-valued neutrosophic graphs

In this section, we consider a BSVNG $G=(V,E,\mu^+,\gamma^+,\sigma^+,\mu^-,\gamma^-,\sigma^-)$, then we define $(\mu_1^+,\gamma_1^+,\sigma_1^+) : V\to[0,1], (\mu_1^-,\gamma_1^-,\sigma_1^-) : V\to[-1,0]$ and prove that $(\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-)$ is a bipolar single-valued neutrosophic set (BSVNS).

Definition 4.1

Let $G=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-}\right)$ be a dominating BSVNG where $\mu_1^{+}:V\to[0,1]$, denotes a positive degree of membership, $\gamma_1^{+}:V\to[0,1]$, denotes a positive degree of indeterminacy, $\sigma_1^{+}:V\to[0,1]$, denotes a positive degree of non-membership, $\mu_1^{-}:V\to[-1,0]$, denotes a negative degree of membership, $\gamma_1^{-}:V\to[-1,0]$, denotes a negative degree of indeterminacy and $\sigma_1^{-}:V\to[-1,0]$, denotes a negative degree of non-membership defined such as

$$\mu_1^{+}(v_m)=\min_{v_n}\left[\mu^{+}(v_m,v_n)\right],\gamma_1^{+}(v_m)=\max_{v_n}\left[\gamma^{+}(v_m,v_n)\right],\sigma_1^{+}(v_m)=\max_{v_n}\left[\sigma^{+}(v_m,v_n)\right],$$
$$\mu_1^{-}(v_m)=\max_{v_n}\left[\mu^{-}(v_m,v_n)\right],\gamma_1^{-}(v_m)=\min_{v_n}\left[\gamma^{-}(v_m,v_n)\right],\sigma_1^{-}(v_m)=\min_{v_n}\left[\sigma^{+}(v_m,v_n)\right].$$

Definition 4.2

Let G be a dominating BSVNG. Let $x,y\in V$, we state that x dominates y in G if there exists strong arc from x to y. A subset $D^N\subseteq V$ is called dominating set in G if for each $y\in V-D^N$, there exists x in D^N such that x dominates y .

Definition 4.3

A dominating set D^N of BSVNG is said to be minimal dominating set if for any $x\in D^N, D^N\setminus\{x\}$ is not a dominating set.

Definition 4.4

The minimum cardinality among all minimal dominating sets is called a domination number of G and denoted by $\gamma^N(G)$.

Definition 4.5

Let $G=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-}\right)$ be a dominating BSVNG. The adjacency matrix of a dominating BSVNG G is defined as, $A_{D^N}(G)=\left[d_{jk}\right]$ where

$$d_{jk}=\begin{cases}\begin{pmatrix}\mu_{jk}^{+},\gamma_{jk}^{+},\sigma_{jk}^{+}\\ \mu_{jk}^{-},\gamma_{jk}^{-},\sigma_{jk}^{-}\end{pmatrix} & if\ \left(v_j,v_k\right)\in E\\ \begin{pmatrix}1,1,1,\\ -1,-1,-1\end{pmatrix} & if\ j=k\ and\ v_j\in D^N\\ \begin{pmatrix}0,0,0,\\ 0,0,0\end{pmatrix} & otherwise\end{cases}$$

This dominating bipolar single-valued neutrosophic adjacency matrix, $A_{D^N}(G)$ can be written as $A_{D^N}(G)=\left(\mu_{D^N}^{+}(G),\gamma_{D^N}^{+}(G),\sigma_{D^N}^{+}(G),\mu_{D^N}^{-}(G),\gamma_{D^N}^{-}(G),\sigma_{D^N}^{-}(G)\right)$ where

$$\mu_{D^N}^{+}(G)=\begin{cases}\mu_{jk}^{+} & if\ \left(v_j,v_k\right)\in E\\ 1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases},\qquad \gamma_{D^N}^{+}(G)=\begin{cases}\gamma_{jk}^{+} & if\ \left(v_j,v_k\right)\in E\\ 1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases},$$

$$\sigma^{+}_{D^N}(G)=\begin{cases}\sigma^{+}_{jk} & if\ (v_j,v_k)\in E\\ 1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases},\qquad \mu^{-}_{D^N}(G)=\begin{cases}\mu^{-}_{jk} & if\ (v_j,v_k)\in E\\ -1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases},$$

$$\gamma^{-}_{D^N}(G)=\begin{cases}\gamma^{-}_{jk} & if\ (v_j,v_k)\in E\\ -1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases},\qquad \sigma^{-}_{D^N}(G)=\begin{cases}\sigma^{-}_{jk} & if\ (v_j,v_k)\in E\\ -1 & if\ i=j\ and\ v_j\in D^N\\ 0 & otherwise\end{cases}.$$

Definition 4.6

The spectrum of adjacency matrix of a dominating BSVNG G is defined as $\left(S^{\mu^+}_{D^N},S^{\gamma^+}_{D^N},S^{\sigma^+}_{D^N},S^{\mu^-}_{D^N},S^{\gamma^-}_{D^N},S^{\sigma^-}_{D^N}\right)$, where $S^{\mu^+}_{D^N},S^{\gamma^+}_{D^N},S^{\sigma^+}_{D^N},S^{\mu^-}_{D^N},S^{\gamma^-}_{D^N},S^{\sigma^-}_{D^N}$ are the sets of eigenvalues of $\mu^{+}_{D^N}(G),\gamma^{+}_{D^N}(G),\sigma^{+}_{D^N}(G),\mu^{-}_{D^N}(G),$ $\gamma^{-}_{D^N}(G)$, and $\sigma^{-}_{D^N}(G)$, respectively.

Definition 4.7

The energy of a dominating BSVNG $G=\left(V,E,\mu^+,\gamma^+,\sigma^+,\mu^-,\gamma^-,\sigma^-,\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-\right)$ is defined as

$$E_{D^N}(G)=\left(E\left(\mu^{+}_{D^N}(G)\right),E\left(\gamma^{+}_{D^N}(G)\right),E\left(\sigma^{+}_{D^N}(G)\right),E\left(\mu^{-}_{D^N}(G)\right),E\left(\gamma^{-}_{D^N}(G)\right),E\left(\sigma^{-}_{D^N}(G)\right)\right)$$
$$=\left(\sum_{p=1}^{n}|\zeta_p|,\sum_{p=1}^{n}|\tau_p|,\sum_{p=1}^{n}|\upsilon_p|,\sum_{p=1}^{n}|\vartheta_p|,\sum_{p=1}^{n}|\xi_p|,\sum_{p=1}^{n}|\varepsilon_p|\right)$$

where $S^{\mu^+}_{D^N}=\{\zeta_p\}_{p=1}^{n},S^{\gamma^+}_{D^N}=\{\tau_p\}_{p=1}^{n},S^{\sigma^+}_{D^N}=\{\upsilon_p\}_{p=1}^{n},S^{\mu^-}_{D^N}=\{\vartheta_p\}_{p=1}^{n},S^{\gamma^-}_{D^N}=\{\xi_p\}_{p=1}^{n}$ and $S^{\sigma^-}_{D^N}=\{\varepsilon_p\}_{p=1}^{n}$.

Example 4.8

Consider the BSVNG $G=(V,E)$ where $V=\{v_1,v_2,v_3,v_4,v_5\}$, $E=\{v_1v_2,v_1v_5,v_2v_3,v_2v_5,v_3v_4,v_4v_5\}$ as shown in Figure 2. Then, the above dominating BSVNG can be written as

$$G=\left(V,E,\mu^+,\gamma^+,\sigma^+,\mu^-,\gamma^-,\sigma^-,\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-\right)$$

where $\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-$ are given by $\mu_1^+:V\to[0,1]$, $\gamma_1^+:V\to[0,1]$, $\sigma_1^+:V\to[0,1]$, $\mu_1^-:V\to[-1,0]$, $\gamma_1^-:V\to[-1,0]$, and $\sigma_1^-:V\to[-1,0]$ such that

$$\mu_1^+(v_1)=\min\left[\mu^+(v_1,v_2),\mu^+(v_1,v_5)\right]=\min[0.1,0.5]=0.1$$
$$\mu_1^+(v_2)=\min\left[\mu^+(v_2,v_1),\mu^+(v_2,v_3),\mu^+(v_2,v_5)\right]=\min[0.1,0.2,0.4]=0.1$$
$$\mu_1^+(v_3)=\min\left[\mu^+(v_3,v_2),\mu^+(v_3,v_4)\right]=\min[0.2,0.3]=0.2$$
$$\mu_1^+(v_4)=\min\left[\mu^+(v_4,v_3),\mu^+(v_4,v_5)\right]=\min[0.3,0.2]=0.2$$
$$\mu_1^+(v_5)=\min\left[\mu^+(v_5,v_1),\mu^+(v_5,v_2),\mu^+(v_5,v_4)\right]=\min[0.5,0.4,0.2]=0.2$$
$$\gamma_1^+(v_1)=\max\left[\gamma^+(v_1,v_2),\gamma^+(v_1,v_5)\right]=\max[0.3,0.2]=0.3$$
$$\gamma_1^+(v_2)=\max\left[\gamma^+(v_2,v_1),\gamma^+(v_2,v_3),\gamma^+(v_2,v_5)\right]=\max[0.3,0.6,0.5]=0.6$$
$$\gamma_1^+(v_3)=\max\left[\gamma^+(v_3,v_2),\gamma^+(v_3,v_4)\right]=\max[0.4,0.1]=0.4$$
$$\gamma_1^+(v_4)=\max\left[\gamma^+(v_4,v_3),\gamma^+(v_4,v_5)\right]=\max[0.1,0.2]=0.2$$
$$\gamma_1^+(v_5)=\max\left[\gamma^+(v_5,v_1),\gamma^+(v_5,v_2),\gamma^+(v_5,v_4)\right]=\max[0.2,0.5,0.3]=0.5$$

$$\sigma_1^+(v_1)=\max\left[\sigma^+(v_1,v_2),\sigma^+(v_1,v_5)\right]=\max\left[0.2,0.5\right]=0.5$$
$$\sigma_1^+(v_2)=\max\left[\sigma^+(v_2,v_1),\sigma^+(v_2,v_3),\sigma^+(v_2,v_5)\right]=\max\left[0.5,0.2,0.1\right]=0.5$$
$$\sigma_1^+(v_3)=\max\left[\sigma^+(v_3,v_2),\sigma^+(v_3,v_4)\right]=\max\left[0.2,0.5\right]=0.5$$
$$\sigma_1^+(v_4)=\max\left[\sigma^+(v_4,v_3),\sigma^+(v_4,v_5)\right]=\max\left[0.5,0.3\right]=0.5$$
$$\sigma_1^+(v_5)=\max\left[\sigma^+(v_5,v_1),\sigma^+(v_5,v_2),\sigma^+(v_5,v_4)\right]=\max\left[0.3,0.1,0.3\right]=0.3$$
$$\mu_1^-(v_1)=\max\left[\mu^-(v_1,v_2),\mu^-(v_1,v_5)\right]=\max\left[-0.2,-0.3\right]=-0.2$$
$$\mu_1^-(v_2)=\max\left[\mu^-(v_2,v_1),\mu^-(v_2,v_3),\mu^-(v_2,v_5)\right]=\max\left[-0.2,-0.3,-0.2\right]=-0.2$$
$$\mu_1^-(v_3)=\max\left[\mu^-(v_3,v_2),\mu^-(v_3,v_4)\right]=\max\left[-0.3,-0.4\right]=-0.3$$
$$\mu_1^-(v_4)=\max\left[\mu^-(v_4,v_3),\mu^-(v_4,v_5)\right]=\max\left[-0.4,-0.2\right]=-0.2$$
$$\mu_1^-(v_5)=\max\left[\mu^-(v_5,v_1),\mu^-(v_5,v_2),\mu^-(v_5,v_4)\right]=\max\left[-0.3,-0.2,-0.2\right]=-0.2$$
$$\gamma_1^-(v_1)=\min\left[\gamma^-(v_1,v_2),\gamma^-(v_1,v_5)\right]=\min\left[-0.5,-0.3\right]=-0.5$$
$$\gamma_1^-(v_2)=\min\left[\gamma^-(v_2,v_1),\gamma^-(v_2,v_3),\gamma^-(v_2,v_5)\right]=\min\left[-0.5,-0.4,-0.4\right]=-0.5$$
$$\gamma_1^-(v_3)=\min\left[\gamma^-(v_3,v_2),\gamma^-(v_3,v_4)\right]=\min\left[-0.4,-0.1\right]=-0.4$$
$$\gamma_1^-(v_4)=\min\left[\gamma^-(v_4,v_3),\gamma^-(v_4,v_5)\right]=\min\left[-0.1,-0.2\right]=-0.2$$
$$\gamma_1^-(v_5)=\min\left[\gamma^-(v_5,v_1),\gamma^-(v_5,v_2),\gamma^-(v_5,v_4)\right]=\min\left[-0.3,-0.4,-0.2\right]=-0.4$$
$$\sigma_1^-(v_1)=\min\left[\sigma^-(v_1,v_2),\sigma^-(v_1,v_5)\right]=\min\left[-0.6,-0.2\right]=-0.6$$
$$\sigma_1^-(v_2)=\min\left[\sigma^-(v_2,v_1),\sigma^-(v_2,v_3),\sigma^-(v_2,v_5)\right]=\min\left[-0.6,-0.2,-0.6\right]=-0.6$$
$$\sigma_1^-(v_3)=\min\left[\sigma^-(v_3,v_2),\sigma^-(v_3,v_4)\right]=\min\left[-0.2,-0.7\right]=-0.7$$
$$\sigma_1^-(v_4)=\min\left[\sigma^-(v_4,v_3),\sigma^-(v_4,v_5)\right]=\min\left[-0.7,-0.1\right]=-0.7$$
$$\sigma_1^-(v_5)=\min\left[\sigma^-(v_5,v_1),\sigma^-(v_5,v_2),\sigma^-(v_5,v_4)\right]=\min\left[-0.2,-0.6,-0.1\right]=-0.6$$

Here, v_1 dominates v_2 because

$$\mu^+(v_1v_2)\le\mu_1^+(v_1)\wedge\mu_1^+(v_2)\qquad \mu^-(v_1v_2)\le\mu_1^-(v_1)\wedge\mu_1^-(v_2)$$
$$0.1\le 0.1\wedge 0.1\qquad -0.2\le -0.2\wedge -0.2$$
$$\gamma^+(v_1v_2)\le\gamma_1^+(v_1)\wedge\gamma_1^+(v_2)\qquad \gamma^-(v_1v_2)\le\gamma_1^-(v_1)\wedge\gamma_1^-(v_2)$$
$$0.3\le 0.3\wedge 0.6\qquad -0.5\le -0.5\wedge -0.5$$
$$\sigma^+(v_1v_2)\le\sigma_1^+(v_1)\wedge\sigma_1^+(v_2)\qquad \sigma^-(v_1v_2)\le\sigma_1^-(v_1)\wedge\sigma_1^-(v_2)$$
$$0.5\le 0.5\wedge 0.5\qquad -0.6\le -0.6\wedge -0.6$$

Also, v_4 dominates v_5 because

$$\mu^+(v_4v_5)\le\mu_1^+(v_4)\wedge\mu_1^+(v_5)\qquad \mu^-(v_4v_5)\le\mu_1^-(v_4)\wedge\mu_1^-(v_5)$$
$$0.2\le 0.2\wedge 0.2\qquad -0.4\le -0.4\wedge -0.2$$
$$\gamma^+(v_4v_5)\le\gamma_1^+(v_4)\wedge\gamma_1^+(v_5)\qquad \gamma^-(v_4v_5)\le\gamma_1^-(v_4)\wedge\gamma_1^-(v_5)$$
$$0.2\le 0.2\wedge 0.5\qquad -0.4\le -0.2\wedge -0.4$$
$$\sigma^+(v_4v_5)\le\sigma_1^+(v_4)\wedge\sigma_1^+(v_5)\qquad \sigma^-(v_4v_5)\le\sigma_1^-(v_4)\wedge\sigma_1^-(v_5)$$
$$0.3\le 0.5\wedge 0.3\qquad -0.7\le -0.7\wedge -0.6$$

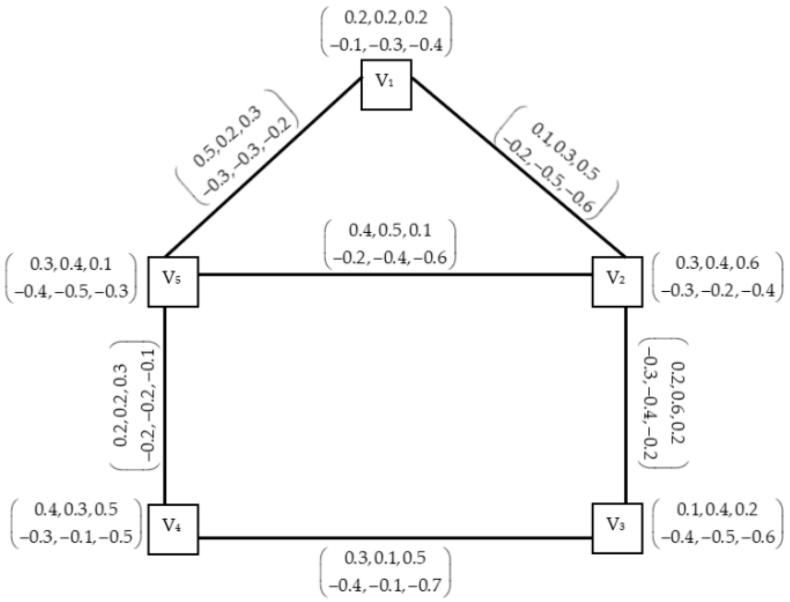

Figure 2. Dominating BSVNG

Thus, $D^N = \{v_1, v_4\}$ is a dominating set because every vertex in $V - D^N = \{v_2, v_3, v_5\}$, is dominated by at least one vertex in D^N and $|D^N| = 2$ is sum of dominating elements. The adjacency matrix of dominating BSVNG is given below

$$A_{D^N}(G) = \begin{bmatrix} \begin{pmatrix} 1,1,1 \\ -1,-1,-1 \end{pmatrix} & \begin{pmatrix} 0.1,0.3,0.5 \\ -0.2,-0.5,-0.6 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.5,0.2,0.3 \\ -0.3,-0.3,-0.2 \end{pmatrix} \\ \begin{pmatrix} 0.1,0.3,0.5 \\ -0.2,-0.5,-0.6 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.2,0.6,0.2 \\ -0.3,-0.4,-0.2 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.4,0.5,0.1 \\ -0.2,-0.4,-0.6 \end{pmatrix} \\ \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.2,0.6,0.2 \\ -0.3,-0.4,-0.2 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.3,0.1,0.5 \\ -0.4,-0.1,-0.7 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} \\ \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.3,0.1,0.5 \\ -0.4,-0.1,-0.7 \end{pmatrix} & \begin{pmatrix} 1,1,1 \\ -1,-1,-1 \end{pmatrix} & \begin{pmatrix} 0.2,0.2,0.3 \\ -0.2,-0.2,-0.1 \end{pmatrix} \\ \begin{pmatrix} 0.5,0.2,0.3 \\ -0.3,-0.3,-0.2 \end{pmatrix} & \begin{pmatrix} 0.4,0.5,0.1 \\ -0.2,-0.4,-0.6 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.2,0.2,0.3 \\ -0.2,-0.2,-0.1 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} \end{bmatrix}$$

This can be written in six different matrices as:

$$A\left(\mu^{+}_{D^N}(G)\right) = \begin{bmatrix} 1 & 0.1 & 0 & 0 & 0.5 \\ 0.1 & 0 & 0.2 & 0 & 0.4 \\ 0 & 0.2 & 0 & 0.3 & 0 \\ 0 & 0 & 0.3 & 1 & 0.2 \\ 0.5 & 0.4 & 0 & 0.2 & 0 \end{bmatrix} \qquad A\left(\mu^{-}_{D^N}(G)\right) = \begin{bmatrix} -1 & -0.2 & 0 & 0 & -0.3 \\ -0.2 & 0 & -0.3 & 0 & -0.2 \\ 0 & -0.3 & 0 & -0.4 & 0 \\ 0 & 0 & -0.4 & -1 & -0.2 \\ -0.3 & -0.2 & 0 & -0.2 & 0 \end{bmatrix}$$

$$A\left(\gamma^{+}_{D^N}(G)\right)=\begin{bmatrix}1 & 0.3 & 0 & 0 & 0.2\\ 0.3 & 0 & 0.6 & 0 & 0.5\\ 0 & 0.6 & 0 & 0.1 & 0\\ 0 & 0 & 0.1 & 1 & 0.2\\ 0.2 & 0.5 & 0 & 0.2 & 0\end{bmatrix} \qquad A\left(\gamma^{-}_{D^N}(G)\right)=\begin{bmatrix}-1 & -0.5 & 0 & 0 & -0.3\\ -0.5 & 0 & -0.4 & 0 & -0.4\\ 0 & -0.4 & 0 & -0.1 & 0\\ 0 & 0 & -0.1 & -1 & -0.2\\ -0.3 & -0.4 & 0 & -0.2 & 0\end{bmatrix}$$

$$A\left(\sigma^{+}_{D^N}(G)\right)=\begin{bmatrix}1 & 0.5 & 0 & 0 & 0.3\\ 0.5 & 0 & 0.2 & 0 & 0.1\\ 0 & 0.2 & 0 & 0.5 & 0\\ 0 & 0 & 0.5 & 1 & 0.3\\ 0.3 & 0.1 & 0 & 0.3 & 0\end{bmatrix} \qquad A\left(\sigma^{-}_{D^N}(G)\right)=\begin{bmatrix}-1 & -0.6 & 0 & 0 & -0.2\\ -0.6 & 0 & -0.2 & 0 & -0.6\\ 0 & -0.2 & 0 & -0.7 & 0\\ 0 & 0 & -0.7 & -1 & -0.1\\ -0.2 & -0.6 & 0 & -0.1 & 0\end{bmatrix}$$

Since,

$$Spec\left(A\left(\mu^{+}_{D^N}(G)\right)\right)=\{-0.530,-0.080,0.239,1.069,1.301\}$$
$$Spec\left(A\left(\gamma^{+}_{D^N}(G)\right)\right)=\{-0.801,-0.035,0.557,1.019,1.261\}$$
$$Spec\left(A\left(\sigma^{+}_{D^N}(G)\right)\right)=\{-0.398,-0.192,0.042,1.181,1.367\}$$
$$Spec\left(A\left(\mu^{-}_{D^N}(G)\right)\right)=\{-1.233,-1.081,-0.190,0.068,0.436\}$$
$$Spec\left(A\left(\gamma^{-}_{D^N}(G)\right)\right)=\{-1.385,-1.025,-0.272,0.073,0.609\}$$
$$Spec\left(A\left(\sigma^{-}_{D^N}(G)\right)\right)=\{-1.477,-1.322,-0.225,0.295,0.730\}$$

Therefore,

$$Spec\left(A_{D^N}(G)\right)=\left\{\begin{array}{c}(-0.530,-0.801,-0.398,-1.233,-1.385,-1.477),\\ (-0.080,-0.035,-0.192,-1.081,-1.025,-1.322),\\ (0.239,0.557,0.042,-0.190,-0.272,-0.225),\\ (1.069,1.019,1.181,0.068,0.073,0.295),\\ (1.301,1.261,1.367,0.436,0.609,0.730)\end{array}\right\}$$

The energy of BSVNG is

$$\begin{aligned}E_{D^N}(G)&=\left(E\left(\mu^{+}_{D^N}(G)\right),E\left(\gamma^{+}_{D^N}(G)\right),E\left(\sigma^{+}_{D^N}(G)\right),E\left(\mu^{-}_{D^N}(G)\right),E\left(\gamma^{-}_{D^N}(G)\right),E\left(\sigma^{-}_{D^N}(G)\right)\right)\\ &=\left(\sum_{p=1}^{n}|\zeta_p|,\sum_{p=1}^{n}|\tau_p|,\sum_{p=1}^{n}|\upsilon_p|,\sum_{p=1}^{n}|\vartheta_p|,\sum_{p=1}^{n}|\xi_p|,\sum_{p=1}^{n}|\varepsilon_p|\right)\\ &=(3.219,3.673,3.180,3.008,3.364,4.049)\end{aligned}$$

5. Dominating energy in various operation of BSVNG

5.1 Dominating energy in complement of bipolar single-valued neutrosophic graphs

Definition 5.1

The complement of BSVNG $G=(V,E)$ is a BSVNG $\bar{G}=(\bar{V},\bar{E})$ where

$$\bar{\mu}_{1i}^{+}=\mu_{1i}^{+},\bar{\gamma}_{1i}^{+}=\gamma_{1i}^{+},\bar{\sigma}_{1i}^{+}=\sigma_{1i}^{+},$$

$$\bar{\mu}_{1i}^{-}=\mu_{1i}^{-},\bar{\gamma}_{1i}^{-}=\gamma_{1i}^{-},\bar{\sigma}_{1i}^{-}=\sigma_{1i}^{-},\forall i=1,2,\dots,n$$

$$\bar{\mu}_{2ij}^{+}=\mu_{1i}^{+}\mu_{1j}^{+}-\mu_{2ij}^{+},\bar{\gamma}_{2ij}^{+}=\gamma_{1i}^{+}\gamma_{1j}^{+}-\gamma_{2ij}^{+},\bar{\sigma}_{2ij}^{+}=\sigma_{1i}^{+}\sigma_{1j}^{+}-\sigma_{2ij}^{+},$$

$$\bar{\mu}_{2ij}^{-}=\mu_{1i}^{-}\mu_{1j}^{-}-\mu_{2ij}^{-},\bar{\gamma}_{2ij}^{-}=\gamma_{1i}^{-}\gamma_{1j}^{-}-\gamma_{2ij}^{-},\bar{\sigma}_{2ij}^{-}=\sigma_{1i}^{-}\sigma_{1j}^{-}-\sigma_{2ij}^{-},\forall i,j=1,2,\dots,n.$$

Example 5.2

First, we find the dominating energy of BSVNG $G=(V,E)$ as shown in Figure 3. Consider a dominating BSVNG $G=(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-})$ where $V=\{v_1,v_2,v_3,v_4\}$, and $\mu_1^{+}:V\to[0,1],\gamma_1^{+}:V\to[0,1],\sigma_1^{+}:V\to[0,1],\mu_1^{-}:V\to[-1,0],\gamma_1^{-}:V\to[-1,0],\sigma_1^{-}:V\to[-1,0]$ such that

$$\begin{aligned}\mu_1^{+}(v_1)&=\min[\mu^{+}(v_1v_2),\mu^{+}(v_1v_4)]\\&=\min[0.1,0.1]=0.1\\ \mu_1^{+}(v_2)&=\min[\mu^{+}(v_2v_1),\mu^{+}(v_2v_3)]\\&=\min[0.1,0.3]=0.1\\ \mu_1^{+}(v_3)&=\min[\mu^{+}(v_3v_2),\mu^{+}(v_3v_4)]\\&=\min[0.3,0.1]=0.1\\ \mu_1^{+}(v_4)&=\min[\mu^{+}(v_4v_1),\mu^{+}(v_4v_3)]\\&=\min[0.1,0.1]=0.1\\ \gamma_1^{+}(v_1)&=\max[\gamma^{+}(v_1v_2),\gamma^{+}(v_1v_4)]\\&=\max[0.5,0.5]=0.5\\ \gamma_1^{+}(v_2)&=\max[\gamma^{+}(v_2v_1),\gamma^{+}(v_2v_3)]\\&=\max[0.5,0.4]=0.5\\ \gamma_1^{+}(v_3)&=\max[\gamma^{+}(v_3v_2),\gamma^{+}(v_3v_4)]\\&=\max[0.4,0.4]=0.4\\ \gamma_1^{+}(v_4)&=\max[\gamma^{+}(v_4v_1),\gamma^{+}(v_4v_3)]\\&=\max[0.5,0.4]=0.5\\ \sigma_1^{+}(v_1)&=\max[\sigma^{+}(v_1v_2),\sigma^{+}(v_1v_4)]\\&=\max[0.6,0.6]=0.6\\ \sigma_1^{+}(v_2)&=\max[\sigma^{+}(v_2v_1),\sigma^{+}(v_2v_3)]\\&=\max[0.6,0.7]=0.7\\ \sigma_1^{+}(v_3)&=\max[\sigma^{+}(v_3v_2),\sigma^{+}(v_3v_4)]\\&=\max[0.7,0.6]=0.7\\ \sigma_1^{+}(v_4)&=\max[\sigma^{+}(v_4v_1),\sigma^{+}(v_4v_3)]\\&=\max[0.6,0.6]=0.6\end{aligned}$$

$$\begin{aligned}\mu_1^{-}(v_1)&=\max[\mu^{-}(v_1v_2),\mu^{-}(v_1v_4)]\\&=\max[-0.2,-0.3]=-0.2\\ \mu_1^{-}(v_2)&=\max[\mu^{-}(v_2v_1),\mu^{-}(v_2v_3)]\\&=\max[-0.2,-0.2]=-0.2\\ \mu_1^{-}(v_3)&=\max[\mu^{-}(v_3v_2),\mu^{-}(v_3v_4)]\\&=\max[-0.2,-0.2]=-0.2\\ \mu_1^{-}(v_4)&=\max[\mu^{-}(v_4v_1),\mu^{-}(v_4v_3)]\\&=\max[-0.3,-0.2]=-0.2\\ \gamma_1^{-}(v_1)&=\min[\gamma^{-}(v_1v_2),\gamma^{-}(v_1v_4)]\\&=\min[-0.6,-0.6]=-0.6\\ \gamma_1^{-}(v_2)&=\min[\gamma^{-}(v_2v_1),\gamma^{-}(v_2v_3)]\\&=\min[-0.6,-0.6]=-0.6\\ \gamma_1^{-}(v_3)&=\min[\gamma^{-}(v_3v_2),\gamma^{-}(v_3v_4)]\\&=\min[-0.6,-0.6]=-0.6\\ \gamma_1^{-}(v_4)&=\min[\gamma^{-}(v_4v_1),\gamma^{-}(v_4v_3)]\\&=\min[-0.6,-0.6]=-0.6\\ \sigma_1^{-}(v_1)&=\min[\sigma^{-}(v_1v_2),\sigma^{-}(v_1v_4)]\\&=\min[-0.7,-0.7]=-0.7\\ \sigma_1^{-}(v_2)&=\min[\sigma^{-}(v_2v_1),\sigma^{-}(v_2v_3)]\\&=\min[-0.7,-0.7]=-0.7\\ \sigma_1^{-}(v_3)&=\min[\sigma^{-}(v_3v_2),\sigma^{-}(v_3v_4)]\\&=\min[-0.7,-0.5]=-0.7\\ \sigma_1^{-}(v_4)&=\min[\sigma^{-}(v_4v_1),\sigma^{-}(v_4v_3)]\\&=\min[-0.7,-0.5]=-0.7\end{aligned}$$

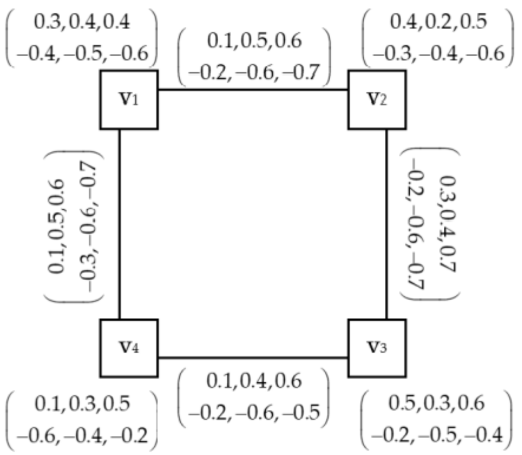

Figure 3: $G=(V,E)$

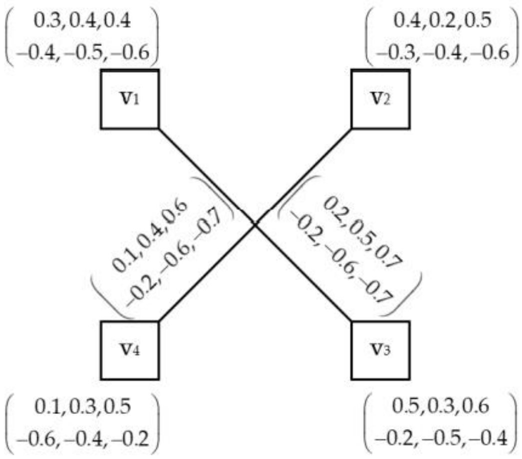

Figure 4: $\bar{G}=(\bar{V},\bar{E})$

Here, v_2 dominates v_1 because

$$\mu^+(v_2v_1)\le\mu_1^+(v_2)\wedge\mu_1^+(v_1) \qquad \mu^-(v_2v_1)\le\mu_1^-(v_2)\wedge\mu_1^-(v_1)$$
$$0.1\le 0.1\wedge 0.1 \qquad -0.3\le -0.2\wedge -0.2$$
$$\gamma^+(v_2v_1)\le\gamma_1^+(v_2)\wedge\gamma_1^+(v_1) \qquad \gamma^-(v_2v_1)\le\gamma_1^-(v_2)\wedge\gamma_1^-(v_1)$$
$$0.5\le 0.5\wedge 0.5 \qquad -0.6\le -0.6\wedge -0.6$$
$$\sigma^+(v_2v_1)\le\sigma_1^+(v_2)\wedge\sigma_1^+(v_1) \qquad \sigma^-(v_2v_1)\le\sigma_1^-(v_2)\wedge\sigma_1^-(v_1)$$
$$0.6\le 0.7\wedge 0.6 \qquad -0.7\le -0.7\wedge -0.7$$

Also, v_1 dominates v_4 because

$$\mu^+(v_1v_4)\le\mu_1^+(v_1)\wedge\mu_1^+(v_4) \qquad \mu^-(v_1v_4)\le\mu_1^-(v_1)\wedge\mu_1^-(v_4)$$
$$0.1\le 0.1\wedge 0.1 \qquad -0.3\le -0.2\wedge -0.2$$
$$\gamma^+(v_1v_4)\le\gamma_1^+(v_1)\wedge\gamma_1^+(v_4) \qquad \gamma^-(v_1v_4)\le\gamma_1^-(v_1)\wedge\gamma_1^-(v_4)$$
$$0.5\le 0.5\wedge 0.5 \qquad -0.6\le -0.6\wedge -0.6$$
$$\sigma^+(v_1v_4)\le\sigma_1^+(v_1)\wedge\sigma_1^+(v_4) \qquad \sigma^-(v_1v_4)\le\sigma_1^-(v_1)\wedge\sigma_1^-(v_4)$$
$$0.6\le 0.6\wedge 0.6 \qquad -0.7\le -0.7\wedge -0.7$$

Thus, $D^N=\{v_1,v_2\}$ is a dominating set because every vertex in $V-D^N=\{v_3,v_4\}$, is dominated by at least one vertex in D^N and $|D^N|=2$ is sum of dominating elements. The adjacency matrix of dominating BSVNG is given below;

$$A_{D^N}(G)=\begin{bmatrix} \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.1,0.5,0.6\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.1,0.5,0.6\\-0.3,-0.6,-0.7\end{pmatrix} \\ \begin{pmatrix}0.1,0.5,0.6\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.3,0.4,0.7\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.3,0.4,0.7\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.1,0.4,0.6\\-0.2,-0.6,-0.5\end{pmatrix} \\ \begin{pmatrix}0.1,0.5,0.6\\-0.3,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.1,0.4,0.6\\-0.2,-0.6,-0.5\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \end{bmatrix}$$

This can be written in six different matrices as:

$$A\left(\mu^{+}_{D^N}(G)\right)=\begin{bmatrix}1 & 0.1 & 0 & 0.1\\ 0.1 & 1 & 0.3 & 0\\ 0 & 0.3 & 0 & 0.1\\ 0.1 & 0 & 0.1 & 0\end{bmatrix} \qquad A\left(\mu^{-}_{D^N}(G)\right)=\begin{bmatrix}-1 & -0.2 & 0 & -0.3\\ -0.2 & -1 & -0.2 & 0\\ 0 & -0.2 & 0 & -0.2\\ -0.3 & 0 & -0.2 & 0\end{bmatrix}$$

$$A\left(\gamma^{+}_{D^N}(G)\right)=\begin{bmatrix}1 & 0.5 & 0 & 0.5\\ 0.5 & 1 & 0.4 & 0\\ 0 & 0.4 & 0 & 0.4\\ 0.5 & 0 & 0.4 & 0\end{bmatrix} \qquad A\left(\gamma^{-}_{D^N}(G)\right)=\begin{bmatrix}-1 & -0.6 & 0 & -0.6\\ -0.6 & -1 & -0.6 & 0\\ 0 & -0.6 & 0 & -0.6\\ -0.6 & 0 & -0.6 & 0\end{bmatrix}$$

$$A\left(\sigma^{+}_{D^N}(G)\right)=\begin{bmatrix}1 & 0.6 & 0 & 0.6\\ 0.6 & 1 & 0.7 & 0\\ 0 & 0.7 & 0 & 0.6\\ 0.6 & 0 & 0.6 & 0\end{bmatrix} \qquad A\left(\sigma^{-}_{D^N}(G)\right)=\begin{bmatrix}-1 & -0.7 & 0 & -0.7\\ -0.7 & -1 & -0.7 & 0\\ 0 & -0.7 & 0 & -0.5\\ -0.7 & 0 & -0.5 & 0\end{bmatrix}$$

Since,

$$\begin{aligned}
Spec\left(A\left(\mu^{+}_{D^N}(G)\right)\right)&=\{0.942,0.058,1.152,-0.152\},\\
Spec\left(A\left(\gamma^{+}_{D^N}(G)\right)\right)&=\{-0.588,0.238,0.688,1.662\},\\
Spec\left(A\left(\sigma^{+}_{D^N}(G)\right)\right)&=\{0.721,0.279,1.922,-0.922\},\\
Spec\left(A\left(\mu^{-}_{D^N}(G)\right)\right)&=\{-0.139,-0.861,0.262,-1.262\},\\
Spec\left(A\left(\gamma^{-}_{D^N}(G)\right)\right)&=\{-0.319,-0.681,0.881,-1.881\},\\
Spec\left(A\left(\sigma^{-}_{D^N}(G)\right)\right)&=\{-0.706,0.906,-2.022,-0.178\}.
\end{aligned}$$

Therefore, the dominating energy of BSVNG is

$$\begin{aligned}
E_{D^N}(G)&=\left(E\left(\mu^{+}_{D^N}(G)\right),E\left(\gamma^{+}_{D^N}(G)\right),E\left(\sigma^{+}_{D^N}(G)\right),E\left(\mu^{-}_{D^N}(G)\right),E\left(\gamma^{-}_{D^N}(G)\right),E\left(\sigma^{-}_{D^N}(G)\right)\right)\\
&=\left(\sum_{p=1}^{n}|\zeta_p|,\sum_{p=1}^{n}|\tau_p|,\sum_{p=1}^{n}|\upsilon_p|,\sum_{p=1}^{n}|\vartheta_p|,\sum_{p=1}^{n}|\xi_p|,\sum_{p=1}^{n}|\varepsilon_p|\right)\\
&=\left(2.304,3.176,3.844,2.524,3.762,3.812\right).
\end{aligned}$$

Now, we find the dominating energy of complement of BSVNG $\bar{G}=\left(\bar{V},\bar{E}\right)$ as shown in Figure 4;

Consider a dominating BSVNG $\bar{G}=\left(V,E,\bar{\mu}^{+},\bar{\gamma}^{+},\bar{\sigma}^{+},\bar{\mu}^{-},\bar{\gamma}^{-},\bar{\sigma}^{-},\bar{\mu}_1^{+},\bar{\gamma}_1^{+},\bar{\sigma}_1^{+},\bar{\mu}_1^{-},\bar{\gamma}_1^{-},\bar{\sigma}_1^{-}\right)$ where $V=\{v_1,v_2,v_3,v_4\}$ and given, $\bar{\mu}_1^{+}:V\to[0,1]$, $\bar{\gamma}_1^{+}:V\to[0,1]$, $\bar{\sigma}_1^{+}:V\to[0,1]$, $\bar{\mu}_1^{-}:V\to[-1,0]$, $\bar{\gamma}_1^{-}:V\to[-1,0]$, $\bar{\sigma}_1^{-}:V\to[-1,0]$ where

$$\begin{aligned}
\bar{\mu}_1^{+}(v_1)&=\min\left[\bar{\mu}^{+}(v_1v_3)\right]=\min[0.2]=0.2 & \bar{\mu}_1^{-}(v_1)&=\max\left[\bar{\mu}^{-}(v_1v_3)\right]=\max[-0.2]=-0.2\\
\bar{\mu}_1^{+}(v_2)&=\min\left[\bar{\mu}^{+}(v_2v_4)\right]=\min[0.1]=0.1 & \bar{\mu}_1^{-}(v_2)&=\max\left[\bar{\mu}^{-}(v_2v_4)\right]=\max[-0.2]=-0.2\\
\bar{\mu}_1^{+}(v_3)&=\min\left[\bar{\mu}^{+}(v_3v_1)\right]=\min[0.2]=0.2 & \bar{\mu}_1^{-}(v_3)&=\max\left[\bar{\mu}^{-}(v_3v_1)\right]=\max[-0.2]=-0.2\\
\bar{\mu}_1^{+}(v_4)&=\min\left[\bar{\mu}^{+}(v_4v_2)\right]=\min[0.1]=0.1 & \bar{\mu}_1^{-}(v_4)&=\max\left[\bar{\mu}^{-}(v_4v_2)\right]=\max[-0.2]=-0.2
\end{aligned}$$

$$\bar{\gamma}_1^+(v_1) = \max\left[\bar{\gamma}^+(v_1v_3)\right] = \max[0.5] = 0.5 \qquad \bar{\gamma}_1^-(v_1) = \min\left[\bar{\gamma}^-(v_1v_3)\right] = \min[-0.6] = -0.6$$
$$\bar{\gamma}_1^+(v_2) = \max\left[\bar{\gamma}^+(v_2v_4)\right] = \max[0.4] = 0.4 \qquad \bar{\gamma}_1^-(v_2) = \min\left[\bar{\gamma}^-(v_2v_4)\right] = \min[-0.6] = -0.6$$
$$\bar{\gamma}_1^+(v_3) = \max\left[\bar{\gamma}^+(v_3v_1)\right] = \max[0.5] = 0.5 \qquad \bar{\gamma}_1^-(v_3) = \min\left[\bar{\gamma}^-(v_3v_1)\right] = \min[-0.6] = -0.6$$
$$\bar{\gamma}_1^+(v_4) = \max\left[\bar{\gamma}^+(v_4v_2)\right] = \max[0.4] = 0.4 \qquad \bar{\gamma}_1^-(v_4) = \min\left[\bar{\gamma}^-(v_4v_2)\right] = \min[-0.6] = -0.6$$
$$\bar{\sigma}_1^+(v_1) = \max\left[\bar{\sigma}^+(v_1v_3)\right] = \max[0.7] = 0.7 \qquad \bar{\sigma}_1^-(v_1) = \min\left[\bar{\sigma}^-(v_1v_3)\right] = \min[-0.7] = -0.7$$
$$\bar{\sigma}_1^+(v_2) = \max\left[\bar{\sigma}^+(v_2v_4)\right] = \max[0.6] = 0.6 \qquad \bar{\sigma}_1^-(v_2) = \min\left[\bar{\sigma}^-(v_2v_4)\right] = \min[-0.7] = -0.7$$
$$\bar{\sigma}_1^+(v_3) = \max\left[\bar{\sigma}^+(v_3v_1)\right] = \max[0.7] = 0.7 \qquad \bar{\sigma}_1^-(v_3) = \min\left[\bar{\sigma}^-(v_3v_1)\right] = \min[-0.7] = -0.7$$
$$\bar{\sigma}_1^+(v_4) = \max\left[\bar{\sigma}^+(v_4v_2)\right] = \max[0.6] = 0.6 \qquad \bar{\sigma}_1^-(v_4) = \min\left[\bar{\sigma}^-(v_4v_2)\right] = \min[-0.7] = -0.7$$

Here, v_1 dominates v_3 because

$$\bar{\mu}^+(v_1v_3) \le \bar{\mu}_1^+(v_1) \wedge \bar{\mu}_1^+(v_3) \qquad \bar{\mu}^-(v_1v_3) \le \bar{\mu}_1^-(v_1) \wedge \bar{\mu}_1^-(v_3)$$
$$0.2 \le 0.2 \wedge 0.2 \qquad -0.2 \le -0.2 \wedge -0.2$$
$$\bar{\gamma}^+(v_1v_3) \le \bar{\gamma}_1^+(v_1) \wedge \bar{\gamma}_1^+(v_3) \qquad \bar{\gamma}^-(v_1v_3) \le \bar{\gamma}_1^-(v_1) \wedge \bar{\gamma}_1^-(v_3)$$
$$0.5 \le 0.5 \wedge 0.5 \qquad -0.6 \le -0.6 \wedge -0.6$$
$$\bar{\sigma}^+(v_1v_3) \le \bar{\sigma}_1^+(v_1) \wedge \bar{\sigma}_1^+(v_3) \qquad \bar{\sigma}^-(v_1v_3) \le \bar{\sigma}_1^-(v_1) \wedge \bar{\sigma}_1^-(v_3)$$
$$0.7 \le 0.7 \wedge 0.7 \qquad -0.7 \le -0.7 \wedge -0.7$$

Also, v_2 dominates v_4 because

$$\bar{\mu}^+(v_2v_4) \le \bar{\mu}_1^+(v_2) \wedge \bar{\mu}_1^+(v_4) \qquad \bar{\mu}^-(v_1v_4) \le \bar{\mu}_1^-(v_1) \wedge \bar{\mu}_1^-(v_4)$$
$$0.1 \le 0.1 \wedge 0.1 \qquad -0.2 \le -0.2 \wedge -0.2$$
$$\bar{\gamma}^+(v_2v_4) \le \bar{\gamma}_1^+(v_2) \wedge \bar{\gamma}_1^+(v_4) \qquad \bar{\gamma}^-(v_1v_4) \le \bar{\gamma}_1^-(v_1) \wedge \bar{\gamma}_1^-(v_4)$$
$$0.4 \le 0.4 \wedge 0.4 \qquad -0.6 \le -0.6 \wedge -0.6$$
$$\bar{\sigma}^+(v_2v_4) \le \bar{\sigma}_1^+(v_2) \wedge \bar{\sigma}_1^+(v_4) \qquad \bar{\sigma}^-(v_1v_4) \le \bar{\sigma}_1^-(v_1) \wedge \bar{\sigma}_1^-(v_4)$$
$$0.6 \le 0.6 \wedge 0.6 \qquad -0.7 \le -0.7 \wedge -0.7$$

Thus, $D^N = \{v_1, v_2\}$ is a dominating set because every vertex in $V - D^N = \{v_3, v_4\}$, is dominated by at least one vertex in D^N and $|D^N| = 2$ is sum of dominating elements. The adjacency matrix of dominating of complement BSVNG is given below;

$$A_{D^N}(\bar{G}) = \begin{bmatrix} \begin{pmatrix} 1,1,1 \\ -1,-1,-1 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.2,0.5,0.7 \\ -0.2,-0.6,-0.7 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} \\ \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 1,1,1 \\ -1,-1,-1 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.1,0.4,0.6 \\ -0.2,-0.6,-0.7 \end{pmatrix} \\ \begin{pmatrix} 0.2,0.5,0.7 \\ -0.2,-0.6,-0.7 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} \\ \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0.1,0.4,0.6 \\ -0.2,-0.6,-0.7 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} & \begin{pmatrix} 0,0,0 \\ 0,0,0 \end{pmatrix} \end{bmatrix}$$

This can be written in six different matrices as:

$$A\left(\bar{\mu}_{D^N}^{+}\left(\bar{G}\right)\right)=\begin{bmatrix}1&0&0.2&0\\0&1&0&0.1\\0.2&0&0&0\\0&0.1&0&0\end{bmatrix}\qquad A\left(\bar{\mu}_{D^N}^{-}\left(\bar{G}\right)\right)=\begin{bmatrix}-1&0&-0.2&0\\0&-1&0&-0.2\\-0.2&0&0&0\\0&-0.2&0&0\end{bmatrix}$$

$$A\left(\bar{\gamma}_{D^N}^{+}\left(\bar{G}\right)\right)=\begin{bmatrix}1&0&0.5&0\\0&1&0&0.4\\0.5&0&0&0\\0&0.4&0&0\end{bmatrix}\qquad A\left(\bar{\gamma}_{D^N}^{-}\left(\bar{G}\right)\right)=\begin{bmatrix}-1&0&-0.6&0\\0&-1&0&-0.6\\-0.6&0&0&0\\0&-0.6&0&0\end{bmatrix}$$

$$A\left(\bar{\sigma}_{D^N}^{+}\left(\bar{G}\right)\right)=\begin{bmatrix}1&0&0.7&0\\0&1&0&0.6\\0.7&0&0&0\\0&0.6&0&0\end{bmatrix}\qquad A\left(\bar{\sigma}_{D^N}^{-}\left(\bar{G}\right)\right)=\begin{bmatrix}-1&0&-0.7&0\\0&-1&0&-0.7\\-0.7&0&0&0\\0&-0.7&0&0\end{bmatrix}$$

Since,

$$Spec\left(A\left(\bar{\mu}_{D^N}^{+}\left(\bar{G}\right)\right)\right)=\left\{-0.0099,-0.0385,1.0099,1.0385\right\},$$
$$Spec\left(A\left(\bar{\gamma}_{D^N}^{+}\left(\bar{G}\right)\right)\right)=\left\{-0.1403,-0.2071,1.1403,1.2071\right\},$$
$$Spec\left(A\left(\bar{\sigma}_{D^N}^{+}\left(\bar{G}\right)\right)\right)=\left\{-0.2810,-0.3602,1.2810,1.3602\right\},$$
$$Spec\left(A\left(\bar{\mu}_{D^N}^{-}\left(\bar{G}\right)\right)\right)=\left\{0.0385,0.0385,-1.0385,-1.0385\right\},$$
$$Spec\left(A\left(\bar{\gamma}_{D^N}^{-}\left(\bar{G}\right)\right)\right)=\left\{0.2810,0.2810,-1.2810,-1.2810\right\},$$
$$Spec\left(A\left(\bar{\sigma}_{D^N}^{-}\left(\bar{G}\right)\right)\right)=\left\{0.3602,0.3602,-1.3602,-1.3602\right\}.$$

Therefore, the dominating energy of complement of BSVNG is;

$$\begin{aligned}E_{D^N}\left(\bar{G}\right)&=\left(E\left(\bar{\mu}_{D^N}^{+}\left(\bar{G}\right)\right),E\left(\bar{\gamma}_{D^N}^{+}\left(\bar{G}\right)\right),E\left(\bar{\sigma}_{D^N}^{+}\left(\bar{G}\right)\right),E\left(\bar{\mu}_{D^N}^{-}\left(\bar{G}\right)\right),E\left(\bar{\gamma}_{D^N}^{-}\left(\bar{G}\right)\right),E\left(\bar{\sigma}_{D^N}^{-}\left(\bar{G}\right)\right)\right)\\&=\left(\sum_{p=1}^{n}\left|\zeta_p\right|,\sum_{p=1}^{n}\left|\tau_p\right|,\sum_{p=1}^{n}\left|\upsilon_p\right|,\sum_{p=1}^{n}\left|\vartheta_p\right|,\sum_{p=1}^{n}\left|\xi_p\right|,\sum_{p=1}^{n}\left|\varepsilon_p\right|\right)\\&=\left(2.0968,2.6948,3.2824,2.1540,3.1240,3.4408\right).\end{aligned}$$

5.2 Dominating energy in union of bipolar single-valued neutrosophic graphs

Definition 5.3

Let $G_1=\left(V_1,E_1\right)$ and $G_2=\left(V_2,E_2\right)$ be two BSVNG with $V_1\cap V_2=\varnothing$ and $G=G_1\cup G_2=\left(V_1\cup V_2,E_1\cup E_2\right)$ be the union of G_1 and G_2. Then, the union of BSVNG G_1 and G_2 is defined by

$$\left(\mu_1^{+}\cup\mu_1^{'+}\right)(v)=\begin{cases}\mu_1^{+}(v) & \text{if } v\in V_1-V_2\\ \mu_1^{'+}(v) & \text{if } v\in V_2-V_1\end{cases},\qquad \left(\mu_1^{-}\cup\mu_1^{'-}\right)(v)=\begin{cases}\mu_1^{-}(v) & \text{if } v\in V_1-V_2\\ \mu_1^{'-}(v) & \text{if } v\in V_2-V_1\end{cases},$$

$$\left(\gamma_1^{+}\cup\gamma_1^{'+}\right)(v)=\begin{cases}\gamma_1^{+}(v) & \text{if } v\in V_1-V_2\\ \gamma_1^{'+}(v) & \text{if } v\in V_2-V_1\end{cases},\qquad \left(\gamma_1^{-}\cup\gamma_1^{'-}\right)(v)=\begin{cases}\gamma_1^{-}(v) & \text{if } v\in V_1-V_2\\ \gamma_1^{'-}(v) & \text{if } v\in V_2-V_1\end{cases},$$

$$\left(\sigma_1^{+}\cup\sigma_1^{'+}\right)(v)=\begin{cases}\sigma_1^{+}(v) & \text{if } v\in V_1-V_2\\ \sigma_1^{'+}(v) & \text{if } v\in V_2-V_1\end{cases},\qquad \left(\sigma_1^{-}\cup\sigma_1^{'-}\right)(v)=\begin{cases}\sigma_1^{-}(v) & \text{if } v\in V_1-V_2\\ \sigma_1^{'-}(v) & \text{if } v\in V_2-V_1\end{cases},$$

where $\left(\mu_1^+,\gamma_1^+,\sigma_1^+\right)$ and $\left(\mu_1^{'+},\gamma_1^{'+},\sigma_1^{'+}\right)$ denote the vertex of truth-membership, indeterminacy-membership and falsity-membership of G_1 and G_2, respectively. Moreover, the membership values of edges are given as follows;

$$\left(\mu_2^+\cup\mu_2^{'+}\right)\left(v_iv_j\right)=\begin{cases}\mu_2^+\left(e_{ij}\right) & \text{if } e_{ij}\in E_1-E_2\\ \mu_2^{'+}\left(e_{ij}\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},\qquad \left(\mu_2^-\cup\mu_2^{'-}\right)\left(v_iv_j\right)=\begin{cases}\mu_2^-\left(v_iv_j\right) & \text{if } e_{ij}\in E_1-E_2\\ \mu_2^{'-}\left(v_iv_j\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},$$

$$\left(\gamma_2^+\cup\gamma_2^{'+}\right)\left(v_iv_j\right)=\begin{cases}\gamma_2^+\left(e_{ij}\right) & \text{if } e_{ij}\in E_1-E_2\\ \gamma_2^{'+}\left(e_{ij}\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},\qquad \left(\gamma_2^-\cup\gamma_2^{'-}\right)\left(v_iv_j\right)=\begin{cases}\gamma_2^-\left(e_{ij}\right) & \text{if } e_{ij}\in E_1-E_2\\ \gamma_2^{'-}\left(e_{ij}\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},$$

$$\left(\sigma_2^+\cup\sigma_2^{'+}\right)\left(v_iv_j\right)=\begin{cases}\sigma_2^+\left(e_{ij}\right) & \text{if } e_{ij}\in E_1-E_2\\ \sigma_2^{'+}\left(e_{ij}\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},\qquad \left(\sigma_2^-\cup\sigma_2^{'-}\right)\left(v_iv_j\right)=\begin{cases}\sigma_2^-\left(e_{ij}\right) & \text{if } e_{ij}\in E_1-E_2\\ \sigma_2^{'-}\left(e_{ij}\right) & \text{if } e_{ij}\in E_2-E_1\end{cases},$$

where $\left(\mu_2^+,\gamma_2^+,\sigma_2^+\right)$ and $\left(\mu_2^{'+},\gamma_2^{'+},\sigma_2^{'+}\right)$ denote the edge of truth-membership, indeterminacy-membership and falsity-membership of G_1 and G_2, respectively.

Example 5.4

First, we find the dominating energy of BSVNG $G_1=\left(V_1,E_1\right)$ as shown in Figure 5. Consider a dominating BSVNG $G_1=\left(V_1,E_1,\mu^+,\gamma^+,\sigma^+,\mu^-,\gamma^-,\sigma^-,\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-\right)$ where $V_1=\left\{v_1,v_2,v_3,v_4\right\}$, $\mu_1^+:V_1\to\left[0,1\right],\gamma_1^+:V_1\to\left[0,1\right],\sigma_1^+:V_1\to\left[0,1\right],\mu_1^-:V_1\to\left[-1,0\right],\gamma_1^-:V_1\to\left[-1,0\right],\sigma_1^-:V_1\to\left[-1,0\right]$ such that

$$\begin{aligned}\mu_1^+\left(v_1\right)&=\min\left[\mu^+\left(v_1v_2\right),\mu^+\left(v_1v_4\right)\right]\\&=\min\left[0.2,0.2\right]=0.2\\ \mu_1^+\left(v_2\right)&=\min\left[\mu^+\left(v_2v_1\right),\mu^+\left(v_2v_3\right)\right]\\&=\min\left[0.2,0.4\right]=0.2\\ \mu_1^+\left(v_3\right)&=\min\left[\mu^+\left(v_3v_2\right),\mu^+\left(v_3v_4\right)\right]\\&=\min\left[0.4,0.2\right]=0.2\\ \mu_1^+\left(v_4\right)&=\min\left[\mu^+\left(v_4v_1\right),\mu^+\left(v_4v_3\right)\right]\\&=\min\left[0.2,0.2\right]=0.2\end{aligned}\qquad \begin{aligned}\mu_1^-\left(v_1\right)&=\max\left[\mu^-\left(v_1v_2\right),\mu^-\left(v_1v_4\right)\right]\\&=\max\left[-0.2,-0.3\right]=-0.2\\ \mu_1^-\left(v_2\right)&=\max\left[\mu^-\left(v_2v_1\right),\mu^-\left(v_2v_3\right)\right]\\&=\max\left[-0.2,-0.2\right]=-0.2\\ \mu_1^-\left(v_3\right)&=\max\left[\mu^-\left(v_3v_2\right),\mu^-\left(v_3v_4\right)\right]\\&=\max\left[-0.2,-0.3\right]=-0.2\\ \mu_1^-\left(v_4\right)&=\max\left[\mu^-\left(v_4v_1\right),\mu^-\left(v_4v_3\right)\right]\\&=\max\left[-0.3,-0.3\right]=-0.3\end{aligned}$$

$$\begin{aligned}\gamma_1^+\left(v_1\right)&=\max\left[\gamma^+\left(v_1v_2\right),\gamma^+\left(v_1v_4\right)\right]\\&=\max\left[0.7,0.6\right]=0.7\\ \gamma_1^+\left(v_2\right)&=\max\left[\gamma^+\left(v_2v_1\right),\gamma^+\left(v_2v_3\right)\right]\\&=\max\left[0.7,0.5\right]=0.7\\ \gamma_1^+\left(v_3\right)&=\max\left[\gamma^+\left(v_3v_2\right),\gamma^+\left(v_3v_4\right)\right]\\&=\max\left[0.5,0.6\right]=0.6\\ \gamma_1^+\left(v_4\right)&=\max\left[\gamma^+\left(v_4v_1\right),\gamma^+\left(v_4v_3\right)\right]\\&=\max\left[0.6,0.6\right]=0.6\end{aligned}\qquad \begin{aligned}\gamma_1^-\left(v_1\right)&=\min\left[\gamma^-\left(v_1v_2\right),\gamma^-\left(v_1v_4\right)\right]\\&=\min\left[-0.6,-0.6\right]=-0.6\\ \gamma_1^-\left(v_2\right)&=\min\left[\gamma^-\left(v_2v_1\right),\gamma^-\left(v_2v_3\right)\right]\\&=\min\left[-0.6,-0.5\right]=-0.6\\ \gamma_1^-\left(v_3\right)&=\min\left[\gamma^-\left(v_3v_2\right),\gamma^-\left(v_3v_4\right)\right]\\&=\min\left[-0.5,-0.6\right]=-0.6\\ \gamma_1^-\left(v_4\right)&=\min\left[\gamma^-\left(v_4v_1\right),\gamma^-\left(v_4v_3\right)\right]\\&=\min\left[-0.6,-0.6\right]=-0.6\end{aligned}$$

$$\begin{aligned}\sigma_1^+(v_1) &= \max\left[\sigma^+(v_1v_2), \sigma^+(v_1v_4)\right] \\ &= \max\left[0.8, 0.4\right] = 0.8 \\ \sigma_1^+(v_2) &= \max\left[\sigma^+(v_2v_1), \sigma^+(v_2v_3)\right] \\ &= \max\left[0.8, 0.8\right] = 0.8 \\ \sigma_1^+(v_3) &= \max\left[\sigma^+(v_3v_2), \sigma^+(v_3v_4)\right] \\ &= \max\left[0.8, 0.7\right] = 0.8 \\ \sigma_1^+(v_4) &= \max\left[\sigma^+(v_4v_1), \sigma^+(v_4v_3)\right] \\ &= \max\left[0.4, 0.7\right] = 0.7\end{aligned}$$

$$\begin{aligned}\sigma_1^-(v_1) &= \min\left[\sigma^-(v_1v_2), \sigma^-(v_1v_4)\right] \\ &= \min\left[-0.7, -0.7\right] = -0.7 \\ \sigma_1^-(v_2) &= \min\left[\sigma^-(v_2v_1), \sigma^-(v_2v_3)\right] \\ &= \min\left[-0.7, -0.7\right] = -0.7 \\ \sigma_1^-(v_3) &= \min\left[\sigma^-(v_3v_2), \sigma^-(v_3v_4)\right] \\ &= \min\left[-0.7, -0.8\right] = -0.8 \\ \sigma_1^-(v_4) &= \min\left[\sigma^-(v_4v_1), \sigma^-(v_4v_3)\right] \\ &= \min\left[-0.7, -0.8\right] = -0.8\end{aligned}$$

Here, v_1 dominates v_2 because

$$\begin{aligned}\mu^+(v_1v_2) &\le \mu_1^+(v_1) \wedge \mu_1^+(v_2) \\ 0.2 &\le 0.2 \wedge 0.2 \\ \gamma^+(v_1v_2) &\le \gamma_1^+(v_1) \wedge \gamma_1^+(v_2) \\ 0.7 &\le 0.7 \wedge 0.7 \\ \sigma^+(v_1v_2) &\le \sigma_1^+(v_1) \wedge \sigma_1^+(v_2) \\ 0.8 &\le 0.8 \wedge 0.8\end{aligned}$$

$$\begin{aligned}\mu^-(v_1v_2) &\le \mu_1^-(v_1) \wedge \mu_1^-(v_2) \\ -0.2 &\le -0.2 \wedge -0.2 \\ \gamma^-(v_1v_2) &\le \gamma_1^-(v_1) \wedge \gamma_1^-(v_2) \\ -0.6 &\le -0.6 \wedge -0.6 \\ \sigma^-(v_1v_2) &\le \sigma_1^-(v_1) \wedge \sigma_1^-(v_2) \\ -0.7 &\le -0.7 \wedge -0.7\end{aligned}$$

Also, v_3 dominates v_4 because

$$\begin{aligned}\mu^+(v_3v_4) &\le \mu_1^+(v_3) \wedge \mu_1^+(v_4) \\ 0.2 &\le 0.2 \wedge 0.2 \\ \gamma^+(v_3v_4) &\le \gamma_1^+(v_3) \wedge \gamma_1^+(v_4) \\ 0.6 &\le 0.6 \wedge 0.6 \\ \sigma^+(v_3v_4) &\le \sigma_1^+(v_3) \wedge \sigma_1^+(v_4) \\ 0.7 &\le 0.8 \wedge 0.7\end{aligned}$$

$$\begin{aligned}\mu^-(v_1v_4) &\le \mu_1^-(v_1) \wedge \mu_1^-(v_4) \\ -0.2 &\le -0.2 \wedge -0.2 \\ \gamma^-(v_1v_4) &\le \gamma_1^-(v_1) \wedge \gamma_1^-(v_4) \\ -0.6 &\le -0.6 \wedge -0.6 \\ \sigma^-(v_1v_4) &\le \sigma_1^-(v_1) \wedge \sigma_1^-(v_4) \\ -0.7 &\le -0.7 \wedge -0.7\end{aligned}$$

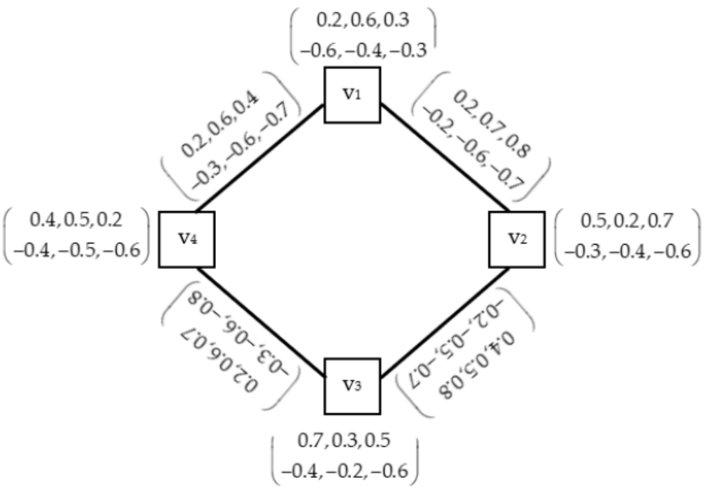

Figure 5. $G_1 = (V_1, E_1)$

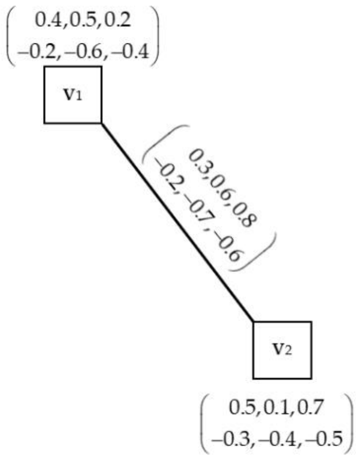

Figure 6. $G_2 = (V_2, E_2)$

Thus, $D^N=\{v_1,v_3\}$ is a dominating set because every vertex in $V-D^N=\{v_2,v_4\}$, is dominated by at least one vertex in D^N and $\left|D^N\right|=2$ is sum of dominating elements. The adjacency matrix of dominating BSVNG $G_1=(V_1,E_1)$ is given below;

$$A_{D^N}(G_1)=\begin{bmatrix} \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.2,0.7,0.8\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.4\\-0.3,-0.6,-0.7\end{pmatrix} \\ \begin{pmatrix}0.2,0.7,0.8\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.8\\-0.2,-0.5,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.8\\-0.2,-0.5,-0.7\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.7\\-0.3,-0.6,-0.8\end{pmatrix} \\ \begin{pmatrix}0.2,0.6,0.4\\-0.3,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.7\\-0.3,-0.6,-0.8\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \end{bmatrix}$$

This can be written in six different matrices as:

$$A\left(\mu^+_{D^N}(G_1)\right)=\begin{bmatrix}1&0.2&0&0.2\\0.2&0&0.4&0\\0&0.4&1&0.2\\0.2&0&0.2&0\end{bmatrix} \qquad A\left(\mu^-_{D^N}(G_1)\right)=\begin{bmatrix}-1&-0.2&0&-0.3\\-0.2&0&-0.2&0\\0&-0.2&-1&-0.3\\-0.3&0&-0.3&0\end{bmatrix}$$

$$A\left(\gamma^+_{D^N}(G_1)\right)=\begin{bmatrix}1&0.7&0&0.6\\0.7&0&0.5&0\\0&0.5&1&0.6\\0.6&0&0.6&0\end{bmatrix} \qquad A\left(\gamma^-_{D^N}(G_1)\right)=\begin{bmatrix}-1&-0.6&0&-0.6\\-0.6&0&-0.5&0\\0&-0.5&-1&-0.6\\-0.6&0&-0.6&0\end{bmatrix}$$

$$A\left(\sigma^+_{D^N}(G_1)\right)=\begin{bmatrix}1&0.8&0&0.4\\0.8&0&0.8&0\\0&0.8&1&0.7\\0.4&0&0.7&0\end{bmatrix} \qquad A\left(\sigma^-_{D^N}(G_1)\right)=\begin{bmatrix}-1&-0.7&0&-0.7\\-0.7&0&-0.7&0\\0&-0.7&-1&-0.8\\-0.7&-0.7&-0.8&0\end{bmatrix}$$

Since,

$$Spec\left(A\left(\mu^+_{D^N}(G_1)\right)\right)=\{1.2240,1.0058,-0.2240,-0.0058\},$$
$$Spec\left(A\left(\gamma^+_{D^N}(G_1)\right)\right)=\{1.8039,1.0098,-0.8039,-0.0098\},$$
$$Spec\left(A\left(\sigma^+_{D^N}(G_1)\right)\right)=\{1.9869,1.0377,-0.9869,-0.0377\},$$
$$Spec\left(A\left(\mu^-_{D^N}(G_1)\right)\right)=\{-1.2141,-1.0000,0.2141,0.0000\},$$
$$Spec\left(A\left(\gamma^-_{D^N}(G_1)\right)\right)=\{-1.7559,-1.0027,0.7559,0.0027\},$$
$$Spec\left(A\left(\sigma^-_{D^N}(G_1)\right)\right)=\{-2.3055,1.0355,-1.0023,0.0023\}.$$

Therefore, the dominating energy of BSVNG $G_1=(V_1,E_1)$ is;

$$\begin{aligned} E_{D^N}(G_1)&=\left(E\left(\mu^+_{D^N}(G_1)\right),E\left(\gamma^+_{D^N}(G_1)\right),E\left(\sigma^+_{D^N}(G_1)\right),E\left(\mu^-_{D^N}(G_1)\right),E\left(\gamma^-_{D^N}(G_1)\right),E\left(\sigma^-_{D^N}(G_1)\right)\right)\\ &=\left(\sum_{p=1}^{n}\left|\zeta_p\right|,\sum_{p=1}^{n}\left|\tau_p\right|,\sum_{p=1}^{n}\left|\upsilon_p\right|,\sum_{p=1}^{n}\left|\vartheta_p\right|,\sum_{p=1}^{n}\left|\xi_p\right|,\sum_{p=1}^{n}\left|\varepsilon_p\right|\right)\\ &=(2.4596,3.6274,4.0492,2.4282,3.5172,4.3456). \end{aligned}$$

Also, we find the dominating energy of BSVNG $G_2=(V_2,E_2)$ as shown in Figure 6. Consider a dominating BSVNG $G_2=(V_2,E_2,\mu^+,\gamma^+,\sigma^+,\mu^-,\gamma^-,\sigma^-,\mu_1^+,\gamma_1^+,\sigma_1^+,\mu_1^-,\gamma_1^-,\sigma_1^-)$ where $V_2=\{v_1,v_2\}$, $\mu_1^+:V_2\to[0,1],\gamma_1^+:V_2\to[0,1],\sigma_1^+:V_2\to[0,1],\mu_1^-:V_2\to[-1,0],\gamma_1^-:V_2\to[-1,0],\sigma_1^-:V_2\to[-1,0]$ such that

$$\mu_1^+(v_1)=\min[\mu^+(v_1v_2)]=\min[0.3]=0.3 \qquad \mu_1^-(v_1)=\max[\mu^-(v_1v_2)]=\max[-0.2]=-0.2$$
$$\mu_1^+(v_2)=\min[\mu^+(v_2v_1)]=\min[0.3]=0.3 \qquad \mu_1^-(v_2)=\max[\mu^-(v_2v_1)]=\max[-0.2]=-0.2$$
$$\gamma_1^+(v_1)=\max[\gamma^+(v_1v_2)]=\max[0.6]=0.6 \qquad \gamma_1^-(v_1)=\min[\gamma^-(v_1v_2)]=\min[-0.7]=-0.7$$
$$\gamma_1^+(v_2)=\max[\gamma^+(v_2v_1)]=\max[0.6]=0.6 \qquad \gamma_1^-(v_2)=\min[\gamma^-(v_2v_1)]=\min[-0.7]=-0.7$$
$$\sigma_1^+(v_1)=\max[\sigma^+(v_1v_2)]=\max[0.8]=0.8 \qquad \sigma_1^-(v_1)=\min[\sigma^-(v_1v_2)]=\min[-0.6]=-0.6$$
$$\sigma_1^+(v_2)=\max[\sigma^+(v_2v_1)]=\max[0.8]=0.8 \qquad \sigma_1^-(v_2)=\min[\sigma^-(v_2v_1)]=\min[-0.6]=-0.6$$

Here, v_1 dominates v_2 because

$$\mu^+(v_1v_2)\le\mu_1^+(v_1)\wedge\mu_1^+(v_2) \qquad \mu^-(v_1v_2)\le\mu_1^-(v_1)\wedge\mu_1^-(v_2)$$
$$0.3\le 0.3\wedge 0.3 \qquad -0.2\le -0.2\wedge -0.2$$
$$\gamma^+(v_1v_2)\le\gamma_1^+(v_1)\wedge\gamma_1^+(v_2) \qquad \gamma^-(v_1v_2)\le\gamma_1^-(v_1)\wedge\gamma_1^-(v_2)$$
$$0.6\le 0.6\wedge 0.6 \qquad -0.7\le -0.7\wedge -0.7$$
$$\sigma^+(v_1v_2)\le\sigma_1^+(v_1)\wedge\sigma_1^+(v_2) \qquad \sigma^-(v_1v_2)\le\sigma_1^-(v_1)\wedge\sigma_1^-(v_2)$$
$$0.8\le 0.8\wedge 0.8 \qquad -0.6\le -0.6\wedge -0.6$$

$V_2=\{v_1,v_2\};D^N=\{v_1\};V_2-D^N=\{v_2\};|D^N|=1$ is sum of dominating element. Then, we have the adjacency matrix of dominating BSVNG $G_2=(V_2,E_2)$ is given below;

$$A_{D^N}(G_2)=\begin{bmatrix}\begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.3,0.6,0.8\\-0.2,-0.7,-0.6\end{pmatrix}\\ \begin{pmatrix}0.3,0.6,0.8\\-0.2,-0.7,-0.6\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix}\end{bmatrix}$$

where

$$A(\mu_{D^N}^+(G_2))=\begin{bmatrix}1&0.3\\0.3&0\end{bmatrix},A(\gamma_{D^N}^+(G_2))=\begin{bmatrix}1&0.6\\0.6&0\end{bmatrix},A(\sigma_{D^N}^+(G_2))=\begin{bmatrix}1&0.8\\0.8&0\end{bmatrix},$$
$$A(\mu_{D^N}^-(G_2))=\begin{bmatrix}-1&-0.2\\-0.2&0\end{bmatrix},A(\gamma_{D^N}^-(G_2))=\begin{bmatrix}-1&-0.7\\-0.7&0\end{bmatrix},A(\sigma_{D^N}^-(G_2))=\begin{bmatrix}-1&-0.6\\-0.6&0\end{bmatrix}.$$

Since,

$$Spec(A(\mu_{D^N}^+(G_2)))=\{1.0831,-0.0831\},Spec(A(\gamma_{D^N}^+(G_2)))=\{1.2810,-0.2810\},$$
$$Spec(A(\sigma_{D^N}^+(G_2)))=\{1.4434,-0.4434\},Spec(A(\mu_{D^N}^-(G_2)))=\{-1.0385,0.0385\},$$
$$Spec(A(\gamma_{D^N}^-(G_2)))=\{-1.3602,0.3602\},Spec(A(\sigma_{D^N}^-(G_2)))=\{-1.2810,0.2810\}.$$

Therefore, the dominating energy of BSVNG $G_2=(V_2,E_2)$ is;

$$E_{D^N}(G_2)=\left(E(\mu_{D^N}^+(G_2)),E(\gamma_{D^N}^+(G_2)),E(\sigma_{D^N}^+(G_2)),E(\mu_{D^N}^-(G_2)),E(\gamma_{D^N}^-(G_2)),E(\sigma_{D^N}^-(G_2))\right)$$
$$=\left(\sum_{p=1}^{n}|\zeta_p|,\sum_{p=1}^{n}|\tau_p|,\sum_{p=1}^{n}|\upsilon_p|,\sum_{p=1}^{n}|\vartheta_p|,\sum_{p=1}^{n}|\xi_p|,\sum_{p=1}^{n}|\varepsilon_p|\right)$$
$$=(1.1662,1.5620,1.8868,1.077,1.7204,1.5620).$$

Now, we find the dominating energy of union of two BSVNG $G_1 \cup G_2$ as shown in Figure 7.

$$\mu_1^+(v_1) = \min\left[\mu^+(v_1v_2), \mu^+(v_1v_4)\right] = \min\left[0.2, 0.2\right] = 0.2 \qquad \mu_1^-(v_1) = \max\left[\mu^-(v_1v_2), \mu^-(v_1v_4)\right] = \max\left[-0.2, -0.3\right] = -0.2$$

$$\mu_1^+(v_2) = \min\left[\mu^+(v_2v_1), \mu^+(v_2v_3)\right] = \min\left[0.2, 0.4\right] = 0.2 \qquad \mu_1^-(v_2) = \max\left[\mu^-(v_2v_1), \mu^-(v_2v_3)\right] = \max\left[-0.2, -0.2\right] = -0.2$$

$$\mu_1^+(v_3) = \min\left[\mu^+(v_3v_2), \mu^+(v_3v_4)\right] = \min\left[0.4, 0.2\right] = 0.2 \qquad \mu_1^-(v_3) = \max\left[\mu^-(v_3v_2), \mu^-(v_3v_4)\right] = \max\left[-0.2, -0.3\right] = -0.2$$

$$\mu_1^+(v_4) = \min\left[\mu^+(v_4v_1), \mu^+(v_4v_3)\right] = \min\left[0.2, 0.2\right] = 0.2 \qquad \mu_1^-(v_4) = \max\left[\mu^-(v_4v_1), \mu^-(v_4v_3)\right] = \max\left[-0.3, -0.3\right] = -0.3$$

$$\mu_1^+(u_1) = \min\left[\mu^+(u_1u_2)\right] = \min\left[0.3\right] = 0.3 \qquad \mu_1^-(u_1) = \max\left[\mu^-(u_1u_2)\right] = \max\left[-0.2\right] = -0.2$$

$$\mu_1^+(u_2) = \min\left[\mu^+(u_2u_1)\right] = \min\left[0.3\right] = 0.3 \qquad \mu_1^-(u_2) = \max\left[\mu^-(u_2u_1)\right] = \max\left[-0.2\right] = -0.2$$

$$\gamma_1^+(v_1) = \max\left[\gamma^+(v_1v_2), \gamma^+(v_1v_4)\right] = \max\left[0.7, 0.6\right] = 0.7 \qquad \gamma_1^-(v_1) = \min\left[\gamma^-(v_1v_2), \gamma^-(v_1v_4)\right] = \min\left[-0.6, -0.6\right] = -0.6$$

$$\gamma_1^+(v_2) = \max\left[\gamma^+(v_2v_1), \gamma^+(v_2v_3)\right] = \max\left[0.7, 0.5\right] = 0.7 \qquad \gamma_1^-(v_2) = \min\left[\gamma^-(v_2v_1), \gamma^-(v_2v_3)\right] = \min\left[-0.6, -0.5\right] = -0.6$$

$$\gamma_1^+(v_3) = \max\left[\gamma^+(v_3v_2), \gamma^+(v_3v_4)\right] = \max\left[0.5, 0.6\right] = 0.6 \qquad \gamma_1^-(v_3) = \min\left[\gamma^-(v_3v_2), \gamma^-(v_3v_4)\right] = \min\left[-0.5, -0.6\right] = -0.6$$

$$\gamma_1^+(v_4) = \max\left[\gamma^+(v_4v_1), \gamma^+(v_4v_3)\right] = \max\left[0.6, 0.6\right] = 0.6 \qquad \gamma_1^-(v_4) = \min\left[\gamma^-(v_4v_1), \gamma^-(v_4v_3)\right] = \min\left[-0.6, -0.6\right] = -0.6$$

$$\gamma_1^+(u_1) = \max\left[\gamma^+(u_1u_2)\right] = \max\left[0.6\right] = 0.6 \qquad \gamma_1^-(u_1) = \min\left[\gamma^-(u_1u_2)\right] = \min\left[-0.7\right] = -0.7$$

$$\gamma_1^+(u_2) = \max\left[\gamma^+(u_2u_1)\right] = \max\left[0.6\right] = 0.6 \qquad \gamma_1^-(u_2) = \min\left[\gamma^-(u_2u_1)\right] = \min\left[-0.7\right] = -0.7$$

$$\sigma_1^+(v_1) = \max\left[\sigma^+(v_1v_2), \sigma^+(v_1v_4)\right] = \max\left[0.8, 0.4\right] = 0.8 \qquad \sigma_1^-(v_1) = \min\left[\sigma^-(v_1v_2), \sigma^-(v_1v_4)\right] = \min\left[-0.7, -0.7\right] = -0.7$$

$$\sigma_1^+(v_2) = \max\left[\sigma^+(v_2v_1), \sigma^+(v_2v_3)\right] = \max\left[0.8, 0.8\right] = 0.8 \qquad \sigma_1^-(v_2) = \min\left[\sigma^-(v_2v_1), \sigma^-(v_2v_3)\right] = \min\left[-0.7, -0.7\right] = -0.7$$

$$\sigma_1^+(v_3) = \max\left[\sigma^+(v_3v_2), \sigma^+(v_3v_4)\right] = \max\left[0.8, 0.7\right] = 0.8 \qquad \sigma_1^-(v_3) = \min\left[\sigma^-(v_3v_2), \sigma^-(v_3v_4)\right] = \min\left[-0.7, -0.8\right] = -0.8$$

$$\sigma_1^+(v_4) = \max\left[\sigma^+(v_4v_1), \sigma^+(v_4v_3)\right] = \max\left[0.4, 0.7\right] = 0.7 \qquad \sigma_1^-(v_4) = \min\left[\sigma^-(v_4v_1), \sigma^-(v_4v_3)\right] = \min\left[-0.7, -0.8\right] = -0.8$$

$$\sigma_1^+(u_1) = \max\left[\sigma^+(u_1u_2)\right] = \max\left[0.8\right] = 0.8 \qquad \sigma_1^-(u_1) = \min\left[\sigma^-(u_1u_2)\right] = \min\left[-0.6\right] = -0.6$$

$$\sigma_1^+(u_2) = \max\left[\sigma^+(u_2u_1)\right] = \max\left[0.8\right] = 0.8 \qquad \sigma_1^-(u_2) = \min\left[\sigma^-(u_2u_1)\right] = \min\left[-0.6\right] = -0.6$$

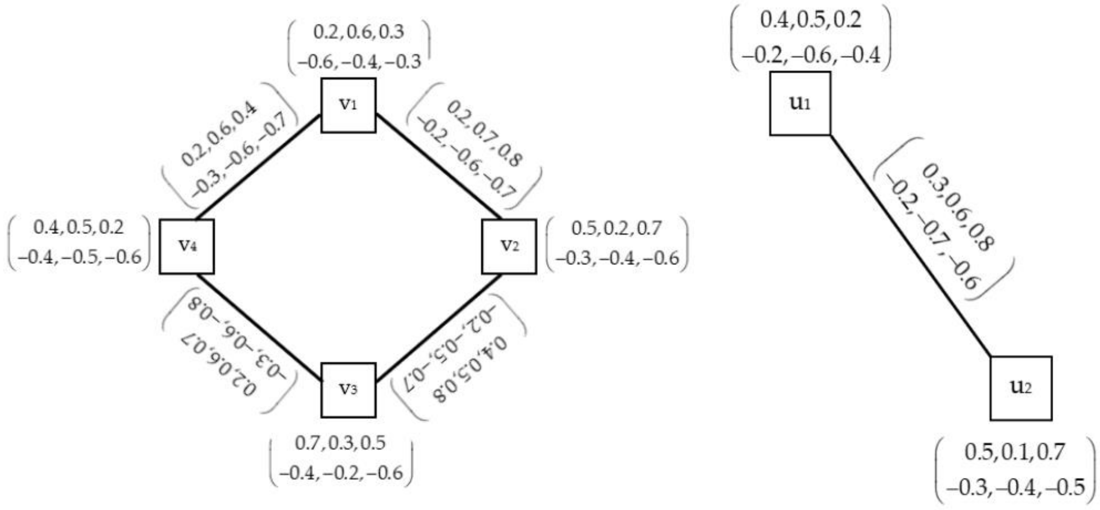

Figure 7. $G_1 \cup G_2$

Here, v_1 dominates v_2 because

$$\mu^+(v_1v_2) \le \mu_1^+(v_1) \wedge \mu_1^+(v_2) \qquad \mu^-(v_1v_2) \le \mu_1^-(v_1) \wedge \mu_1^-(v_2)$$
$$0.2 \le 0.2 \wedge 0.2 \qquad -0.2 \le -0.2 \wedge -0.2$$
$$\gamma^+(v_1v_2) \le \gamma_1^+(v_1) \wedge \gamma_1^+(v_2) \qquad \gamma^-(v_1v_2) \le \gamma_1^-(v_1) \wedge \gamma_1^-(v_2)$$
$$0.7 \le 0.7 \wedge 0.7 \qquad -0.6 \le -0.6 \wedge -0.6$$
$$\sigma^+(v_1v_2) \le \sigma_1^+(v_1) \wedge \sigma_1^+(v_2) \qquad \sigma^-(v_1v_2) \le \sigma_1^-(v_1) \wedge \sigma_1^-(v_2)$$
$$0.8 \le 0.8 \wedge 0.8 \qquad -0.7 \le -0.7 \wedge -0.7$$

Also, v_3 dominates v_4 because

$$\mu^+(v_3v_4) \le \mu_1^+(v_3) \wedge \mu_1^+(v_4) \qquad \mu^-(v_1v_4) \le \mu_1^-(v_1) \wedge \mu_1^-(v_4)$$
$$0.2 \le 0.2 \wedge 0.2 \qquad -0.2 \le -0.2 \wedge -0.2$$
$$\gamma^+(v_3v_4) \le \gamma_1^+(v_3) \wedge \gamma_1^+(v_4) \qquad \gamma^-(v_1v_4) \le \gamma_1^-(v_1) \wedge \gamma_1^-(v_4)$$
$$0.6 \le 0.6 \wedge 0.6 \qquad -0.6 \le -0.6 \wedge -0.6$$
$$\sigma^+(v_3v_4) \le \sigma_1^+(v_3) \wedge \sigma_1^+(v_4) \qquad \sigma^-(v_1v_4) \le \sigma_1^-(v_1) \wedge \sigma_1^-(v_4)$$
$$0.7 \le 0.8 \wedge 0.7 \qquad -0.7 \le -0.7 \wedge -0.7$$

Also, u_1 dominates u_2 because

$$\mu^+(u_1u_2) \le \mu_1^+(u_1) \wedge \mu_1^+(u_2) \qquad \mu^-(u_1u_2) \le \mu_1^-(u_1) \wedge \mu_1^-(u_2)$$
$$0.3 \le 0.3 \wedge 0.3 \qquad -0.2 \le -0.2 \wedge -0.2$$
$$\gamma^+(u_1u_2) \le \gamma_1^+(u_1) \wedge \gamma_1^+(u_2) \qquad \gamma^-(u_1u_2) \le \gamma_1^-(u_1) \wedge \gamma_1^-(u_2)$$
$$0.6 \le 0.6 \wedge 0.6 \qquad -0.7 \le -0.7 \wedge -0.7$$
$$\sigma^+(u_1u_2) \le \sigma_1^+(u_1) \wedge \sigma_1^+(u_2) \qquad \sigma^-(u_1u_2) \le \sigma_1^-(u_1) \wedge \sigma_1^-(u_2)$$
$$0.8 \le 0.8 \wedge 0.8 \qquad -0.6 \le -0.6 \wedge -0.6$$

$V = \{v_1, v_2, v_3, v_4, u_1, u_2\}; D^N = \{v_1, v_3, u_1\}; V - D^N = \{v_2, v_4, u_2\}; \left|D^N\right| = 3$ is sum of dominating element. Then, we have the adjacency matrix of dominating BSVNG $G_1 \cup G_2$ is given below;

$$A_{D^N}\left(G_1 \cup G_2\right)=$$

$$\begin{bmatrix} \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.2,0.7,0.8\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.4\\-0.3,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0.2,0.7,0.8\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.8\\-0.2,-0.5,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.8\\-0.2,-0.5,-0.7\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.7\\-0.3,-0.6,-0.8\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0.2,0.6,0.4\\-0.3,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.2,0.6,0.7\\-0.3,-0.6,-0.8\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \\ \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.3,0.6,0.8\\-0.2,-0.7,-0.6\end{pmatrix} \\ \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.3,0.6,0.8\\-0.2,-0.7,-0.6\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} \end{bmatrix}$$

where

$$A\left(\mu^+_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}1&0.2&0&0.2&0&0\\0.2&0&0.4&0&0&0\\0&0.4&1&0.2&0&0\\0.2&0&0.2&0&0&0\\0&0&0&0&1&0.3\\0&0&0&0&0.3&0\end{bmatrix} \quad A\left(\mu^-_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}-1&-0.2&0&-0.3&0&0\\-0.2&0&-0.2&0&0&0\\0&-0.2&-1&-0.3&0&0\\-0.3&0&-0.3&0&0&0\\0&0&0&0&-1&-0.2\\0&0&0&0&-0.2&0\end{bmatrix}$$

$$A\left(\gamma^+_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}1&0.7&0&0.6&0&0\\0.7&0&0.5&0&0&0\\0&0.5&1&0.6&0&0\\0.6&0&0.6&0&0&0\\0&0&0&0&1&0.6\\0&0&0&0&0.6&0\end{bmatrix} \quad A\left(\gamma^-_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}-1&-0.6&0&-0.6&0&0\\-0.6&0&-0.5&0&0&0\\0&-0.5&-1&-0.6&0&0\\-0.6&0&-0.6&0&0&0\\0&0&0&0&-1&-0.7\\0&0&0&0&-0.7&0\end{bmatrix}$$

$$A\left(\sigma^+_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}1&0.8&0&0.4&0&0\\0.8&0&0.8&0&0&0\\0&0.8&1&0.7&0&0\\0.4&0&0.7&0&0&0\\0&0&0&0&1&0.8\\0&0&0&0&0.8&0\end{bmatrix} \quad A\left(\sigma^-_{D^N}\left(G_1 \cup G_2\right)\right)=\begin{bmatrix}-1&-0.7&0&-0.7&0&0\\-0.7&0&-0.7&0&0&0\\0&-0.7&-1&-0.8&0&0\\-0.7&0&-0.8&0&0&0\\0&0&0&0&-1&-0.6\\0&0&0&0&-0.6&0\end{bmatrix}$$

Since,

$$Spec\left(A\left(\mu^+_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{1.2239, 1.0831, 1.0058, -0.2239, -0.0831, -0.0058\right\},$$

$$Spec\left(A\left(\gamma^+_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{1.8039, 1.2810, 1.0098, -0.8039, -0.2810, -0.0098\right\},$$

$$Spec\left(A\left(\sigma^+_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{1.9662, 1.4434, 1.0295, -0.9662, -0.4434, -0.0295\right\},$$

$$Spec\left(A\left(\mu^-_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{-1.2141, -1.0385, -1.0000, 0.2141, 0.0385, 0.0000\right\},$$

$$Spec\left(A\left(\gamma^-_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{-1.7559, -1.3602, -1.0027, 0.7559, 0.3602, 0.0027\right\},$$

$$Spec\left(A\left(\sigma^-_{D^N}\left(G_1 \cup G_2\right)\right)\right)=\left\{-2.0355, -1.2810, 1.0355, -1.0023, 0.2810, 0.0023\right\}.$$

Therefore, the dominating energy of union BSVNG $G_1 \cup G_2$ is;

$$E_{D^N}(G_1 \cup G_2) = \begin{pmatrix} E\left(\mu^+_{D^N}(G_1 \cup G_2)\right), E\left(\gamma^+_{D^N}(G_1 \cup G_2)\right), E\left(\sigma^+_{D^N}(G_1 \cup G_2)\right), E\left(\mu^-_{D^N}(G_1 \cup G_2)\right), \\ E\left(\gamma^-_{D^N}(G_1 \cup G_2)\right), E\left(\sigma^-_{D^N}(G_1 \cup G_2)\right) \end{pmatrix}$$
$$= \left(\sum_{p=1}^{n}|\zeta_p|, \sum_{p=1}^{n}|\tau_p|, \sum_{p=1}^{n}|\upsilon_p|, \sum_{p=1}^{n}|\vartheta_p|, \sum_{p=1}^{n}|\xi_p|, \sum_{p=1}^{n}|\varepsilon_p|\right)$$
$$= (3.6256, 5.1894, 5.8782, 3.5052, 5.2376, 5.6376).$$

5.3 Dominating energy in join of bipolar single-valued neutrosophic graphs

Definition 5.5

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two BSVNG. The join of two BSVNG $\hat{G} = (\hat{V}, \hat{E}) = G_1 + G_2$ such that $\hat{V} = V_1 + V_2 = V_1 \cup V_2$, defined by

$$\left(\mu_1^+ + \mu_1^{'+}\right)(v) = \left(\mu_1^+ \cup \mu_1^{'+}\right)(v) \text{ if } v \in V_1 \cup V_2; \qquad \left(\mu_1^- + \mu_1^{'-}\right)(v) = \left(\mu_1^- \cup \mu_1^{'-}\right)(v) \text{ if } v \in V_1 \cup V_2;$$
$$\left(\gamma_1^+ + \gamma_1^{'+}\right)(v) = \left(\gamma_1^+ \cup \gamma_1^{'+}\right)(v) \text{ if } v \in V_1 \cup V_2; \qquad \left(\gamma_1^- + \gamma_1^{'-}\right)(v) = \left(\gamma_1^- \cup \gamma_1^{'-}\right)(v) \text{ if } v \in V_1 \cup V_2;$$
$$\left(\sigma_1^+ + \sigma_1^{'+}\right)(v) = \left(\sigma_1^+ \cup \sigma_1^{'+}\right)(v) \text{ if } v \in V_1 \cup V_2; \qquad \left(\sigma_1^- + \sigma_1^{'-}\right)(v) = \left(\sigma_1^- \cup \sigma_1^{'-}\right)(v) \text{ if } v \in V_1 \cup V_2.$$

and $\hat{E} = E_1 + E_2 = E_1 \cup E_2$ be the set of all edges joining the vertices of G_1 and G_2.

Example 5.6

Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two BSVNG as shown in Figure 8 and 9, respectively. Now, we find the dominating energy of join BSVNG $\hat{G}(\hat{V}, \hat{E}) = G_1 + G_2$, as shown in Figure 10. Consider a dominating BSVNG $\hat{G} = (\hat{V}, \hat{E}, \mu^+, \gamma^+, \sigma^+, \mu^-, \gamma^-, \sigma^-, \mu_1^+, \gamma_1^+, \sigma_1^+, \mu_1^-, \gamma_1^-, \sigma_1^-)$ where $\hat{V} = (v_1, v_2, v_3, v_4, u_1, u_2)$ and $\mu_1^+ : \hat{V} \to [0,1], \gamma_1^+ : \hat{V} \to [0,1], \sigma_1^+ : \hat{V} \to [0,1], \mu_1^- : \hat{V} \to [-1,0], \gamma_1^- : \hat{V} \to [-1,0]$, $\sigma_1^- : \hat{V} \to [-1,0]$ such that

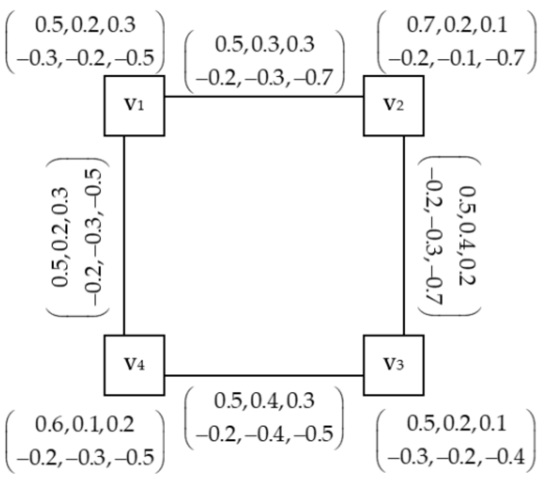

Figure 8. $G_1 = (V_1, E_1)$

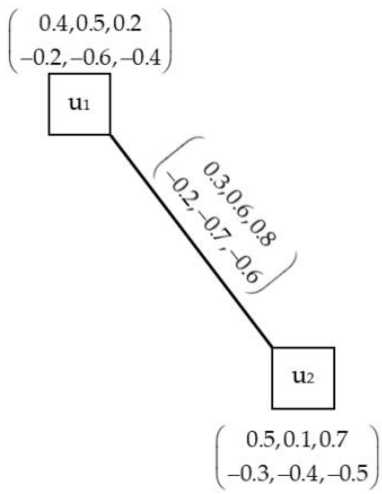

Figure 9. $G_2 = (V_2, E_2)$

$$\mu_1^+(v_1) = \min\left[\mu^+(v_1v_2), \mu^+(v_1v_4), \mu^+(v_1u_1), \mu^+(v_1u_2)\right] = \min(0.5, 0.5, 0.4, 0.5) = 0.4$$
$$\mu_1^+(v_2) = \min\left[\mu^+(v_2v_1), \mu^+(v_2v_3), \mu^+(v_2u_1), \mu^+(v_2u_2)\right] = \min(0.5, 0.5, 0.4, 0.5) = 0.4$$
$$\mu_1^+(v_3) = \min\left[\mu^+(v_3v_2), \mu^+(v_3v_4), \mu^+(v_3u_1), \mu^+(v_3u_2)\right] = \min(0.5, 0.5, 0.4, 0.5) = 0.4$$
$$\mu_1^+(v_4) = \min\left[\mu^+(v_4v_1), \mu^+(v_4v_3), \mu^+(v_4u_1), \mu^+(v_4u_2)\right] = \min(0.4, 0.5, 0.4, 0.5) = 0.4$$
$$\mu_1^+(u_1) = \min\left[\mu^+(u_1v_1), \mu^+(u_1v_2), \mu^+(u_1v_3), \mu^+(u_1v_4), \mu^+(u_1u_2)\right] = \min(0.4, 0.4, 0.4, 0.4, 0.5) = 0.4$$
$$\mu_1^+(u_2) = \min\left[\mu^+(u_2v_1), \mu^+(u_2v_2), \mu^+(u_2v_3), \mu^+(u_2v_4), \mu^+(u_2u_1)\right] = \min(0.5, 0.5, 0.5, 0.5, 0.5) = 0.5$$

$$\gamma_1^+(v_1) = \max\left[\gamma^+(v_1v_2), \gamma^+(v_1v_4), \gamma^+(v_1u_1), \gamma^+(v_1u_2)\right] = \max(0.3, 0.2, 0.5, 0.2) = 0.5$$
$$\gamma_1^+(v_2) = \max\left[\gamma^+(v_2v_1), \gamma^+(v_2v_3), \gamma^+(v_2u_1), \gamma^+(v_2u_2)\right] = \max(0.3, 0.4, 0.5, 0.2) = 0.5$$
$$\gamma_1^+(v_3) = \max\left[\gamma^+(v_3v_2), \gamma^+(v_3v_4), \gamma^+(v_3u_1), \gamma^+(v_3u_2)\right] = \max(0.4, 0.4, 0.5, 0.2) = 0.5$$
$$\gamma_1^+(v_4) = \max\left[\gamma^+(v_4v_1), \gamma^+(v_4v_3), \gamma^+(v_4u_1), \gamma^+(v_4u_2)\right] = \max(0.2, 0.4, 0.5, 0.1) = 0.5$$
$$\gamma_1^+(u_1) = \max\left[\gamma^+(u_1v_1), \gamma^+(u_1v_2), \gamma^+(u_1v_3), \gamma^+(u_1v_4), \gamma^+(u_1u_2)\right] = \max(0.5, 0.5, 0.5, 0.5, 0.2) = 0.5$$
$$\gamma_1^+(u_2) = \max\left[\gamma^+(u_2v_1), \gamma^+(u_2v_2), \gamma^+(u_2v_3), \gamma^+(u_2v_4), \gamma^+(u_2u_1)\right] = \max(0.2, 0.2, 0.2, 0.1, 0.2) = 0.2$$

$$\sigma_1^+(v_1) = \max\left[\sigma^+(v_1v_2), \sigma^+(v_1v_4), \sigma^+(v_1u_1), \sigma^+(v_1u_2)\right] = \max(0.3, 0.3, 0.3, 0.7) = 0.7$$
$$\sigma_1^+(v_2) = \max\left[\sigma^+(v_2v_1), \sigma^+(v_2v_3), \sigma^+(v_2u_1), \sigma^+(v_2u_2)\right] = \max(0.3, 0.2, 0.2, 0.7) = 0.7$$
$$\sigma_1^+(v_3) = \max\left[\sigma^+(v_3v_2), \sigma^+(v_3v_4), \sigma^+(v_3u_1), \sigma^+(v_3u_2)\right] = \max(0.2, 0.3, 0.2, 0.7) = 0.7$$
$$\sigma_1^+(v_4) = \max\left[\sigma^+(v_4v_1), \sigma^+(v_4v_3), \sigma^+(v_4u_1), \sigma^+(v_4u_2)\right] = \max(0.3, 0.3, 0.2, 0.7) = 0.7$$
$$\sigma_1^+(u_1) = \max\left[\sigma^+(u_1v_1), \sigma^+(u_1v_2), \sigma^+(u_1v_3), \sigma^+(u_1v_4), \sigma^+(u_1u_2)\right] = \max(0.3, 0.2, 0.2, 0.2, 0.3) = 0.3$$
$$\sigma_1^+(u_2) = \max\left[\sigma^+(u_2v_1), \sigma^+(u_2v_2), \sigma^+(u_2v_3), \sigma^+(u_2v_4), \sigma^+(u_2u_1)\right] = \max(0.7, 0.7, 0.7, 0.7, 0.3) = 0.7$$

$$\mu_1^-(v_1) = \max\left[\mu^-(v_1v_2), \mu^-(v_1v_4), \mu^-(v_1u_1), \mu^-(v_1u_2)\right] = \max(-0.2, -0.2, -0.2, -0.3) = -0.2$$
$$\mu_1^-(v_2) = \max\left[\mu^-(v_2v_1), \mu^-(v_2v_3), \mu^-(v_2u_1), \mu^-(v_2u_2)\right] = \max(-0.2, -0.2, -0.2, -0.2) = -0.2$$
$$\mu_1^-(v_3) = \max\left[\mu^-(v_3v_2), \mu^-(v_3v_4), \mu^-(v_3u_1), \mu^-(v_3u_2)\right] = \max(-0.2, -0.2, -0.2, -0.3) = -0.2$$
$$\mu_1^-(v_4) = \max\left[\mu^-(v_4v_1), \mu^-(v_4v_3), \mu^-(v_4u_1), \mu^-(v_4u_2)\right] = \max(-0.2, -0.2, -0.2, -0.2) = -0.2$$
$$\mu_1^-(u_1) = \max\left[\mu^-(u_1v_1), \mu^-(u_1v_2), \mu^-(u_1v_3), \mu^-(u_1v_4), \mu^-(u_1u_2)\right] = \max(-0.2, -0.2, -0.2, -0.2, -0.2) = -0.2$$
$$\mu_1^-(u_2) = \max\left[\mu^-(u_2v_1), \mu^-(u_2v_2), \mu^-(u_2v_3), \mu^-(u_2v_4), \mu^-(u_2u_1)\right] = \max(-0.3, -0.2, -0.3, -0.2, -0.2) = -0.2$$

$$\gamma_1^-(v_1) = \min\left[\gamma^-(v_1v_2), \gamma^-(v_1v_4), \gamma^-(v_1u_1), \gamma^-(v_1u_2)\right] = \min(-0.3, -0.3, -0.6, -0.4) = -0.6$$
$$\gamma_1^-(v_2) = \min\left[\gamma^-(v_2v_1), \gamma^-(v_2v_3), \gamma^-(v_2u_1), \gamma^-(v_2u_2)\right] = \min(-0.3, -0.3, -0.6, -0.4) = -0.6$$
$$\gamma_1^-(v_3) = \min\left[\gamma^-(v_3v_2), \gamma^-(v_3v_4), \gamma^-(v_3u_1), \gamma^-(v_3u_2)\right] = \min(-0.3, -0.4, -0.6, -0.4) = -0.6$$
$$\gamma_1^-(v_4) = \min\left[\gamma^-(v_4v_1), \gamma^-(v_4v_3), \gamma^-(v_4u_1), \gamma^-(v_4u_2)\right] = \min(-0.3, -0.4, -0.6, -0.4) = -0.6$$
$$\gamma_1^-(u_1) = \min\left[\gamma^-(u_1v_1), \gamma^-(u_1v_2), \gamma^-(u_1v_3), \gamma^-(u_1v_4), \gamma^-(u_1u_2)\right] = \min(-0.6, -0.6, -0.6, -0.6, -0.7) = -0.7$$
$$\gamma_1^-(u_2) = \min\left[\gamma^-(u_2v_1), \gamma^-(u_2v_2), \gamma^-(u_2v_3), \gamma^-(u_2v_4), \gamma^-(u_2u_1)\right] = \min(-0.4, -0.4, -0.4, -0.4, -0.7) = -0.7$$

$$\sigma_1^-\left(v_1\right)=\min\left[\sigma^-\left(v_1v_2\right),\sigma^-\left(v_1v_4\right),\sigma^-\left(v_1u_1\right),\sigma^-\left(v_1u_2\right)\right]=\min\left(-0.7,-0.5,-0.5,-0.5\right)=-0.7$$
$$\sigma_1^-\left(v_2\right)=\min\left[\sigma^-\left(v_2v_1\right),\sigma^-\left(v_2v_3\right),\sigma^-\left(v_2u_1\right),\sigma^-\left(v_2u_2\right)\right]=\min\left(-0.7,-0.7,-0.7,-0.7\right)=-0.7$$
$$\sigma_1^-\left(v_3\right)=\min\left[\sigma^-\left(v_3v_2\right),\sigma^-\left(v_3v_4\right),\sigma^-\left(v_3u_1\right),\sigma^-\left(v_3u_2\right)\right]=\min\left(-0.7,-0.5,-0.4,-0.5\right)=-0.7$$
$$\sigma_1^-\left(v_4\right)=\min\left[\sigma^-\left(v_4v_1\right),\sigma^-\left(v_4v_3\right),\sigma^-\left(v_4u_1\right),\sigma^-\left(v_4u_2\right)\right]=\min\left(-0.5,-0.5,-0.5,-0.5\right)=-0.5$$
$$\sigma_1^-\left(u_1\right)=\min\left[\sigma^-\left(u_1v_1\right),\sigma^-\left(u_1v_2\right),\sigma^-\left(u_1v_3\right),\sigma^-\left(u_1v_4\right),\sigma^-\left(u_1u_2\right)\right]=\min\left(-0.5,-0.7,-0.4,-0.5,-0.6\right)=-0.7$$
$$\sigma_1^-\left(u_2\right)=\min\left[\sigma^-\left(u_2v_1\right),\sigma^-\left(u_2v_2\right),\sigma^-\left(u_2v_3\right),\sigma^-\left(u_2v_4\right),\sigma^-\left(u_2u_1\right)\right]=\min\left(-0.5,-0.7,-0.5,-0.5,-0.6\right)=-0.7$$

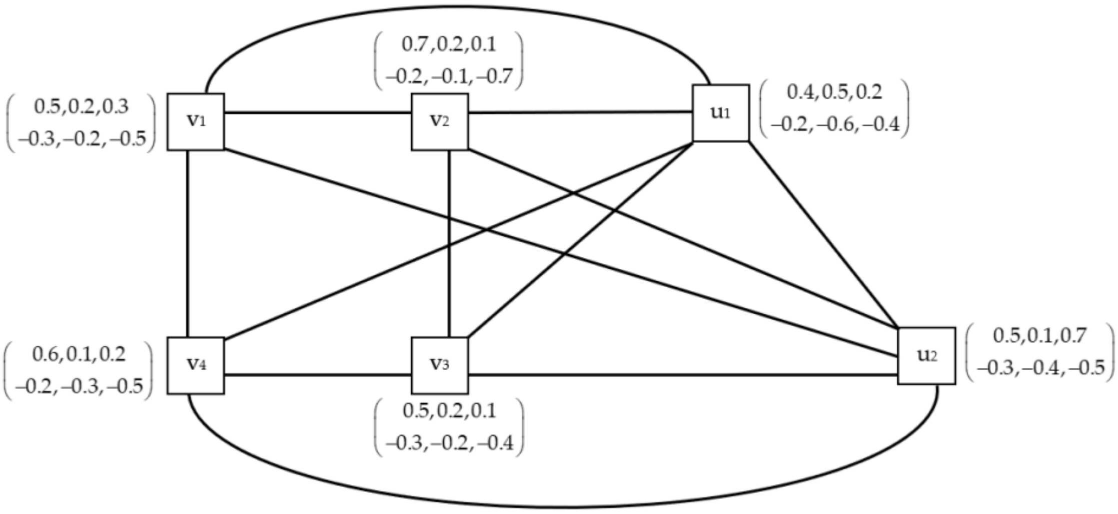

Figure 10. G_1+G_2

$V=\{v_1,v_2,v_3,v_4,u_1,u_2\};D^N=\{v_1,v_2,v_3,v_4,u_1\};V-D^N=\{u_2\};\left|D^N\right|=5$ is sum of dominating element. Then, we have the adjacency matrix of dominating BSVNG G_1+G_2 is given below;

$$A_{D^N}\left(G_1+G_2\right)=$$
$$\begin{bmatrix}
\begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.5,0.3,0.3\\-0.2,-0.3,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.3\\-0.2,-0.3,-0.5\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.3\\-0.2,-0.6,-0.5\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.7\\-0.3,-0.4,-0.5\end{pmatrix}\\
\begin{pmatrix}0.5,0.3,0.3\\-0.2,-0.3,-0.7\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.5,0.4,0.2\\-0.2,-0.3,-0.7\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.7\\-0.2,-0.4,-0.7\end{pmatrix}\\
\begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.5,0.4,0.2\\-0.2,-0.3,-0.7\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.5,0.4,0.3\\-0.2,-0.4,-0.5\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.4\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.7\\-0.3,-0.4,-0.5\end{pmatrix}\\
\begin{pmatrix}0.5,0.2,0.3\\-0.2,-0.3,-0.5\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix} & \begin{pmatrix}0.5,0.4,0.3\\-0.2,-0.4,-0.5\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.5\end{pmatrix} & \begin{pmatrix}0.5,0.1,0.7\\-0.2,-0.4,-0.5\end{pmatrix}\\
\begin{pmatrix}0.4,0.5,0.3\\-0.2,-0.6,-0.5\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.7\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.4\end{pmatrix} & \begin{pmatrix}0.4,0.5,0.2\\-0.2,-0.6,-0.5\end{pmatrix} & \begin{pmatrix}1,1,1\\-1,-1,-1\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.3\\-0.2,-0.3,-0.5\end{pmatrix}\\
\begin{pmatrix}0.5,0.2,0.7\\-0.3,-0.4,-0.5\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.7\\-0.2,-0.4,-0.7\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.7\\-0.3,-0.4,-0.5\end{pmatrix} & \begin{pmatrix}0.5,0.1,0.7\\-0.2,-0.4,-0.5\end{pmatrix} & \begin{pmatrix}0.5,0.2,0.3\\-0.2,-0.3,-0.5\end{pmatrix} & \begin{pmatrix}0,0,0\\0,0,0\end{pmatrix}
\end{bmatrix}$$

where

$$A\left(\mu_{D^N}^{+}\left(G_1+G_2\right)\right)=\begin{bmatrix}1 & 0.5 & 0 & 0.5 & 0.4 & 0.5\\0.5 & 1 & 0.5 & 0 & 0.4 & 0.5\\0 & 0.5 & 1 & 0.5 & 0.4 & 0.5\\0.5 & 0 & 0.5 & 1 & 0.4 & 0.5\\0.4 & 0.4 & 0.4 & 0.4 & 1 & 0.5\\0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0\end{bmatrix}$$

$$A\left(\mu_{D^N}^{-}\left(G_1+G_2\right)\right)=\begin{bmatrix}-1 & -0.2 & 0 & -0.2 & -0.2 & -0.3\\-0.2 & -1 & -0.2 & 0 & -0.2 & -0.2\\0 & -0.2 & -1 & -0.2 & -0.2 & -0.3\\-0.2 & 0 & -0.4 & -1 & -0.2 & -0.2\\-0.2 & -0.2 & -0.2 & -0.2 & -1 & -0.2\\-0.3 & -0.2 & -0.3 & -0.2 & -0.2 & 0\end{bmatrix}$$

$$A\left(\gamma_{D^N}^{+}\left(G_1+G_2\right)\right)=\begin{bmatrix}1 & 0.3 & 0 & 0.2 & 0.5 & 0.2\\0.3 & 1 & 0.4 & 0 & 0.5 & 0.2\\0 & 0.4 & 1 & 0.4 & 0.5 & 0.2\\0.2 & 0 & 0.4 & 1 & 0.5 & 0.1\\0.5 & 0.5 & 0.5 & 0.5 & 1 & 0.2\\0.2 & 0.2 & 0.2 & 0.1 & 0.2 & 0\end{bmatrix}$$

$$A\left(\gamma_{D^N}^{-}\left(G_1+G_2\right)\right)=\begin{bmatrix}-1 & -0.3 & 0 & -0.3 & -0.6 & -0.4\\-0.3 & -1 & -0.3 & 0 & -0.6 & -0.4\\0 & -0.3 & -1 & -0.4 & -0.6 & -0.4\\-0.3 & 0 & -0.4 & -1 & -0.6 & -0.4\\-0.6 & -0.6 & -0.6 & -0.6 & -1 & -0.3\\-0.4 & -0.4 & -0.4 & -0.4 & -0.3 & 0\end{bmatrix}$$

$$A\left(\sigma_{D^N}^{+}\left(G_1+G_2\right)\right)=\begin{bmatrix}1 & 0.3 & 0 & 0.3 & 0.3 & 0.7\\0.3 & 1 & 0.2 & 0 & 0.2 & 0.7\\0 & 0.2 & 1 & 0.3 & 0.2 & 0.7\\0.3 & 0 & 0.3 & 1 & 0.2 & 0.7\\0.3 & 0.2 & 0.2 & 0.2 & 1 & 0.3\\0.7 & 0.7 & 0.7 & 0.7 & 0.3 & 0\end{bmatrix}$$

$$A\left(\sigma_{D^N}^{-}\left(G_1+G_2\right)\right)=\begin{bmatrix}-1 & -0.7 & 0 & -0.5 & -0.5 & -0.5\\-0.7 & -1 & -0.7 & 0 & -0.7 & -0.7\\0 & -0.7 & -1 & -0.5 & -0.4 & -0.5\\-0.5 & 0 & -0.5 & -1 & -0.5 & -0.5\\-0.5 & -0.7 & -0.4 & -0.5 & -1 & -0.5\\-0.5 & -0.7 & -0.5 & -0.5 & -0.5 & 0\end{bmatrix}$$

Since,

$$Spec\left(A\left(\mu_{D^N}^{+}\left(G_1+G_2\right)\right)\right)=\{0,1,-0.436,0.558,2.877\},$$
$$Spec\left(A\left(\gamma_{D^N}^{+}\left(G_1+G_2\right)\right)\right)=\{-0.080,0.270,0.339,0.955,1.068,2.448\},$$
$$Spec\left(A\left(\sigma_{D^N}^{+}\left(G_1+G_2\right)\right)\right)=\{-0.826,0.439,0.806,0.955,1.062,2.564\},$$
$$Spec\left(A\left(\mu_{D^N}^{-}\left(G_1+G_2\right)\right)\right)=\{-1,-1.838,-0.770,-0.565,0.173\},$$
$$Spec\left(A\left(\gamma_{D^N}^{-}\left(G_1+G_2\right)\right)\right)=\{-0.954,-0.346,-2.809,-1.050,-0.207,0.366\},$$
$$Spec\left(A\left(\sigma_{D^N}^{-}\left(G_1+G_2\right)\right)\right)=\{-1,-3.280,-1.006,-0.421,0.213,0.495\}.$$

Therefore, the dominating energy of union BSVNG G_1+G_2 is;

$$\begin{aligned}E_{D^N}\left(G_1+G_2\right)&=\begin{pmatrix}E\left(\mu_{D^N}^{+}\left(G_1+G_2\right)\right),E\left(\gamma_{D^N}^{+}\left(G_1+G_2\right)\right),E\left(\sigma_{D^N}^{+}\left(G_1+G_2\right)\right),E\left(\mu_{D^N}^{-}\left(G_1+G_2\right)\right),\\E\left(\gamma_{D^N}^{-}\left(G_1+G_2\right)\right),E\left(\sigma_{D^N}^{-}\left(G_1+G_2\right)\right)\end{pmatrix}\\&=\left(\sum_{p=1}^{n}\left|\zeta_p\right|,\sum_{p=1}^{n}\left|\tau_p\right|,\sum_{p=1}^{n}\left|\upsilon_p\right|,\sum_{p=1}^{n}\left|\vartheta_p\right|,\sum_{p=1}^{n}\left|\xi_p\right|,\sum_{p=1}^{n}\left|\varepsilon_p\right|\right)\\&=\left(4.871,5.160,6.652,4.346,5.732,6.415\right).\end{aligned}$$

Theorem 5.7 Let $G=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-}\right)$ be a dominating BSVNG with *n* vertices. Let $D^N=\{z_1,z_2,\ldots,z_k\}$ be a dominating set. If

$$\zeta_1,\zeta_2,\ldots\zeta_n,\tau_1,\tau_2,\ldots\tau_n,\upsilon_1,\upsilon_2,\ldots\upsilon_n,\vartheta_1,\vartheta_2,\ldots,\vartheta_n,\xi_1,\xi_2,\ldots,\xi_n,\varepsilon_1,\varepsilon_2,\ldots,\varepsilon_n$$

are the eigen values of adjacency matrix $A\left(\mu_{D^N}^{+}(G)\right),A\left(\gamma_{D^N}^{+}(G)\right),A\left(\sigma_{D^N}^{+}(G)\right),A\left(\mu_{D^N}^{-}(G)\right),\ A\left(\gamma_{D^N}^{-}(G)\right),A\left(\sigma_{D^N}^{-}(G)\right)$ respectively, then

1. $\sum_{p=1}^{n}\zeta_p=\left|D^N\right|,\sum_{p=1}^{n}\tau_p=\left|D^N\right|,\sum_{p=1}^{n}\upsilon_p=\left|D^N\right|,\sum_{p=1}^{n}\vartheta_p=-\left|D^N\right|,\sum_{p=1}^{n}\xi_p=-\left|D^N\right|,\sum_{p=1}^{n}\varepsilon_p=-\left|D^N\right|$

2. $$\sum_{p=1}^{n}\left(\zeta_p\right)^2=\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+},\sum_{p=1}^{n}\left(\tau_p\right)^2=\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{+}\gamma_{qn}^{+},$$
$$\sum_{p=1}^{n}\left(\upsilon_p\right)^2=\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{+}\sigma_{qn}^{+},\sum_{p=1}^{n}\left(\vartheta_p\right)^2=\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{-}\mu_{qn}^{-},$$
$$\sum_{p=1}^{n}\left(\xi_p\right)^2=\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{-}\gamma_{qn}^{-},\sum_{p=1}^{n}\left(\varepsilon_p\right)^2=\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{-}\sigma_{qn}^{-}.$$

Proof.

1. By the trace property of matrices, where the sum of eigen values is equal to its trace, we have
$$\sum_{p=1}^{n}\zeta_p=\sum\mu_{pp}^{+}=\left|D^N\right|.$$

Analogously, we can show that
$$\sum_{p=1}^{n}\tau_p=\left|D^N\right|,\sum_{p=1}^{n}\upsilon_p=\left|D^N\right|,\sum_{p=1}^{n}\vartheta_p=-\left|D^N\right|,\sum_{p=1}^{n}\xi_p=-\left|D^N\right|,\sum_{p=1}^{n}\varepsilon_p=-\left|D^N\right|.$$

2. Equivalently, the sum of square of eigenvalues of $\left(\mu_{D^N}^{+}(G)\right)$ is equal to the trace of $\left(\mu_{D^N}^{+}(G)\right)^2$
$$\begin{aligned}tr\left(A\left(\mu_{D^N}^{+}(G)\right)^2\right)&=\sum_{p=1}^{n}\left(\zeta_p\right)^2\\&=\mu_{11}^{+}\mu_{11}^{+}+\mu_{12}^{+}\mu_{21}^{+}+\ldots+\mu_{1n}^{+}\mu_{n1}^{+}+\mu_{21}^{+}\mu_{12}^{+}+\mu_{22}^{+}\mu_{22}^{+}+\ldots+\mu_{2n}^{+}\mu_{n2}^{+}\\&\quad+\ldots+\mu_{n1}^{+}\mu_{1n}^{+}+\mu_{n2}^{+}\mu_{2n}^{+}+\ldots+\mu_{nn}^{+}\mu_{nn}^{+}\\&=\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}\end{aligned}$$

Analogously, we can show that
$$\sum_{p=1}^{n}\left(\tau_p\right)^2=\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{+}\gamma_{qn}^{+},\sum_{p=1}^{n}\left(\upsilon_p\right)^2=\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{+}\sigma_{qn}^{+},$$
$$\sum_{p=1}^{n}\left(\vartheta_p\right)^2=\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{-}\mu_{qn}^{-},\sum_{p=1}^{n}\left(\xi_p\right)^2=\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{-}\gamma_{qn}^{-},$$
$$\sum_{p=1}^{n}\left(\varepsilon_p\right)^2=\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{-}\sigma_{qn}^{-}.$$

This completes the proof. ■

We now give upper and lower bounds of dominating energy of a BSVNG in terms of the number of vertices and the sum of squares of positive truth-membership, positive indeterminacy-membership, positive falsity-membership, negative truth-membership values, negative indeterminacy-membership values, and negative falsity-membership values of the edges.

Theorem 5.8 Let $G=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-}\right)$ be a dominating BSVNG with *n* vertices. If $D^N=\{z_1,z_2,\ldots,z_k\}$ is the dominating set, then

1. $\sqrt{\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}+n(n-1)|A|^{\frac{2}{n}}}\le E\left(\mu_{D^N}^{+}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}\right)}$

where $|A|$ is the determinant of $\mu_{D^N}^{+}(G)$.

2. $\sqrt{\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{+}\gamma_{qn}^{+}+n(n-1)|B|^{\frac{2}{n}}}\le E\left(\gamma_{D^N}^{+}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{+}\gamma_{qn}^{+}\right)}$

where $|B|$ is the determinant of $\gamma_{D^N}^{+}(G)$.

3. $\sqrt{\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{+}\sigma_{qn}^{+}+n(n-1)|C|^{\frac{2}{n}}}\le E\left(\sigma_{D^N}^{+}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{+}\sigma_{qn}^{+}\right)}$

where $|C|$ is the determinant of $\sigma_{D^N}^{+}(G)$.

4. $\sqrt{\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{-}\mu_{qn}^{-}+n(n-1)|D|^{\frac{2}{n}}}\le E\left(\mu_{D^N}^{-}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{-}\mu_{qn}^{-}\right)}$

where $|D|$ is the determinant of $\mu_{D^N}^{-}(G)$.

5. $\sqrt{\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{-}\gamma_{qn}^{-}+n(n-1)|F|^{\frac{2}{n}}}\le E\left(\gamma_{D^N}^{-}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\gamma_{pq}^{-}\gamma_{qn}^{-}\right)}$

where $|F|$ is the determinant of $\gamma_{D^N}^{-}(G)$.

6. $\sqrt{\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{-}\sigma_{qn}^{-}+n(n-1)|H|^{\frac{2}{n}}}\le E\left(\sigma_{D^N}^{-}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2+2\sum_{1\le p<q\le n}\sigma_{pq}^{-}\sigma_{qn}^{-}\right)}$

where $|H|$ is the determinant of $\sigma_{D^N}^{-}(G)$.

Proof.

By Cauchy Schwarz inequality, $\left(\sum_{p=1}^{n}a_pb_p\right)^2\le\left(\sum_{p=1}^{n}a_p{}^2\right)\left(\sum_{p=1}^{n}b_p{}^2\right)$. Therefore,

Upper bound

If $a_p=1$ and $b_p=|\zeta_p|$, then $\left(\sum_{p=1}^{n}|\zeta_p|\right)^2\le\left(\sum_{p=1}^{n}1\right)\left(\sum_{p=1}^{n}\zeta_p{}^2\right)$. Thus,

$$\left(E\left(\mu_{D^N}^{+}(G)\right)\right)^2\le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}\right)$$

$$\left(E\left(\mu_{D^N}^{+}(G)\right)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2+2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}\right)}.$$

Lower bound

$$\left(E\left(\mu^{+}_{D^N}(G)\right)\right)^2=\left(\sum_{p=1}^{n}\left|\zeta_p\right|\right)^2$$
$$=\left(\sum_{p=1}^{n}\left|\mu^{+}_{pp}\right|^2+2\sum_{1\le p<q\le n}\left|\mu^{+}_{pq}\right|\left|\mu^{+}_{qn}\right|\right)$$
$$=\left(\sum_{p=1}^{n}\left(\mu^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{+}_{pq}\mu^{+}_{qn}\right)+\frac{2n(n-1)}{2}\underset{1\le p<q\le n}{AM}\left\{\left|\zeta_p\right|\left|\zeta_q\right|\right\}.$$

Since $\underset{1\le p<q\le n}{AM}\left\{\left|\zeta_p\right|\left|\zeta_q\right|\right\}\ge\underset{1\le p<q\le n}{GM}\left\{\left|\zeta_p\right|\left|\zeta_q\right|\right\}$, hence

$$\left(E\left(\mu^{+}_{D^N}(G)\right)\right)\ge\sqrt{\sum_{p=1}^{n}\left(\mu^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{+}_{pq}\mu^{+}_{qn}+n(n-1)\underset{1\le p<q\le n}{GM}\left\{\left|\zeta_p\right|\left|\zeta_q\right|\right\}}$$

where

$$\underset{1\le p<q\le n}{GM}\left\{\left|\zeta_p\right|\left|\zeta_q\right|\right\}=\left(\prod_{1\le p<q\le n}\left|\zeta_p\right|\left|\zeta_q\right|\right)^{\frac{2}{n(n-1)}}$$
$$=\left(\prod_{p=1}\left|\zeta_p\right|^{n-1}\right)^{\frac{2}{n(n-1)}}$$
$$=\left(\prod_{p=1}\left|\zeta_p\right|\right)^{\frac{2}{n}}$$
$$=\left|A\right|^{\frac{2}{n}}.$$

Therefore,

$$\left(E\left(\mu^{+}_{D^N}(G)\right)\right)\ge\sqrt{\sum_{p=1}^{n}\left(\mu^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{+}_{pq}\mu^{+}_{qn}+n(n-1)\left|A\right|^{\frac{2}{n}}}.$$

Combining these bounds, we have

1. $\sqrt{\sum_{p=1}^{n}\left(\mu^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{+}_{pq}\mu^{+}_{qn}+n(n-1)\left|A\right|^{\frac{2}{n}}}\le E\left(\mu^{+}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\mu^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{+}_{pq}\mu^{+}_{qn}\right)}.$

Analogously, we can show that

2. $\sqrt{\sum_{p=1}^{n}\left(\gamma^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\gamma^{+}_{pq}\gamma^{+}_{qn}+n(n-1)\left|B\right|^{\frac{2}{n}}}\le E\left(\gamma^{+}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\gamma^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\gamma^{+}_{pq}\gamma^{+}_{qn}\right)},$

3. $\sqrt{\sum_{p=1}^{n}\left(\sigma^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\sigma^{+}_{pq}\sigma^{+}_{qn}+n(n-1)\left|C\right|^{\frac{2}{n}}}\le E\left(\sigma^{+}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\sigma^{+}_{pp}\right)^2+2\sum_{1\le p<q\le n}\sigma^{+}_{pq}\sigma^{+}_{qn}\right)},$

4. $\sqrt{\sum_{p=1}^{n}\left(\mu^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{-}_{pq}\mu^{-}_{qn}+n(n-1)\left|D\right|^{\frac{2}{n}}}\le E\left(\mu^{-}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\mu^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\mu^{-}_{pq}\mu^{-}_{qn}\right)},$

5. $\sqrt{\sum_{p=1}^{n}\left(\gamma^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\gamma^{-}_{pq}\gamma^{-}_{qn}+n(n-1)\left|F\right|^{\frac{2}{n}}}\le E\left(\gamma^{-}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\gamma^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\gamma^{-}_{pq}\gamma^{-}_{qn}\right)},$

6. $\sqrt{\sum_{p=1}^{n}\left(\sigma^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\sigma^{-}_{pq}\sigma^{-}_{qn}+n(n-1)\left|H\right|^{\frac{2}{n}}}\le E\left(\sigma^{-}_{D^N}(G)\right)\le\sqrt{n\left(\sum_{p=1}^{n}\left(\sigma^{-}_{pp}\right)^2+2\sum_{1\le p<q\le n}\sigma^{-}_{pq}\sigma^{-}_{qn}\right)}.$

This completes the proof. ∎

Theorem 5.9 Let $G=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-}\right)$ be a BSVNG and

$$A(G)=\left(\mu^{+}(G),\gamma^{+}(G),\sigma^{+}(G),\mu^{-}(G),\gamma^{-}(G),\sigma^{-}(G)\right)$$

be the adjacency matrix of G. Let $G_1=\left(V,E,\mu^{+},\gamma^{+},\sigma^{+},\mu^{-},\gamma^{-},\sigma^{-},\mu_1^{+},\gamma_1^{+},\sigma_1^{+},\mu_1^{-},\gamma_1^{-},\sigma_1^{-}\right)$ be a dominating BSVNG of G and $A_{D^N}(G)=\left(\mu_{D^N}^{+}(G),\gamma_{D^N}^{+}(G),\sigma_{D^N}^{+}(G),\mu_{D^N}^{-}(G),\gamma_{D^N}^{-}(G),\sigma_{D^N}^{-}(G)\right)$ be the adjacency matrix of a dominating BSVNG G_1. Then

1. $E\left(\mu_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2 + E\left(\mu^{+}(G)\right)^2\right)$,
2. $E\left(\gamma_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2 + E\left(\gamma^{+}(G)\right)^2\right)$,
3. $E\left(\sigma_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2 + E\left(\sigma^{+}(G)\right)^2\right)$,
4. $E\left(\mu_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2 + E\left(\mu^{-}(G)\right)^2\right)$,
5. $E\left(\gamma_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2 + E\left(\gamma^{-}(G)\right)^2\right)$,
6. $E\left(\sigma_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2 + E\left(\sigma^{-}(G)\right)^2\right)$.

Proof.

$$E\left(\mu^{+}(G)\right)^2 \ge 2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+} + n(n-1)|A|^{\frac{2}{n}} \ge 2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}$$

$$\text{(i.e.) } 2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+} \le E\left(\mu^{+}(G)\right)^2 \qquad (1)$$

Now,

$$E\left(\mu_{D^N}^{+}(G)\right)^2 \le n\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2 + 2n\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}$$

$$E\left(\mu_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2 + 2\sum_{1\le p<q\le n}\mu_{pq}^{+}\mu_{qn}^{+}\right)$$

$$\le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{+}\right)^2 + E\left(\mu^{+}(G)\right)^2\right) \qquad (\text{by Eq (1)})$$

Analogously, we can show that

2. $E\left(\gamma_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{+}\right)^2 + E\left(\gamma^{+}(G)\right)^2\right)$,
3. $E\left(\sigma_{D^N}^{+}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{+}\right)^2 + E\left(\sigma^{+}(G)\right)^2\right)$,
4. $E\left(\mu_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\mu_{pp}^{-}\right)^2 + E\left(\mu^{-}(G)\right)^2\right)$,
5. $E\left(\gamma_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\gamma_{pp}^{-}\right)^2 + E\left(\gamma^{-}(G)\right)^2\right)$,
6. $E\left(\sigma_{D^N}^{-}(G)\right)^2 \le n\left(\sum_{p=1}^{n}\left(\sigma_{pp}^{-}\right)^2 + E\left(\sigma^{-}(G)\right)^2\right)$.

This completes the proof. ■

6. Conclusion

As part of this study, we explored a few graph-theoretic concepts and integrated a hypothesis of dominating energy with the idea of BSVNG. Specifically, in this study, we developed a new concept of adjacency matrix, as well as the spectrum of the adjacency matrix of the dominating in BSVNG. Hence, we computed the energy of dominating BSVNG. Apart from that, various operations regarding this domination have been illustrated. With appropriate instances, the complement, union and join of dominating energy in BSVNG have been examined. Finally, certain theorems regarding the dominating energy in BSVNG are established. In view of Akram et al. [49], the terms and notions discussed in this study can be extended in the framework of double domination energy in BSVNG.

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