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A Contemporary approach on Generalised NB Closed Sets in Neutrosophic Binary Topological Spaces

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Abstract. The paper, concentrated on the introduction of neutrosophic binary αgs-closed sets and neutrosophic binary αgs-open sets in Neutrosophic binary topological Space. The properties of neutrosophic binary αgs-closed sets and neutrosophic binary αgs-open sets have been studied. Furthermore, we examined its relationship with other framed neutrosophic binary sets. Also, some of the theorems were contemplated and the contrary part have been analyzed using examples.

Keywords: Neutrosophic Binary αgs-closed sets, Neutrosophic Binary αgs-open sets.

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1. **Introduction**

Neutrosophic Topological Space was initiated and formulated by A.A.Salama [14] in 2012 by using Smarandache [10] neutrosophic sets which was introduced in 2002, a generalisation of intuitionistic fuzzy sets. The neutrosophic sets consists of the degree of truth membership, degree of indeterminancy and the degree of false membership which are self-supporting and defines uncertainity. Neutrosophic α closed sets were introduced by I.Arockiarani [1] in 2017. In 2016, P.Ishwarya [12] defined the neutrosophic semi open and closed sets in neutrosophic topological space. Also, they studied the neutrosophic semi interior and closure operators. Neutrosophic α-generalized closed sets was established by R. Dhavaseelan et al., V.K.Shanthi [15] in 2018 initiated neutrosophic generalised semi closed sets. Furthermore, neutrosophic αgs closed sets were initiated by V.Banu Priya [2] in 2019. S.N.Jothi [5] in 2011 introduced the topology between two universal sets which is defined to be binary topology. The binary topology is a binary structure from $\dot{\mathcal{U}}$ to $\dot{\mathcal{V}}$ which consists of ordered pairs $(\dot{S}, \dot{Q})$ where $A \subseteq \dot{\mathcal{U}}$ and $B \subseteq \dot{\mathcal{V}}$. In continuation, S.S.Surekha, J.Elekiah and G.Sindhu [16] in 2022 introduced Neutrosophic Binary Topological Space which consists of two universal sets and each universal set contain its own truth, indeterminancy and false membership values. Also, in 2022, S.S.Surekha and G.sindhu [17] formulated binary αgs closed sets in binary topological space. In this article, we defined Neutrosophic Binary regular, semi and α open and closed sets. Also, Neutrosophic Binary αgs closed sets was defined and some characteristics have been framed.

2. Motivation

Neutrosophic Topological Space was formulated by A.A.Salama [14] in 2012. The neutrosophic sets consists of the degree of truth membership, degree of indeterminancy and the degree of false membership. S.N.Jothi [5] in 2011 introduced the binary topology. Later, S.S.Surekha, J.Elekiah and G.Sindhu [16] in 2022 introduced Neutrosophic Binary Topological Space consisting of two universal sets and each has its own truth, indeterminancy and false membership values. Also, in 2022, S.S.Surekha and G.sindhu [17] formulated binary αgs-closed sets in binary topological space. Also, neutrosophic αgs closed sets were initiated by V.Banu Priya [2] in 2019. Hence these implications motivated the researcher to investigate the role of neutrosophic binary αgs closed and open sets in neutrosophic binary topological spaces.

3. Preliminaries

Definition 3.1. [16] A Neutrosophic binary topology is a binary structure consisting of two universal sets $\dot{\mathcal{U}}$ and $\dot{\mathcal{V}}$ where $\mathcal{MN} \subseteq P(\dot{\mathcal{U}}) \times P(\dot{\mathcal{V}})$ and it satisfies the following conditions:

(1) $(\dot{0}_{\dot{\mathcal{U}}}, \dot{0}_{\dot{\mathcal{V}}}) \in \mathcal{MN}$ and $(\dot{1}_{\dot{\mathcal{U}}}, \dot{1}_{\dot{\mathcal{V}}}) \in \mathcal{MN}$.
(2) $(\dot{S}_1 \cap \dot{Q}_2, \dot{S}_1 \cap \dot{Q}_2) \in \mathcal{MN}$ whenever $(\dot{S}_1, \dot{Q}_1) \in \mathcal{MN}$ and $(\dot{S}_2, \dot{Q}_2) \in \mathcal{MN}$.
(3) If $(\dot{S}_\alpha, \dot{Q}_\alpha)_{\alpha \in S}$ is a family of members of $\mathcal{MN}$, then $(\cup_{\alpha \in S}\dot{S}_\alpha, \cup_{\alpha \in S}\dot{Q}_\alpha) \in \mathcal{MN}$.

The triplet $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ is called Neutrosophic Binary Topological space.

Definition 3.2. [16] $(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}})$ can be defined as
(0_1) $0_{\dot{\mathcal{U}}} = \{< \dot{U}, 0, 0, 1 >: \dot{u} \in \dot{\mathcal{U}}\}$, $0_{\dot{\mathcal{V}}} = \{< \dot{V}, 0, 0, 1 >: \dot{v} \in \dot{\mathcal{V}}\}$
(0_2) $0_{\dot{\mathcal{U}}} = \{< \dot{U}, 0, 1, 1 >: \dot{u} \in \dot{\mathcal{U}}\}$, $0_{\dot{\mathcal{V}}} = \{< \dot{V}, 0, 1, 1 >: \dot{v} \in \dot{\mathcal{V}}\}$
(0_3) $0_{\dot{\mathcal{U}}} = \{< \dot{U}, 0, 1, 0 >: \dot{u} \in \dot{\mathcal{U}}\}$, $0_{\dot{\mathcal{V}}} = \{< \dot{V}, 0, 1, 0 >: \dot{v} \in \dot{\mathcal{V}}\}$

(0_4) $0_{\dot{\mathcal{U}}} = \{<\dot{U},0,0,0>: \dot{u} \in \dot{\mathcal{U}}\}$, $0_{\dot{\mathcal{V}}} = \{<\dot{V},0,0,0>: \sqsubseteq \in \dot{\mathcal{V}}\}$

$(1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}})$ can be defined as

(1_1) $1_{\dot{\mathcal{U}}} = \{<\dot{U},1,0,0>: \dot{u} \in \dot{\mathcal{U}}\}$, $1_{\dot{\mathcal{V}}} = \{<\dot{V},1,0,0>: \dot{v} \in \dot{\mathcal{V}}\}$

(1_2) $1_{\dot{\mathcal{U}}} = \{<\dot{U},1,0,1>: \dot{u} \in \dot{\mathcal{U}}\}$, $1_{\dot{\mathcal{V}}} = \{<\dot{V},1,0,1>: \dot{v} \in \dot{\mathcal{V}}\}$

(1_3) $1_{\dot{\mathcal{U}}} = \{<\dot{U},1,1,0>: \dot{u} \in \dot{\mathcal{U}}\}$, $1_{\dot{\mathcal{V}}} = \{<\dot{V},1,1,0>: \dot{v} \in \dot{\mathcal{V}}\}$

(1_4) $1_{\dot{\mathcal{U}}} = \{<\dot{U},1,1,1>: \dot{u} \in \dot{\mathcal{U}}\}$, $1_{\dot{\mathcal{V}}} = \{<\dot{V},1,1,1>: \dot{v} \in \dot{\mathcal{V}}\}$

Definition 3.3. [16] Let $(\dot{S},\dot{Q}) = \{<\mu_S,\sigma_S,\gamma_S>, <\mu_Q,\sigma_Q,\gamma_Q>\}$ be a neutrosophic binary set on $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$, then the complement of the set $\tilde{C}(\dot{S},\dot{Q})$ may be defined as

$$(\tilde{C_1}) \quad \tilde{C}(\dot{S},\dot{Q}) = \{\dot{U}, <1-\mu_{\dot{S}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}), 1-\gamma_{\dot{S}}(\dot{U})>: \dot{u} \in \dot{U}, <\dot{V}, 1-\mu_{\dot{Q}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}), 1-\gamma_{\dot{Q}}(\dot{V})>: \dot{v} \in \dot{V}\}$$

$$(\tilde{C_2}) \quad \tilde{C}(\dot{S},\dot{Q}) = \{\dot{U}, <\gamma_{\dot{S}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}), \mu_{\dot{S}}(\dot{U})>: \dot{u} \in \dot{U}, <\dot{V}, \gamma_{\dot{Q}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}), \mu_{\dot{Q}}(\dot{V})>: \dot{v} \in \dot{V}\}$$

$$(\tilde{C_3}) \quad \tilde{C}(\dot{S},\dot{Q}) = \{\dot{U}, <\gamma_{\dot{S}}(\dot{U}), 1-\sigma_{\dot{S}}(\dot{U}), \mu_{\dot{S}}(\dot{U})>: \dot{u} \in \dot{U}, <\dot{V}, \gamma_{\dot{Q}}(\dot{V}), 1-\sigma_{\dot{Q}}(\dot{V}), \mu_{\dot{Q}}(\dot{V})>: \dot{v} \in \dot{V}\}$$

Definition 3.4. [16] Let $(\dot{S},\dot{Q})$ and $(\dot{T},\dot{R})$ be two neutrosophic binary sets.
Then $(\dot{S},\dot{Q}) \subseteq (\dot{T},\dot{R})$ can be defined as

(1) $(\dot{S},\dot{Q}) \subseteq (\dot{T},\dot{R}) \iff \mu_{\dot{S}}(\dot{U}) \leq \mu_{\dot{T}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}) \leq \sigma_{\dot{T}}(\dot{U}), \gamma_{\dot{S}}(\dot{U}) \geq \gamma_{\dot{T}}(\dot{U}) \forall \dot{u} \in \dot{U}$
$\mu_{\dot{Q}}(\dot{V}) \leq \mu_{\dot{R}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}) \leq \sigma_{\dot{R}}(\dot{V}), \gamma_{\dot{Q}}(\dot{V}) \geq \gamma_{\dot{R}}(\dot{V}) \forall \dot{v} \in \dot{V}$

(2) $(\dot{S},\dot{Q}) \subseteq (\dot{T},\dot{R}) \iff \mu_{\dot{S}}(\dot{U}) \leq \mu_{\dot{T}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}) \geq \sigma_{\dot{T}}(\dot{U}), \gamma_{\dot{S}}(\dot{U}) \geq \gamma_{\dot{T}}(\dot{U}) \forall \dot{u} \in \dot{U}$
$\mu_{\dot{Q}}(\dot{V}) \leq \mu_{\dot{R}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}) \geq \sigma_{\dot{R}}(\dot{V}), \gamma_{\dot{Q}}(\dot{V}) \geq \gamma_{\dot{R}}(\dot{V}) \forall \dot{v} \in \dot{V}$

Definition 3.5. [16] Let $(\dot{S},\dot{Q})$ and $(\dot{T},\dot{R})$ be two neutrosophic binary sets.
(1) $(\dot{S},\dot{Q}) \cap (\dot{T},\dot{R})$ can be defined as

$$(\dot{S},\dot{Q}) \cap (\dot{T},\dot{R}) = \{ <\dot{U}, \mu_{\dot{S}}(\dot{U}) \wedge \mu_{\dot{T}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}) \wedge \sigma_{\dot{T}}(\dot{U}), \gamma_{\dot{S}}(\dot{U}) \vee \gamma_{\dot{T}}(\dot{U})> <\dot{V}, \mu_{\dot{Q}}(\dot{V}) \wedge \mu_{\dot{R}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}) \wedge \sigma_{\dot{R}}(\dot{V}), \gamma_{\dot{Q}}(\dot{V}) \vee \gamma_{\dot{R}}(\dot{V})>\}$$

$$(\dot{S},\dot{Q}) \cap (\dot{T},\dot{R}) = \{ <\dot{U}, \mu_{\dot{S}}(\dot{U}) \wedge \mu_{\dot{T}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}) \vee \sigma_{\dot{T}}(\dot{U}), \gamma_{\dot{S}}(\dot{U}) \vee \gamma_{\dot{T}}(\dot{U})> <\dot{V}, \mu_{\dot{Q}}(\dot{V}) \wedge \mu_{\dot{R}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}) \vee \sigma_{\dot{R}}(\dot{V}), \gamma_{\dot{Q}}(\dot{V}) \vee \gamma_{\dot{R}}(\dot{V})>\}$$

(2) $(\dot{S},\dot{Q}) \cup (\dot{T},\dot{R})$ can be defined as

$$(\dot{S},\dot{Q}) \cup (\dot{T},\dot{R}) = \{ <\dot{U}, \mu_{\dot{S}}(\dot{U}) \vee \mu_{\dot{T}}(\dot{U}), \sigma_{\dot{S}}(\dot{U}) \vee \sigma_{\dot{T}}(\dot{U}), \gamma_{\dot{S}}(\dot{U}) \wedge \gamma_{\dot{T}}(\dot{U})> <\dot{V}, \mu_{\dot{Q}}(\dot{V}) \vee \mu_{\dot{R}}(\dot{V}), \sigma_{\dot{Q}}(\dot{V}) \vee \sigma_{\dot{R}}(\dot{V}), \gamma_{\dot{Q}}(\dot{V}) \wedge \gamma_{\dot{R}}(\dot{V})>\}$$

$$(\dot{S},\dot{Q})\cap(\dot{T},\dot{R})=\{<\dot{U},\mu_{\dot{S}}(\dot{U})\vee\mu_{\dot{T}}(\dot{U}),\sigma_{\dot{S}}(\dot{U})\wedge\sigma_{\dot{T}}(\dot{U}),\gamma_{\dot{S}}(\dot{U})\wedge\gamma_{\dot{T}}(\dot{U})>$$
$$<\dot{V},\mu_{\dot{Q}}(\dot{V})\vee\mu_{\dot{R}}(\dot{V}),\sigma_{\dot{Q}}(\dot{V})\wedge\sigma_{\dot{R}}(\dot{V}),\gamma_{\dot{Q}}(\dot{V})\wedge\gamma_{\dot{R}}(\dot{V})>\}$$

Definition 3.6. [16] Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then,
$(\dot{S},\dot{Q})^{1^{\star}_{N}}=\cap\{S_\alpha:(S_\alpha,Q_\alpha)$ is neutrosophic binary closed and $(\dot{S},\dot{Q})\subseteq(S_\alpha,Q_\alpha)\}$
$(\dot{S},\dot{Q})^{2^{\star}_{N}}=\cap\{Q_\alpha:(S_\alpha,Q_\alpha)$ is neutrosophic binary closed and $(\dot{S},\dot{Q})\subseteq(S_\alpha,Q_\alpha)\}$.
The ordered pair $((\dot{S},\dot{Q})^{1^{\star}_{N}},(\dot{S},\dot{Q})^{2^{\star}_{N}})$ is called the neutrosophic binary closure of $(\dot{S},\dot{Q})$.

Definition 3.7. [16] Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then,
$(\dot{S},\dot{Q})^{1^{0}_{N}}=\cup\{S_\alpha:(S_\alpha,Q_\alpha)$ is neutrosophic binary open and $(S_\alpha,Q_\alpha)\subseteq(\dot{S},\dot{Q})\}$
$(\dot{S},\dot{Q})^{2^{0}_{N}}=\cup\{Q_\alpha:(S_\alpha,Q_\alpha)$ is neutrosophic binary open and $(S_\alpha,Q_\alpha)\subseteq(\dot{S},\dot{Q})\}$.
The ordered pair $((\dot{S},\dot{Q})^{1^{0}_{N}},(\dot{S},\dot{Q})^{2^{0}_{N}})$ is called the neutrosophic binary interior of $(\dot{S},\dot{Q})$.

Definition 3.8. Let $(\dot{\mathcal{U}},\tau_N)$ be a Neutrosophic topological space. Then the subset A is said to be

(1) neutrosophic α open [1] if $A\subseteq Nint(Ncl(Nint(A)))$.
(2) neutrosophic semi open [12] if $A\subseteq Ncl(Nint(A))$.

Definition 3.9. [1] Let $(\dot{\mathcal{U}},\tau_N)$ be a Neutrosophic Topological Space. Let A be the subset of neutrosophic topological space. The intersection of all the neutrosophic α closed sets which contains A is called the neutrosophic α closure of A and is denoted by $N\alpha cl(A)$.

Definition 3.10. [2] Let $(\dot{\mathcal{U}},\tau_N)$ be a Neutrosophic topological space. Then the subset A is said to be neutrosophic αgs closed if $N\alpha cl(A)\subseteq U$ whenever $A\subseteq U$ and U is neutrosophic semi open.

4. Neutrosophic Binary αgs-closed sets

Definition 4.1. Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then $(\dot{S},\dot{Q})$ is called

(1) Neutrosophic binary regular open if $(\dot{S},\dot{Q})=\mathcal{N}^{\mathbf{b}}int(\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q}))$ and the complement of neutrosophic binary regular open sets are called as neutrosophic binary regular closed (shortly $\mathcal{N}^{\mathbf{b}}$-regular closed).
(2) Neutrosophic binary semiopen if $(\dot{S},\dot{Q})\subseteq\mathcal{N}^{\mathbf{b}}cl(\mathcal{N}^{\mathbf{b}}int(\dot{S},\dot{Q}))$ and the complement of neutrosophic binary semi open sets are called as neutrosophic binary semi closed sets (shortly $\mathcal{N}^{\mathbf{b}}$-semi closed).

(3) Neutrosophic binary α open if $(\dot{S},\dot{Q}) \subseteq \mathcal{N}^{\mathbf{b}}int(\mathcal{N}^{\mathbf{b}}cl(\mathcal{N}^{\mathbf{b}}int(\dot{S},\dot{Q})))$ and the complement of neutrosophic binary α open sets are called as neutrosophic binary α closed sets (shortly $\mathcal{N}^{\mathbf{b}}$-α closed).

Definition 4.2. Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then,
$(\dot{S},\dot{T})^{1\star_N}_{\alpha} = \cap\{S_\alpha : (S_\alpha,T_\alpha)$ is neutrosophic binary α closed and $(\dot{S},\dot{T}) \subseteq (S_\alpha,T_\alpha)\}$
$(\dot{S},\dot{T})^{2\star_N}_{\alpha} = \cap\{T_\alpha : (S_\alpha,T_\alpha)$ is neutrosophic binary α closed and $(\dot{S},\dot{T}) \subseteq (S_\alpha,T_\alpha)\}$.
The ordered pair $((\dot{S},\dot{T})^{1\star_N}_{\alpha},(\dot{S},\dot{T})^{2\star_N}_{\alpha})$ is called the neutrosophic binary α closure of $(\dot{S},\dot{Q})$ and is denoted by $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q})$.

Definition 4.3. Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then,
$(\dot{S},\dot{T})^{1^0_N}_{\alpha} = \cup\{S_\alpha : (S_\alpha,B_\alpha)$ is neutrosophic binary α open and $(S_\alpha,T_\alpha) \subseteq (\dot{S},\dot{T})\}$
$(\dot{S},\dot{T})^{2^0_N}_{\alpha} = \cup\{T_\alpha : (S_\alpha,T_\alpha)$ is neutrosophic binary α open and $(S_\alpha,T_\alpha) \subseteq (\dot{S},\dot{T})\}$.
The ordered pair $((\dot{S},\dot{T})^{1^0_N}_{\alpha},(\dot{S},\dot{T})^{2^0_N}_{\alpha})$ is called the neutrosophic binary α interior of $(\dot{S},\dot{T})$ and it is denoted by $\mathcal{N}^{\mathbf{b}}\alpha int(\dot{S},\dot{T})$.

Example 4.4. Let $\dot{\mathcal{U}} = \{a_1,a_2,a_3\}$ and $\dot{\mathcal{V}} = \{b_1,b_2,b_3\}$. Let

$$(A_1,A_2) = \{<\dot{\mathcal{U}},(O.4,0.5,0.2),(0.3,0.2,0.1),(0.9,0.6,0.8)>, <\dot{\mathcal{V}},(0.2,0.4,0.5),(0.1,0.1,0.2),(0.6,0.5,0.8)>\}$$
$$(B_1,B_2) = \{<\dot{\mathcal{U}},(0.5,0.6,0.2),(0.4,0.3,0.1),(0.7,0.6,0.7)>, <\dot{\mathcal{V}},(0.3,0.5,0.4),(0.3,0.2,0.1),(0.7,0.5,0.6)>\}.$$

The Neutrosophic Binary Topological space is given by $\mathcal{M}_N = \{(0_{\dot{\mathcal{U}}},0_{\dot{\mathcal{V}}}),(1_{\dot{\mathcal{U}}},1_{\dot{\mathcal{V}}}),(A_1,A_2),(B_1,B_2)\}$. Let

$$(C_1,C_2) = \{<\dot{\mathcal{U}},(0.5,0.6,0.1),(0.4,0.3,0.1),(0.9,0.8,0.5)>, <\dot{\mathcal{V}},(0.3,0.4,0.5),(0.9,0.3,0.1),(0.7,0.6,0.7)>\}.$$

Clearly, (C_1,C_2) is Neutrosophic Binary semi open. Also, it is Neutrosophic Binary alpha open.

Theorem 4.5. *In a neutrosophic binary topological space* $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$,

(1) *Every* $\mathcal{N}^{\mathbf{b}}$*-regular closed sets are* $\mathcal{N}^{\mathbf{b}}$ *closed sets.*
(2) *Every* $\mathcal{N}^{\mathbf{b}}$*-semi closed sets are* $\mathcal{N}^{\mathbf{b}}\alpha$ *closed sets.*

Proof. (1) Since, $(\dot{S},\dot{Q})$ is Neutrosophic Binary-regular closed set, we have $(\dot{S},\dot{Q}) = \mathcal{N}^{\mathbf{b}}cl(\mathcal{N}^{\mathbf{b}}int(\dot{S},\dot{Q}))$. Obviously, $(\dot{S},\dot{Q})$ is Neutrosophic Binary closed set.
(2) Since, $(\dot{S},\dot{Q})$ is Neutrosophic Binary Semi closed, we have $\mathcal{N}^{\mathbf{b}}int(\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q})) \subseteq (\dot{S},\dot{Q})$. This implies, $\mathcal{N}^{\mathbf{b}}cl(\mathcal{N}^{\mathbf{b}}int(\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q}))) \subseteq \mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q})$. Since, every neutrosophic binary semi closed sets are closed, we have $(\dot{S},\dot{Q}) = \mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q})$. Therefore,

$\mathcal{N}^{\mathbf{b}}cl(\mathcal{N}^{\mathbf{b}}int(\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q}))) \subseteq (\dot{S},\dot{Q})$. Hence, $(\dot{S},\dot{Q})$ is neutrosophic binary α closed in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. □

Remark 4.6. Since, every $\mathcal{N}^{\mathbf{b}}$-semi closed sets are $\mathcal{N}^{\mathbf{b}}\alpha$ closed sets, it is obvious that $\mathcal{N}^{\mathbf{b}}scl(\dot{S},\dot{Q}) \subseteq \mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q})$.

Definition 4.7. Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Let $(\dot{S},\dot{T}) \subseteq (\dot{\mathcal{U}},\dot{\mathcal{V}})$. Then $(\dot{S},\dot{T})$ is called a Neutrosophic Binary α generalised semiclosed set (shortly $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set) if $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{T}) \subseteq (P,V)$ whenever (P,V) is Neutrosophic Binary Semiopen.

Theorem 4.8. *The union of two $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set is also a $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set.*

Proof. Let $(\dot{S},\dot{Q})$ and $(\dot{T},\dot{R})$ be two $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Then by definition 4.7, we have $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q}) \subseteq (\dot{P},\dot{V})$ whenever $(\dot{S},\dot{Q}) \subseteq (\dot{P},\dot{V})$ and $(\dot{P},\dot{V})$ is $\mathcal{N}^{\mathbf{b}}$ semiopen. Also, $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{T},\dot{R}) \subseteq (\dot{P},\dot{V})$ whenever $(\dot{T},\dot{R}) \subseteq (\dot{P},\dot{V})$ and $(\dot{P},\dot{V})$ is $\mathcal{N}^{\mathbf{b}}$ semiopen. This implies $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q}) \cup \mathcal{N}^{\mathbf{b}}\alpha cl(\dot{T},\dot{R}) \subseteq (\dot{P},\dot{V}) \implies \mathcal{N}^{\mathbf{b}}\alpha cl[(\dot{S},\dot{Q}) \cup (\dot{T},\dot{R})] \subseteq (\dot{P},\dot{V})$. Therefore, $(\dot{S},\dot{Q}) \cup (\dot{T},\dot{R})$ is a $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. □

Remark 4.9. The intersection of two $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed sets need not be a $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set.

It is demonstrated by the following example.

Example 4.10. Let $\dot{\mathcal{U}} = \{a_1,a_2,a_3\}$ and $\dot{\mathcal{V}} = \{b_1,b_2,b_3\}$ be the universe. Let $\mathcal{M}_N = \{(0_{\dot{\mathcal{U}}},0_{\dot{\mathcal{V}}}),(1_{\dot{\mathcal{U}}},1_{\dot{\mathcal{V}}}),(A_1,A_2),(B_1,B_2),(C_1,C_2),(D_1,D_2)\}$ be the neutrosophic binary topological space. Here

$$(A_1,A_2) = \{ <\dot{\mathcal{U}},(0.4,0.5,0.2),(0.3,0.5,0.1),(0.9,0.6,0.8)>, <\dot{\mathcal{V}},(0.2,0.5,0.5),(0.1,0.5,0.2),(0.6,0.5,0.8)>\}$$
$$(B_1,B_2) = \{ <\dot{\mathcal{U}},(0.5,0.6,0.2),(0.4,0.5,0.1),(0.7,0.6,0.7)>, <\dot{\mathcal{V}},(0.3,0.5,0.4),(0.3,0.5,0.1),(0.7,0.5,0.6)>\}$$
$$(C_1,C_2) = \{ <\dot{\mathcal{U}},(0.5,0.5,0.2),(0.4,0.5,0.1),(0.9,0.6,0.7)>, <\dot{\mathcal{V}},(0.3,0.5,0.4),(0.3,0.5,0.1),(0.7,0.5,0.6)>\}$$
$$(D_1,D_2) = \{ <\dot{\mathcal{U}},(0.4,0.6,0.2),(0.3,0.5,0.1),(0.7,0.6,0.8)>, <\dot{\mathcal{V}},(0.2,0.5,0.5),(0.1,0.5,0.2),(0.6,0.5,0.8)>\}$$

$$Let\ (\dot{S},\dot{Q}) = \{ <\dot{\mathcal{U}},(0.3,0.4,0.2),(0.3,0.1,0.1),(0.6,0.4,0.9)>,$$
$$<\dot{\mathcal{V}},(0.2,0.3,0.6),(0.1,0.1,0.3),(0.4,0.4,0.9)>\}\ and$$
$$(\dot{T},\dot{R}) = \{ <\dot{\mathcal{U}},(0.4,0.5,0.2),(0.3,0.5,0.1),(0.9,0.4,0.8)>,$$
$$<\dot{\mathcal{V}},(0.2,0.5,0.5),(0.1,0.5,0.2),(0.6,0.5,0.8)>\}$$

be two $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed sets in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. The intersection of the two subsets

$$(\dot{S},\dot{Q})\cap(\dot{T},\dot{R}) = \{ <\dot{\mathcal{U}},(0.3,0.5,0.2),(0.3,0.5,0.1),(0.6,0.4,0.9)>,$$
$$<\dot{\mathcal{V}},(0.2,0.5,0.6),(0.1,0.5,0.3),(0.4,0.5,0.9)>\}$$

which is not $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$.

Theorem 4.11. *In a neutrosophic binary topological space $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$, every $\mathcal{N}^{\mathbf{b}}$-closed sets are $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set.*

Proof. Let $(\dot{S},\dot{Q})$ be a $\mathcal{N}^{\mathbf{b}}$-closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Let us consider a neutrosophic binary set $(\dot{S},\dot{Q})\subseteq(\dot{P},\dot{V})$ where $(\dot{P},\dot{V})$ is neutrosophic binary semiopen in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Since $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q})\subseteq\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q})$ and also $(\dot{S},\dot{Q})$ is neutrosophic binary closed set, we have $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q})\subseteq\mathcal{N}^{\mathbf{b}}cl(\dot{S},\dot{Q})=(\dot{S},\dot{Q})\subseteq(\dot{P},\dot{V})$ which implies $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S},\dot{Q})\subseteq(\dot{P},\dot{V})$ whenever $(\dot{P},\dot{V})$ is neutrosophic binary semiopen. Hence $(\dot{S},\dot{Q})$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set. □

Remark 4.12. The converse of the above theorem need not be true as illustrated by the following example.

Example 4.13. Let $\dot{\mathcal{U}}=\{a_1,a_2\}$ and $\dot{\mathcal{V}}=\{b_1,b_2\}$ be the universe.
Let $\mathcal{MN}=\{(0_{\dot{\mathcal{U}}},0_{\dot{\mathcal{V}}}),(1_{\dot{\mathcal{U}}},1_{\dot{\mathcal{V}}}),(V,W)\}$ be the neutrosophic binary topological space.
$(V,W)=\{<\dot{\mathcal{U}},(0.7,0.5,0.3),(0.6,0.5,0.4)>,<\dot{\mathcal{V}},(0.6,0.5,0.4),(0.7,0.5,0.3)>\}$.
$(\dot{S},\dot{Q})=\{<\dot{\mathcal{U}},(0.2,0.5,0.8),(0.3,0.5,0.7)>,<\dot{\mathcal{V}},(0.3,0.5,0.8),(0.2,0.4,0.7)>\}$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed set but not $\mathcal{N}^{\mathbf{b}}$ closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$.

Theorem 4.14. *Every $\mathcal{N}^{\mathbf{b}}$-Regular closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$.*

Proof. Let $(\dot{S},\dot{Q})$ be a $\mathcal{N}^{\mathbf{b}}$-regular closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. By theorem 4.5, every $\mathcal{N}^{\mathbf{b}}$-regular closed set is $\mathcal{N}^{\mathbf{b}}$-closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. This implies, $(\dot{S},\dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$-closed set. Also, by theorem 4.11, we have $(\dot{S},\dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$-αgs-closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. □

Remark 4.15. The converse of the above theorem need not be true as proved in the following example.

Example 4.16. Let $\dot{\mathcal{U}} = \{a_1, a_2\}$ and $\dot{\mathcal{V}} = \{b_1, b_2\}$ be the universe.
Let $\mathcal{MN} = \{(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}}), (1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}}), (V, W)\}$ be the neutrosophic binary topological space.
$(V, W) = \{< \dot{\mathcal{U}}, (0.6, 0.5, 0.4), (0.6, 0.5, 0.4) >, < \dot{\mathcal{V}}, 0.7, 0.5, 0.3), (0.8, 0.5, 0.4)\}$.
Let $(\dot{S}, \dot{Q}) = \{< \dot{\mathcal{U}}, (0.3, 0.5, 0.7), (0.2, 0.5, 0.8) >, < \dot{\mathcal{V}}, (0.4, 0.5, 0.8), (0.3, 0.5, 0.7) >\}$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed but not $\mathcal{N}^{\mathbf{b}}$ regular closed in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.

Theorem 4.17. *Every $\mathcal{N}^{\mathbf{b}}$-αgs closed set is $\mathcal{N}^{\mathbf{b}}$-semi closed in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.*

Proof. Let $(\dot{S}, \dot{Q})$ be a $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. By remark 4.6, we have $\mathcal{N}^{\mathbf{b}}scl(\dot{S}, \dot{Q}) \subseteq \mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S}, \dot{Q})$. Since $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed set, we have $\mathcal{N}^{\mathbf{b}}\alpha cl(\dot{S}, \dot{Q}) \subseteq (\dot{P}, \dot{V})$, where $(\dot{P}, \dot{V})$ is $\mathcal{N}^{\mathbf{b}}$- semiopen. This implies $\mathcal{N}^{\mathbf{b}}Scl(\dot{S}, \dot{Q}) \subseteq (\dot{P}, \dot{V})$. Hence, $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$-semi closed. □

Remark 4.18. The converse of the above theorem need not be true as illustrated in the following example.

Example 4.19. Let $\dot{\mathcal{U}} = \{a_1, a_2\}$ and $\dot{\mathcal{V}} = \{b_1, b_2\}$ be the universe.
Let $\mathcal{MN} = \{(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}}), (1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}}), (V, W)\}$ be the neutrosophic binary topological space where $(V, W) = \{< \dot{\mathcal{U}}, (0.6, 0.5, 0.2), (0.3, 0.5, 0.2) >, < \dot{\mathcal{V}}, (0.7, 0.5, 0.1), (0.3, 0.5, 0.3) > \}$.
Let $(\dot{S}, \dot{Q}) = \{< \dot{\mathcal{U}}, (0.1, 0.5, 0.7), (0.1, 0.5, 0.5) >, < \dot{\mathcal{V}}, (0.2, 0.5, 0.6), (0.2, 0.5, 0.7) >\}$ is $\mathcal{N}^{\mathbf{b}}$ semi closed but not $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.

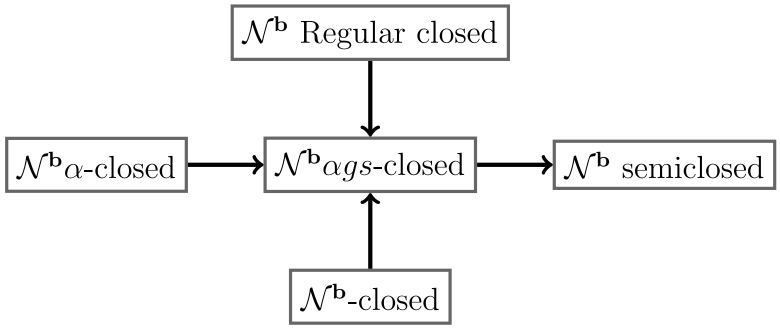
Theorem 4.20. *Every $\mathcal{N}^{\mathbf{b}}$-α closed set is $\mathcal{N}^{\mathbf{b}}$-αgs closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.*

Proof. Let $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ be a neutrosophic binary topological space. Let $(\dot{S}, \dot{Q})$ be a $\mathcal{N}^{\mathbf{b}}$-α closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Let $(\dot{S}, \dot{Q}) \subseteq (\dot{P}, \dot{V})$ where $(\dot{P}, \dot{V})$ is $\mathcal{N}^{\mathbf{b}}$ semiopen. Since $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$- α closed set, we have $\mathcal{N}^{\mathbf{b}}$-$\alpha cl(\dot{S}, \dot{Q}) = (\dot{S}, \dot{Q}) \subseteq (\dot{P}, \dot{V})$ which implies $\mathcal{N}^{\mathbf{b}}$-$\alpha cl(\dot{S}, \dot{Q}) \subseteq (\dot{P}, \dot{V})$. Hence $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$- αgs closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. □

Remark 4.21. The converse of the above theorem need not be true as seen in the following example.

Example 4.22. Let $\dot{\mathcal{U}} = \{a_1, a_2\}$ and $\dot{\mathcal{V}} = \{b_1, b_2\}$ be the universe.
Let $\mathcal{MN} = \{(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}}), (1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}}), (V_1, W_1), (V_2, W_2)\}$ be the neutrosophic binary topological space. Here
$(V_1, W_1) = \{< \dot{\mathcal{U}}, (0.4, 0.5, 0.5), (0.3, 0.5, 0.6) >, < \dot{\mathcal{V}}, (0.3, 0.5, 0.5), (0.4, 0.5, 0.7) >\}$
$(V_2, W_2) = \{< \dot{\mathcal{U}}, (0.3, 0.5, 0.6), (0.2, 0.5, 0.7) >, < \dot{\mathcal{V}}, (0.2, 0.5, 0.6), (0.3, 0.5, 0.7) >\}$.
$(\dot{S}, \dot{Q}) = \{< \dot{\mathcal{U}}, (0.8, 0.5, 0.1), (0.8, 0.5, 0.1) >, < \dot{\mathcal{V}}, (0.7, 0.5, 0.2), (0.6, 0.5, 0.1) >\}$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed set but not $\mathcal{N}^{\mathbf{b}}\alpha$ closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.

Remark 4.23. The following diagram shows the above implications.



5. **Neutrosophic Binary αgs open sets**

Definition 5.1. A neutrosophic binary set $(\dot{S}, \dot{T})$ in a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ is said to be neutrosophic binary αgs-open sets (shortly $\mathcal{N}^{\mathbf{b}}$-αgs open) if the complement $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.

Theorem 5.2. *In a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$, every $\mathcal{N}^{\mathbf{b}}$-open sets are $\mathcal{N}^{\mathbf{b}}\alpha gs$-open sets.*

Proof. Let $(\dot{S}, \dot{Q})$ be the $\mathcal{N}^{\mathbf{b}}$-open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}$-closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then by theorem 4.11,$(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set, which implies $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. □

Remark 5.3. The converse of the above theorem is not true as illustrated in the following example.

Example 5.4. Let $\dot{\mathcal{U}} = \{a_1, a_2\}$ and $\dot{\mathcal{V}} = \{b_1, b_2\}$. The neutrosophic binary topological space is given by $\mathcal{MN} = \{(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}}), (1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}}), (V, W)\}$ where
$(V, W) = \{< \dot{\mathcal{U}}, (0.8, 0.5, 0.2), (0.6, 0.5, 0.4) >, < \dot{\mathcal{V}}, (0.7, 0.5, 0.1), (0.6, 0.5, 0.3) >\}$.
Consider the neutrosophic binary set
$(\dot{S}, \dot{Q}) = \{< \dot{\mathcal{U}}, (0.9, 0.5, 0.1), (0.7, 0.5, 0.3) >, < \dot{\mathcal{V}}, (0.8, 0.5, 0.1), (0.8, 0.5, 0.3) >\}$. Here, $(\dot{S}, \dot{Q})^c$ is neutrosophic binary *alphags* closed set set. This implies that $(\dot{S}, \dot{Q})$ is neutrosophic binary αgs open set. But $(\dot{S}, \dot{Q})$ is not the neutrosophic binary open set.

Theorem 5.5. *In a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$, every $\mathcal{N}^{\mathbf{b}}$- regular open sets are $\mathcal{N}^{\mathbf{b}}\alpha gs$-open sets.*

Proof. Let $(\dot{S}, \dot{Q})$ be the $\mathcal{N}^{\mathbf{b}}$- regular open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}$-regular closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then by theorem 4.14, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set, which implies $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. □

Remark 5.6. The converse of the above theorem is not true as seen in the following example.

Example 5.7. Let $\dot{\mathcal{U}} = \{a_1, a_2\}$ and $\dot{\mathcal{V}} = \{b_1, b_2\}$. The neutrosophic binary topological space is given by $\mathcal{MN} = \{(0_{\dot{\mathcal{U}}}, 0_{\dot{\mathcal{V}}}), (1_{\dot{\mathcal{U}}}, 1_{\dot{\mathcal{V}}}), (V, W)\}$ where
$(V, W) = \{< \dot{\mathcal{U}}, (0.6, 0.5, 0.4), (0.6, 0.5, 0.4) >, < \dot{\mathcal{V}}, (0.7, 0.5, 0.3), (0.6, 0.5, 0.4) >\}$.
Consider the neutrosophic binary set
$(\dot{S}, \dot{Q}) = \{< \dot{\mathcal{U}}, (0.7, 0.5, 0.3), (0.8, 0.5, 0.2) >, < \dot{\mathcal{V}}, (0.8, 0.5, 0.3), (0.8, 0.5, 0.2) >\}$. Here, $(\dot{S}, \dot{Q})^c$ is neutrosophic binary αgs closed set set. This implies that $(\dot{S}, \dot{Q})$ is neutrosophic binary αgs open set. But $(\dot{S}, \dot{Q})$ is not the neutrosophic binary regular open set.

Theorem 5.8. *In a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$, every $\mathcal{N}^{\mathbf{b}}\alpha$- open sets are $\mathcal{N}^{\mathbf{b}}\alpha gs$-open sets.*

Proof. Let $(\dot{S}, \dot{Q})$ be the $\mathcal{N}^{\mathbf{b}}\alpha$- open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\alpha$-closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then by theorem 4.20, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set, which implies $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. □

Remark 5.9. The converse of the above theorem is not true as seen in the example 4.22.

Theorem 5.10. *In a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$, every $\mathcal{N}^{\mathbf{b}}\alpha gs$-open sets are $\mathcal{N}^{\mathbf{b}}$ semi open sets.*

Proof. Let $(\dot{S}, \dot{Q})$ be the $\mathcal{N}^{\mathbf{b}}\alpha gs$- open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-closed set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. Then by theorem 4.17, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}$ semi-closed set, which implies $(\dot{S}, \dot{Q})$ is $\mathcal{N}^{\mathbf{b}}$- semi open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$. □

Remark 5.11. The converse of the above theorem is not true as seen in the example 4.19.

Theorem 5.12. *A neutrosophic binary set $(\dot{S}, \dot{Q})$ of a neutrosophic binary topological space $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ is $\mathcal{N}^{\mathbf{b}}\alpha gs$-open set if and only if $(\dot{E}, \dot{F}) \subseteq \mathcal{N}^{\mathbf{b}}\alpha$-int$(\dot{S}, \dot{Q})$ whenever $(\dot{E}, \dot{F})$ is $\mathcal{N}^{\mathbf{b}}$-semi closed in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ and $(\dot{E}, \dot{F}) \subseteq (\dot{S}, \dot{Q})$.*

Proof. Necessary Part:
Let $(\dot{S}, \dot{Q})$ be a $\mathcal{N}^{\mathbf{b}}$-αgs open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$.
Let $(\dot{E}, \dot{F})$ be a $\mathcal{N}^{\mathbf{b}}$-αgs closed set and also $(\dot{E}, \dot{F}) \subseteq (\dot{S}, \dot{Q})$. This implies, $(\dot{E}, \dot{F})^c$ is $\mathcal{N}^{\mathbf{b}}$-αgs open set in $(\dot{\mathcal{U}}, \dot{\mathcal{V}}, \mathcal{MN})$ and $(\dot{S}, \dot{Q})^c \subseteq (\dot{E}, \dot{F})^c$. Since, $(\dot{S}, \dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}$-αgs closed set, we have $\mathcal{N}^{\mathbf{b}}$-$\alpha cl(\dot{S}, \dot{Q})^c \subseteq (\dot{E}, \dot{F})^c$. Hence, $(\dot{E}, \dot{F}) \subseteq \mathcal{N}^{\mathbf{b}}$-$\alpha int(\dot{S}, \dot{Q})$.

Sufficient Part:
Let $(\dot{E},\dot{F}) \subseteq \mathcal{N}^{\mathbf{b}}\text{-}\alpha int(\dot{S},\dot{Q})$.
This implies, $\mathcal{N}^{\mathbf{b}}\text{-}\alpha cl(\dot{S},\dot{Q})^c \subseteq (\dot{E},\dot{F})^c$ whenever $(\dot{E},\dot{F})^c$ is $\mathcal{N}^{\mathbf{b}}$-semi open. Therefore, $(\dot{S},\dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Hence, $(\dot{S},\dot{Q})$ is $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ open set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. □

Theorem 5.13. *Let $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ be a Neutrosophic Binary Topological Space. Then for every $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ open set $(\dot{S},\dot{Q})$ in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$ and for every Neutrosophic Binary set $(\dot{T},\dot{R})$ in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$, $\mathcal{N}^{\mathbf{b}}\alpha int(\dot{S},\dot{Q}) \subseteq (\dot{T},\dot{R}) \subseteq (\dot{S},\dot{Q}) \implies (\dot{T},\dot{R})$ is a $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ open set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$.*

Proof. By hypothesis, $(\dot{S},\dot{Q})^c \subseteq (\dot{T},\dot{R})^c \subseteq (\mathcal{N}^{\mathbf{b}}\alpha int(\dot{S},\dot{Q}))^c$. Let $(\dot{T},\dot{R})^c \subseteq (\dot{P},\dot{V})$ and $(\dot{P},\dot{V})$ be a $\mathcal{N}^{\mathbf{b}}$-semiopen set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Since $(\dot{S},\dot{Q})^c \subseteq (\dot{T},\dot{R})^c$, we have $(\dot{S},\dot{Q})^c \subseteq (\dot{P},\dot{V})$. But $(\dot{S},\dot{Q})^c$ is $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ closed. Therefore, $\mathcal{N}^{\mathbf{b}}\alpha cl((\dot{S},\dot{Q})^c) \subseteq (\dot{P},\dot{V})$. Also, $(\dot{T},\dot{R})^c \subseteq (\mathcal{N}^{\mathbf{b}}\alpha int(\dot{S},\dot{Q}))^c = \mathcal{N}^{\mathbf{b}}\alpha cl((\dot{S},\dot{Q})^c)$.That implies,
$\mathcal{N}^{\mathbf{b}}\alpha cl((\dot{T},\dot{R})^c) \subseteq \mathcal{N}^{\mathbf{b}}\alpha cl((\dot{S},\dot{Q})^c) \subseteq (\dot{P},\dot{V})$. Therefore, $(\dot{T},\dot{R})^c$ is $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ closed set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. Hence $(\dot{T},\dot{R})$ is $\mathcal{N}^{\mathbf{b}}\text{-}\alpha gs$ open set in $(\dot{\mathcal{U}},\dot{\mathcal{V}},\mathcal{MN})$. □

6. Conclusion

The Neutrosophic Binary αgs closed and open sets were introduced in this article. Also,its relationship with other sets in Neutrosophic Binary Topological Spaces were analyzed. The characteristics of such sets are closely examined and studied with the examples. In future, the decision making problems in real life will be analyzed using the neutrosophic binary sets.

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