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Water Quality Evaluation Using Generalized Correlation Coefficient for M-Polar Neutrosophic Hypersoft Sets

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Abstract: Decision-making is a complex issue, especially for attributes being more than one and further bifurcated. Correlation analysis plays an important role in decision-making problems. For neutrosophic hypersoft sets (NHSSs), they have bifurcated sub-attributes so that we cannot compare the attributive values. Thus, Correlation Coefficient (CC) should be a good tool for decision-making in NHSSs. Moreover, in decision-making problems, most of the times opinion of more than one expert is involved. For dealing this, m-polar values can be better used. The basic purpose of this paper is to propose the concept of CC and Weighted CC (WCC) for m-polar NHSSs with some aggregation operators, theorems, and propositions. Algorithms, based on CC and WCC are also been proposed to solve decision-making problems. Two case studies have been solved by applying the proposed algorithms. The results obtained are compared with existing approaches. The experiment and comparison results reveal the validity and superiority of the proposed methods. They are more accurate and precise. In the future, the proposed methods can be applied to case studies, in which attributes are more than one and further bifurcated along with more than one decision-maker. They can be extended for several existing approaches, like TOPSIS, VIKOR, AHP, and many others.

Keywords: Aggregation operators, Correlation Coefficient (CC); Multi-Criteria Decision-Making (MCDM); Neutrosophic Hypersoft Sets (NHSSs); Weighted Correlation Coefficients (WCC).
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1. Introduction

Fuzzy Set (FS) theory with the concept of membership was proposed by Zadeh [1]. Nowadays, this theory is at its boom that a gadget used for the ease in our life or even the luxury we feel can be based on the FS theory. The FS had been extended to the new types of set structures. For more accuracy, falsity value is considered, and so the FS was extended to an Intuitionistic FS (IFS) by Atanassov [2] that has membership and non-membership values. A generalization of IFS was given by Yager and Abbasov [3] as Pythagorean FS (PFS). These FS, IFS and PFS had various applications, such as [4-6].

For an extension of FS, IFS and PFS for dealing a scientific gadget with truth membership, falsity membership, and indeterminacy membership, Smarandache [7] proposed a new concept of Neutrosophic Set (NS). The NS added an indeterminacy membership and then extended IFS to truth membership, indeterminacy membership, and falsity membership with a triple (T, I, F) component of memberships. This concept is important because indeterminacy exists extraordinarily in application systems. The NS with (T, I, F) memberships was used by a Decision-Maker (DM) and applied to Multi-Criteria Decision-Making (MCDM) problems. More extensions of NS can refer Awang et al. [8].

On the other hand, Molodtsov [9] first proposed Soft Set (SS) in 1999 in which the SS is a mapping from attributes to the power set of a universal set. The SS can be used for handling issues of indefinite circumstances with a parameterized family of the power set of the universal set. Afterwards, Ali et al. [10] and Cagman and Enginoglu [11] offered new operations and applications of soft sets in a decision making. By combining NS with soft set, Maji [12] proposed Neutrosophic Soft Set (NSS). By extending SS so that it is usable in the cases when attributes are more bi-furcated, Smarandache [13] came up with a new set structure known as HyperSoft Set (HSS). Basically, HSS is a mapping from the product of attributes which are further bi-furcated to the power set of universal set. To deal with truthiness, indeterminacy, and falsity, NHSS was considered in Saqlain et al. [14] where they also applied NHSS to TOPSIS using accuracy function. Saqlain et al. [15] gave similarity measures for NHSSs and Jafar *et al.* [16] proposed trigonometric similarity measures for NHSSs with application to renewable energy source selection.

On the other hand, the importance of bipolarity cannot be ignored in various real-life problems. Bipolarity can give positive and negative information for an object. Zhang [17] first considered a bipolar FS (BFS) for handling fuzziness with bipolarity. The BFS assigns each alternative to a positive membership degree and a negative membership degree between 0 and 1. Alghamdi et al. [18] applied BFS in multi-criteria decision-making and Zhang [19] applied BFS to quantum intelligence machinery. Furthermore, Akram et al. [20] considered m-polar FS and used it in decision making where the m-polar FS is an extension of BFS.

The joint connection between two variables may be used to analyze the interdependence of two or more variables. The correlation analysis can be used as a connection measure which is important in statistics and engineering. The correlation coefficient (CC) between random variables is generally used in correlation analysis. The CC for IFSs was first presented by Gerstenkorn and J. Mafiko [21], and then Bustince and Burillo [22] presented CC for the interval-valued IFSs. Ye [23] proposed CC for the single-valued NS (SVNS) along with an algorithm to solve decision-making problems. Samad et al. [24] considered the CC for NHSSs and applied it to the selection of an effective hand sanitizer to reduce covid-19 effects. Saqlain [25] proposed interval-valued, m-polar and m-polar interval-valued neutrosophic hypersoft set, Irfan et al. [26] later developed the similarity measures of m-polar NHSSs (m-p-NHSSs). However, there is no any CC method for m-p-NHSSs. In this paper, we propose the generalized CC for m-p-NHSSs. Thus, we fill the research gap of the CC methods for m-p-NHSSs. We then use the proposed CC to create the algorithms to solve multi-criteria decision-making (MCDM) problems under the m-p-NHSSs environment. In future, this can be used to create a high machine IQ and hybrid intelligent system by combining the m-polar hypersoft set with other soft computing techniques like bipolar fuzzy, Pythagorean set, and other hybrid structures. These techniques can be used in image processing, expert systems, and cognitive maps.

The remainder of the paper is organized as follows. In Section 2, some basic definitions are reviewed to understand the rest of the article i.e. SSs, NSs, NSSs, HSSs, NHSSs, and m-p-NSSs. In Section 3, we establish the generalized CC for m-p-NSSs, and then some examples and their desirable properties will be also considered in detail. We next develop an algorithm based on the generalized CC for m-p-NHSSs to solve decision-making problems in Section 4. In Section 5, by using these algorithms, we will solve the decision-making problem (case studies) to the m-p-NHSSs environment. In Section 6, results, discussion, and comparison will be discussed. Finally, the conclusion along with future directions will be presented in the last section.

2. Preliminary Section

In this section, we review essential concepts: Soft Sets (SSs), Neutrosophic Sets (NSs), Neutrosophic Soft Sets (NSSs), Hypersoft set (HSS), Neutrosophic Hypersoft Set (NHSSs), and m-polar NHSSs (m-p-NHSSs).

Definition 2.1 [9]. Assume $\mathbb{E}$ is a set of parameters, and $\mathbb{U}$ is a universe set. Suppose the power set of $\mathbb{U}$ is denoted by P($\mathbb{U}$), and $\mathbb{A}\subseteq\mathbb{E}$. A Soft Set (SS) over $\mathbb{U}$ is a pair (ζ, $\mathbb{A}$) where $\zeta:\ \mathbb{A}\rightarrow\ \mathbb{P}(\mathbb{U})$ is the mapping of the given set $\mathbb{A}$. To put it another way, the SS (ζ, $\mathbb{A}$) over $\mathbb{U}$ is said to be parameterized subset of $\mathbb{U}$. For $\mathbb{A}$ and $\zeta(\boldsymbol{e})$, the SS of e-approximate or e-elements might be considered (ζ, $\mathbb{A}$), and so (ζ, $\mathbb{A}$) can be given as;

$$(\zeta,\mathbb{A}) = \{\zeta(\boldsymbol{e}) \in \mathbb{P}(\mathbb{U}):\ \boldsymbol{e}\in\mathbb{E}, \zeta(\boldsymbol{e}) = \emptyset \ \text{ if } \ \boldsymbol{e}\neq\mathbb{A}\}$$

Definition 2.2 [12]. Assume that $\mathbb{U}$ is a universe set and a collection of attributes that apply to $\mathbb{U}$ is the set of attributes. Suppose that P($\mathbb{U}$) represents the collection of Neutrosophic values of $\mathbb{U}$. A pair (ζ, $\mathbb{A}$) is said to be a Neutrosophic SS (NSS) over $\mathbb{U}$ where ζ is a mapping with $\zeta:\ \mathbb{A}\rightarrow\mathbb{P}\ (\mathbb{U})$.

Definition 2.3 [13]. Suppose that the universe set and its power set are given as $\mathbb{U}$ and $\mathbb{P}(\mathbb{U})$, respectively. Let $\boldsymbol{\mathcal{K}} = \boldsymbol{\mathcal{K}_1}, \boldsymbol{\mathcal{K}_2}, \dots, \boldsymbol{\mathcal{K}_n}$ for $n \geq 1$ where $\boldsymbol{\mathcal{K}_i}$ specifies the collection of attributes and sub-attributes that are included in them with $\boldsymbol{\mathcal{K}_i} \cap \boldsymbol{\mathcal{K}_j} = \emptyset$, $i \neq j$ for $i, j \in \{1,2,3 \dots n\}$. Let $\boldsymbol{\mathcal{K}_1}\times\boldsymbol{\mathcal{K}_2}\times\dots\times\boldsymbol{\mathcal{K}_n}=\mathbb{A}$. Then, a pair $(\zeta\ , \boldsymbol{\mathcal{K}_1}\times\boldsymbol{\mathcal{K}_2}\times\dots\times\boldsymbol{\mathcal{K}_n})$ is called a hypersoft set (HSS) over $\mathbb{U}$ defined as ζ: $\boldsymbol{\mathcal{K}_1}\times\boldsymbol{\mathcal{K}_2}\times\dots\times\boldsymbol{\mathcal{K}_n}=\mathbb{A}\rightarrow\ \mathbb{P}(\mathbb{U})$. It is also described as $(\zeta,\mathbb{A})=\{(a,\zeta_{\mathbb{A}}(a)):\ a\in\ \mathbb{A},\zeta_{\mathbb{A}}(a)\in\mathbb{P}(\mathbb{U})\}$.

Definition 2.4 [14]. Suppose $\mathbb{U}$ and P($\mathbb{U}$) are a universal set and power set, respectively. Assumed the well define attributes are $\mathbb{L}^1,\mathbb{L}^2,\dots,\mathbb{L}^m$ with corresponding attributive values $\mathbb{I}^1,\mathbb{I}^2,\dots,\mathbb{I}^m for\ \ m\geq 1$ such that $\mathbb{L}^j\cap\ \mathbb{L}^k=\emptyset\ \ for\ \ j\neq k\ \ and\ \ j,k\in\{1,2,\dots,m\}$ and the relation is $\mathbb{L}^1\times\mathbb{L}^2\times\dots\times\mathbb{L}^m=\delta$. The pair of (ζ, δ) is known as a Neutrosophic HSS (NHSS) over $\mathbb{U}$ with $\zeta:\mathbb{L}^1\times\mathbb{L}^2\times\dots\times\mathbb{L}^m\rightarrow$ P($\mathbb{U}$) and $\zeta(\mathbb{L}^1\times\mathbb{L}^2\times\dots\times\mathbb{L}^m)=\{<x,T(\zeta(\delta)),\mathrm{I}(\zeta(\delta)),\mathrm{F}(\zeta(\delta))>, x\in\mathbb{U}\ \}$, where T is the truthiness, I is the indeterminacy, and F is the falsity membership value with T, I, F: $\mathbb{U}\rightarrow[0,1]$ and also $0\leq\ T(\zeta(\delta))+\mathrm{I}(\zeta(\delta))+\mathrm{F}(\zeta(\delta))\leq 3$.

Definition 2.5 [15]. Let $\mathbb{U}$ be a universe set and let $P(\mathbb{U})$ be the power set of $\mathbb{U}$. Let $\mathbb{E}$ be a set of attributes and consider $\mathbb{A} \subseteq \mathbb{E}$. The pair $(\zeta, \mathbb{A})$ is called multi-valued NHSS (MVNHSS) over $\mathbb{U}$ where ζ is a mapping with $\zeta: \mathbb{A} \rightarrow \mathbb{P}(\mathbb{U})$ and $(\zeta, \mathbb{A}) = \left\{\frac{\langle \mathbb{T}^x(\zeta(\mathbb{A})), \mathbb{I}^y(\zeta(\mathbb{A})), F^z(\zeta(\mathbb{A}))\rangle}{u}, \mathcal{U} \in \mathrm{U}\right\}$, where $\mathbb{T}^x(\zeta(\mathbb{A})) \subseteq [0,1]$, $\mathbb{I}^y(\zeta(\mathbb{A})) \subseteq [0,1]$ and $F^z(\zeta(\mathbb{A})) \subseteq [0,1]$ are the multi-valued numbers and they are given as

$$\mathbb{T}^x(\zeta(\mathbb{A})) = \mathbb{T}^1(\zeta(\mathbb{A})), \mathbb{T}^2(\zeta(\mathbb{A})), \ldots, \mathbb{T}^x(\zeta(\mathbb{A}))$$
$$\mathbb{I}^y(\zeta(\mathbb{A})) = \mathbb{I}^1(\zeta(\mathbb{A})), \mathbb{I}^2(\zeta(\mathbb{A})), \ldots, \mathbb{I}^y(\zeta(\mathbb{A}))$$
$$F^z((\mathbb{A})) = F^1(\zeta(\mathbb{A})), F^2(\zeta(\mathbb{A})), \ldots, F^z(\zeta(\mathbb{A}))$$

$\mathbb{T}(\zeta(\mathbb{A})), \mathbb{I}(\zeta(\mathbb{A}))$, and $\mathrm{F}(\zeta(\mathbb{A}))$ represent the truthiness, indeterminacy and falsity of $\mathcal{U}$ to $\mathbb{A}$, respectively.

Definition 2.6 [25]. Let $\mathbb{U} = \{u_1, u_2, \ldots, u_n\}$ be a universe set and $\mathbb{P}(\mathbb{U})$ be the power set of $\mathbb{U}$. Let $\mathbb{L}_1, \mathbb{L}_2, \ldots, \mathbb{L}_b$ for b≥1 be b well-defined attributes whose corresponding attribute values are $\mathbb{L}_1^1, \mathbb{L}_2^2, \ldots, \mathbb{L}_b^n$, respectively, and their relation is $\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \times \mathbb{L}_b^z$ where $a,b,c,\ldots,z = 1,2,\ldots,n$. Then, the pair $(\zeta, \mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \times \mathbb{L}_b^z)$ is called to be a m-polar PHSS (m-p-NHSS) over $\mathbb{U}$ where ζ is a mapping with $\zeta: \mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \times \mathbb{L}_b^z \rightarrow \mathbb{P}(\mathbb{U})$; $\zeta(\mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \times \mathbb{L}_b^z) = \{< u, \mathbb{T}_l^i(u), \mathbb{I}_l^j(u), \mathrm{F}_l^k(u) >: u \in \mathbb{U}; \ell \in \mathbb{L}_1^a \times \mathbb{L}_2^b \times \ldots \times \mathbb{L}_b^z$ where $\mathrm{i,j,k} = 1,2,\ldots,\mathrm{n}\}$ and $0 \leq \sum_{i=1}^{p} \mathbb{T}_l^i(u) \leq 1$, $0 \leq \sum_{j=1}^{q} \mathbb{I}_l^j(u) \leq 1$, $0 \leq \sum_{k=1}^{r} \zeta_l^k(u) \leq 1$, where $\mathbb{T}_l^i(u) \subseteq [0,1], \mathbb{I}_l^j(u) \subseteq [0,1]$, and $\zeta_l^k(u) \subseteq [0,1]$ are the numbers with $0 \leq \sum_{i=1}^{p} \mathbb{T}_l^i(u) + \sum_{j=1}^{q} \mathbb{I}_l^j(u) + \sum_{k=1}^{r} \mathrm{F}_l^k(u) \leq 3$.

For convenience, we assume that

$$\mathbb{T}_l^i(u) = \mathbb{T}_{l1}^1(u), \mathbb{T}_{l2}^2(u), \mathbb{T}_{l3}^3(u), \ldots, \mathbb{T}_{lp}^p(u)$$
$$\mathbb{I}_l^j(u) = \mathbb{I}_{l1}^1(\mathrm{u}), \mathbb{I}_{l2}^2(\mathrm{u}), \mathbb{I}_{l3}^3(\mathrm{u}), \ldots, \mathbb{I}_{lq}^q(\mathrm{u})$$
$$F_l^k(u) = \mathrm{F}_{l1}^1(u), \mathrm{F}_{l2}^2(u), \mathrm{F}_{l3}^3(u), \ldots, \mathrm{F}_{lr}^r(u)$$

3. Calculations

In this section, we propose informational energies, generalized CC and aggregation operators for m-polar NHSSs (m-p-NHSSs).

Definition 3.1. Informational energies for m-p-NHSSs

Let $(\wp, \ddot{A})$ *and* $((Q, \ddot{B}))$ be two m-p-NHSSs with

$$(\wp, \ddot{A}) = \left\{\left(v_i, \tau_{\wp(\check{d}_k)}(v_i)^i, \mathfrak{I}_{\wp(\check{d}_k)}(v_i)^j, \mathfrak{G}_{\wp(\check{d}_k)}(v_i)^k\right) \middle| v_i \in u\right\}$$

$$(Q, \ddot{B}) = \left\{\left(v_i, \tau_{Q(\check{d}_k)}(v_i)^i, \mathfrak{I}_{Q(\check{d}_k)}(v_i)^j, \mathfrak{G}_{Q(\check{d}_k)}(v_i)^k\right) \middle| v_i \in u\right\}.$$

Then, their informational energies are defined as

$$\varsigma_{m-p-NHSS}(\wp, \ddot{A}) = \sum_{k=1}^{m} \sum_{i=1}^{n} \left(\sum_{i=1}^{p} \left(\tau_{\wp(\check{d}_k)i}^{i}(v_i)\right)^2 + \sum_{j=1}^{q} \left(\mathfrak{I}_{\wp(\check{d}_k)j}^{j}(v_i)\right)^2 + \sum_{k=1}^{r} \left(\mathfrak{G}_{\wp(\check{d}_k)k}^{k}(v_i)\right)^2 \right) \quad (3.1)$$

$$\varsigma_{m-p-NHSS}(Q,\ddot{B}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\tau^{i}_{Q(\breve{d}_k)i}\ (v_i)\right)^2 + \sum_{j=1}^{q}\left(\mathfrak{J}^{j}_{Q(\breve{d}_k)j}\ (v_i)\right)^2 + \sum_{k=1}^{r}\left(\mathfrak{G}^{k}_{Q(\breve{d}_k)k}\ (v_i)\right)^2\right) \quad (3.2)$$

Definition 3.2. Covariance for two m-p-NHSSs

Let $(\wp,\ddot{A})$ ***and*** $((Q,\ddot{B}))$ be two m-p-NHSSs with

$$(\wp,\ddot{A}) = \left\{\left(v_i,\tau_{\wp(\breve{d}_k)}(v_i)^i,\mathfrak{J}_{\wp(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{\wp(\breve{d}_k)}(v_i)^k\right)\middle| v_i \in u\right\}$$

$$(Q,\ddot{B}) = \left\{\left(v_i,\tau_{Q(\breve{d}_k)}(v_i)^i,\mathfrak{J}_{Q(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{Q(\breve{d}_k)}(v_i)^k\right)\middle| v_i \in u\right\}.$$

Then, the covariance between $(\wp,\ddot{A})$ ***and*** $((Q,\ddot{B}))$ is defined as

$$C_{m-p-NHSS}\left((\wp,\ddot{A}),(Q,\ddot{B})\right) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\tau^{i}_{\wp(\breve{d}_k)i}\ (v_i) * \mathfrak{J}^{j}_{Q(\breve{d}_k)j}\ (v_i)\right) + \sum_{j=1}^{q}\left(\mathfrak{J}^{j}_{\wp(\breve{d}_k)j}\ (v_i) * \mathfrak{J}^{j}_{Q(\breve{d}_k)j}\ (v_i)\right) + \sum_{k=1}^{r}\left(\mathfrak{G}^{k}_{\wp(\breve{d}_k)k}\ (v_i) * \mathfrak{G}^{k}_{Q(\breve{d}_k)k}\ (v_i)\right)\right) \quad (3.3)$$

Definition 3.3. Correlation coefficient for two m-p-NHSSs

Let $(\wp,\ddot{\mathrm{A}})$ **and** $((Q,\ddot{\mathrm{B}}))$ be two m-p-NHSSs with

$$(\wp,\ddot{\mathrm{A}}) = \left\{\left(\mathrm{v_i},\tau_{\wp(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{i}},\mathfrak{J}_{\wp(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{j}},\mathfrak{G}_{\wp(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{k}}\right)\middle|\mathrm{v_i} \in \mathrm{u}\right\}$$

$(\mathrm{Q},\ddot{\mathrm{B}}) = \left\{\left(\mathrm{v_i},\tau_{Q(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{i}},\mathfrak{J}_{Q(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{j}},\mathfrak{G}_{Q(\mathrm{d_k})}(\mathrm{v_i})^{\mathrm{k}}\right)\middle|\mathrm{v_i} \in \mathrm{u}\right\}$. Then, CC between them is defined as

$$\delta_{m-p-NHSS}((\wp,A^{\cdots}),(Q,B^{\cdots})) = \frac{c_{m-p-NHSS}\ ((\wp,A^{\cdots}),(Q,B^{\cdots}))}{\sqrt{\varsigma_{m-p-NHSS}((\wp,\ddot{A})} * \sqrt{\varsigma_{m-p-NHSS}((Q,\ddot{B})}} \quad (3.4)$$

Example 3.4. Let $U = \{s^1,s^2,s^3,s^4,s^5\}$ be the set of nominated schools and consider the set of attributes with $E = \{\textit{teaching standard},\textit{organization},\textit{ongoing evaluation},\textit{goals}\}$ Let $A \subseteq E$ with $A = \{A_1,A_2,A_3,A_4\}$ such that $A_1 = \textbf{Teaching standard}$, $A_2 = \textbf{Organization}$, $A_3 = \textit{ongoing evaluation}, A_4 =$ **goals**. These attributes are further bifurcated as $A_1{}^a \rightarrow A_1 \rightarrow$ teaching standard → (High, mediocre, low); $A_2{}^b \rightarrow A_2 \rightarrow$ organization → (good, average, poor); $A_3{}^c \rightarrow A_3 \rightarrow$ ongoing evaluation → (yes, no); $A_4{}^d \rightarrow A_4 \rightarrow$ Goals → (effective, committed, up to date). Define a mapping with **F**(high, average, yes, effective)= $\{s^1,s^5\}$. Then, $(\wp,\ddot{A}) =$

$$\left\{\begin{pmatrix} s^1 < A_1^a\{(0.4,0.3,0.2),(0.3,0.2,0.4),(0.1,0.3,0.5)\},A_2^b\{(0.5,0.4,0.2),(0.4,0.3,0.5),(0.1,0.4,0.5)\}, \\ A_3^c\{(0.6,0.2,0.1),(0.3,0.4,0.2),(0.1,0.3,0.4)\},A_4^d\{(0.2,0.3,0.1),(0.4,0.1,0.2),(0.1,0.2,0.4)\} > \\ s^5 < A_1^a\{(0.3,0.2,0.4),(0.4,0.2,0.3),(0.1,0.3,0.2)\},A_2^b\{(0.4,0.3,0.2),(0.4,0.1,0.5),(0.1,0.3,0.5)\}, \\ A_3^c\{(0.6,0.3,0.1),(0.3,0.1,0.2),(0.1,0.2,0.4)\},A_4^d\{(0.3,0.2,0.1),(0.4,0.3,0.2),(0.1,0.3,0.4)\} > \end{pmatrix}\right\}$$

Also, $B \subseteq E$, $B = \{B_1,B_2,B_3,B_4\}$. Further, bi-furcated attributes of "B" are $B_1{}^a \rightarrow B_1 \rightarrow$ teaching standard → (High, mediocre, low); $B_2{}^b \rightarrow B_2 \rightarrow$ organization → (good, average, poor); $B_3{}^c \rightarrow B_3 \rightarrow$ ongoing evaluation → (yes, no); $B_4{}^d \rightarrow B_4 \rightarrow$ Goals → (effective, committed, up-to-date). Consider another mapping **G** (high, good, yes, up-to-date)= $\{s^2, s^3\}$. Then, we have

$(Q,\ddot{B}) =$

$$\left\{\begin{pmatrix} s^2 < B_1^a\{(0.5,0.3,0.2),(0.4,0.2,0.1),(0.1,0.3,0.4)\}, B_2^b\{(0.5,0.3,0.2),(0.4,0.3,0.2),(0.1,0.3,0.5)\}, \\ B_3^c\{(0.6,0.1,0.2),(0.3,0.4,0.1),(0.1,0.3,0.2)\}, B_4^d\{(0.4,0.3,0.1),(0.4,0.3,0.1),(0.1,0.2,0.5)\} > \\ s^3 < B_1^a\{(0.4,0.2,0.3),(0.4,0.2,0.3),(0.1,0.3,0.4)\}, B_2^b\{(0.4,0.1,0.2),(0.4,0.1,0.3),(0.1,0.3,0.4)\}, \\ B_3^c\{(0.6,0.2,0.1),(0.3,0.4,0.2),(0.1,0.2,0.5)\}, B_4^d\{(0.3,0.2,0.4),(0.4,0.1,0.2),(0.1,0.2,0.4)\} > \end{pmatrix}\right\}$$

Thus, we have $\boldsymbol{\delta_{m-p-NHSS}}(\wp,\ddot{A}) = 7.26$; $\boldsymbol{\delta_{m-p-NHSS}}(Q,\ddot{B}) = 6.78$, and $\boldsymbol{\delta_{m-p-NHSS}}\left((\wp,\ddot{A})(Q,\ddot{B})\right) = \frac{6.54}{\sqrt{7.26}*\sqrt{6.78}} = \mathbf{0.93} \in [\mathbf{0},\mathbf{1}]$.It shows that $(\wp,\ddot{A})$ and $(Q,\ddot{B})$ have a good positive relation.

Proposition 3.5. Let $(\wp,\ddot{A}) = \left\{\left(v_i, \tau_{\wp(\tilde{d}_k)}(v_i)^i, \mathfrak{I}_{\wp(\tilde{d}_k)}(v_i)^j, \mathfrak{G}_{\wp(\tilde{d}_k)}(v_i)^k\right) \middle| v_i \in u\right\}$ and $(Q,\ddot{B}) = \left\{\left(v_i, \tau_{Q(\tilde{d}_k)}(v_i)^i, \mathfrak{I}_{Q(\tilde{d}_k)}(v_i)^j, \mathfrak{G}_{Q(\tilde{d}_k)}(v_i)^k\right) \middle| v_i \in u\right\}$ be two m-p-NHSSs and let $\mathcal{C}_{m-p-NHSS}((\wp,\ddot{A}),(Q,\ddot{B}))$ be a CC between them. It satisfies the following properties:

1. $\mathcal{C}_{m-p-NHSS}(\wp,\ddot{A}),(\wp,\ddot{A}) = \varsigma_{m-p-NHSS}(\wp,\ddot{A})$.
2. $\mathcal{C}_{m-p-NHSS}(Q,\ddot{B}),(Q,\ddot{B}) = \varsigma_{m-p-NHSS}(Q,\ddot{B}))$.

Theorem 3.6. Let $(\wp,\ddot{A}) = \{(v_i, T_{\wp(d_k)}(v_i)^i, I_{\wp(d_k)}(v_i)^j, C_{\wp(d_k)}(v_i)^k) \mid v_i \in U\}$ and

$(\mathbf{Q},\ddot{\mathbf{B}}) = \{(\mathbf{v_i}, \mathbf{T_{Q(d_k)}}(\mathbf{v_i})^{\mathbf{i}}, \mathbf{I_{Q(d_k)}}(\mathbf{v_i})^{\mathbf{j}}, \mathbf{C_{Q(d_k)}}(\mathbf{v_i})^{\mathbf{k}}) \mid \mathbf{v_i} \in \mathbf{U}\}$ be two m-p-NHSSs, then CC between them satisfies the following properties:

$0 \leq \boldsymbol{\delta_{m-p-NHSS}}\left((\wp,\ddot{A}),(Q,\ddot{B})\right) \leq 1$

$\boldsymbol{\delta_{m-p-NHSS}}\left((\wp,\ddot{A}),(Q,\ddot{B})\right) = \boldsymbol{\delta_{m-p-NHSS}}\left((\wp,\ddot{A}),(Q,\ddot{B})\right)$

iff $\left((\wp,\ddot{A}) = (Q,\ddot{B})\right)$.

If $T_{\wp(d_k)}(v_i)^i = T_{Q(d_k)}(v_i)^i$, $I_{\wp(d_k)}(v_i)^j = I_{Q(d_k)}(v_i)^j$, and

$C_{\wp(d_k)}(v_i)^k = C_{Q(d_k)}(v_i)^k$, then $\boldsymbol{\delta_{m-p-NHSS}}((\wp,\ddot{A}), (Q,\ddot{B})) = 1$.

Whenever experts regulate distinctive weights for every alternative, the choice might be dissimilar. So, it is precisely to plot the weights for experts preceding assembling a decision. Assume the weights of experts can be expressed as $\Omega = \{\Omega_1, \Omega_2, \Omega_3, \ldots, \Omega_m\}^T$, where $\Omega_k > 0$, $\sum_{k=1}^m \Omega_k = \mathbf{1}$. Similarly, assume that the weights for sub-attributes are as follows $\gamma = \{\gamma_1, \gamma_2, \gamma_3, \ldots, \gamma_n\}^T$, where $\gamma_i > 0$, $\sum_{i=1}^n \gamma_i = \mathbf{1}$.

Definition 3.7. Weighted CC for two m-p-NHSSs

Let $(\wp,\ddot{A}) = \{(v_i, T_{\wp(d_k)}(v_i)^i, I_{\wp(d_k)}(v_i)^j, C_{\wp(d_k)}(v_i)^k) \mid v_i \in U\}$ and $(Q,B^{\cdot\cdot}) = \{(v_i, T_{Q(d_k)}(v_i)^i, I_{Q(d_k)}(v_i)^j, C_{Q(d_k)}(v_i)^k) \mid v_i \in U\}$ be two m-p-NHSSs, then, a weighted CC (WCC) among them is expressed as $\boldsymbol{\delta_{m-p-wNHSS}}((\wp,\ddot{A}),(Q,\ddot{B}))$ and defined as follows:

$$\boldsymbol{\delta_{m-p-wNHSS}}((\wp,\ddot{A}),(Q,\ddot{B})) = \frac{\mathcal{C}_{m-p-wNHSS}((\wp,A^{\cdot\cdot}),(Q,B^{\cdot\cdot}))}{\sqrt{\varsigma_{m-p-wNHSS}(\wp,A^{\cdot\cdot})} * \sqrt{\varsigma_{m-p-wNHSS}(Q,B^{\cdot\cdot})}} \quad (3.5)$$ i.e.

$$\delta_{m-p-wNHSS}\left((\wp,\ddot{A}),(Q,\ddot{B})\right)=$$

$$\frac{\sum_{k=1}^{m}\Omega_k\left(\left(\left(\begin{array}{c}\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\mathcal{T}^{i}_{\wp(\breve{d}_k)i}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\sum_{i=1}^{n}\gamma_i\mathcal{T}^{i}_{Q(\breve{d}_k)i}(v_i)\right)+\\ \sum_{j=1}^{q}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\mathcal{I}^{j}_{\wp(\breve{d}_k)j}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\ \mathcal{I}^{j}_{Q(\breve{d}_k)j}(v_i)\right)+\sum_{k=1}^{r}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\ \mathfrak{C}^{k}_{\wp(\breve{d}_k)k}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\ \mathfrak{C}^{k}_{Q(\breve{d}_k)k}(v_i)\right)\end{array}\right)\right)\right)}{\sqrt{\sum_{k=1}^{m}\Omega_k\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\gamma_i\mathcal{T}^{i}_{\wp(\breve{d}_k)i}(v_i)\right)^2+\sum_{j=1}^{q}\left(\sum_{i=1}^{n}\gamma_i\mathcal{I}^{j}_{\wp(\breve{d}_k)j}(v_i)\right)^2+\sum_{k=1}^{r}\left(\sum_{i=1}^{n}\gamma_i\mathfrak{C}^{k}_{\wp(\breve{d}_k)k}(v_i)\right)^2\right)\right)}\sqrt{\sum_{k=1}^{m}\Omega_k\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\gamma_i\mathcal{T}^{i}_{Q(\breve{d}_k)i}(v_i)\right)^2+\sum_{j=1}^{q}\left(\sum_{i=1}^{n}\gamma_i\mathcal{I}^{j}_{Q(\breve{d}_k)j}(v_i)\right)^2+\sum_{k=1}^{r}\left(\sum_{i=1}^{n}\gamma_i\mathfrak{C}^{k}_{\wp(\breve{d}_k)k}(v_i)\right)^2\right)\right)}}$$

Theorem 3.8. Let $(\wp,\ddot{A})=\left\{\left(v_i,\tau_{\wp(\breve{d}_k)}(v_i)^i,\mathfrak{I}_{\wp(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{\wp(\breve{d}_k)}(v_i)^k\right)\middle|v_i\in u\right\}$ and

$(Q,\ddot{B})=\left\{\left(v_i,\tau_{Q(\breve{d}_k)}(v_i)^i,\mathfrak{I}_{Q(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{Q(\breve{d}_k)}(v_i)^k\right)\middle|v_i\in u\right\}$ be two m-p-NHSSs, then WCC between them satisfies the following properties:

$0\ \leq\ \delta_{m-p-wNHSS}((\wp,\ddot{A})(Q,\ddot{B}))\ \leq 1$

$$\delta_{m-p-wNHSS}\left((\wp,\ddot{A})(Q,\ddot{B})\right)=\ \delta_{m-p-wNHSS}((Q,\ddot{B}),(\wp,\ddot{A}))\ \ iff\ \ (\wp,\ddot{A})=(Q,\ddot{B})$$

$T_{\wp(d_k)}(v_i)^i=T_{Q(d_k)}(v_i)$, $I_{\wp(d_k)}(v_i)^j=\ I_{Q(d_k)}(v_i)^j$ and $C_{\wp(d_k)}(v_i)^k=\ C_{Q(d_k)}(v_i)^k$
then $\delta_{m-p-wNHSS}\left((\wp,\ddot{A}),(Q,\ddot{B})\right)=1$

Proposition 3.9. Let $(\wp,\ddot{A})=\left\{\left(v_i,\tau_{\wp(\breve{d}_k)}(v_i)^i,\mathfrak{I}_{\wp(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{\wp(\breve{d}_k)}(v_i)^k\right)\middle|v_i\in u\right\}$

Consider $J_{d_k}=\ \langle {T_{F(d_{ij})}}^i,{I_{F(d_{ij})}}^j,{C_{F(d_{ij})}}^k\rangle$, $J_{d_{11}}=\ \langle {T_{F(d_{11})}}^i,{I_{F(d_{11})}}^j,{C_{F(d_{11})}}^k\rangle$ and $J_{d_{12}}=$ $\langle {T_{F(d_{11})}}^i,{I_{F(d_{12})}}^j,{C_{F(d_{12})}}^k\rangle$ as three m-p-NHSSs and α be a positive real number, by algebraic norms, then

$${J_{d_{11}}}^i\oplus {J_{\breve{d}_{12}}}^i=\langle {T_{F(d_{11})}}^i+{T_{F(d_{12})}}^i-{T_{F(d_{11})}}^i{T_{F(d_{12})}}^i,{J_{F(d_{11})}}^j{J_{F(d_{12})}}^j,\qquad {C_{F(d_{11})}}^k{C_{F(d_{12})}}^k\rangle$$

${J_{d_{11}}}^i\otimes {J_{\breve{d}_{12}}}^i=\langle {T_{F(d_{11})}}^i{T_{F(d_{12})}}^i,{J_{F(d_{11})}}^j+{J_{F(d_{12})}}^j-{J_{F(d_{11})}}^j{J_{F(d_{12})}}^j,\ {C_{F(d_{11})}}^k+{C_{F(d_{12})}}^k-$ ${C_{F(d_{11})}}^k{C_{F(d_{12})}}^k\rangle$

$\alpha J_{d_k}=\langle 1-\left(1-{T_{d_k}}^i\right)^\alpha,{J_{d_k}}^{j^\alpha},{C_{d_k}}^{k^\alpha}\rangle$

$$J\ \ {}_{d_k}^{i^\alpha}=\langle,1-\left(1-{J_{d_k}}^j\right)^\alpha,1-\left(1-{C_{d_k}}^k\right)^\alpha\rangle$$

Definition 3.10. Aggregate operator for m-p-NHSSs

Let $(\wp,\ddot{A})=\left\{\left(v_i,\tau_{\wp(\breve{d}_k)}(v_i)^i,\mathfrak{I}_{\wp(\breve{d}_k)}(v_i)^j,\mathfrak{G}_{\wp(\breve{d}_k)}(v_i)^k\right)\middle|v_i\in u\right\}$ and $J_{d_k}=$ $\langle {T_{F(d_{ij})}}^i,{I_{F(d_{ij})}}^j,{C_{F(d_{ij})}}^k\rangle$ be an m-p-NHSS. Ω_i and γ_j are weight vector for expert's and sub-attributes of the considered attributes correspondingly along with specified circumstances $\Omega_i>0$, $\sum_{i=1}^{n}\Omega_i=1$, $\gamma_j>0$, $\sum_{j=1}^{m}\gamma_j=1$. Then m-p-NHSS aggregate operator is defined as $\mathbf{m-PNHSWA}$:

$\Delta^{n} \rightarrow \Delta$ where $\left(\mathfrak{J}_{\breve{d}_{11}}, \mathfrak{J}_{\breve{d}_{12}}, \ldots, \mathfrak{J}_{\breve{d}_{nm}}\right) = \oplus^{m}_{j=1} \boldsymbol{\gamma}_j \left(\oplus_{i=1}^{n} \Omega_i \mathfrak{J}_{\breve{d}_{ij}}\right) = \left\langle \mathbf{1} - \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\mathbf{1} - \mathcal{T}_{\breve{d}_{ij}}{}^{i}\right)^{\Omega_i}\right)^{\gamma_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\mathcal{J}_{\breve{d}_{ij}}{}^{j}\right)^{\Omega_i}\right)^{\gamma_j}, \prod_{j=1}^{m}\left(\prod_{i=1}^{n}\left(\mathfrak{C}_{\breve{d}_{ij}}{}^{k}\right)^{\Omega_i}\right)^{\gamma_j}\right\rangle$ (3.6)

4. Proposed Algorithms

In this section, we develop the algorithm based on Correlation Coefficient (CC) and Weighted Correlation Coefficient (WCC) under m-PNHSs and utilize the proposed approach for decision making in real life problems.

Algorithm 4.1.

The proposed algorithm 4.1, can be used solve MCDM problems based on CC of m-PNHSs and shown in Figure 1.

Step 1: Select Hypersoft sets $(\wp, \ddot{A})$ and $(\boldsymbol{Q}, \ddot{\boldsymbol{B}})$

Step 2: Construction of m-PNHSs by assigning m-PNHSN to each sub-attribute and solve them to get SVNHSs.

Step 3: Find the informational energies of the selected m-PNHSs using the formula;

$$\varsigma_{m-PNHSs}(\wp, \ddot{\boldsymbol{A}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\mathcal{T}^{i}_{\wp(\breve{d}_k)i}(\boldsymbol{v}_i)\right)^2 + \sum_{j=1}^{q}\left(\mathcal{J}^{j}_{\wp(\breve{d}_k)j}(\boldsymbol{v}_i)\right)^2 + \sum_{k=1}^{r}\left(\mathfrak{C}^{k}_{\wp(\breve{d}_k)k}(\boldsymbol{v}_i)\right)^2\right)$$

Step 4: Calculate the correlation between the selected m-PNHS sets $(\wp, \ddot{A})$ and $(\boldsymbol{Q}, \ddot{\boldsymbol{B}})$ by using the formula;

$$\mathcal{C}_{m-PNHSs}\ (\wp, \ddot{\boldsymbol{A}}), (\boldsymbol{Q}, \ddot{\boldsymbol{B}}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\mathcal{T}^{i}_{\wp(\breve{d}_k)i}(\boldsymbol{v}_i) * \mathcal{T}^{i}_{\mathcal{Q}(\breve{d}_k)i}(\boldsymbol{v}_i)\right) + \sum_{j=1}^{q}\left(\mathcal{J}^{j}_{\wp(\breve{d}_k)j}(\boldsymbol{v}_i) * \mathcal{J}^{j}_{\mathcal{Q}(\breve{d}_k)j}(\boldsymbol{v}_i)\right) + \sum_{k=1}^{r}\left(\mathfrak{C}^{k}_{\wp(\breve{d}_k)k}(\boldsymbol{v}_i) * \mathfrak{C}^{k}_{\mathcal{Q}(\breve{d}_k)k}(\boldsymbol{v}_i)\right)\right)$$

Step 5: Calculate correlation coefficients of the selected m-PNHS sets $(\wp, \ddot{\boldsymbol{A}})$ and $(\wp, \ddot{\boldsymbol{B}})$ by using the formula;

$$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\wp, \ddot{\boldsymbol{A}}), (\boldsymbol{Q}, \ddot{\boldsymbol{B}}) = \frac{\mathcal{C}_{\mathbf{m-PNHSs}}((\wp, \ddot{\boldsymbol{A}}), (\boldsymbol{Q}, \ddot{\boldsymbol{B}}))}{\sqrt{\varsigma_{\mathbf{m-PNHSs}}(\wp, \ddot{A})} * \sqrt{\varsigma_{\mathbf{m-PNHSs}}(\mathbf{Q}, \ddot{\boldsymbol{B}})}}$$

Step 6: Arrange the alternatives in descending order of the CC values.

Step 7: Rank the alternatives from largest to smallest CC values.

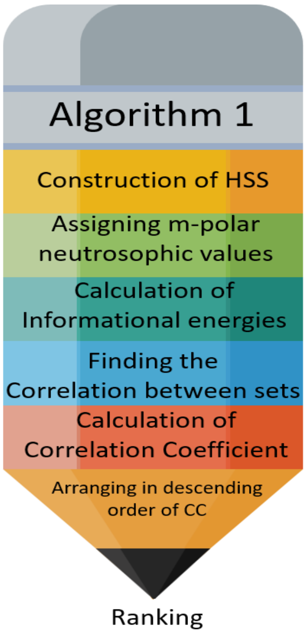

Figure 1. Algorithm based on Correlation Coefficient form-PNHSs

Algorithm 4.2.

The proposed algorithm 4.2, can be used solve MCDM problems based on WCC of m-PNHSs and shown in Figure 2.

Step 1: Construction of Hypersoft set and sub-attribute parameters.

Step 2: Assigning m-PNHSNs to the selected sets.

Step 3: Find the weighted informational energies for m-PNHSs using the formula;

$$\varsigma_{m-PWNHSs}(\wp,\ddot{A}) = \sum_{k=1}^{m}\Omega_k\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\gamma_i\,\mathcal{T}^{i}_{\wp(\breve{a}_k)_i}(v_i)\right)^2 + \sum_{j=1}^{q}\left(\sum_{i=1}^{n}\gamma_i\,\mathcal{I}^{j}_{\wp(\breve{a}_k)_j}(v_i)\right)^2 + \sum_{k=1}^{r}\left(\sum_{i=1}^{n}\gamma_i\,\mathfrak{C}^{k}_{\wp(\breve{a}_k)k}(v_i)\right)^2\right)\right)$$

Step 4. Calculate the Weighted Correlation between two m-PNHSs by using the formula;

$$\mathcal{C}_{m-PWNHSs}((\wp,\ddot{A}),(\boldsymbol{Q},\ddot{\boldsymbol{B}})) =$$

$$\sum_{k=1}^{m} \Omega_k \left(\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{T}^{i}_{\wp(\breve{a}_k)i}(v_i) * \sum_{i=1}^{n}\sqrt{\gamma_i}\sum_{i=1}^{n}\gamma_i\,\mathcal{T}^{i}_{\mathcal{Q}(\breve{a}_k)i}(v_i)\right) + \sum_{j=1}^{q}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{I}^{j}_{\wp(\breve{a}_k)j}(v_i) * \sum_{i=1}^{n}\sqrt{\gamma_i}\;\;\mathcal{I}^{j}_{\mathcal{Q}(\breve{a}_k)j}(v_i)\right) + \sum_{k=1}^{r}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\;\;\mathfrak{C}^{k}_{\wp(\breve{a}_k)k}(v_i) * \sum_{i=1}^{n}\sqrt{\gamma_i}\;\;\mathfrak{C}^{k}_{\mathcal{Q}(\breve{a}_k)k}(v_i)\right)\right)\right)\right)$$

Step 5: Calculate the WCC between two m-PNHSs by using the formula;

$$\boldsymbol{\delta}_{m-PWNHSs}((\wp,\ddot{A}),(\boldsymbol{Q},\ddot{\boldsymbol{B}}))= \frac{\mathcal{C}_{m-PWNHSs}((\wp,\ddot{A}),(\mathbf{Q},\ddot{\boldsymbol{B}}))}{\sqrt{\varsigma_{m-PWNHSs}(\boldsymbol{Q},\ddot{A})} * \sqrt{\varsigma_{m-PWNHSs}(\boldsymbol{Q},\ddot{\boldsymbol{B}})}}$$

Step 6: Arrange the alternative in descending order of WCC and rank the alternative from highest to the lowest.

5. Experiment

Lahore Garrison University (LGU) wanted to hire a teacher in Mathematics department, let $\mathbb{P} = \{\mathbb{P}^{1}, \mathbb{P}^{2}, \mathbb{P}^{3}, \mathbb{P}^{4}\}$ be a set of candidates (alternatives) who has been shortlisted for the interview. The Interview committee consists of four decision-makers (DM), $\boldsymbol{\mathcal{D}} = \{\boldsymbol{\sigma_1}, \boldsymbol{\sigma_2}, \boldsymbol{\sigma_3}, \boldsymbol{\sigma_4}\}$. The committee will decide the criteria (attributes) to fill up the said post which are $\boldsymbol{\mathcal{L}} = \{\boldsymbol{\ell_1} = \boldsymbol{Experience}, \boldsymbol{\ell_2} = \boldsymbol{Dealing\ \ skills}, \boldsymbol{\ell_3} = \boldsymbol{Qualification}\}$ be the set of attributes and their corresponding sub-attributes are given by;

$$\boldsymbol{\ell_1} = \boldsymbol{Experience} = \{\boldsymbol{a_{11}} = \boldsymbol{less\ \ than\ \ 20}, \boldsymbol{a_{12}} = \boldsymbol{more\ \ than\ \ 20}\ \}$$

Figure 2. Algorithm based on Weighted Correlation Coefficient for m-PNHSs

$$\boldsymbol{\ell_2 = Dealing\ \ skills} = \left\{ \begin{matrix} \boldsymbol{a_{21} = Good\ \ Communication\ \ skills,} \\ \boldsymbol{a_{22} = Good\ \ Teaching\ \ skills,} \\ \boldsymbol{a_{23} = Certified\ \ skills} \end{matrix} \right\},$$

$$\boldsymbol{\ell_3 = Qualifaication} = \ \{\boldsymbol{a_{31} = M.Phil., a_{32} = PhD. a_{33} = Post\ \ Doctorate}\}$$

Solved example using Algorithm 4.1

Assume case study formulated above. The DM will assign values in term of m-PNHS numbers, based on their knowledge and expertise to each candidate.

Step 1: Define a mapping, and select Hypersoft set.

$$\boldsymbol{F}:\ \boldsymbol{\ell_1} \times\ \ \boldsymbol{\ell_2} \times \boldsymbol{\ell_3} = \boldsymbol{\mathcal{L}'} \rightarrow \boldsymbol{P}(\beth) = \mathbb{P}^1, \mathbb{P}^2$$

Step 2: Assigning values to the selected Hypersoft set in term of m-polar Neutrosophic number by considering m=3 as presented in Table 1-10.

Table 1. Neutrosophic m-polar values for alternative ℵ

ℵ	δ_1	δ_2	δ_3	δ_4

$\check{a}_1$	(<0.2,0.1,0.2>,<0.1,0.2, 0.1>, <0.3,0.2,0.1>)	(<0.2,0.2,0.2>,<0.1,0.0, 0.2>, <0.2,0.3,0.1>)	(<0.2,0.1,0.1>,<0.2,0.2, 0.2>, <0.2,0.3,0.1>)	(<0.1,0.2,0.0>,<0.3,0.2, 0.2>, <0.1,0.1,0.1>)
$\check{a}_2$	(<0.1,0.0,0.2>,<0.4,0.2, 0.1 ,<0.1,0.2,0.0>)	(<0.3,0.4,0.0>,<0.2,0.0, 0.1>,<0.2,0.1,0.3>)	(<0.2,0.1,0.1>,<0.1,0.3, 0.1>,<0.1,0.2,0.2>)	(<0.1,0.2,0.1>,<0.2,0.1, 0.2>,<0.0,0.3,0.4>)
$\check{a}_3$	(<0.3,0.1,0.2>,<0.1,0.0, 0.2>,<0.2,0.1,0.1>)	(<0.2,0.1,0.1>,<0.1,0.0, 0.3>,<0.2,0.1,0.2>)	(<0.1,0.2,0.3>,<0.1,0.2, 0.0>,<0.1,0.3,0.0>)	(<0.1,0.2,0.1>,<0.2,0.1, 0.2>,<0.2,0.1,0.3>)
$\check{a}_4$	(<0.1,0.2,0.1>,<0.2,0.1, 0.2>,<0.3,0.1,0.2>)	(<0.2,0.4,0.2>,<0.1,0.0, 0.1>,<0.2,0.1,0.2>)	(<0.2,0.1,0.2>,<0.1,0.1, 0.1>,<0.2,0.2,0.2>)	(<0.2,0.3,0.2>,<0.0,0.2, 0.0>,<0.2,0.2,0.2>)
$\check{a}_5$	(<0.1,0.0,0.2>,<0.3,0.0, 0.1>,<0.3,0.1,0.1>)	(<0.2,0.2,0.2>,<0.2,0.1, 0.2>,<0.2,0.1,0.1>)	(<0.2,0.2,0.0>,<0.3,0.2, 0.0>,<0.3,0.2,0.2>)	(<0.2,0.2,0.2>,<0.1,0.1, 0.1>,<0.1,0.2,0.1>)
$\check{a}_6$	(<0.2,0.2,0.2>,<0.1,0.1, 0.1>,<0.2,0.1,0.1>)	(<0.2,0.1,0.2>,<0.1,0.3, 0.1>,<0.1,0.2,0.0>)	(<0.2,0.2,0.1>,<0.4,0.0, 0.1>,<0.2,0.0,0.1>)	(<0.1,0.0,0.1>,<0.3,0.1, 0.4>,<0.2,0.1,0.1>)
$\check{a}_7$	(<0.3,0.1,0.3>,<0.2,0.0, 0.1>,<0.2,0.1,0.1>)	(<0.1,0.0,0.1>,<0.4,0.1, 0.3>,<0.2,0.1,0.1>)	(<0.2,0.0,0.2>,<0.3,0.3, 0.2>,<0.0,0.2,0.0>)	(<0.2,0.0,0.2>,<0.2,0.1, 0.1>,<0.2,0.1,0.2>)
$\check{a}_8$	(<0.2,0.1,0.2>,<0.2,0.4, 0.2>,<0.1,0.0,0.1>)	(<0.1,0.3,0.3>,<0.0,0.2, 0.0>,<0.2,0.1,0.3>)	(<0.0,0.1,0.1>,<0.3,0.3, 0.2>,<0.1,0.2,0.1>)	(<0.1,0.2,0.3>,<0.2,0.0, 0.1>,<0.2,0.2,0.2>)

Table 2. Neutrosophic m-polar values for alternative $\mathbb{P}^1$

$\mathbb{P}^1$	δ_1	δ_2	δ_3	δ_4
$\check{a}_1$	(<0.2,0.1,0.1>,<0.2,0.1, 0.2>,<0.2,0.1,0.1>)	(<0.1,0.2,0.1>,<0.2,0.1, 0.2>,<0.3,0.1,0.3>)	(<0.1,0.2,0.3>,<0.0,0.1, 0.2>,<0.3,0.1,0.1>)	(<0.2,0.2,0.1>,<0.1,0.1, 0.2>,<0.4,0.1,0.2>)
$\check{a}_2$	(<0.1,0.2,0.1>,<0.2,0.1, 0.2>,<0.3,0.1,0.1>)	(<0.2,0.2,0.3>,<0.0,0.1, 0.1>,<0.1,0.1,0.0>)	(<0.3,0.2,0.2>,<0.0,0.1, 0.1>,<0.1,0.1,0.2>)	(<0.1,0.2,0.1>,<0.1,0.1, 0.2>,<0.2,0.1,0.2>)
$\check{a}_3$	(<0.2,0.2,0.2>,<0.0,0.1, 0.2>,<0.1,0.1,0.0>)	(<0.1,0.2,0.2>,<0.2,0.1, 0.2>,<0.1,0.1,0.0>)	(<0.3,0.2,0.3>,<0.0,0.1, 0.1>,<0.1,0.1,0.2>)	(<0.1,0.2,0.1>,<0.2,0.4, 0.2>,<0.0,0.1,0.1>)
$\check{a}_4$	(<0.1,0.2,0.1>,<0.2,0.1, 0.3>,<0.0,0.1,0.1>)	(<0.2,0.2,0.3>,<0.1,0.1, 0.0>,<0.1,0.1,0.2>)	(<0.1,0.2,0.0>,<0.2,0.1, 0.2>,<0.2,0.1,0.2>)	(<0.4,0.2,0.1>,<0.0,0.1, 0.1>,<0.1,0.1,0.0>)

$\breve{a}_5$	(<0.1,0.1,0.1>,<0.2,0.3,0.2>,<0.0,0.1,0.2>)	(<0.3,0.2,0.3>,<0.1,0.1,0.0>,<0.2,0.1,0.2>)	(<0.1,0.2,0.1>,<0.1,0.1,0.2>,<0.1,0.1,0.0>)	(<0.1,0.2,0.3>,<0.1,0.1,0.1>,<0.3,0.0,0.2>)
$\breve{a}_6$	(<0.2,0.2,0.1>,<0.2,0.1,0.2>,<0.1,0.0,0.2>)	(<0.1,0.2,0.1>,<0.2,0.4,0.2>,<0.0,0.1,0.1>)	(<0.1,0.2,0.2>,<0.2,0.1,0.1>,<0.2,0.1,0.2>)	(<0.1,0.2,0.1>,<0.2,0.1,0.2>,<0.3,0.1,0.2>)
$\breve{a}_7$	(<0.4,0.2,0.1>,<0.0,0.1,0.1>,<0.2,0.1,0.0>)	(<0.1,0.2,0.1>,<0.2,0.1,0.2>,<0.3,0.1,0.2>)	(<0.1,0.2,0.2>,<0.1,0.1,0.2>,<0.3,0.1,0.2>)	(<0.1,0.2,0.3>,<0.2,0.1,0.0>,<0.3,0.1,0.1>)
$\breve{a}_8$	(<0.3,0.2,0.1>,<0.0,0.1,0.2>,<0.2,0.1,0.2>)	(<0.1,0.2,0.1>,<0.2,0.1,0.2>,<0.2,0.1,0.1>)	(<0.1,0.2,0.3>,<0.0,0.1,0.2>,<0.3,0.0,0.2>)	(<0.1,0.2,0.0>,<0.2,0.1,0.2>,<0.2,0.1,0.2>)

Table 3. Neutrosophic m-polar values for alternative $\mathbb{P}^2$

$\mathbb{P}^2$	δ_1	δ_2	δ_3	δ_4
$\breve{a}_1$	(<0.2,0.1,0.1>,<0.2,0.2,0.2>,<0.5,0.1,0.1>)	<0.2,0.2,0.1>,<0.3,0.1,0.1>, <0.0,0.1,0.1>)	<0.0,0.1,0.3>,<0.4,0.1,0.1>,<0.2,0.1,0.1>)	<0.1,0.2,0.1>,<0.2,0.1,0.2>, <0.3,0.3,0.2>)
$\breve{a}_2$	(<0.1,0.2,0.1>,<0.2,0.2,0.2>,<0.3,0.1,0.1>)	<0.2,0.2,0.3>,<0.1,0.1,0.1>, <0.2,0.1,0.1>)	<0.1,0.2,0.1>,<0.2,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.1>,<0.3,0.0,0.2>, <0.3,0.3,0.1>)
$\breve{a}_3$	(<0.2,0.0,0.0>,<0.3,0.4,0.1>,<0.1,0.1,0.3>)	<0.0,0.2,0.1>,<0.5,0.1,0.1>, <0.3,0.2,0.1>)	<0.1,0.2,0.2>,<0.2,0.2,0.1>,<0.0,0.1,0.1>)	<0.1,0.1,0.0>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)
$\breve{a}_4$	(<0.1,0.2,0.2>,<0.2,0.0,0.3>,<0.0,0.1,0.1>)	<0.1,0.2,0.1>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)	<0.1,0.0,0.1>,<0.2,0.1,0.2>,<0.2,0.3,0.1>)	<0.2,0.2,0.0>,<0.2,0.1,0.1>, <0.0,0.1,0.1>)
$\breve{a}_5$	(<0.1,0.1,0.1>,<0.2,0.3,0.1>,<0.2,0.1,0.2>)	<0.1,0.2,0.1>,<0.2,0.1,0.1>, <0.0,0.1,0.1>)	<0.2,0.2,0.1>,<0.3,0.1,0.1>,<0.3,0.3,0.1>)	<0.2,0.0,0.3>,<0.3,0.1,0.1>, <0.0,0.1,0.1>)
$\breve{a}_6$	(<0.1,0.0,0.1>,<0.2,0.4,0.2>,<0.1,0.2,0.2>)	<0.0,0.1,0.1>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)	<0.2,0.1,0.2>,<0.2,0.2,0.1>,<0.3,0.2,0.2>)	<0.1,0.0,0.1>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)
$\breve{a}_7$	(<0.4,0.2,0.2>,<0.0,0.1,0.1>,<0.2,0.1,0.0>)	<0.2,0.2,0.0>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)	<0.1,0.1,0.0>,<0.3,0.1,0.1>,<0.3,0.2,0.1>)	<0.2,0.2,0.1>,<0.2,0.2,0.1>, <0.0,0.1,0.1>)
$\breve{a}_8$	(<0.0,0.2,0.1>,<0.4,0.1,0.2>,<0.2,0.1,0.2>)	<0.2,0.1,0.1>,<0.1,0.3,0.1>, <0.3,0.1,0.2>)	<0.2,0.2,0.3>,<0.0,0.1,0.1>,<0.2,0.1,0.1>)	<0.0,0.1,0.1>,<0.3,0.1,0.1>, <0.3,0.2,0.1>)

Table 4. Neutrosophic m-polar values for alternative $\mathbb{P}^3$

$\mathbb{P}^3$	δ_1	δ_2	δ_3	δ_4

$\breve{a}_1$	<0.2,0.2,0.3>,<0.0,0.1,0.1>,<0.3,0.1,0.1>)	<0.3,0.2,0.2>,<0.2,0.2,0.0>,<0.1,0.1,0.1>)	<0.2,0.0,0.0>,<0.2,0.0,0.2>,<0.3,0.1,0.1>)	<0.3,0.4,0.1>,<0.2,0.0,0.2>,<0.0,0.1,0.1>)
$\breve{a}_2$	<0.2,0.4,0.2>,<0.1,0.1,0.0>,<0.1,0.1,0.1>)	<0.2,0.2,0.3>,<0.2,0.1,0.2>,<0.1,0.1,0.1>)	<0.2,0.2,0.3>,<0.1,0.1,0.1>,<0.4,0.2,0.1>)	<0.5,0.1,0.1>,<0.1,0.0,0.1>,<0.3,0.1,0.1>)
$\breve{a}_3$	<0.2,0.3,0.2>,<0.2,0.1,0.2>,<0.1,0.1,0.0>)	<0.2,0.2,0.2>,<0.0,0.2,0.2>,<0.0,0.1,0.1>)	<0.2,0.2,0.2>,<0.2,0.0,0.1>,<0.0,0.1,0.1>)	<0.1,0.0,0.1>,<0.2,0.1,0.2>,<0.4,0.1,0.1>)
$\breve{a}_4$	<0.3,0.3,0.2>,<0.1,0.0,0.1>,<0.3,0.1,0.1>)	<0.1,0.4,0.3>,<0.2,0.2,0.1>,<0.1,0.1,0.1>)	<0.2,0.2,0.1>,<0.2,0.1,0.2>,<0.4,0.2,0.1>)	<0.2,0.3,0.1>,<0.1,0.1,0.2>,<0.0,0.1,0.1>)
$\breve{a}_5$	<0.2,0.2,0.3>,<0.0,0.1,0.1>,<015,0.0,0.1>)	<0.2,0.0,0.0>,<0.1,0.2,0.2>,<0.4,0.1,0.1>)	<0.5,0.2,0.1>,<0.0,0.2,0.0>,<0.0,0.1,0.1>)	<0.1,0.1,0.1>,<0.1,0.1,0.2>,<0.2,0.2,0.1>)
$\breve{a}_6$	<0.4,0.2,0.2>,<0.1,0.0,0.1>,<0.2,0.1,0.1>)	<0.1,0.,20.1>,<0.3,0.1,0.1>,<0.2,0.2,0.2>)	<0.2,0.4,0.2>,<0.0,0.0,0.2>,<0.2,0.1,0.1>)	<0.1,0.1,0.1>,<0.3,0.2,0.2>,<0.3,0.2,0.0>)
$\breve{a}_7$	<0.1,0.1,0.1>,<0.2,0.3,0.2>,<0.3,0.1,0.1>)	<0.2,0.1,0.3>,<0.1,0.1,0.1>,<0.1,0.0,0.1>)	<0.1,0.1,0.0>,<0.1,0.3,0.1>,<0.4,0.1,0.1>)	<0.1,0.6,0.1>,<0.0,0.0,0.2>,<0.2,0.2,0.1>)
$\breve{a}_8$	<0.2,0.4,0.2>,<0.0,0.1,0.1>,<0.1,0.1,0.1>)	<0.2,0.2,0.2>,<0.1,0.2,0.0>,<0.0,0.1,0.1>)	<0.2,0.5,0.1>,<0.0,0.1,0.1>,<0.1,0.1,0.1>)	<0.2,0.3,0.1>,<0.1,0.1,0.1>,<0.0,0.1,0.1>)

Table 5. Neutrosophic m-polar values for alternative $\mathbb{P}^4$

$\mathbb{P}^4$	δ_1	δ_2	δ_3	δ_4
$\breve{a}_1$	<0.1,0.1,0.3>,<0.2,0.1,0.1>,<0.0,0.1,0.1>)	<0.1,0.2,0.2>,<0.5,0.1,0.1>,<0.2,0.1,0.1>)	<0.2,0.2,0.0>,<0.2,0.1,0.1>,<0.3,0.1,0.1>)	<0.1,0.0,0.1>,<0.3,0.1,0.1>,<0.3,0.2,0.1>)
$\breve{a}_2$	<0.2,0.2,0.2>,<0.1,0.0,0.1>,<0.3,0.1,0.3>)	<0.2,0.2,0.3>,<0.3,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.2>,<0.1,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.3>,<0.2,0.1,0.1>,<0.1,0.1,0.1>)
$\breve{a}_3$	<0.2,0.2,0.3>,<0.2,0.1,0.1>,<0.1,0.1,0.1>)	<0.2,0.2,0.4>,<0.0,0.1,0.1>,<0.1,0.1,0.1>)	<0.2,0.0,0.3>,<0.3,0.1,0.1>,<0.0,0.1,0.1>)	<0.2,0.2,0.1>,<0.2,0.2,0.1>,<0.1,0.1,0.1>)
$\breve{a}_4$	<0.3,0.2,0.3>,<0.2,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.0>,<0.1,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.1>,<0.4,0.1,0.1>,<0.3,0.3,0.1>)	<0.2,0.2,0.2>,<0.3,0.1,0.1>,<0.2,0.1,0.1>)
$\breve{a}_5$	<0.2,0.2,0.3>,<0.2,0.1,0.1>,<0.2,0.1,0.1>)	<0.1,0.1,0.3>,<0.1,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.3>,<0.0,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.4>,<0.2,0.1,0.1>,<0.1,0.1,0.1>)

$\breve{\alpha}_6$	<0.2,0.2,0.2>,<0.3,0.1,0.1>,<0.2,0.1,0.1>)	<0.1,0.2,0.1>,<0.1,0.1,0.1>,<0.4,0.1,0.1>)	<0.2,0.2,0.2>,<0.3,0.1,0.1>,<0.0,0.1,0.1>)	<0.2,0.2,0.2>,<0.1,0.1,0.1>,<0.0,0.1,0.1>)
$\breve{\alpha}_7$	<0.1,0.2,0.0>,<0.5,0.1,0.1>,<0.2,0.1,0.1>)	<0.2,0.4,0.2>,<0.4,0.1,0.1>,<0.1,0.1,0.1>)	<0.2,0.2,0.3>,<0.0,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.2,0.3>,<0.3,0.1,0.1>,<0.2,0.1,0.1>)
$\breve{\alpha}_8$	<0.2,0.2,0.3>,<0.2,0.1,0.1>,<0.3,0.1,0.1>)	<0.2,0.1,0.3>,<0.1,0.1,0.1>,<0.0,0.1,0.1>)	<0.2,0.2,0.1>,<0.1,0.1,0.1>,<0.3,0.2,0.1>)	<0.2,0.2,0.0>,<0.2,0.1,0.1>,<0.3,0.1,0.1>)

Table 6. Neutrosophic values for alternative ℵ

ℵ	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
δ_1	$(\mathbf{0.5, 0.4, 0.}$	$(\mathbf{0.3, 0.7, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.3, 0.4, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.7, 0.3, 0.}$	$(\mathbf{0.5, 0.8, 0}$
δ_2	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.7, 0.3, 0.}$	$(\mathbf{0.4, 0.4, 0.}$	$(\mathbf{0.8, 0.2, 0.}$	$(\mathbf{0.6, 0.5, 0.}$	$(\mathbf{0.5, 0.5, 0.}$	$(\mathbf{0.2, 0.8, 0.}$	$(\mathbf{0.7, 0.2, 0}$
δ_3	$(\mathbf{0.4, 0.6, 0}$	$(\mathbf{0.4, 0.4, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.5, 0.3, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.5, 0.5, 0.}$	$(\mathbf{0.4, 0.8, 0.}$	$(\mathbf{0.2, 0.8, 0}$
δ_4	$(\mathbf{0.3, 0.7, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.2, 0.8, 0.}$	$(\mathbf{0.4, 0.4, 0.}$	$(\mathbf{0.6, 0.3, 0}$

Table 7. Neutrosophic values for alternative $\mathbb{P}^{\mathbf{1}}$

$\mathbb{P}^1$	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
δ_1	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.4, 0.6, 0.}$	$(\mathbf{0.3, 0.7, 0.}$	$(\mathbf{0.5, 0.5, 0.}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.6, 0.3, 0.}$
δ_2	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.5, 0.5, 0.}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.8, 0.2, 0.}$	$(\mathbf{0.4, 0.8, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.4, 0.5, 0.}$
δ_3	$(\mathbf{0.6, 0.3, 0}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.8, 0.2, 0.}$	$(\mathbf{0.3, 0.5, 0.}$	$(\mathbf{0.4, 0.4, 0.}$	$(\mathbf{0.5, 0.4, 0.}$	$(\mathbf{0.5, 0.4, 0.}$	$(\mathbf{0.7, 0.2, 0.}$
δ_4	$(\mathbf{0.5, 0.4, 0.}$	$(\mathbf{0.4, 0.4, 0.}$	$(\mathbf{0.4, 0.8, 0.}$	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.6, 0.3, 0.}$	$(\mathbf{0.3, 0.5, 0.}$

Table 8. Neutrosophic values for alternative $\mathbb{P}^{\mathbf{2}}$

$\mathbb{P}^2$	$\breve{\alpha}_1$	$\breve{\alpha}_2$	$\breve{\alpha}_3$	$\breve{\alpha}_4$	$\breve{\alpha}_5$	$\breve{\alpha}_6$	$\breve{\alpha}_7$	$\breve{\alpha}_8$
δ_1	$(\mathbf{0.4, 0.6, 0.}$	$(\mathbf{0.3, 0.6, 0}$	$(\mathbf{0.2, 0.8, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.3, 0.6, 0}$	$(\mathbf{0.2, 0.8, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.3, 0.7, 0}$
δ_2	$(\mathbf{0.5, 0.5, 0.}$	$(\mathbf{0.7, 0.2, 0}$	$(\mathbf{0.3, 0.7, 0}$	$(\mathbf{0.4, 0.5, 0}$	$(\mathbf{0.4, 0.4, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.4, 0.5, 0}$	$(\mathbf{0.4, 0.5, 0}$
δ_3	$(\mathbf{0.4, 0.6,}$	$(\mathbf{0.4, 0.4, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.7, 0.2, 0}$

δ_4	$(\mathbf{0.4, 0.5, 0.}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.4, 0.4, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$

Table 9. Neutrosophic values for alternative $\mathbb{P}^{\mathbf{3}}$

$\mathbb{P}^3$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	$(\mathbf{0.7, 0.2, 0.}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.7, 0.5, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.7, 0.2, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.3, 0.7, 0}$	$(\mathbf{0.8, 0.2, 0}$
δ_2	$(\mathbf{0.7, 0.4, 0.}$	$(\mathbf{0.7, 0.5, 0}$	$(\mathbf{0.6, 0.4, 0}$	$(\mathbf{0.8, 0.5, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.4, 0.5, 0}$	$(\mathbf{0.6, 0.3, 0}$	$(\mathbf{0.6, 0.3, 0}$
δ_3	$(\mathbf{0.4, 0.4,}$	$(\mathbf{0.7, 0.3, 0}$	$(\mathbf{0.6, 0.3, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.8, 0.2, 0}$
δ_4	$(\mathbf{0.8, 0.4, 0.}$	$(\mathbf{0.7, 0.2, 0}$	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.6, 0.4, 0}$	$(\mathbf{0.4, 0.4, 0}$	$(\mathbf{0.3, 0.7, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.6, 0.3, 0}$

Table 10. Neutrosophic values for alternative $\mathbb{P}^{\mathbf{4}}$

$\mathbb{P}^4$	$\breve{a}_1$	$\breve{a}_2$	$\breve{a}_3$	$\breve{a}_4$	$\breve{a}_5$	$\breve{a}_6$	$\breve{a}_7$	$\breve{a}_8$
δ_1	$(\mathbf{0.5, 0.4, 0}$	$(\mathbf{0.6, 0.2, 0}$	$(\mathbf{0.7, 0.4, 0}$	$(\mathbf{0.8, 0.4, 0}$	$(\mathbf{0.7, 0.4, 0}$	$(\mathbf{0.6, 0.5, 0}$	$(\mathbf{0.3, 0.7, 0}$	$(\mathbf{0.7, 0.4, 0}$
δ_2	$(\mathbf{0.5, 0.7, 0}$	$(\mathbf{0.7, 0.5, 0}$	$(\mathbf{0.8, 0.2, 0}$	$(\mathbf{0.4, 0.3, 0}$	$(\mathbf{0.5, 0.3, 0}$	$(\mathbf{0.4, 0.3, 0}$	$(\mathbf{0.8, 0.6, 0}$	$(\mathbf{0.6, 0.3, 0}$
δ_3	$(\mathbf{0.4, 0.4, 0}$	$(\mathbf{0.6, 0.3, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.5, 0.6, 0}$	$(\mathbf{0.7, 0.2, 0}$	$(\mathbf{0.6, 0.5, 0}$	$(\mathbf{0.7, 0.2, 0}$	$(\mathbf{0.5, 0.3, 0}$
δ_4	$(\mathbf{0.2, 0.5, 0}$	$(\mathbf{0.7, 0.4, 0}$	$(\mathbf{0.5, 0.5, 0}$	$(\mathbf{0.6, 0.5, 0}$	$(\mathbf{0.8, 0.4, 0}$	$(\mathbf{0.6, 0.3, 0}$	$(\mathbf{0.7, 0.5, 0}$	$(\mathbf{0.4, 0.4, 0}$

Step 3: Find informational energies of $\aleph$ ***and*** $\mathbb{P}^{\mathbf{2}}$ using the formula;

$$\varsigma_{m-PNHSs}(\wp,\ddot{A}) = \sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\mathcal{T}^{i}_{\wp(\breve{a}_k)i}(v_i)\right)^2 + \sum_{j=1}^{q}\left(\mathcal{I}^{j}_{\wp(\breve{a}_k)j}(v_i)\right)^2 + \sum_{k=1}^{r}\left(\mathfrak{C}^{k}_{\wp(\breve{a}_k)k}(v_i)\right)^2\right)$$

and we get,

$\varsigma_{m-PNHSs}(\aleph) = 23.7$

$\varsigma_{m-PNHSs}(\mathbb{P}^1) = 21.8$

Step 4: Now we'll calculate correlation by using the formula;

$$\mathcal{C}_{m-PNHSs}((\wp,\ddot{A}),(\mathcal{Q},\ddot{B})) =$$

$$\sum_{k=1}^{m}\sum_{i=1}^{n}\left(\sum_{i=1}^{p}\left(\mathcal{T}^{i}_{\wp(\breve{a}_k)i}(v_i) * \mathcal{T}^{i}_{\mathcal{Q}(\breve{a}_k)i}(v_i)\right) + \sum_{j=1}^{q}\left(\mathcal{I}^{j}_{\wp(\breve{a}_k)j}(v_i) * \mathcal{I}^{j}_{\mathcal{Q}(\breve{a}_k)j}(v_i)\right) + \sum_{k=1}^{r}\left(\mathfrak{C}^{k}_{\wp(\breve{a}_k)k}(v_i) * \mathfrak{C}^{k}_{\mathcal{Q}(\breve{a}_k)k}(v_i)\right)\right)$$

$= \mathcal{C}_{m-PNHSs}((\aleph, \mathbb{P}^1) = 19.95$

Step 5: Now we'll find CC by using the formula;

$$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^1) = \frac{\mathcal{C}_{m-PNHSs}((\aleph,\mathbb{P}^1)}{\sqrt{\varsigma_{m-PNHSs}(\aleph)} * \sqrt{\varsigma_{m-PNHSs}(\mathbb{P}^1)}} = \frac{\mathbf{19.95}}{\sqrt{23.7} * \sqrt{21.8}} = \mathbf{0.877}$$

Repeating the step 3, and step 4 to calculate CC of the given candidates;

$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^1) = \mathbf{0.877}$,

$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^2) = \mathbf{0.885}$,

$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^3) = \mathbf{0.774}$,

$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^4) = \mathbf{0.880}$,

Step 6: Arrange the CC values in descending order,

$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^2) = \mathbf{0.885} > \boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^4) = \mathbf{0.880} > \boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^1) = \mathbf{0.877} >$
$\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^3) = \mathbf{0.774}$

Step 7: Rank the alternatives from largest to smallest CC and informational energy values, from above results, $\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^2) = \mathbf{0.885}$ has highest CC. Therefore, the position of Mathematics teacher at LGU can be filled by hiring $\mathbb{P}^2$ alternative.

Real Application for Water quantity evaluation (Problem formulation)

Water that is safe to drink is a basic requirement for good health. Water supply organizations place a high priority on quantity-related issues while paying little attention to drinking water quality concerns. We must supply safe drinking water (not necessarily excellent tasting) as well as appetizing food (pleasing to drink). The following four criteria are used to characterize the quality of drinking water: Physical, chemical, microbiological, and radiological. Due to water quality and quantity difficulties in Pakistan, access to clean drinking water is one of the country's public health concerns. A large percentage of the country's drinking water (almost 70%) originates from underground aquifers. Toxic metals such as arsenic, iron, and mercury have been found in some places due to bacterial contamination. Fluorides are a serious danger to the country's water quality. Microbial pollution of drinking water has been identified as a major source of sickness and mortality among Pakistanis, particularly youngsters, who are particularly sensitive. Water contamination is estimated to be the cause of 30% of all diseases and 40% of all fatalities in the country. Unfortunately, the drinking water quality issue in the country receives little attention, and most people consume water without understanding if it is safe or hazardous for them. The drinking water standards in Pakistan were evaluated by the ministry of health, the Government of Pakistan, and the World Health Organization (WHO). Pakistan adheres to WHO drinking water quality norms and standards (Pak-EPA-2008, And the Gazette of Pakistan 2010) and the data is listed in Table 11.

Table 11. Comparison of National and International water quality standards

Parameters	Pakistan standards (mg/L)	WHO standards (mg/L)

Color	$\leq \mathbf{15}$ TCU	$\leq \mathbf{15}$ TCU
Odor	**resinous / fragrant**	**resinous / fragrant**
Turbidity	5 NTU	5 NTU
pH	6.5-8.5	6.5-8.5
Chloride	250	250
Fluoride	$\leq \mathbf{1.5}$	1.5
Lead	$\leq \mathbf{0.05}$	0.01
Manganese	$\leq \mathbf{0.5}$	0.5
Zinc	5.0	3
Arsenic	0.05	0.01
Magnesium	$\leq \mathbf{100}$	30
Calcium	$\leq \mathbf{100}$	60-120
Sulfate	$<$ 250	$\leq \mathbf{250}$
Sodium	100	$\leq \mathbf{200}$
Iron	0.3	0.3

Consider U = $\{\boldsymbol{S_1},\ \boldsymbol{S_2}, \boldsymbol{S_3}\}$ are there samples of water, we've to check which sample of water is safe for drinking purposes according to world health organization standards and we have taken a WHO standard parameter and represented with $\boldsymbol{\omega}$ ideal water (safe to drink). Consider the parameters describe above in Table 11. P = $\{\wp_1 =$ Color, $\wp_2 =$ Turbidity, $\wp_3 =$ pH, $\wp_4 =$ odour, $\wp_5 =$ Chloride, $\wp_6 = \boldsymbol{Fluroide}$, $\wp_7 =$ Magnesium, $\wp_8 =$ Calcium, $\wp_9 =$ Sulphate, $\wp_{10} =$ Sodium, $\wp_{11} =$Iron, $\wp_{12} =$ Arsenic, $\wp_{13} =$Manganese , $\wp_{14} =$ Lead, $\wp_{15} =$Zinc$\}$. These attributes are further divided as:

$\wp_1 =$ Color $\rightarrow \{\boldsymbol{a_{11}} \leq \mathbf{15}$ TCU, $\boldsymbol{a_{11}} \geq \mathbf{15}\ \boldsymbol{TCU}\}$

$\wp_2 =$ Turbidity $\rightarrow \{\boldsymbol{a_{21}} \leq \mathbf{5}$ **NTU** $, \boldsymbol{a_{22}} > \mathbf{5}\boldsymbol{NTU}\}$

$\wp_3 =$ pH $\rightarrow \{\ \mathbf{6.5} \leq \boldsymbol{a_{31}} \leq \mathbf{8.5}\ , \boldsymbol{a_{32}} = \boldsymbol{other}\}$

$\wp_4 =$ Odour $\rightarrow \{\boldsymbol{a_{41}} = \mathbf{resinous}\ , \boldsymbol{a_{42}} = \ \mathbf{fragrant}\}$

$\wp_5 =$ Chloride$\rightarrow \{\boldsymbol{a_{51}} = \ \mathbf{250mg/L},\ \boldsymbol{a_{52}} > \ \mathbf{250\ \ mg/\ L}\}$

$\wp_6 =$Fluoride$\rightarrow \left\{\boldsymbol{a_{61}} \leq \mathbf{1.5}\frac{\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{62}} > \mathbf{1.5}\ \ \boldsymbol{mg/L}\right\}$

$\wp_7 =$ Magnesium $\rightarrow \left\{\boldsymbol{a_{71}} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}},\ \boldsymbol{a_{72}} > \ \ \mathbf{100}\quad \boldsymbol{mg/L}\right\}$

$\wp_8 =$ Calcium $\rightarrow \left\{\boldsymbol{a_{81}} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{82}} > \ \ \mathbf{100}\ \ \boldsymbol{mg/L}\right\}$

$\wp_9 =$ Sulfate $\rightarrow \left\{\boldsymbol{a_{91}} \leq \frac{\mathbf{250}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{92}} > \ \ \mathbf{250}\ \ \boldsymbol{mg/L}\right\}$

$\wp_{10} =$ Sodium$\rightarrow \left\{\boldsymbol{a_{10}} = \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{10,2}} > \mathbf{100}\boldsymbol{mg/L}\right\}$

$\wp_{11} =$ Iron $\rightarrow \left\{\boldsymbol{a_{11,1}} = \frac{\mathbf{0.3}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{11,2}} > \mathbf{0.3}\boldsymbol{mg/L}\right\}$

$\wp_{12} =$ Arsenic $\rightarrow \left\{\boldsymbol{a_{12,1}} < \frac{\mathbf{0.05}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{12,2}} > \ \ \mathbf{0.05}\boldsymbol{mg/L}\right\}$

$\wp_{13} =$ Manganese $\rightarrow \left\{\boldsymbol{a_{13,1}} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{13,2}} > \ \ \mathbf{100}\boldsymbol{mg/L}\right\}$

$\wp_{14} =$ Lead $\rightarrow \left\{\boldsymbol{a_{14,1}} < \frac{\mathbf{0.05}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{14,2}}\ \ > \mathbf{0.05}\boldsymbol{mg/L}\right\}$

$\wp_{15} =$ Zinc $\left\{\boldsymbol{a_{15,1}} < \frac{\mathbf{5}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{a_{15,2}} > \ \ \mathbf{5}\boldsymbol{mg/L}\right\}$

The ideal water $\boldsymbol{\omega} = \boldsymbol{F}\Big(\mathbf{Color} \leq \mathbf{15TCU}, \mathbf{Turbidity} = \mathbf{5\ NTU}, \mathbf{PH} = \mathbf{6.5-8.5}, \mathbf{Odour} = \mathbf{Acceptable}, \mathbf{Chloride} = \mathbf{250mg/L}, \mathbf{Fluoride} \leq \mathbf{1.5}\frac{\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Magnesium} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Calcium} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Sulfate} \leq \frac{\mathbf{250}\boldsymbol{mg}}{\boldsymbol{L}}, \boldsymbol{Sodium} = \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Iron} = \frac{\mathbf{0.3}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Arsenic} < \frac{\mathbf{0.05}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Manganese} \leq \frac{\mathbf{100}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Lead} < \frac{\mathbf{0.05}\boldsymbol{mg}}{\boldsymbol{L}}, \mathbf{Zinc} < \ \mathbf{5}\boldsymbol{mg/L}\Big)$ (a)

And $\boldsymbol{\mathcal{DM}} = \{\boldsymbol{\mathcal{DM}_1}\ , \boldsymbol{\mathcal{DM}_2}\}$ and $\boldsymbol{\Omega} = \{\ \boldsymbol{\Omega_1} = \ \mathbf{0.6}\ ,\ \boldsymbol{\Omega_1} = \ \mathbf{0.4}\ \}^{\boldsymbol{T}}$ be the set of decision makers and their weights respectively.

Solved Example using Algorithm 4.2

Step 1: Define a mapping, and select Hypersoft set;

$$\boldsymbol{F}\colon \wp_1 \times \ \wp_2 \times \wp_3 \times \ldots\ldots \times \wp_{15} = \boldsymbol{\gamma}' \rightarrow \boldsymbol{P}(\aleph) = \boldsymbol{S}^1, \boldsymbol{S}^2, \boldsymbol{S}^3$$

compute the Weighted Correlation Coefficient (WCC) between $\boldsymbol{\delta_{m-PWNHSS}(\omega, S_1)}$, $\boldsymbol{\delta_{m-PWNHSs}(\omega, S_2)}$, $\boldsymbol{\delta_{m-PWNHSs}(\omega, S_3)}$, to check that whether taken sample of water has positive correlation with the safe water $\boldsymbol{\omega}$, or not, if yes then it means the sample of water which is taken is safe to use for drinking purposes if the value of correlation coefficient is less than 0.50 then it means that water requires treatment before use, check the Truthiness, Indeterminacy and falsity values of sample regarding each attribute, those attributes which has unbalance ratio according to National standard for safe water ($\boldsymbol{\omega}$). Now, 1st we'll find $\boldsymbol{\delta_{m-PNHSs}(\omega, S_1)}$ i.e. (Correlation coefficient between $\boldsymbol{\omega}$(safe water) and $\boldsymbol{S_1}$(Sample 1). Let $\{\boldsymbol{\omega}\}$, and $\{\boldsymbol{S_1}\}$ be the two sets having sub-attributes, Step 2: We construct m-PNHSs in the form of m-PNHSNs by assigning Neutrosophic values to the selected alternatives of Hypersoft set i.e. $\boldsymbol{S^1}, \boldsymbol{S^2}, \boldsymbol{S^3}$. Also, DM will assign m=3 neutrosophic values to the ideal water i.e. $\boldsymbol{\omega}$ and shown in Table 12-15. Their simplified Neutrosophic form is shown in Table 16-19.

Table 12. Neutrosophic m-polar values for alternative $\boldsymbol{\omega}$

ω	$\mathcal{DM}_1$	$\mathcal{DM}_2$
$Color \leq$ 15TCU	(<0.2,0.4,0.3>,<0.02,0.01,0.02>,<0.03,0.01,0.01>)	(<0.20,0.40,0.25>,<0.10,0.5,0.0>,<0.0,0.1,0.0>)
$Turbidity = 5\ NTU$	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)
$pH = 6.5-8.5$	(<0.3,0.3,0.3>,<0.02,0.01,0.02>,<0.02,0.02,0.01)	(<0.20,0.40,0.25>,<0.1,0.,0.0>,<0.03,0.02,0.0>)
Odor = $resinous$	(<0.25,0.25,0.35>,<0.1,0.0,0.0>,<0.03,0.01,0.01>)	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<<0.0,0.1,0.0>)
$Chloride = 250\ mg/\ L$	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<<0.03,0.01,0.01>)	(<0.20,0.40,0.25>,<0.1,0.0,0.0>,<0.01,0.01,0.03>)
Fluoride $\leq$ 1.5mg/L	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<<0.0,0.1,0.0>)	(<0.2,0.4,0.1>,<0.1,0.0,0.0>,<0.1,0.1,0.0>)
$Magnesium \leq 100mg/L$	(<0.2,0.4,0.2>,<0.03,0.01,0.01>,<0.0,0.1,0.0>)	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<0.0,0.1,0.0>)
Calcium $\leq$ 100 mg/L	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.1,0.0>)	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<0.0,0.1,0.0>)
$Sulfate \leq 250mg/L$	(<0.2,0.4,0.2>,<0.03,0.01,0.01>,<0.0,0.1,0.0>)	(<0.2,0.4,0.2>,<0.1,0.0,0.0>,<0.0,0.1,0.0>)
$Sodium = 100mg/L$	(<0.0,0.4,0.2>,<0.0,0.1,0.1>,<0.0,0.1,0.1>)	(<0.2,0.4,0.0>,<0.1,0.0,0.0>,<0.2,0.1,0.0>)
$Iron = 0.3mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.1,0.0>)	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.1,0.0>)
$Arsenic < 0.05mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.1,0.0>)	(<0.2,0.4,0.1>,<0.1,0.0,0.0>,<0.1,0.1,0.0>)

Manganese ≤ 100mg/L	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.0,0.10,0.05>)	(<0.2,0.3,0.2>,<0.0,0.1,0.0>,<0.0,0.1,0.0>)
$Lead < \ 0.05mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.1,0.1,0.0>)	(<0.20,0.40,0.05>,<0.1,0.1,0.0>,<0.0,0.10, 0.05>)
$Zinc < \ 5mg/L$	(<0.20,0.40,0.15>,<0.1,0.1,0.0>,<0.0,0.1, 0.0>)	(<0.2,0.4,0.1>,<0.1,0.0,0.0>,<<0.0,0.1,0.0>)

Table 13. Neutrosophic m-polar values for alternative $\boldsymbol{S_1}$

S_1	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color$ ≤15TCU	(<0.2,0.4,0.3>,<0.02,0.01,0.02>,<0.03,0.01,0.01>)	(<0.20,0.40,0.25>,<0.0,0.10,0.05>,<0.1,0.0,0.0>)
Turbidity ≥5NTU	(<0.2,0.1,0.3>,<0.0,0.1,0.0>,<0.0,0.1,0.1>)	(<0.20,0.40,0.05>,<0.0,0.10,0.05>,<0.1,0.0,0.0>)
$pH = \ 6.5 - 8.5$	(<0.2,0.4,0.3>,<0.02,0.01,0.02>,<0.03,0.01,0.01>)	(<0.20,0.40,0.25>,<0.0,0.1,0.0>,<0.0,0.0,0.05>)
Odor = $fragrant$	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)
$Chloride > \ 250 \ mg / \ L$	(<0.2,0.3,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)	(<0.2,0.3,0.1>,<0.0,0.1,0.0>,<0.2,0.0,0.1>)
Fluoride > 1.5mg/L	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
$Magnesium > \ 100mg/L$	(<0.20,0.40,0.05>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
Calcium > 100 mg/L	(<0.2,0.3,0.1>,<0.1,0.1,0.0>,<0.1,0.0,0.1>)	(<0.2,0.3,0.1>,<0.05,0.10,0.0>,<0.1,0.0,0.1>)
$Sulfate \leq \ 250mg/L$	(<0.2,0.4,0.2>,<0.01,0.02,0.02>,<0.0,0.0,0.1>)	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
$Sodium = \ 100mg/L$	(<0.2,0.3,0.1>,<0.0,0.1,0.1>,<0.1,0.0,0.1>)	(<0.2,0.3,0.1>,<0.0,0.1,0.0>,<0.1,0.1,0.1>)

$Iron = 0.3mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.1,0.0,0.0>)
$Arsenic < 0.05mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
Manganese ≤ 100mg/L	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.10,0.0,0.05>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)
$Lead < 0.05mg/L$	(<0.2,0.3,0.1>,<0.1,0.1,0.0>,<0.1,0.0,0.1>)	(<0.20,0.40,0.05>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)
$Zinc < 5mg/L$	(<0.20,0.40,0.15>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)

Table 14. Neutrosophic m-polar values for alternative $\boldsymbol{S_2}$

S_2	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color$ >15TCU	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)	(<0.25,0.25,0.25>,<0.05,0.05,0.05>,<0.0,0.0,0.1>)
$Turbidity > 5\ NTU$	(<0.20,0.40,0.05>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)	(<0.25,0.25,0.15>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)
$pH = 6.5-8.5$	(<0.2,0.4,0.3>,<0.02,0.01,0.02>,<0.02,0.01,0.02>)	(<0.35,0.35,0.15>,<0.0,0.1,0.0>,<0.02,0.02,0.01>)
Odor = fragrant	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.01,0.02,0.02>)	(<0.25,0.25,0.25>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
$Chloride > 250\ mg/L$	(<0.20,0.20,0.25>,<0.05,0.05,0.05>,<0.0,0.1,0.1>)	(<0.25,0.25,0.15>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
Fluoride >1.5mg/L	(<0.30,0.30,0.15>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.05,0.05,0.05>)
$Magnesium \leq 100mg/L$	(<0.3,0.3,0.2>,<0.02,0.02,0.01>,<0.0,0.0,0.1>)	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
Calcium ≤ 100 mg/L	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.1,0.1>)	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
$Sulfate > 250mg/L$	(<0.2,0.2,0.2>,<0.0,0.1,0.1>,<0.1,0.0,0.1>)	(<0.2,0.2,0.2>,<0.0,0.1,0.0>,<0.1,0.1,0.1>)

$Sodium > 100mg/L$	(<0.15,0.15,0.25>,<0.1,0.1,0.0>,<0.0,0.2,0.1>)	(<0.2,0.2,0.2>,<0.0,0.1,0.0>,<0.1,0.1,0.1>)
$Iron > \ 0.3mg/L$	(<0.2,0.2,0.2>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.25,0.25,0.15>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)
$Arsenic < \ 0.05mg/L$	(<0.2,0.4,0.1>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.1,0.1>)
Manganese ≤ 100mg/L	(<0.25,0.25,0.25>,<0.0,0.1,0.0>,<0.05,0.05, 0.05>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
$Lead < \ 0.05mg/L$	(<0.2,0.2,0.2>,<0.1,0.1,0.0>,<0.1,0.0,0.1>)	(<0.25,0.25,0.15>,<0.1,0.1,0.0>,<0.05,0.05, 0.05>)
$Zinc < \ 5mg/L$	(<0.25,0.25,0.25>,<0.1,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)

Table 15. Neutrosophic m-polar values for alternative $\boldsymbol{S_3}$

S_3	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color \leq$ to 15TCU	(<0.3,0.3,0.3>,<0.02,0.01,0.02>,<0.01,0.03, 0.01>)	(<0.20,0.40,0.25>,<0.05,0.10,0.0>,<0.1,0.0, 0.0>)
$Turbidity = \ 5\ NTU$	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.1,0.0,0.0>)
$pH \ = \ 6.5-8.5$	(<0.2,0.4,0.3>,<0.02,0.01,0.02>,<0.01,0.02, 0.02>)	(<0.20,0.40,0.25>,<0.0,0.1,0.0>,<0.01,0.02, 0.02>)
Odor =resinous	(<0.20,0.40,0.25>,<0.0,0.1,0.0>,<0.02,0.01, 0.02>)	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)
$Chloride > \ 250\ mg\ /\ L$	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.01,0.0,0.01>)	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.10,0.10, 0.05>)
Fluoride ≤1.5mg/L	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
$Magnesium > \ 100mg/L$	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
Calcium >100 mg/L	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)	(<0.20,0.40,0.25>,<0.0,0.1,0.0>,<0.10,0.05, 0.0>)

$Sulfate \leq 250mg/L$	(<0.2,0.4,0.2>,<0.02,0.01,0.02>,<0.0,0.0,0.1>)	(<0.2,0.4,0.2>,<0.0,0.1,0.0>,<0.0,0.0,0.1>)
$Sodium > 100mg/L$	(<0.20,0.30,0.05>,<0.0,0.1,0.1>,<0.1,0.0,0.0>)	(<0.2,0.3,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
$Iron = 0.3mg/L$	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.0,0.0,0.1>)	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.1,0.0,0.0>)
$Arsenic < 0.05mg/L$	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.1,0.0,0.0>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.1>)
Manganese ≤100mg/L	(<0.20,0.40,0.15>,<0.0,0.1,0.0>,<0.10,0.0,0.05>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.1,0.0,0.0>)
$Lead < 0.05mg/L$	(<0.2,0.3,0.1>,<0.1,0.1,0.0>,<0.1,0.0,0.1>)	(<0.20,0.40,0.05>,<0.1,0.1,0.0>,<0.10,0.05, 0.>)
$Zinc > 5mg/L$	(<0.2,0.4,0.1>,<0.0,0.1,0.1>,<0.10,0.0,0.05>)	(<0.2,0.4,0.1>,<0.0,0.1,0.0>,<0.2,0.1,0.2>)

Table 16. Neutrosophic values for alternative $\boldsymbol{\omega}$

ω	$\mathcal{DM}_1$	$\mathcal{DM}_2$
$Color \leq$ 15TCU	(0.9,0.05,0.05)	(0.85,0.15,0.1)
$Turbidity = 5\ NTU$	(0.8,0.1,0.1)	(0.7,0.2,0.1)
$pH = 6.5 - 8.5$	(0.9,0.05,0.05)	(0.85,0.1,0.05)
Odor = $resinous$	(0.85,0.1,0.05)	(0.8,0.1,0.1)
$Chloride = 250\ mg/\ L$	(0.8,0.1,0.05)	(0.85,0.1,0.05)
Fluoride ≤ 1.5mg/L	(0.8,0.1,0.1)	(0.7,0.1,0.2)
$Magnesium \leq 100mg/L$	(0.8,0.05,0.1)	(0.8,0.1,0.1)
Calcium ≤ 100 mg/L	(0.7,0.1,0.2)	(0.8,0.1,0.1)
$Sulfate \leq 250mg/L$	(0.8,0.05,0.1)	(0.8,0.1,0.1)
$Sodium = 100mg/L$	(0.6,0.2,0.2)	(0.6,0.1,0.3)

$Iron = 0.3mg/L$	(0.7,0.2,0.1)	(0.7,0.2,0.1)
$Arsenic < 0.05mg/L$	(0.7,0.2,0.1)	(0.7,0.1,0.2)
Manganese $\leq$ 100mg/L	(0.75,0.1,0.15)	(0.7,0.1,0.1)
$Lead < 0.05mg/L$	(0.6,0.2,0.2)	(0.65,0.2,0.15)
$Zinc < 5mg/L$	(0.75,0.2,0.1)	(0.7,0.1,0.1)

Table 17. Neutrosophic values for alternative $\boldsymbol{S_1}$

S_1	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color$ ≤15TCU	(0.9,0.05,0.05)	(0.85,0.15,0.1)
Turbidity ≥5NTU	(0.6,0.1,0.2)	(0.65,0.15,0.1)
$pH = 6.5 - 8.5$	(0.9,0.05,0.05)	(0.85,0.1,0.05)
Odor = $fragrant$	(0.7,0.1,0.2)	(0.7,0.1,0.1)
$Chloride > 250\ mg/\ L$	(0.6,0.1,0.2)	(0.6,0.1,0.3)
Fluoride > 1.5mg/L	(0.7,0.1,0.1)	(0.65,0.1,0.2)
$Magnesium > 100mg/L$	(0.65,0.1,0.2)	(0.7,0.1,0.1)
Calcium > 100 mg/L	(0.6,0.2,0.2)	(0.6,0.15,0.2)
$Sulfate \leq 250mg/L$	(0.8,0.05,0.1)	(0.8,0.1,0.1)
$Sodium = 100mg/L$	(0.6,0.2,0.2)	(0.6,0.1,0.3)
$Iron = 0.3mg/L$	(0.7,0.2,0.1)	(0.7,0.2,0.1)
$Arsenic < 0.05mg/L$	(0.7,0.2,0.1)	(0.7,0.1,0.2)
Manganese $\leq$ 100mg/L	(0.75,0.1,0.15)	(0.7,0.1,0.1)
$Lead < 0.05mg/L$	(0.6,0.2,0.2)	(0.65,0.2,0.15)

$Zinc < \quad 5mg/L$	(0.75,0.2,0.1)	(0.7,0.1,0.1)

Table 18. Neutrosophic values for alternative $\boldsymbol{S_2}$

S_2	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color$ >15TCU	(0.7,0.1,0.1)	(0.75,0.15,0.1)
$Turbidity > \quad 5 \quad NTU$	(0.65,0.1,0.1)	(0.65,0.2,0.1)
$pH \quad = \quad 6.5 - 8.5$	(0.9,0.05,0.05)	(0.85,0.1,0.05)
Odor = fragrant	(0.75,0.1,0.05)	(0.75,0.1,0.1)
$Chloride > \quad 250 \quad mg/ \quad L$	(0.65,0.15,0.2)	(0.65,0.1,0.2)
Fluoride >1.5mg/L	(0.75,0.1,0.1)	(0.7,0.1,0.15)
$Magnesium \quad \leq 100mg/L$	(0.8,0.05,0.1)	(0.8,0.1,0.1)
Calcium $\leq$ 100 mg/L	(0.7,0.1,0.2)	(0.8,0.1,0.1)
$Sulfate > \quad 250mg/L$	(0.6,0.2,0.2)	(0.6,0.1,0.3)
$Sodium > 100mg/L$	(0.55,0.2,0.3)	(0.6,0.1,0.3)
$Iron > \quad 0.3mg/L$	(0.6,0.2,0.1)	(0.65,0.2,0.1)
$Arsenic < \quad 0.05mg/L$	(0.7,0.2,0.1)	(0.7,0.1,0.2)
Manganese $\leq$ 100mg/L	(0.75,0.1,0.15)	(0.7,0.1,0.1)
$Lead < \quad 0.05mg/L$	(0.6,0.2,0.2)	(0.65,0.2,0.15)
$Zinc < \quad 5mg/L$	(0.75,0.2,0.1)	(0.7,0.1,0.1)

Table 19. Neutrosophic values for alternative $\boldsymbol{S_3}$

S_3	$D\mathcal{M}_1$	$D\mathcal{M}_2$
$Color \leq \quad 15$TCU	(0.9,0.05,0.05)	(0.85,0.15,0.1)

$Turbidity = 5\ NTU$	(0.8,0.1,0.1)	(0.7,0.2,0.1)
$pH = 6.5 - 8.5$	(0.9,0.05,0.05)	(0.85,0.1,0.05)
Odor =resinous	(0.85,0.1,0.05)	(0.8,0.1,0.1)
$Chloride > 250\ mg/\ L$	(0.65,0.1,0.02)	(0.65,0.1,0.25)
Fluoride ≤1.5mg/L	(0.8,0.1,0.1)	(0.7,0.1,0.2)
$Magnesium > 100mg/L$	(0.7,0.1,0.1)	(0.7,0.1,0.2)
Calcium >100 mg/L	(0.7,0.1,0.2)	(0.75,0.1,0.15)
$Sulfate \leq 250mg/L$	(0.8,0.05,0.1)	(0.8,0.1,0.1)
$Sodium > 100mg/L$	(0.55,0.2,0.1)	(0.6,0.1,0.2)
$Iron = 0.3mg/L$	(0.7,0.2,0.1)	(0.7,0.2,0.1)
$Arsenic < 0.05mg/L$	(0.7,0.2,0.1)	(0.7,0.1,0.2)
Manganese ≤100mg/L	(0.75,0.1,0.15)	(0.7,0.1,0.1)
$Lead < 0.05mg/L$	(0.6,0.2,0.2)	(0.65,0.2,0.15)
$Zinc > 5mg/L$	(0.7,0.2,0.15)	(0.7,0.1,0.15)

Step 3: Find informational energies of m-PNHSs using the formula:

$$\varsigma_{m-PWNHSs}(\wp,\ddot{A}) =$$

$$\sum_{k=1}^{m}\Omega_k\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\gamma_i\mathcal{T}^{i}_{\wp(\breve{a}_k)i}(v_i)\right)^2+\sum_{j=1}^{q}\left(\sum_{i=1}^{n}\gamma_i\mathcal{I}^{j}_{\wp(\breve{a}_k)j}(v_i)\right)^2+\sum_{k=1}^{r}\left(\sum_{i=1}^{n}\gamma_i\mathfrak{C}^{k}_{\wp(\breve{a}_k)k}(v_i)\right)^2\right)\right)$$

we'll find the weighted informational energies for $\boldsymbol{\omega}$ consider,

$\boldsymbol{DM} = \{\boldsymbol{DM_1}\ ,\boldsymbol{DM_2}\ \}$ be the set of decision makers $\{\boldsymbol{\Omega_1} = \boldsymbol{0.6}\ ,\boldsymbol{\Omega_2} = \boldsymbol{0.4}\}^{\boldsymbol{T}}$, who assign weights to the sub-attributes. i.e. $\boldsymbol{\gamma} = \{\boldsymbol{\gamma_1 = 0.06, \gamma_2 = 0.065, \gamma_3 = 0.065, \gamma_4 = 0.06, \gamma_5 = 0.05, \gamma_6 = 0.05, \gamma_7 = 0.06, \gamma_6 = 0.06, \gamma_8 = 0.05, \gamma_9 = 0.05, \gamma_{10} = 0.065, \gamma_{11} = 0.06, \gamma_{12} = 0.06, \gamma_{13} = 0.06, \gamma_{14} = 0.065, \gamma_{15} = 0.06}\}$

The overall sum of the attributives values of the selected samples are listed below;

$$\varsigma_{m-PWNHSs}(\boldsymbol{\omega}) = 0.5473655$$

$$\varsigma_{m-PWNHSs}(\boldsymbol{S_1}) = 0.48561235$$

Step 4: Now we'll calculate correlation by using the formula:

$$\mathcal{C}_{m-PWNHSs}((\wp,\ddot{A}),(\boldsymbol{Q},\ddot{\boldsymbol{B}})) =$$

$$\sum_{k=1}^{m}\Omega_k\left(\left(\left(\sum_{i=1}^{p}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{T}^{i}_{\wp(\check{a}_k)i}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{T}^{i}_{Q(\check{a}_k)i}(v_i)\right)\right.\right.\right.$$
$$+\sum_{j=1}^{q}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{I}^{j}_{\wp(\check{a}_k)j}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathcal{I}^{j}_{Q(\check{a}_k)j}(v_i)\right)$$
$$\left.\left.\left.+\sum_{k=1}^{r}\left(\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathfrak{C}^{k}_{\wp(\check{a}_k)k}(v_i)*\sum_{i=1}^{n}\sqrt{\gamma_i}\,\mathfrak{C}^{k}_{Q(\check{a}_k)k}(v_i)\right)\right)\right)\right)$$

$$\mathcal{C}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_1}) = 0.50206$$

Step 5: Calculate the WCC between two m-PNHSs by using the formula;

$$\boldsymbol{\delta}_{m-PWNHSs}((\wp,\ddot{A}),(\boldsymbol{Q},\ddot{\boldsymbol{B}})) = \frac{\mathcal{C}_{m-PWNHSs}((\wp,\ddot{A}),(Q,\ddot{B}))}{\sqrt{\varsigma_{m-PWNHSs}(\wp,\ddot{A})}*\sqrt{\varsigma_{m-PWNHSs}(Q,\ddot{B})}}$$

$$\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_1}) = \frac{\mathbf{0.50206}}{\sqrt{\mathbf{0.547365}}*\sqrt{\mathbf{0.48561235}}}$$

$$\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_1}) = 0.9743$$

repeating the algorithm for sample $\boldsymbol{S_2}$ ***and*** $\boldsymbol{S_3}$, we get;

$$\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_2}) = 0.8645$$

$$\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_3}) = 0.9571$$

Step 6: Arrange alternatives in descending order of values obtained in step 5.

$$\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_1}) > \boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_3}) > \boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}, \boldsymbol{S_2})$$

Which means that $\boldsymbol{S_1}$ is the best choice. The sample $\boldsymbol{S_2}$ and $\boldsymbol{S_3}$ are also safe for drinking purposes, since the value of their weighted correlation coefficient is positive and above 0.50.

Note: We can also use the above method to analyze the ranking of mineral water, for optimal choice (e.g. Aquafina, Nestle, Gourmet etc.), list their parameters, find the Weighted correlation coefficient by computing each alternative with the safe drinking mineral water according to national standard (as taken $\boldsymbol{\omega}$) in the above case study. Analyze the ranking of each alternative, maximum value of weighted correlation coefficient would decide the best choice.

Result Discussion

Molodtsov's SS theory was highly beneficial in solving decision-making issues, but it only deals with attributes of alternatives about characteristics, thus direct comparison of two sets of variables was easy. If these attributes are further bi-furcated (Hypersoft set structure) and DM wants to analyze the comparison between two sets then it can be done with the help of correlation coefficients, in this regard [25] introduces the idea of correlation coefficient of NHSS. The decision-making in SVNHSS is limited to a single expert/decision-maker, there is a possibility that we will not arrive at the optimal solution. To cope with multi-valued numbers, Saqlain *et. al.* [15] present the idea of m-polar NHSS, since if there is more than one expert/decision-maker, decision making becomes more accurate, unlike SVNHSS. We solve two case studies using the proposed techniques: the first was based on the selection of a suitable mathematics teacher, and the second was based on the determination of drinking water quality. Using the proposed technique, decision-making becomes more accurate because more than one expert is involved, and each expert assign truthiness, indeterminacy, and falsity values based on his/her knowledge and expertise.

The first case study was the selection of mathematics instructor at LGU. The Algorithm 1, of m-PNHSs was used to address this decision-making dilemma. Attributes/parameters provided by the university administration were tabulated in a column, and each attribute/parameter was valued by multiple experts based on each candidate's academic and interview reliability. Finally, we computed overall performance value of each candidate using the proposed/developed CC of m-PNHSs. The calculated results are, $\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^2) = \mathbf{0.885}$ $> \boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^4) = \mathbf{0.880}$ $>$ $\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^1)$=0.877 $>$ $\boldsymbol{\delta}_{\mathbf{m-PNHSs}}(\aleph, \mathbb{P}^3)$=0.774 which shows that $\mathbb{P}^2$is the most suitable alternative, therefore $\mathbb{P}^2$ is the best alternative for the position of mathematics teacher at LGU.

The second case study included determining the quality of drinking water using the WCC of m-PNHSS. Different water quality experts have assigned Truthiness, indeterminacy, and falsity values to various parameters (e.g. color, odor, turbidity, pH, Sodium, Magnesium, Iron, Chloride, Fluoride, Lead, Manganese, Calcium, Iron, Zinc, Arsenic, and so on) to achieve an ideal/safe drinking water while keeping in mind the National and International water quality standards. Samples of drinking water were analyzed by several water quality experts and they have assigned different values of Truthiness, Indeterminacy, and Falsity for each present parameter in the given sample of water. Finally, we used the WCC m-PNHSS to compare the results provided by experts for the given sample to the values provided by experts for an ideal/safe drinking water. The WCC of the m-PNHSS determines whether or not the water sample is safe to consume. If the WCC value is closer to 1 or 100 percent, it is safe to drink; if it is less than 0.50 or 50 percent, it is dangerous for drinking and requires treatment before being used for drinking. The results we obtain after applying the WCC proposed technique are; $\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}\ , \boldsymbol{S}_1) = \mathbf{0.9743}$ $>$ $\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}\ , \boldsymbol{S}_3) = \mathbf{0.9571}$ $>$ $\boldsymbol{\delta}_{m-PWNHSs}(\boldsymbol{\omega}\ , \boldsymbol{S}_2) = \mathbf{0.8645}$ shows that all the samples are safe for the drinking and their ranking as well. The presented approaches can be used to pick the best mineral water in the future. Because some local businesses offer mineral water, but it is conceivable that it is unsafe to drink, we may use the presented approach to determine which mineral water is the best and safest to consume.

The Advantages / Limitations of the proposed result

The fuzzy soft set theory is not particularly efficient in selecting the ideal object of a decision-making issue that possesses some attributes which are further divided, however m-polar neutrosophic hypersoft set theory can be employed. The advantages of the proposed theory are;

1. This new method's specialty is that it may answer any MADM problem including a big number of decision-makers very quickly along with a simple computing approach.
2. The proposed operators are consistent and accurate when compared to existing approaches for MADM problems in a neutrosophic context, demonstrating their applicability.

3. The suggested method analyses the interrelationships of qualities in practical application; while existing approaches cannot.

6. Conclusions

The correlation coefficient (CC) and weighted correlation coefficient (WCC) of the m-polar neutrosophic hypersoft set (m-PNHSs) are established in this article, as well as some basic properties of the developed correlation coefficient (CC) and weighted correlation coefficient (WCC) under m-PNHSs. The algorithm using CC and WCC are developed to solve MCDM problems. Finally, two case studies have been addressed. We gain greater accuracy in decision making using CC and WCC of m-PNHSs (proposed approach), especially in selecting the best alternative because of numerous experts' viewpoints. Unlike the linguistic method, when a single person makes the decision and the alternative is chosen solely on the basis of that person's knowledge and experience. The proposed concept may be used to handle decision-making difficulties in the education system, the medical field, engineering, and economics, and among other fields.

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