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Tangent and Cotangent Similarity Measures of Pentapartitioned Neutrosophic Pythagorean Set in Virtual Education During Covid Pandemic

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Abstract: In this paper, a new tangent and cotangent similarity measures between two Pentapartitioned Neutrosophic Pythagorean [PNP] sets with truth membership, falsity membership, ignorance and contradiction membership as dependent Neutrosophic component is proposed and its properties are investigated. The unknown membership alone will be considered as independent Neutrosophic components. Also, the weighted similarity measures are also studied with a decision making problem.

Keywords: PNP set, Tangent similarity measure, cotangent similarity measure.

1. Introduction

Traditionally, the teaching and learning method uses several exercises fixing, sending and evaluating ideas and information about a subject. Learning is that the method of getting relative permanent changes in understanding, attitude, knowledge, information, capability and skill through expertise. A modification are often set or involuntary, to raised or worse learning. The training method is an enclosed cognitive event. To assist this teaching and learning method, it is necessary the utilization of a laptoop tool ready to stimulate these changes. Also, it is necessary that it will operate as validation and serving tool to the college students.

The COVID-19 pandemic has caused important disruption with in the domain of education, that is considered as essential determinant for economic progress of any country. Even developed countries are waging a battle against COVID-19 for minimizing the impact on their economy because of prolonged lockdown. Education sector isn't an exception, and method of educational delivery has been grossly affected. There has been unforeseen and impetuous transition from real classroom to on-line and virtual teaching methodology across the world. There's an enormous question on the sustainability of online mode of teaching post-pandemic and its percussions on world education market. Impact of lockdown on the teaching—learning method has been studied in present paper with the objective to assess the quality of online classes and challenges associated with them. The paper proposes about the benefits of social media in virtual education among College Students

In order to deal with uncertainties, the thought of fuzzy sets and fuzzy set operations was introduced by Zadeh [17]. The speculation of fuzzy topological space was studied and developed by C.L. Chang [3]. The paper of Chang sealed the approach for the subsequent growth of the various fuzzy topological ideas. Since then a lot of attention has been paid to generalize the fundamental ideas of general topology in fuzzy setting and therefore a contemporary theory of fuzzy topology has been developed. Atanassov and plenty of researchers [1] worked on intuitionistic fuzzy sets within the literature. Florentine Smarandache [15] introduced the idea of Neutrosophic set in 1995 that provides the information of neutral thought by introducing the new issue referred to as uncertainty within the set. Thus neutrosophic set was framed and it includes the parts of truth membership function(T), indeterminacy membership function(I), and falsity membership function(F) severally. Neutrosophic sets deals with non normal interval of]−0 1+[. Pentapartitioned neutrosophic set and its properties were introduced by Rama Malik and Surpati Pramanik [14]. In this case, indeterminacy is divided into three components: contradiction, ignorance, and an unknown membership function. The concept of Pentapartitioned neutrosophic pythagorean sets was initiated by R. Radha and A. Stanis Arul Mary[9].

Similarity measure is an important topic in the current fuzzy, Pythagorean , Neutrosophic and different hybrid environments. Recently, the improved correlation coefficients of Pentapartitioned Neutrosophic Pythagorean sets and Quadripartitioned Neutrosophic Pythagorean sets was introduced by R. Radha and A. Stanis Arul Mary. Pranamik and Mondal [5,6]has also proposed weighted similarity measures based on tangent function and cotangent function and its application on medical diagnosis. In this paper, the weighted similarity measures of Tangent and Cotangent functions has been applied to PNP sets in virtual education during Covid Pandemic.

2. Preliminaries

2.1 Definition [15]

Let X be a universe. A Neutrosophic set A on X can be defined as follows:

$$A = \{< x, T_A(x), I_A(x), F_A(x) >: x \in X\}$$

Where $T_A, I_A, F_A: U \to [0,1]$ and $0 \le T_A(x) + I_A(x) + F_A(x) \le 3$

2.2 Definition [9]

Let X be a universe. A Pentapartitioned neutrosophic pythagorean [PNP] set A with T, F, C and U as dependent neutrosophic components and I as independent component for A on X is an object of the form

$$A = \{< x, T_A, C_A, I_A, U_A, F_A >: x \in X\}$$

Where $T_A + F_A \leq 1, C_A + U_A \leq 1$ *and*

$$(T_A)^2 + (C_A)^2 + (I_A)^2 + (U_A)^2 + (F_A)^2 \leq 3$$

Here, $T_A(x)$ is the truth membership, $C_A(x)$ is contradiction membership, $U_A(x)$ is ignorance membership, $F_A(x)$ is the false membership and $I_A(x)$ is an unknown membership.

2.3 Definition [14]

Let P be a non-empty set. A Pentapartitioned neutrosophic set A over P characterizes each element p in P a truth -membership function T_A, a contradiction membership function C_A, an ignorance membership function G_A, unknown membership function U_A and a false membership function F_A, such that for each p in P

$$T_A + C_A + G_A + U_A + F_A \leq 5$$

2.4 Definition [9]

The complement of a pentapartitioned neutrosophic pythagorean set A on R Denoted by A^C or A^* and is defined as

$A^C = \{< x, F_A(x), U_A(x), 1 - G_A(x), C_A(x), T_A(x) > : x \in X\}$

2.5 Definition [9]

Let A = $< x, T_A(x), C_A(x), G_A(x), U_A(x), F_A(x) >$ and

B = $< x, T_B(x), C_B(x), G_B(x), U_B(x), F_B(x) >$ are pentapartitioned neutrosophic pythagorean sets.

Then

A ∪ B = $< x, max(T_A(x), T_B(x)), max(C_A(x), C_B(x)), \min(G_A(x), G_B(x)),$

$\min(U_A(x), U_B(x)), \min(F_A(x), F_B(x)), >$

A ∩ B = $< x, min(T_A(x), T_B(x)), min(C_A(x), C_B(x)), \max(G_A(x), G_B(x))$

$$, max(U_A(x), U_B(x)), max(F_A(x), F_B(x)) >$$

2.6 Definition[9]

A PNP topology on a nonempty set R is a family of a PNP sets in R satisfying the following axioms

1) $0,1 \in r$
2) $R_1 \cap R_2 \in r$ for any $R_1, R_2 \in r$
3) $\cup R_i \in r$ for any $R_i: i \in I \subseteq r$

The complement R^* of PNP open set (PNPOS, in short) in PNP topological space [PNPTS] (R,r), is called a PNP closed set [PNPCS].

3. Tangent and Cotangent Similarity Measures of PNP Sets

3.1 Definition

Let P = {(r, $B1_P(r), B2_P(r), B3_P(r), B4_P(r), B5_P(r)): r \in R$} and

Q = {(r, $B1_Q(r)$, $B2_Q(r)$, $B3_Q(r)$, $B4_Q(r)$, $B5_Q)(r)$: $r \in R$} be two Pentapartitioned Neutrosophic Pythagorean numbers with B1 and B5, B2 and B4 as dependent Neutrosophic components. Now tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as follows

$$T_{PNP}(P,Q) = \frac{1}{n}\sum_{i=1}^{n}[1 - \tan(\frac{\pi}{20}[|B1_P^2(r_i) - B1_Q^2(r_i)| + |B2_P^2(r_i) - B2_Q^2(r_i)| + |B3_P^2(r_i) - B3_Q^2(r_i)| + |B4_P^2(r_i) - B4_Q^2(r_i)| + |B5_P^2 - B5_Q^2|(r_i)])]$$

3.2 Theorem

The defined tangent similarity measure $T_{PNP}(P,Q)$ between PNP set P and Q satisfies the following properties

1. $0 \le T_{PNP}(P,Q) \le 1$;
2. $T_{PNP}(P,Q) = 1$ iff P = Q;
3. $T_{PNP}(P,Q) = T_{PNP}(Q,P)$;
4. If T is a PNP set in R and P $\subseteq Q \subseteq T$ then

$T_{PNP}(P,T) \le T_{PNP}(P,Q)$ and $T_{PNP}(P,T) \le T_{PNP}(Q,T)$.

Proof

1) As the truth membership, contradiction membership, ignorance membership, falsity membership and the unknown membership function of the PNP sets and the value of the tangent function also is within [0,1].

Hence $0 \le T_{PNP}(P,Q) \le 1$.

2) For any two PNP sets P and Q if P = Q, this implies $B1_P(r_i) = B1_Q(r_i)$, $B2_P(r_i) = B2_Q(r_i)$, $B3_P(r_i) = B3_Q(r_i)$, $B4_P(r_i) = B4_Q(r_i)$ and $B5_P(r_i) = B5_Q(r_i)$.

Hence $|B1_P^2(r_i) - B1_Q^2(r_i)| = 0, |B2_P^2(r_i) - B2_Q^2(r_i)| = 0, |B3_P^2(r_i) - B3_Q^2(r_i)| = 0,$ $|B4_P^2(r_i) - B4_Q^2(r_i)| = 0$ and $|B5_P^2(r_i) - B5_Q^2(r_i)|$.

Thus $T_{PNP}(P,Q) = 1$.

Conversely, if $T_{PNP}(P,Q) = 1$, then $|B1_P^2(r_i) - B1_Q^2(r_i)| = 0, |B2_P^2(r_i) - B2_Q^2(r_i)| = 0$, $|B3_P^2(r_i) - B3_Q^2(r_i)| = 0$, $|B4_P^2(r_i) - B4_Q^2(r_i)| = 0$ and $|B5_P^2(r_i) - B5_Q^2(r_i)|$ since tan(0) = 0. So we can write $B1_P(r_i) = B1_Q(r_i)$, $B2_P(r_i) = B2_Q(r_i)$, $B3_P(r_i) = B3_Q(r_i)$, $B4_P(r_i) = B4_Q(r_i)$ and $B5_P(r_i) = B5_Q(r_i)$.

Hence P = Q.

3) The Proof is obvious

4) If P $\subseteq Q \subseteq T$ then $B1_P(r_i) \le B1_Q(r_i) \le B1_T(r_i)$, $B2_P(r_i) \le B2_Q(r_i) \le B2_T(r_i)$, $B3_P(r_i) \le B3_Q(r_i) \le B3_T(r_i)$, $B4_P(r_i) \le B4_Q(r_i) \le B4_T(r_i)$ and $B5_P(r_i) \le B5_Q(r_i) \le B5_T(r_i)$.

$$|B1_P^2(r_i) - B1_Q^2(r_i)| \le |B1_P^2(r_i) - B1_T^2(r_i)|,$$
$$|B1_Q^2(r_i) - B1_T^2(r_i)| \le |B1_P^2(r_i) - B1_T^2(r_i)|,$$

$$|B2^2_P(r_i) - B2^2_Q(r_i)| \le |B2^2_P(r_i) - B2^2_T(r_i)|,$$
$$|B2^2_Q(r_i) - B2^2_T(r_i)| \le |B2^2_P(r_i) - B2^2_T(r_i)|,$$
$$|B3^2_P(r_i) - B3^2_Q(r_i)| \le |B3^2_P(r_i) - B3^2_T(r_i)|,$$
$$|B3^2_Q(r_i) - B3^2_T(r_i)| \le |B3^2_P(r_i) - B3^2_T(r_i)|,$$
$$|B4^2_P(r_i) - B4^2_Q(r_i)| \le |B4^2_P(r_i) - B4^2_T(r_i)|,$$
$$|B4^2_Q(r_i) - B4^2_T(r_i)| \le |B4^2_P(r_i) - B4^2_T(r_i)|,$$
$$|B5^2_P(r_i) - B5^2_Q(r_i)| \le |B5^2_P(r_i) - B5^2_T(r_i)|,$$
$$|B5^2_Q(r_i) - B5^2_T(r_i)| \le |B5^2_P(r_i) - B5^2_T(r_i)|.$$

Thus,

$T_{PNP}(P,T) \le T_{PNP}(P,Q)$ and $T_{PNP}(P,T) \le T_{PNP}(Q,T)$

Since tangent function is increasing in the interval $[0, \frac{\pi}{4}]$.

3.3 Definition

Let P = {(r, $B1_P(r), B2_P(r), B3_P(r), B4_P(r), B5_P(r)): r \in R\}$ and

Q = {(r, $B1_Q(r), B2_Q(r), B3_Q(r), B4_Q(r), B5_Q)(r): r \in R\}$ be two Pentapartitioned Neutrosophic Pythagorean numbers with B1 and B5, B2 and B4 as dependent Neutrosophic components. Now weighted tangent similarity function which measures the similarity between two vectors based only on the direction, ignoring the impact of the distance between them can be presented as follows

$$T_{WPNP}(P,Q) = \sum_{i=1}^{n} w_i [1 - \tan(\frac{\pi}{20}[|B1^2_P(r_i) - B1^2_Q(r_i)| + |B2^2_P(r_i) - B2^2_Q(r_i)| + |B3^2_P(r_i) - B3^2_Q(r_i)| + |B4^2_P(r_i) - B4^2_Q(r_i)| + |B5^2_P - B5^2_Q|(r_i)])]$$

Where $w_i \in [0,1], i = 0,1,2 \ldots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}, i = 0,1,2 \ldots, n$, then $T_{WPNP}(P,Q) = T_{PNP}(P,Q)$.

3.4 Theorem

The defined weighted tangent similarity measure $T_{WPNP}(P,Q)$ between PNP set P and Q satisfies the following properties

1) $0 \le T_{WPNP}(P,Q) \le 1$;
2) $T_{WPNP}(P,Q) = 1$ iff P = Q;
3) $T_{WPNP}(P,Q) = T_{WPNP}(Q,P)$;
4) If T is a PNP set in R and $P \subseteq Q \subseteq T$ then

$T_{WPNP}(P,T) \le T_{WPNP}(P,Q)$ and $T_{WPNP}(P,T) \le T_{WPNP}(Q,T)$.

Proof

1) As the truth membership, contradiction membership, ignorance membership, falsity membership and the unknown membership function of the PNP sets and the value of the tangent function also is within [0,1] and Where $w_i \in [0,1], i = 0,1,2 \ldots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$.

Hence $0 \le T_{WPNP}(P,Q) \le 1$.

2) For any two PNP sets P and Q if P = Q, this implies $B1_P(r_i) = B1_Q(r_i), B2_P(r_i) = B2_Q(r_i)$, $B3_P(r_i) = B3_Q(r_i)$, $B4_P(r_i) = B4_Q(r_i)$ and $B5_P(r_i) = B5_Q(r_i)$.
Hence $|B1^2_P(r_i) - B1^2_Q(r_i)| = 0, |B2^2_P(r_i) - B2^2_Q(r_i)| = 0, |B3^2_P(r_i) - B3^2_Q(r_i)| = 0$,
$|B4^2_P(r_i) - B4^2_Q(r_i)| = 0$ and $|B5^2_P(r_i) - B5^2_Q(r_i)|$.

Thus $T_{WPNP}(P,Q) = 1$.
Conversely, if $T_{WPNP}(P,Q) = 1$, then $|B1^2_P(r_i) - B1^2_Q(r_i)| = 0, |B2^2_P(r_i) - B2^2_Q(r_i)| = 0$, $|B3^2_P(r_i) - B3^2_Q(r_i)| = 0$, $|B4^2_P(r_i) - B4^2_Q(r_i)| = 0$ and $|B5^2_P(r_i) - B5^2_Q(r_i)|$ since tan(0) = 0. So we can write

$B1_P(r_i) = B1_Q(r_i)$, $B2_P(r_i) = B2_Q(r_i)$, $B3_P(r_i) = B3_Q(r_i)$, $B4_P(r_i) = B4_Q(r_i)$ and $B5_P(r_i) = B5_Q(r_i)$.

Hence P = Q.

3) The Proof is obvious

4) If $P \subseteq Q \subseteq T$ then $B1_P(r_i) \leq B1_Q(r_i) \leq B1_T(r_i)$, $B2_P(r_i) \leq B2_Q(r_i) \leq B2_T(r_i)$, $B3_P(r_i) \leq B3_Q(r_i) \leq B3_T(r_i)$, $B4_P(r_i) \leq B4_Q(r_i) \leq B4_T(r_i)$ and $B5_P(r_i) \leq B5_Q(r_i) \leq B5_T(r_i)$ and $\sum_{i=1}^{n} w_i = 1$.

$$|B1^2_P(r_i) - B1^2_Q(r_i)| \leq |B1^2_P(r_i) - B1^2_T(r_i)|,$$
$$|B1^2_Q(r_i) - B1^2_T(r_i)| \leq |B1^2_P(r_i) - B1^2_T(r_i)|,$$
$$|B2^2_P(r_i) - B2^2_Q(r_i)| \leq |B2^2_P(r_i) - B2^2_T(r_i)|,$$
$$|B2^2_Q(r_i) - B2^2_T(r_i)| \leq |B2^2_P(r_i) - B2^2_T(r_i)|,$$
$$|B3^2_P(r_i) - B3^2_Q(r_i)| \leq |B3^2_P(r_i) - B3^2_T(r_i)|,$$
$$|B3^2_Q(r_i) - B3^2_T(r_i)| \leq |B3^2_P(r_i) - B3^2_T(r_i)|,$$
$$|B4^2_P(r_i) - B4^2_Q(r_i)| \leq |B4^2_P(r_i) - B4^2_T(r_i)|,$$
$$|B4^2_Q(r_i) - B4^2_T(r_i)| \leq |B4^2_P(r_i) - B4^2_T(r_i)|,$$
$$|B5^2_P(r_i) - B5^2_Q(r_i)| \leq |B5^2_P(r_i) - B5^2_T(r_i)|,$$
$$|B5^2_Q(r_i) - B5^2_T(r_i)| \leq |B5^2_P(r_i) - B5^2_T(r_i)|.$$

Thus,

$T_{WPNP}(P,T) \leq T_{WPNP}(P,Q)$ and $T_{WPNP}(P,T) \leq T_{WPNP}(Q,T)$
Since tangent function is increasing in the interval $[0, \frac{\pi}{4}]$.

3.5 Definition

Assume that P = {(r, $B1_P(r), B2_P(r), B3_P(r), B4_P(r), B5_P(r)): r \in R$} and
Q = {(r, $B1_Q(r), B2_Q(r), B3_Q(r), B4_Q(r), B5_Q)(r): r \in R$} are two Pentapartitioned Neutrosophic Pythagorean numbers with B1 and B5, B2 and B4 as dependent Neutrosophic components. A cotangent similarity measure between two PNP sets P and Q is proposed as follows

$$COT_{PNP}(P,Q) = \frac{1}{n}\sum_{i=1}^{n} [\cot(\frac{\pi}{20}[5 + |B1^2_P(r_i) - B1^2_Q(r_i)| + |B2^2_P(r_i) - B2^2_Q(r_i)| + |B3^2_P(r_i) - B3^2_Q(r_i)| + |B4^2_P(r_i) - B4^2_Q(r_i)| + |B5^2_P - B5^2_Q|(r_i)])]$$

3.6 Theorem

The cotangent similarity measure $COT_{PNP}(P,Q)$ between PNP set P and Q also satisfies the following properties

1) $0 \leq COT_{PNP}(P,Q) \leq 1$;
2) $COT_{PNP}(P,Q) = 1$ iff P = Q;
3) $COT_{PNP}(P,Q) = COT_{PNP}(Q,P)$;
4) If T is a PNP set in R and $P \subseteq Q \subseteq T$ then

$COT_{PNP}(P,T) \leq COT_{PNP}(P,Q)$ and $COT_{PNP}(P,T) \leq COT_{PNP}(Q,T)$.

3.7 Definition

Assume that $P = \{(r, B1_P(r), B2_P(r), B3_P(r), B4_P(r), B5_P(r)): r \in R\}$ and
$Q = \{(r, B1_Q(r), B2_Q(r), B3_Q(r), B4_Q(r), B5_Q)(r): r \in R\}$ are two Pentapartitioned Neutrosophic Pythagorean numbers with B1 and B5, B2 and B4 as dependent Neutrosophic components. A weighted cotangent similarity measure between two PNP sets P and Q is proposed as follows

$$COT_{WPNP}(P,Q) = \sum_{i=1}^{n} w_i [\cot(\frac{\pi}{20}[5 + |B1_P^2(r_i) - B1_Q^2(r_i)| + |B2_P^2(r_i) - B2_Q^2(r_i)| + |B3_P^2(r_i) - B3_Q^2(r_i)| + |B4_P^2(r_i) - B4_Q^2(r_i)| + |B5_P^2 - B5_Q^2|(r_i)])]$$

Where $w_i \in [0,1], i = 0,1,2 \dots n$ are the weights and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}, i = 0,1,2 \dots, n$, then $COT_{WPNP}(P,Q) = COT_{PNP}(P,Q)$.

3.8 Theorem

The weighted cotangent similarity measure $COT_{PNP}(P,Q)$ between PNP set P and Q also satisfies the following properties

1) $0 \leq COT_{WPNP}(P,Q) \leq 1$;
2) $COT_{WPNP}(P,Q) = 1$ iff P = Q;
3) $COT_{WPNP}(P,Q) = COT_{WPNP}(Q,P)$;
4) If T is a PNP set in R and $P \subseteq Q \subseteq T$ then

$COT_{WPNP}(P,T) \leq COT_{WPNP}(P,Q)$ and $COT_{WPNP}(P,T) \leq COT_{WPNP}(Q,T)$.

Proof

1) As the truth membership, contradiction membership, ignorance membership, falsity membership and the unknown membership function of the PNP sets and the value of the tangent function also is within [0,1] and $\sum_{i=1}^{n} w_i = 1$.
Hence $0 \leq COT_{WPNP}(P,Q) \leq 1$.
2) For any two PNP sets P and Q if P = Q, this implies $B1_P(r_i) = B1_Q(r_i)$, $B2_P(r_i) = B2_Q(r_i)$, $B3_P(r_i) = B3_Q(r_i)$, $B4_P(r_i) = B4_Q(r_i)$ and $B5_P(r_i) = B5_Q(r_i)$.
Hence $|B1_P^2(r_i) - B1_Q^2(r_i)| = 0, |B2_P^2(r_i) - B2_Q^2(r_i)| = 0, |B3_P^2(r_i) - B3_Q^2(r_i)| = 0$, $|B4_P^2(r_i) - B4_Q^2(r_i)| = 0$ and $|B5_P^2(r_i) - B5_Q^2(r_i)|$.
Thus $COT_{WPNP}(P,Q) = 1$.
Conversely, if $T_{WPNP}(P,Q) = 1$, then $|B1_P^2(r_i) - B1_Q^2(r_i)| = 0, |B2_P^2(r_i) - B2_Q^2(r_i)| = 0$, $|B3_P^2(r_i) - B3_Q^2(r_i)| = 0$, $|B4_P^2(r_i) - B4_Q^2(r_i)| = 0$ and $|B5_P^2(r_i) - B5_Q^2(r_i)|$ since tan(0) = 0. So we can write

$B\ 1_P(r_i) = B1_Q(r_i)$, $B\ 2_P(r_i) = B2_Q(r_i)$, $B\ 3_P(r_i) = B3_Q(r_i)$, $B\ 4_P(r_i) = B4_Q(r_i)$ and $B\ 5_P(r_i) = B5_Q(r_i)$.

Hence P = Q.

3) The Proof is obvious

4) If $P \subseteq Q \subseteq T$ then $B1_P(r_i) \le B1_Q(r_i) \le B1_T(r_i)$, $B2_P(r_i) \le B2_Q(r_i) \le B2_T(r_i)$,

$B\ 3_P(r_i) \ge B3_Q(r_i) \ge B3_T(r_i)$, $B\ 4_P(r_i) \ge B4_Q(r_i) \ge B4_T(r_i)$ and $B\ 5_P(r_i) \ge B5_Q(r_i) \ge B5_T(r_i)$ and $\sum_{i=1}^{n} w_i = 1$.

$$|B1_P^2(r_i) - B1_Q^2(r_i)| \le |B1_P^2(r_i) - B1_T^2(r_i)|,$$
$$|B1_Q^2(r_i) - B1_T^2(r_i)| \le |B1_P^2(r_i) - B1_T^2(r_i)|,$$
$$|B2_P^2(r_i) - B2_Q^2(r_i)| \le |B2_P^2(r_i) - B2_T^2(r_i)|,$$
$$|B2_Q^2(r_i) - B2_T^2(r_i)| \le |B2_P^2(r_i) - B2_T^2(r_i)|,$$
$$|B3_P^2(r_i) - B3_Q^2(r_i)| \le |B3_P^2(r_i) - B3_T^2(r_i)|,$$
$$|B3_Q^2(r_i) - B3_T^2(r_i)| \le |B3_P^2(r_i) - B3_T^2(r_i)|,$$
$$|B4_P^2(r_i) - B4_Q^2(r_i)| \le |B4_P^2(r_i) - B4_T^2(r_i)|,$$
$$|B4_Q^2(r_i) - B4_T^2(r_i)| \le |B4_P^2(r_i) - B4_T^2(r_i)|,$$
$$|B5_P^2(r_i) - B5_Q^2(r_i)| \le |B5_P^2(r_i) - B5_T^2(r_i)|,$$
$$|B5_Q^2(r_i) - B5_T^2(r_i)| \le |B5_P^2(r_i) - B5_T^2(r_i)|.$$

The cotangent function is decreasing function within the interval $[0, \frac{\pi}{4}]$.

Hence $\sum_{i=1}^{n} w_i = 1$.

Hence, we can write

$COT_{WPNP}(P,T) \le COT_{WPNP}(P,Q)$ and $COT_{WPNP}(P,T) \le COT_{WPNP}(Q,T)$

4. Decision Making Based on Tangent and Cotangent Similarity Measures

Let $A_1, A_2, \ldots A_m$ be a discrete set of candidates, $C_1, C_2, \ldots C_n$ be the set of criteria for each candidate and D_1, D_2, … . , D_k are the alternatives of each candidate. The decision -maker provides the ranking of alternatives with respect to each candidate. The ranking presents the performance of candidates $A_i (i = 1,2, \ldots m)$ against the criteria C_j $(j = 1,2 \ldots , n)$.The values associated with the alternatives for MADM problem can be presented in the following decision matrix(see Tab 1 and Tab 2). The relation between candidates and attributes are given in Tab 1. The relation between attributes and alternatives are given in the Tab 2.

Table 1 : The relation between candidates and attributes

R_1	C_1	C_2	…	C_n
A_1	a_{11}	a_{12}	…	a_{1n}
A_2	a_{21}	a_{13}	…	a_{2n}
…	…	…	…	…

A_m	a_{m1}	a_{m2}	…	a_{mn}

Table 2 : The relation between attributes and alternatives

R_2	D_1	D_2	…	D_k
C_1	c_{11}	c_{12}	…	c_{1k}
C_2	c_{21}	c_{22}	…	c_{2k}
…	…	…	…	…
C_n	c_{n1}	c_{n2}	…	c_{nk}

Here a_{ij} and c_{ij} are all Pentapartitioned Neutrosophic Pythagorean Fuzzy numbers.
The steps corresponding to Pentapartitioned Neutrosophic Pythagorean number based on tangent and cotangent functions are presented following steps.

Step 1: Determination of the relation between candidates and attributes
The relation between candidate $A_i(i = 1,2, \dots m)$ and the attribute $C_j\,(j = 1,2 \dots n)$ is presented in Table 3.

Table 3 : The relation between candidates and attributes in terms of PNP sets

R_1	C_1	C_2	…	C_n
A_1	$(b1_{11}, b2_{11}, b3_{11}, b4_{11}, b5_{11})$	$(b1_{12}, b2_{12}, b3_{12}, b4_{12}, b5_{12})$	…	$(b1_{1n}, b2_{1n}, b3_{1n}, b4_{1n}, b5_{1n})$
A_2	$(b1_{21}, b2_{21}, b3_{21}, b4_{21}, b5_{21})$	$(b1_{22}, b2_{22}, b3_{22}, b4_{22}, b5_{22})$	…	$(b1_{2n}, b2_{2n}, b3_{2n}, b4_{2n}, b5_{2n})$
…	…	…	…	…
A_m	$(b1_{m1}, b2_{m1}, b3_{m1}, b4_{m1}, b5_{m1})$	$(b1_{m2}, b2_{m2}, b3_{m2}, b4_{m2}, b5_{m2})$	…	$(b1_{mn}, b2_{mn}, b3_{mn}, b4_{mn}, b5_{mn})$

Table 4 : The relation between attributes and alternatives in terms of PNP sets

R_2	D_1	D_2	…	D_k
C_1	$(c1_{11}, cb2_{11}, c3_{11}, c4_{11}, c5_{11})$	$(c1_{12}, c2_{12}, c3_{12}, c4_{12}, c5_{12})$	…	$(c1_{1k}, c2_{1k}, c3_{1k}, c4_{1k}, c5_{1k})$
C_2	$(b1_{21}, b2_{21}, b3_{21}, b4_{21}, b5_{21})$	$(c1_{22}, c2_{22}, c3_{22}, c4_{22}, c5_{22})$	…	$(c1_{2k}, c2_{2k}, c3_{2k}, c4_{2k}, c5_{2k})$
…	…	…	…	…

C_n	$(c1_{n1}, c2_{n1}, c3_{n1}, c4_{n1}, c5_{n1})$	$(c1_{n2}, c2_{n2}, c3_{n2}, c4_{n2}, c5_{n2})$	…	$(c1_{nk}, c2_{nk}, c3_{nk}, c4_{nk}, c5_{nk})$

Step 3: Determination of the relation between attributes and alternatives

Determine the similarity measure between the Tab 3 and Tab 4 using $T_{PNP}(P,Q)$ $T_{WPNP}(P,Q)$, $COT_{PNP}(P,Q)$ and $COT_{WPNP}(P,Q)$.

Step 4: Ranking the alternatives

Ranking the alternatives is prepared based on the descending order of the similarity measures. Highest value reflects the best alternative.

Step 5: End

5. Example

Higher education institutions have faced various challenges in adapting online education to control the pandemic spread of COVID. The present work aims to apply similaty measures between social media and its benefits of students.Let D ={ R1, R2 ,R3} be a set of College student respondents , E = { YouTube, Facebook, WhatsApp, Blog} be Social medias and H ={ Communication Tool, Online Learning, Connecting with experts, Global exposure} be its benefits.The solution strategy is to determine the student regarding the relation between student respondents and its benefits in virtual education (see Tab 5) and the relation between socal media and its benefits in Table 6 . Further we have calculated Tangent and Cotangent similarity measures can be calculated in Table 7 and 8. Also the weighted similarity measures of the tangent and cotangent functions of PNP sets be calculated in Table 9 and 10.

Table 5 : (P1) The relation between respondents and benefits in Virtual Education

P1	*Online Learning*	*Communication Tool*	*Connecting with Experts*	*Global Exposure*
R1	*(0.7,0.2,0.8,0.3,0.3)*	*(0.1,0.2,0.9,0.3,0.7)*	*(0.4,0.2,0.2,0.3,0.6)*	*(0.2,0.2,0.7,0.3,0.8)*
R2	*(0.3,0.2,0.1,0.3,0.5)*	*(0.6,0.2,0.8,0.3,0.4)*	*(0.6,0.2,0.1,0.3,0.4)*	*(0.2,0.2,0.9,0.3,0.7)*
R3	*(0.1,0.2,0.8,0.3,0.5)*	*(0.6,0.2,0.8,0.3,0.4)*	*(0.6,0.2,0.1,0.3,0.4)*	*(0.7,0.2,0.7,0.3,0.3)*

Table 6: (P2) The relation between Social Media and its benefits

P2	*WhatsApp*	*YouTube*	*Facebook*	*Blog*
Online Learning	*(0.4,0.2,0.6.0.3,0.1)*	*(0.1,0.2,0.5,0.3.0.5)*	*(0.2,0.2,0.5,0.3,0.4)*	*(0.2,0.4,0.6,0.7,0.3)*
Communication Tool	*(0.7,0.2,0.9,0.3,0.3)*	*(0.5,0.2,0.9,0.3,0.5)*	*(0.7,0.2,0.1,0.3,0.2)*	*(0.3,0.5,0.8,0.1,0.2)*
Connecting with Experts	*(0.1,0.2,0.5,0.3,0.7)*	*(0.8,0.2.0.1,0.3,0.2)*	*(0.6,0.2,0.8,0.3,0.4)*	*(0.4,0.6,0.2,0.1,0.1)*

Global Exposure	(0.6,0.2,0.3,0.3,0.4)	(0.5,0.2,0.2,0.3,0.5)	(0.6,0.2,0.9,0.3,0.4)	(0.1,0.2,0.3,0.5,0.3)

Table 7: The Tangent Similarity Measure between P1 and P2

Tangent Similarity Measure	WhatsApp	YouTube	Facebook	Blog
R1	**0.9467**	0.9430	0.9148	0.9463
R2	0.9583	**0.9792**	0.9603	0.9350
R3	0.9595	**0.9791**	0.9504	0.9430

Table 8: The Weighted Tangent Similarity Measure between P1 and P2

Weighted Tangent Similarity Measure	WhatsApp	YouTube	Facebook	Blog
R1	**0.9444**	0.9409	0.919	0.9308
R2	0.9547	**0.977**	0.9597	0.9319
R3	0.9635	**0.9771**	0.9565	0.9416

Table 9: The Cotangent Similarity Measure between P1 and P2

Cotangent Similarity Measure	WhatsApp	YouTube	Facebook	Blog
R1	**0.8995**	0.8927	0.8504	0.8706
R2	0.9583	**0.9599**	0.9195	0.8788
R3	0.9244	**0.9488**	0.9092	0.8927

Table 10: The Weighted Cotangent Similarity Measure between P1 and P2

Weighted Cotangent	WhatsApp	YouTube	Facebook	Blog

Similarity Measure				
R1	***0.8957***	*0.889*	*0.8587*	*0.871*
R2	*0.9547*	***0.9561***	*0.9185*	*0.8732*
R3	*0.9308*	***0.9425***	*0.9207*	*0.8904*

The highest similarity measures reflects the benefits of Social Media among College Students. Therefore Student R2 and R3 gains knowledge more from YouTube and R1 from WhatsApp.

6. Conclusion

In this paper, we have proposed tangent and cotangent similarity measures for Pentapartitioned Neutrosophic Pythagorean set with dependent Neutrosophic components and proved some of its basic properties. Furthermore, we have also investigated about the weighted similarity measures in Decision Making and illustrated with an example. In future, we can study about the improved similarity measure for the above set and can be used in Medical Diagnosis, Data mining. Clustering Analysis etc.

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