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On Novel Hellinger Divergence Measure of Neutrosophic Hypersoft Sets in Symptomatic Detection of COVID-19

Himanshu Dhumras[1] and Rakesh Kumar Bajaj[2,*]

[1] Jaypee University of Information Technology, Waknaghat, Solan, HP, INDIA; himanshudhumrash@gmail.com
[2] Jaypee University of Information Technology, Waknaghat, Solan, HP, INDIA; rakesh.bajaj@juit.ac.in

* Correspondence: rakesh.bajaj@juit.ac.in; Tel.: (+91- 9816337725)

Abstract: Numerous decision science processes involves the divergence measure is the most suitable information measure for dealing with the vagueness and impreciseness of the factors affecting the decision-making models. In this manuscript, a new kind of Hellinger information measure for a single-valued neutrosophic hypersoft set along with some important results have been presented and studied in detail. Also, we have presented the implementation of the proposed information measure to deal with the symptomatic detection of COVID-19 with a numerical illustration. In view of the existing methods related to divergence measures, some comparative results and remarks along with some important advantages have also been presented.

Keywords: Single-Valued Neutrosophic Hypersoft Set; Hellinger divergence measure; Decision-making; COVID-19.

1. Introduction

The notion of fuzzy set theory [1], which have been devised in the recent decade by the efforts of the various eminent researchers working in this area. Subsequently, the idea of a "multi-criteria decision-making (*MCDM*)" problem gives a wider range and serves as one of the delightful options in the area of uncertainty and decision sciences. Atanassov [2] devised the notion of an intuitionistic fuzzy set in order to deal with the uncertainty components of indeterminacy/hesitation margin in the inexact/incomplete information which is in the grades of membership and the grades of non-membership degree. In order to cover the incomplete, ambiguous and contradictory information, the concept of the environment of "neutrosophic set" [3] was devised by Samarandache. The neutrosophic set mainly captures the three components of uncertainty, i.e., "degrees of truth, indeterminacy and falsity" which covers a greater span of information and which is applicable in various decision sciences problems [4-8].

In order to utilize the notion of neutrosophic setup in various problems of decision-making are very difficult. To overcome this issue, Wang et al. [9] came up with the notion of "single-valued neutrosophic set *(SVNSS)*" which properly covers all the three uncertainty components and manages the contradictory information. Due to the limitations of the information that humans get or perceive from their environment; all features of the objects portrayed by the *SVNSS* are perfectly acceptable for dealing with the uncertainties. The notion of a *SVNSS* has been expanding swiftly because of its broader area of hypothetical distinction [10-14]. Molodtsov [15] came up with the parametrized idea of "soft set theory" which covers the "parameterization" of the criterions with their respective sub-criterions. Soft sets are of great importance in many problems of decision-making, game theory and artificial intelligence. The idea of a soft set has been further generalized by Maji [16] with the incorporation of a neutrosophic soft set. Then, Samrandache [17] introduces the novel concept of a hypersoft set with the inclusion of sub-attributes of the respective attributes. However, when there are several sub-attributes the theory of hypersoft sets cannot handle such situations. For overcoming such limitations, the idea of neutrosophic hypersoft sets *(NHSS)* was devised with the coutsey of Saqlain et al. [18-21].

In literature, numerous researchers have thoroughly examined the different types of "similarity measures, divergence measures, distance measures, and entropy measures" for various types of fuzzy sets and their extensions because of their practical utility in the many fields of engineering and sciences. The notion of directed divergence measure was first presented by Bhandari and Pal [22] which is the modified version of the information measures as stated by Kullback and Leibler [23]. The "divergence measure" based on exponential measures has been presented by Fan and Xie [24]. Ghosh et al. [25] presented divergence nature of fuzzy measures in the recognition problem of automation. Also, some "divergence measures for intuitionistic fuzzy sets" were given by Shang and Jiang [26]. Hung and Yang [27] introduced the set of axioms for the divergence measures of intuitionistic fuzzy sets by utilizing the Hausdorff metric. Next, Montes et al. [28] developed some major relations between the distance, divergence and dissimilarity measures.

From the above discussions, the degree of indeterminacy and hesitancy is missing from the intuitionistic fuzzy sets which restrict the various experts for assessing the uncertainities. In order to deal with these kind of shortcomings, the surroundings of "neutrosophic sets" is more effective in the various applications of sciences and engineering. Broumi and Smarandache [29] presented the different types of "information measures" for neutrosophic environment. The similarity kind of information measures for *SVNSS* by utilizing the distance measures have been proposed by Majumdar and Samanta [30]. Further, similarity measures for interval neutrosophic sets have been given by Ye [31]. Also, Ye [32] studied the trigonometric similarity measures for single-valued neutrosophic sets and apply them to multi-criteria decision-making problems. The relationships between the different types of information measures with their trigonometric axiomatic definitions have been given by Wu et al. [33]. Also, Thao and Smarandache [34] established a novel divergence measure for neutrosophic sets to solve the problems related to medical and recognition problems. Also, different types of information measures concerning the various extensions of fuzzy sets and fuzzy soft set are already existing in the literature [35-38].

However, there are some "distance and similarity measures" related to neutrosophic hypersoft sets utilized in the *TOPSIS* technique to compute the *MCDM* problems given by Saqlain et al. [39]. Also, "trigonometric similarity measures for neutrosophic hypersoft sets" to solve the renewable energy source selection problem given by Jafar et al. [40]. In addition to these, various other researchers [41-45] have proposed different types of similarity measures and utilized them in various types of pattern recognition problems and other decision-making problems. In the literature, there is similarity/distance measures for both "neutrosophic and single-valued neutrosophic hypersoft sets *(SVNHSS)*" but there are no divergence measures for *SVNHSSs* available. Here, based on the generalized "Hellinger" fuzzy divergence information measure for fuzzy environment given by Ohlan et al. [46], we have presented a novel kind of Hellinger divergence measure for the *SVNHSS* to moderate the research gap in this area.

The remaining of the manuscript is being organized as. Section 2 involves some fundamental notions related to *SVNHSSs* and some basic operations which are already existing in the literature. In Section

3, we present some set-theoretic operations on *SVNHSSs* and a novel notion of the Hellinger divergence measure for two *SVNHSSs*. Also, we establish the validity of the proposed Hellinger divergence measure under the standard axioms. Various important properties related to the Hellinger divergence measure have been studied and discussed in Section 4. Further, by utilizing the proposed divergence measure, the methodology for the symptomatic detection of COVID-19 has been presented in Section 5. An associated numerical example for illustrating the proposed methodology has been solved and presented in Section 6. In Section 7, a brief discussion of results and the proposed algorithmic technique containing some important points on comparative advantages, importance and shortcomings have been presented. In the end, the paper has been concluded in Section 8 with some possible scope for future work.

2. Preliminaries and Fundamental Notions

In this section, some of the fundamental notions in context with the extensions of the neutrosophic set and information measures are presented.

Definition 1. *[47] "Let X be the universal set and $P(X)$ be the power set of X. Consider $k^1, k^2, \ldots, k^n$ for $n \geq 1$ be n well-defined attributes whose corresponding attribute values are respectively the sets $K^1, K^2, \ldots, K^n$ with $K^i \cap K^j = \emptyset$, for $i \neq j$ and $i, j \in \{1,2,..,n\}$, then the pair $(\mathfrak{R}, K^1 \times K^2 \times \ldots \times K^n)$ is said to be Hypersoft set over the set X, where $\mathfrak{R} : K^1 \times K^2 \times \ldots \times K^n \rightarrow P(X)$."*

Definition 2. *[47] "Let X be the universal set and $P(X)$ be the power set of X. Consider $k^1, k^2, \ldots, k^n$ for $n \geq 1$ be n well-defined attributes whose corresponding attribute values are respectively the sets $K^1, K^2, \ldots, K^n$ with $K^i \cap K^j = \emptyset$, for $i \neq j$ and $i, j \in \{1,2,..,n\}$ and their relation $K^1 \times K^2 \times \ldots \times K^n = \Gamma$, then the pair $(\mathfrak{R}, \Gamma)$ is said to be Neutrosophic Hypersoft set (NHSS) over X, where, $\mathfrak{R} : K^1 \times K^2 \times \ldots \times K^n \rightarrow P(X)$ and $\mathfrak{R}(K^1 \times K^2 \times \ldots \times K^n) = \{\langle x, T(\mathfrak{R}(\Gamma)), I(\mathfrak{R}(\Gamma)), F(\mathfrak{R}(\Gamma))\rangle, x \in X\}$; where T is the degree of truthness, I is the degree of indeterminacy and F is the degree of falsity such that T, I, F : $V \rightarrow (0^-, 1^+)$ and satisfies the constraint $0^- \leq T(\mathfrak{R}(\Gamma)) + I(\mathfrak{R}(\Gamma)) + F(\mathfrak{R}(\Gamma) \leq 3^+$.*
While dealing with applications of science and engineering, it becomes very difficult to handle situations under a neutrosophic environment. In order to deal with such situations notion of Single-Valued Neutrosophic HyperSoft sets($SVNHSS$) is very useful and applicable. "

Definition 3. *[48] "Let X be the universal set and $P(X)$ be the power set of X. Consider $k^1, k^2, \ldots, k^n$ for $n \geq 1$ be n well-defined attributes whose corresponding attribute values are respectively the sets $K^1, K^2, \ldots, K^n$ with $K^i \cap K^j = \emptyset$, for $i \neq j$ and $i, j \in \{1,2,..,n\}$ and their relation $K^1 \times K^2 \times \ldots \times K^n = \Gamma$, then the pair $(\mathfrak{R}, \Gamma)$ is said to be a Single-Valued Neutrosophic Hypersoft set ($SVNHSS$) over X, where, $\mathfrak{R} : K^1 \times K^2 \times \ldots \times K^n \rightarrow P(X)$ and $\mathfrak{R}(K^1 \times K^2 \times \ldots \times K^n) = \{\langle x, T(\mathfrak{R}(\Gamma)), I(\mathfrak{R}(\Gamma)), F(\mathfrak{R}(\Gamma))\rangle, x \in X\}$; where T is the degree of truthness, I is the degree of indeterminacy and F is the degree of falsity such that T, I, F : $V \rightarrow [0, 1]$ and satisfies the constraint $0 \leq T(\mathfrak{R}(\Gamma)) + I(\mathfrak{R}(\Gamma)) + F(\mathfrak{R}(\Gamma) \leq 3$. "*

Definition 4. *[49] "Consider A and B be two single-valued neutrosophic sets, then the axiomatic definition of divergence measure are as follows:*

i. $\mathbb{I}(A, B) = \mathbb{I}(B, A)$;
ii. $\mathbb{I}(A, B) \geq 0$ and $\mathbb{I}(A, B) = 0$ iff $A = B$.

iii. $\mathbb{I}(A \cap B,\ B \cap A\ ) \leq\ \mathbb{I}(A, B)\ \ \forall\ \ B \in SVNS(X).$

iv. $\mathbb{I}(A \cup B,\ B \cup A\ ) \leq\ \mathbb{I}(A, B)\ \ \forall\ \ B \in SVNS(X).$ "

3. Binary Operations and Hellinger Divergence Measure of Neutrosophic Hypersoft Sets

In this section, we first propose some binary operations on $\boldsymbol{SVNHSSs}$. We shall denote the collection of $\boldsymbol{SVNHSS}$ on $\boldsymbol{X}$ by $\boldsymbol{SVNHSS(X)}$. Now, for any two $\boldsymbol{SVNHSSs}$ $\boldsymbol{A, B} \in \boldsymbol{SVNHSS(X)}$, analogous to the operations given for the single-valued neutrosophic sets, we define some basic operations as follows:

• "**Union of** A **and** B": $A \cup B = \{x,\ T_{A\cup B}(\mathfrak{R}(\Gamma)),\ I_{A\cup B}(\mathfrak{R}(\Gamma)),\ F_{A\cup B}(\mathfrak{R}(\Gamma))|\ \ x \in X\ \}$

where,

$T_{A\cup B}(\mathfrak{R}(\Gamma))(x) = max\{T_A(\mathfrak{R}(\Gamma))(x),\ T_B(\mathfrak{R}(\Gamma))(x)\}$, $I_{A\cup B}(\mathfrak{R}(\Gamma))(x) = min\{I_A(\mathfrak{R}(\Gamma))(x),$ $I_B(\mathfrak{R}(\Gamma))(x)\}$

and $F_{A\cup B}(\mathfrak{R}(\Gamma))(x) = min\{F_A(\mathfrak{R}(\Gamma))(x),\ F_B(\mathfrak{R}(\Gamma))(x)\}\ \forall\ \ x\ \ \in X.$

• "**Intersection of** A **and** B": $A \cap B = \{x,\ T_{A\cap B}(\mathfrak{R}(\Gamma)),\ I_{A\cap B}(\mathfrak{R}(\Gamma)),\ F_{A\cap B}(\mathfrak{R}(\Gamma))|\ \ x \in X\ \}$

where,

$T_{A\cap B}(\mathfrak{R}(\Gamma))(x) = min\{T_A(\mathfrak{R}(\Gamma))(x),\ T_B(\mathfrak{R}(\Gamma))(x)\}$, $I_{A\cap B}(\mathfrak{R}(\Gamma))(x) = max\{I_A(\mathfrak{R}(\Gamma))(x),$ $I_B(\mathfrak{R}(\Gamma))(x)\}$

and $F_{A\cap B}(\mathfrak{R}(\Gamma))(x) = max\{F_A(\mathfrak{R}(\Gamma))(x),\ F_B(\mathfrak{R}(\Gamma))(x)\}\ \forall\ \ x\ \ \in X.$

•**Containment:** $A \subseteq B$ if and only if

$T_A(\mathfrak{R}(\Gamma))(x) \leq T_B(\mathfrak{R}(\Gamma))(x),\ \ I_A(\mathfrak{R}(\Gamma))(x) \geq$ $I_B(\mathfrak{R}(\Gamma))(x),\ \ F_A(\mathfrak{R}(\Gamma))(x) \geq F_B(\mathfrak{R}(\Gamma))(x)\ \ \forall\ \ x\ \ \in X.$

• "**Complement:** The complement of a neutrosophic hypersoft set A , denoted by $\bar{A}$," defined by

$T_{\bar{A}}(\mathfrak{R}(\Gamma))(x) = 1 -\ \ T_A(\mathfrak{R}(\Gamma))(x),\ I_{\bar{A}}(\mathfrak{R}(\Gamma))(x) = 1 -\ \ I_A(\mathfrak{R}(\Gamma))(x),\ F_{\bar{A}}(\mathfrak{R}(\Gamma))(x) = 1 -\ \ F_A(\mathfrak{R}(\Gamma))(x)$
"

Next, we introduce a novel Hellinger divergence measure for any two $SVNHSS$ with some of its important properties. For any two fuzzy sets, A and B Ohlan et al. [44] presented the generalized form of divergence measure given by Hellinger as follows:

$$d_\gamma(A, B) = \sum_{i=1}^{n}\left(\frac{\left(\sqrt{\mu_A(x_i)} - \sqrt{\mu_B(x_i)}\right)^{2(\gamma+1)}}{\sqrt{\mu_A(x_i)\mu_B(x_i)}} + \frac{\left(\sqrt{\mu_{A^c}(x_i)} - \sqrt{\mu_{B^c}(x_i)}\right)^{2(\gamma+1)}}{\sqrt{\mu_{A^c}(x_i)\mu_{B^c}(x_i)}}\right), \gamma \in\ \ \mathbb{N}. \qquad (1)$$

Now, on similar lines to the above-presented divergence measure given by (1), we introduce the following parameterized divergence measure for single valued neutrosophic hypersoft set:

$$\mathbb{I}_{\gamma}(A,B) = \sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)} - \sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)} - \sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)} - \sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)} - \sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)} - \sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)} - \sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right], \gamma \in \mathbb{N}. \quad (2)$$

Further, to check the validation of the proposed parameterized divergence measure for *SVNHSSs*, we propose the following theorem as follows:

Theorem 1. *The proposed divergence measure* $\mathbb{I}_{\gamma}(A,B)$ *given by* (2) *is a reliable divergence measure for two* $SVNHSSs$.

Proof: In order to check the validation of the proposed divergence measure, we need to check whether (2) satisfies the axioms of the divergence measures given in Section 2.

i. Since (2) holds symmetry for A and B, therefore it is clear that $\mathbb{I}(A,B) = \mathbb{I}(B,A)$.

ii. Also, we observe that $\mathbb{I}(A,B) = 0$ iff

$T_A(\Re(\Gamma))(x) = T_B(\Re(\Gamma))(x)$, $I_A(\Re(\Gamma))(x) = I_B(\Re(\Gamma))(x)$, $F_A(\Re(\Gamma))(x) = F_B(\Re(\Gamma))(x)$ $\forall\ x \in X$.

Now, it remains shown that $\mathbb{I}(A,B) \geq 0$. In order to prove the non-negativity, first we need to prove the convexity of $\mathbb{I}_{\gamma}$. Since, $\mathbb{I}_{\gamma}(A,B)$ is of Csiszar's g-divergence type with generating mapping $g_{\gamma}: (0,\infty) \to \mathbb{R}^{+}$, defined by ,

$$g_{\gamma}(s) = \frac{2^{\gamma}(\sqrt{s}-1)^{2(\gamma+1)}}{(s+1)^{\gamma}} \quad ; \quad g_{\gamma}(1) = 0. \quad (3)$$

Further, differentiate (3) with respect to s two times we get,

$$g_{\gamma}''(s) = \left(\frac{2^{\gamma}}{2}\right)\frac{\left(2s + 2\gamma\sqrt{s} + 2\gamma s^{\frac{3}{2}} + 4\gamma s + s^2 + 1\right)(\gamma+1)(\sqrt{s}-1)^{2\gamma}}{(s+1)^{\gamma+2}s^{3/2}}.$$

Since, $\gamma \in \mathbb{N}$ and $s \in (0,\infty)$, therefore, $g''_{\gamma}(s) \geq 0$ which shows the convexity of $g''_{\gamma}(s)$. Hence, $\mathbb{I}(A,B) \geq 0$.

iii. In order to prove this part, we divide the collection X into two disjoint subsets X_1 and X_2 defined as

$$X_1 = \{x_i \in X |\ T_A(\mathfrak{R}(\Gamma))(x) \geq T_B(\mathfrak{R}(\Gamma))(x) \geq T_C(\mathfrak{R}(\Gamma))(x),\ I_A(\mathfrak{R}(\Gamma))(x) \leq I_B(\mathfrak{R}(\Gamma))(x) \leq I_C(\mathfrak{R}(\Gamma))(x),\ F_A(\mathfrak{R}(\Gamma))(x) \leq F_B(\mathfrak{R}(\Gamma))(x) \leq F_C(\mathfrak{R}(\Gamma))(x)\}; \quad (4)$$

and

$$X_2 = \{x_i \in X |\ T_A(\mathfrak{R}(\Gamma))(x) \geq T_B(\mathfrak{R}(\Gamma))(x) \geq T_C(\mathfrak{R}(\Gamma))(x),\ I_A(\mathfrak{R}(\Gamma))(x) \leq I_B(\mathfrak{R}(\Gamma))(x) \leq I_C(\mathfrak{R}(\Gamma))(x),\ F_A(\mathfrak{R}(\Gamma))(x) \leq F_B(\mathfrak{R}(\Gamma))(x) \leq F_C(\mathfrak{R}(\Gamma))(x)\}. \quad (5)$$

Now, by making use of the definition of neutrosophic sets and (2) in association with (4) and (5), the components of X_1 will vanish and the components of X_2 will only remain on the left-hand side. Hence, the left side will remain with only one term and the right side remain with two terms. The detailed steps of calculation can be shown easily. Therefore, axiom *iii* is satisfied.

iv. The proof of this axiom can be done by the union operation accordingly as the proof of the axiom of *iii*. Therefore, $\mathbb{I}_{\gamma}(A,B)$ is a validated divergence measure between the single-valued neutrosophic hypersoft sets A and B.

4. Properties of Novel Parameterized Neutrosophic Hypersoft Divergence Measure

In this section, we give some of the important properties of the proposed divergence measure in a single-valued neutrosophic environment.

Theorem 2. . *For any A, B and $C \in SVNHSS(X)$, the parametric divergence information measure (2) holds the below mentioned fundamental properties:*

1. "$\mathbb{I}_{\gamma}(A \cup B, A \cap B) = \mathbb{I}_{\gamma}(A,B)$
2. $\mathbb{I}_{\gamma}(A \cup B, A) + \mathbb{I}_{\gamma}(A \cap B, A) = \mathbb{I}_{\gamma}(A,B)$
3. $\mathbb{I}_{\gamma}(A \cup B, C) + \mathbb{I}_{\gamma}(A \cap B, C) = \mathbb{I}_{\gamma}(A,C) + \mathbb{I}_{\gamma}(B,C)$
4. $\mathbb{I}_{\gamma}(A, A \cup B) = \mathbb{I}_{\gamma}(B, A \cap B)$
5. $\mathbb{I}_{\gamma}(A, A \cap B) = \mathbb{I}_{\gamma}(B, A \cup B)$."

Proof: In order to prove the above-stated properties, we divide the set X between two disjoint subsets X_1 & X_2 defined as

$$X_1 = \{x_i \in X |\ T_A(\mathfrak{R}(\Gamma))(x) \geq T_B(\mathfrak{R}(\Gamma))(x) \geq T_C(\mathfrak{R}(\Gamma))(x),\ I_A(\mathfrak{R}(\Gamma))(x) \leq I_B(\mathfrak{R}(\Gamma))(x) \leq I_C(\mathfrak{R}(\Gamma))(x),\ F_A(\mathfrak{R}(\Gamma))(x) \leq F_B(\mathfrak{R}(\Gamma))(x) \leq F_C(\mathfrak{R}(\Gamma))(x)\}; \quad (6)$$

and

$$X_2 = \{x_i \in X \mid T_A(\mathfrak{R}(\Gamma))(x) \geq T_B(\mathfrak{R}(\Gamma))(x) \geq T_C(\mathfrak{R}(\Gamma))(x),\ I_A(\mathfrak{R}(\Gamma))(x) \leq I_B(\mathfrak{R}(\Gamma))(x) \leq I_C(\mathfrak{R}(\Gamma))(x),\ F_A(\mathfrak{R}(\Gamma))(x) \leq F_B(\mathfrak{R}(\Gamma))(x) \leq F_C(\mathfrak{R}(\Gamma))(x)\}. \quad (7)$$

1. $\mathbb{I}_\gamma(A \cup B, A \cap B)$

$$= \sum_{i=1}^{n} 2^\gamma \left[\frac{\left(\sqrt{T_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{T_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) + T_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} + \frac{\left(\sqrt{1-T_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{1-T_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) - T_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} \right]$$

$$+ \sum_{i=1}^{n} 2^\gamma \left[\frac{\left(\sqrt{I_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{I_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) + I_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} + \frac{\left(\sqrt{1-I_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{1-I_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) - I_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} \right]$$

$$+ \sum_{i=1}^{n} 2^\gamma \left[\frac{\left(\sqrt{F_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{F_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) + F_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} + \frac{\left(\sqrt{1-F_{A\cup B}(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{1-F_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_{A\cup B}(\mathfrak{R}(\Gamma))(x_i) - F_{A\cap B}(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} \right]$$

Now, by making use of Equations (6) and (7), we have

$$\mathbb{I}_\gamma(A \cup B, A \cap B) = \sum_{x_i \in X_1}^{n} 2^\gamma \left[\frac{\left(\sqrt{T_B(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{T_A(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_B(\mathfrak{R}(\Gamma))(x_i) + T_A(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} + \frac{\left(\sqrt{1-T_B(\mathfrak{R}(\Gamma))(x_i)} - \sqrt{1-T_A(\mathfrak{R}(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\mathfrak{R}(\Gamma))(x_i) - T_A(\mathfrak{R}(\Gamma))(x_i)}\right)^\gamma} \right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_B(\Re(\Gamma))(x_i)}-\sqrt{I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_B(\Re(\Gamma))(x_i)+I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_B(\Re(\Gamma))(x_i)}-\sqrt{1-I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_B(\Re(\Gamma))(x_i)}-\sqrt{F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_B(\Re(\Gamma))(x_i)+F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_B(\Re(\Gamma))(x_i)}-\sqrt{1-F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i)-F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$=\mathbb{I}_{\gamma}(A,B).$$

2. Proof of this can be done on similar lines as of 1.

3. $\mathbb{I}_{\gamma}(A\cup B,C)+\mathbb{I}_{\gamma}(A\cap B,C)$

$$=" \sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_{A\cup B}(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\right.$$

$$\left.\frac{\left(\sqrt{1-T_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_{A\cup B}(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_{A\cap B}(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_{A\cap B}(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_{A\cup B}(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{1-I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_{A\cup B}(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_{A\cap B}(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{1-I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_{A\cap B}(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_{A\cup B}(\Re(\Gamma))(x_i)+F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_{A\cup B}(\Re(\Gamma))(x_i)}-\sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_{A\cup B}(\Re(\Gamma))(x_i)-F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_{A\cap B}(\Re(\Gamma))(x_i)+F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_{A\cap B}(\Re(\Gamma))(x_i)}-\sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_{A\cap B}(\Re(\Gamma))(x_i)-F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$=\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_B(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_B(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_B(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_B(\Re(\Gamma))(x_i)}-\sqrt{T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_B(\Re(\Gamma))(x_i)+T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_B(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\Re(\Gamma))(x_i)-T_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_B(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_B(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_B(\Re(\Gamma))(x_i)}-\sqrt{1-I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_B(\Re(\Gamma))(x_i)}-\sqrt{I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_B(\Re(\Gamma))(x_i)+I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_B(\Re(\Gamma))(x_i)}-\sqrt{1-I_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_B(\Re(\Gamma))(x_i)}-\sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_B(\Re(\Gamma))(x_i)+F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_B(\Re(\Gamma))(x_i)}-\sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i)-F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{x_i\in X_2}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+ \sum_{x_i \in X_1}^{n} 2^{\gamma} \left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)} - \sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i) + F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)} - \sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i) - F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}} \right]$$

$$+ \sum_{x_i \in X_2}^{n} 2^{\gamma} \left[\frac{\left(\sqrt{F_B(\Re(\Gamma))(x_i)} - \sqrt{F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_B(\Re(\Gamma))(x_i) + F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-F_B(\Re(\Gamma))(x_i)} - \sqrt{1-F_C(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i) - F_C(\Re(\Gamma))(x_i)}\right)^{\gamma}} \right]$$

$$= \mathbb{I}_{\gamma}(A, C) + \mathbb{I}_{\gamma}(B, C)."$$

4. $\mathbb{I}_{\gamma}(\mathrm{A}, \mathrm{A} \cup \mathrm{B})$

$$= "\sum_{i=1}^{n} 2^{\gamma} \left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)} - \sqrt{T_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i) + T_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)} - \sqrt{1-T_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i) - T_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\gamma} \left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)} - \sqrt{I_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i) + I_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)} - \sqrt{1-I_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i) - I_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}} \right]$$

$$+ \sum_{i=1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_{A\cup B}(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$= \sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma}\left[\frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}\right]$$

$$= \mathbb{I}_{\gamma}(B, A\cap B)."$$

5. Proof of this can be done on similar lines.

Theorem 3. *For any A and $B \in SVNHSS(X)$, the parametric divergence information measure (2) holds the below mentioned properties:*

1. "$\mathbb{I}_{\gamma}(\bar{A}\ ,\bar{B}) = \mathbb{I}_{\gamma}(A,B)$
2. $\mathbb{I}_{\gamma}(\overline{A\cup B},\overline{A\cap B}) = \mathbb{I}_{\gamma}(\bar{A}\cap\bar{B},\bar{A}\cup\bar{B}) = \mathbb{I}_{\gamma}(A,B)$
3. $\mathbb{I}_{\gamma}(A,\bar{B}) = \mathbb{I}_{\gamma}(\bar{A},B)$
4. $\mathbb{I}_{\gamma}(A,\bar{B}) + \mathbb{I}_{\gamma}(\bar{A}\ ,\bar{B}) = \mathbb{I}_{\gamma}(A,B) + \mathbb{I}_{\gamma}(\bar{A},B)$"

Proof :

1. Proof of (1) can easily be done with the definition of *complement* and the required results hold.
2. Proof of (2) can be done by making use of (6) and (7) as:
$\mathbb{I}_{\gamma}(\overline{A\cup B},\overline{A\cap B}) =$

$$\sum_{x_i\in X_1}^{n} 2^{\gamma}\frac{\left(\sqrt{1-T_B(\Re(\Gamma))(x_i)}-\sqrt{1-T_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\Re(\Gamma))(x_i)-T_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}+\sum_{x_i\in X_1}^{n} 2^{\gamma}\frac{\left(\sqrt{T_B(\Re(\Gamma))(x_i)}-\sqrt{T_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_B(\Re(\Gamma))(x_i)+T_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-I_B(\Re(\Gamma))(x_i)}-\sqrt{1-I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{I_B(\Re(\Gamma))(x_i)}-\sqrt{I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_B(\Re(\Gamma))(x_i)+I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-F_B(\Re(\Gamma))(x_i)}-\sqrt{1-F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i)-F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{F_B(\Re(\Gamma))(x_i)}-\sqrt{F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_B(\Re(\Gamma))(x_i)+F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$= \mathbb{I}_{\gamma}(A,B).$$

Now, $\mathbb{I}_{\gamma}(\bar{A}\cap\bar{B},\bar{A}\cup\bar{B}) =$

$$\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-T_B(\Re(\Gamma))(x_i)}-\sqrt{1-T_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\Re(\Gamma))(x_i)-T_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \text{"}\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{T_B(\Re(\Gamma))(x_i)}-\sqrt{T_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_B(\Re(\Gamma))(x_i)+T_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_A(\Re(\Gamma))(x_i)-T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-I_B(\Re(\Gamma))(x_i)}-\sqrt{1-I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{I_B(\Re(\Gamma))(x_i)}-\sqrt{I_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_B(\Re(\Gamma))(x_i)+I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_A(\Re(\Gamma))(x_i)-I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-F_B(\Re(\Gamma))(x_i)}-\sqrt{1-F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i)-F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{F_B(\Re(\Gamma))(x_i)}-\sqrt{F_A(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_B(\Re(\Gamma))(x_i)+F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_A(\Re(\Gamma))(x_i)-F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_2}^{n} 2^{\gamma} \frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}"$$

$$= \mathbb{I}_{\gamma}(A,B).$$

Therefore, $\mathbb{I}_{\gamma}(\overline{A\cup B},\overline{A\cap B}) = \mathbb{I}_{\gamma}(\bar{A}\cap\bar{B},\bar{A}\cup\bar{B}) = \mathbb{I}_{\gamma}(A,B)$.

3. $\mathbb{I}_{\gamma}(A,\bar{B})$

$$=\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{T_A(\Re(\Gamma))(x_i)}-\sqrt{1-T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-T_B(\Re(\Gamma))(x_i)-T_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)}-\sqrt{T_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{1-T_A(\Re(\Gamma))(x_i)+T_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+"\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{I_A(\Re(\Gamma))(x_i)}-\sqrt{1-I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-I_B(\Re(\Gamma))(x_i)-I_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)}-\sqrt{I_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{1-I_A(\Re(\Gamma))(x_i)+I_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}$$

$$+\sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{F_A(\Re(\Gamma))(x_i)}-\sqrt{1-F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{2-F_B(\Re(\Gamma))(x_i)-F_A(\Re(\Gamma))(x_i)}\right)^{\gamma}} + \sum_{x_i\in X_1}^{n} 2^{\gamma} \frac{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)}-\sqrt{F_B(\Re(\Gamma))(x_i)}\right)^{2(\gamma+1)}}{\left(\sqrt{1-F_A(\Re(\Gamma))(x_i)+F_B(\Re(\Gamma))(x_i)}\right)^{\gamma}}"$$

$$= \mathbb{I}_{\gamma}(\bar{A},B).$$

4. Proof of (4) can be done by making use of (1) and (3), which satisfies $\mathbb{I}_{\gamma}(A,\bar{B}) + \mathbb{I}_{\gamma}(\bar{A}\ ,\bar{B}) = \mathbb{I}_{\gamma}(A,B) + \mathbb{I}_{\gamma}(\bar{A},B)$.

5. Utilization of the Proposed Parameterized Divergence Measure in the MCDM Problem.

In this section, we propose a methodology for the *MCDM* based on proposed parameterized divergence measures of $SVNHSSs$. The steps of the proposed methodology have been explained with the help of Figure 1 in an abstract way. Consider the set of m alternatives $\{Y_1, Y_2, \dots, Y_n\}$ and n attributes $k^1, k^2, \dots, k^n$ and "whose corresponding attribute values are respectively the sets $K^1, K^2, \dots, K^n$ with $K^i \cap K^j = \emptyset$, for $i \neq j$ and $i,j \in \{1,2,..,n\}$." The set of all possible $SVNHSSs$ are given by $(\Re,\Gamma)$, where $\Gamma = K^1 \times\ K^2 \times \dots \times\ K^n$. The aim of an expert is to choose the best suitable alternative out of the available alternatives which satisfy the n attribute values. The opinions of all the experts have been considered in terms of a matrix representation $\mathrm{H} = [\mathrm{h_{ij}}]_{\mathrm{m\times n}}$ called single-valued neutrosophic hypersoft matrix where $h_{ij} = \left(T(\mathfrak{R}(\Gamma))_{ij}, I(\mathfrak{R}(\Gamma))_{ij}, F(\mathfrak{R}(\Gamma))_{ij}\right)$. The necessary steps involved in the algorithm of the proposed methodology are outlined as follows:

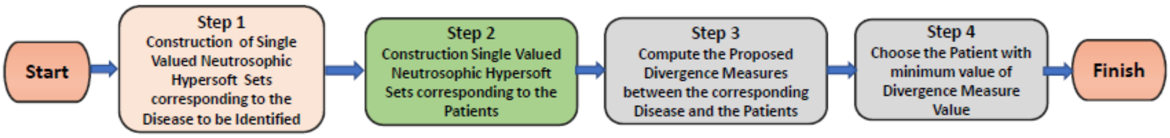

Figure 1: Algorithmic Steps of the proposed Methodology

Step 1: In the first step, construct the single-valued neutrosophic hypersoft decision matrix based on available information.

Step 2: In this step, remove the heterogeneity in the attributes (if any) and convert it into a homogeneous type of attribute. Majorly, there are two types of attributes i.e. cost type and benefit type, we convert the cost type attributes into benefit type. For this, the expert matrix $H = [h_{ij}]_{m\times n}$ is converted into a new expert matrix $H' = [h'_{ij}]_{m\times n}$ where h'_{ij} is given by

$$h'_{ij} = \left(T(\mathfrak{R}(\Gamma))_{ij}, I(\mathfrak{R}(\Gamma))_{ij}, F(\mathfrak{R}(\Gamma))_{ij}\right) = \begin{cases} h_{ij} \ ; & \text{for benefit criteria} \\ h_{ij}{}^{c} \ ; & \text{for cost criteria.} \end{cases}$$

Step 3: In this step, compute the values of the proposed divergence measure of the alternatives $Y_i{}'s$ with respect to the sub-attributes individually.

Step 4: In this step, the ordering of alternatives can be done with the least value of the proposed divergence measure.

6. Use of Proposed Divergence Measures in Symptomatic Detection of COVID-19.

In this section, we shall make use of above-stated methodology for the symptomatic detection of COVID-19 on the basis of divergence measures for $SVNHSSs$. Consider a set of four patients $\{Y_1, Y_2, Y_3, Y_4\}$ in a hospital having symptoms of COVID-19. Suppose there are three stages of characterization of the symptoms as $severe(x^1)$, $mild(x^2)$ and $no(x^3)$. The universal set $X = \{x^1, x^2, x^3\}$. Let $K = \{K^1 = sense\ \ of\ \ taste,\ K^2 = temperature,\ K^3 = chest\ \ pain,\ K^4 = flu\}$ be the set of symptoms that are further classified into sub-attributes:

$K^1 =$ "$sense\ \ of\ \ taste = \{no\ \ taste, can\ \ taste\}$"

$K^2 =$ "$temperature = \{97.5 - 98.5, 98.6 - 99.5, 99.6 - 101.5, 101.6 - 102.5\}$"

$K^3 =$ "$chest\ \ pain = \{shortness\ \ of\ \ breath,\ no\ \ pain,\ normal\ \ pain\ \ angina\}$"

$K^4 =$ "$flu = \{sore\ \ throat, cough,\ strep\ \ throat\}$"

Now, let us define a relation $\mathfrak{R} : K^1 \times\ K^2 \times \ldots \times\ K^n \rightarrow P(X)$ defined as,

$\mathfrak{R}\ (K^1 \times\ K^2 \times \ldots \times\ K^n) = \{\xi = shortness\ \ of\ \ breath, \zeta = 101.3, \varrho = sore\ \ throat, \varsigma = no\ \ taste\}$ is the most prominent sample of the patient for the confirmation of COVID-19.

Step1: Let $(\mathfrak{R}, \Gamma)$ be a $SVNHSS(X)$ for COVID-19 prepared with the help of medical experts as given in Table 1.

Table 1. $SVNHSS(\mathfrak{R}, \Gamma)$ for COVID-19

$(\mathfrak{R}, \Gamma)$	K^1	K^2	K^3	K^4
x^1	$\xi(0.4,0.2,0.3)$	$\zeta(0.3,0.4,0.3)$	$\varrho(0.7,0.1,0.2)$	$\varsigma(0.4,0.2,0.3)$

x^2	$\xi(0.5,0.1,0.3)$	$\zeta(0.1,0.8,0.1)$	$\varrho(0.4,0.3,0.2)$	$\varsigma(0.5,0.2,0.3)$
x^3	$\xi(0.3,0.5,0.1)$	$\zeta(0.1,0.2,0.7)$	$\varrho(0.1,0.6,0.2)$	$\varsigma(0.5,0.4,0.1)$

Next, the $SVNHSSs$ for the patients under consideration are given in Table 2-Table 5.

Table 2. $SVNHSS(\mathfrak{N},\Gamma)$ for the patient Y_1

$(\mathfrak{N},\Gamma)$	K^1	K^2	K^3	K^4
x^1	$\xi(0.5,0.2,0.3)$	$\zeta(0.8,0.1,0.0)$	$\varrho(0.2,0.7,0.1)$	$\varsigma(0.9,0.1,0.0)$
x^2	$\xi(0.3,0.1,0.5)$	$\zeta(0.2,0.8,0.0)$	$\varrho(0.5,0.2,0.2)$	$\varsigma(0.6,0.1,0.2)$
x^3	$\xi(0.4,0.5,0.1)$	$\zeta(0.7,0.2,0.0)$	$\varrho(0.3,0.6,0.1)$	$\varsigma(0.4,0.5,0.1)$

Table 3. $SVNHSS(\mathfrak{N},\Gamma)$ for the patient Y_2

$(\mathfrak{N},\Gamma)$	K^1	K^2	K^3	K^4
x^1	$\xi(0.2,0.6,0.2)$	$\zeta(0.2,0.5,0.3)$	$\varrho(0.6,0.1,0.2)$	$\varsigma(0.7,0.2,0.1)$
x^2	$\xi(0.3,0.4,0.3)$	$\zeta(0.2,0.6,0.2)$	$\varrho(0.4,0.3,0.2)$	$\varsigma(0.5,0.2,0.3)$
x^3	$\xi(0.8,0.1,0.1)$	$\zeta(0.1,0.2,0.7)$	$\varrho(0.1,0.6,0.2)$	$\varsigma(0.8,0.1,0.1)$

Table 4. $SVNHSS(\mathfrak{N},\Gamma)$ for the patient Y_3

$(\mathfrak{N},\Gamma)$	K^1	K^2	K^3	K^4
x^1	$\xi(0.3,0.4,0.3)$	$\zeta(0.2,0.6,0.1)$	$\varrho(0.3,0.6,0.0)$	$\varsigma(0.4,0.2,0.3)$
x^2	$\xi(0.5,0.1,0.3)$	$\zeta(0.1,0.8,0.1)$	$\varrho(0.4,0.3,0.2)$	$\varsigma(0.5,0.2,0.3)$
x^3	$\xi(0.5,0.5,0.0)$	$\zeta(0.2,0.0,0.8)$	$\varrho(0.4,0.5,0.1)$	$\varsigma(0.4,0.4,0.1)$

Table 5. $SVNHSS(\mathfrak{N},\Gamma)$ for the patient Y_4

$(\mathfrak{N},\Gamma)$	K^1	K^2	K^3	K^4
x^1	$\xi(0.9,0.0,0.1)$	$\zeta(0.2,0.6,0.1)$	$\varrho(0.6,0.1,0.2)$	$\varsigma(0.5,0.2,0.2)$
x^2	$\xi(0.3,0.5,0.2)$	$\zeta(0.4,0.0,0.6)$	$\varrho(0.2,0.3,0.5)$	$\varsigma(0.7,0.2,0.1)$
x^3	$\xi(0.4,0.3,0.3)$	$\zeta(0.4,0.2,0.4)$	$\varrho(0.3,0.6,0.1)$	$\varsigma(0.0,0.1,0.9)$

Step 2: Since all the attributes are of benefit type, so there is no need for normalization of attributes.

Step 3: In this step, we shall make use of the proposed divergence measure to compute the values of the divergence measure for different patients. Now, by applying the proposed divergence measure (1), we get $\mathbb{I}_\gamma(\mathfrak{N},Y_1)=0.3457$ for the patient Y_1, $\mathbb{I}_\gamma(\mathfrak{N},Y_2)=0.6243$ for the patient Y_2, $\mathbb{I}_\gamma(\mathfrak{N},Y_3)=0.4892$ for the patient Y_3 and $\mathbb{I}_\gamma(\mathfrak{N},Y_4)=0.8657$ for the patient Y_4.

Step 4: Now, the minimum value of the divergence measure is 0.3457 which is for the patient Y_1, hence out of all four patients, Y_1 is suffering from COVID-19 on the basis of symptomatic detection.

7. Discussion on Results and Methodology

In this section, we briefly present a discussion of the proposed methodology and the obtained results by mentioning some remarks on comparative advantages, importance and limitations. The important discussion points on the notions of neutrosophic hypersoft set and its Hellinger divergence measure are as follows:

- The computed value of the divergence measure is more deterministic as compared with other obtained values.

- The utilization of a neutrosophic hypersoft set and its measure have a superiority to dealing incorporating a broader notion of applicability in uncertain situations of a direct parameter and sub-parameterization due to the hypersoft feature.
- The hypersoft sets which are already existing in the literature – "intuitionistic fuzzy hypersoft set, Pythagorean fuzzy Hypersoft set, and Neutrosophic hypersoft set" due to the rejection and abstain components being excluded, each have their own shortcomings.
- The methodology implementing the proposed Hellinger divergence measure be effectively and consistently applied to different group strategic MCDM issues as well as in a broader framework.

The following characteristic comparison table (Table 1) represents the added-on advantages of the proposed methodology over the existing ones:

Table 1: Characteristic Comparison Table

Authors	Divergence Measures	Truthiness	Indeterminacy	Falsity	Sub-Attributes
Ohlan et al. [46]	"Fuzzy Sets"	✓	×	×	×
Kadian et al. [50]	"Intuitionistic Fuzzy Sets"	✓	×	✓	×
Montes et al.[51]	"Picture Fuzzy Sets"	✓	×	✓	×
Proposed	"Single-valued Neutrosophic Hypersoft Sets"	✓	✓	✓	✓

8. Conclusions & Scope for Future Work

The Hellinger divergence measure for $SVNHSSs$ has been successfully presented along with some important deliberations and properties. In literature, the Hellinger divergence measure for $SVNHSSs$ is novel and utilized to propose a new methodology for the symptomatic detection of COVID-19. The necessary steps of the proposed methodology have been illustrated successfully. The obtained results based on the proposed methodology are found to be efficient and consistent. In the future, the utility distribution can further be incorporated in the Hellinger divergence measure to propose a `useful' Hellinger divergence measure for $SVNHSSs$, and eventually, the total ambiguity and hybrid ambiguity can be discussed with due applications. Also, the notion of expert sets can be appended with the proposed Hellinger divergence measure.

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