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Neutrosophic 2−normed spaces and generalized summability

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Abstract. The aim of present paper is to introduce the concept of neutrosophic 2−norm space (briefly abbreviated as $N-2-NS$) and study statistical summability in these spaces. We construct examples to demonstrate that statistical convergence is stronger method than usual convergence. Finally, we define statistically Cauchy sequence, statistical completeness and obtain the Cauchy convergence criteria.

Keywords: Neutrosophic norm spaces, statistical convergence, statistical completeness, neutrosophic 2-norm spaces.

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1. **Introduction**

Summability method is primarily concerned with the assignment of a limit in some generalized form to those sequences which do not converge in the usual sense. Over the years, many summability methods have been developed. One among these is developed by Henry Fast[6] and Schoenberg [20] independently by use of the natural density δ of subsets of $\mathbb{N}$ and called it as statistical convergence. For any set $K \subseteq \mathbb{N}$, the natural density of K is denoted by $\delta(K)$ and is defined by $\lim_n \frac{1}{n}|\{k \leq n : k \in K\}|$ provided the limit exists. Using δ, statistical convergent can be defined as follows.

"A sequence $x = (x_k)$ of numbers is said to be statistical convergent to L if for each $\epsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n}\left|\{k \leq n : |x_k - L| \geq \epsilon\}\right| = 0;$$

or equivalently $\delta\big(\{k \leq n : |x_k - L| \geq \epsilon\}\big) = 0$. In this case, we write $S - \lim_k x_k = L$". Over the years, statistical convergence and related concepts have been further explored by numerous authors in different directions. For some interesting works on statistical convergence in this concern, we refer [5], [8], [11]-[13], and [19].

Apart from this, fuzzy sets were introduced by Zadeh [22] in 1965 as a generalization of crisp sets and turned out to be a very effective tool to deal with those situations which can not be fit in the framework of classical sets. These sets have wide applications in many areas of science and technology, especially in control engineering, artifical intelegence, robotics and many more to achieve better solutions. During development phase of fuzzy sets, many interesting generalizations of these sets have been appeared in the literature. For instance: intuitionistic fuzzy sets [1], vague fuzzy sets [4], interval-valued fuzzy sets [21], neutrosophic sets [15], etc. These sets have been further used to define some new kind of spaces such as fuzzy normed spaces [7], intuitionistic fuzzy metric spaces [17], intuitionistic fuzzy 2-normed spaces [16], intuitionistic fuzzy topological spaces [18] and neutrosophic normed spaces ([2], [3]). Recently, these spaces have been explored from sequence spaces point of view and linked with summability theory. Many summability method such as statistical convergence, ideal convergence, and lacunary statistical convergence have been developed. For an extensive view in this direction, we refer to the reader [10], [12]-[14]. In present work, we define a generalized neutrosophic normed space which we call neutrosophic $2-$norm space and introduce the convergence structure in these spaces. Later, we define statistical convergence, statistical Cauchy sequences in a neutrosophic $2-$norm space and develop some of their properties.

2. **Prelimanaries**

This section record a few definitions and outcomes that will be required in present study. Through out this work, $\mathbb{R}^+$ will denote the open interval $(0, \infty)$ and $\mathbb{N}$, the set of positive integers.

Definition 2.1 [11] Let $I = [0, 1]$. A function $\circ : I \times I \to I$ is said to be a $t-$norm for all $f, g, h, i \in I$ we have:

(i) $f \circ g = g \circ f$;

(ii)$f \circ (g \circ h) = (f \circ g) \circ h$;

(iii) $\circ$ is continuous;

(iv) $f \circ 1 = f$ for every $f \in [0,\ 1]$ and

(v) $f \circ g \leq h \circ i$ whenever $f \leq h$ and $g \leq i$.

Definition 2.2 [11] Let $I = [0, 1]$. A function $\diamond : I \times I \to I$ is said to be a continuous triangular conorm or $t-$conorm for all $f, g, h, i \in I$ we have:

(i) $f \diamond g = g \diamond f$;

(ii)$f \diamond (g \diamond h) = (f \diamond g) \circ h$;

(iii) $\diamond$ is continuous;

(iv) $f \diamond 0 = f$ for every $f \in [0,1]$

(v) $f \diamond g \leq h \diamond i$ whenever $f \leq h$ and $g \leq i$.

Kirişci and Şimşek [13] recently defined NNS as follows.

Definition 2.3 [13] Let F is a vector space, $N = \{\langle \vartheta, \mathcal{H}(\vartheta), \mathcal{I}(\vartheta), \mathcal{J}(\vartheta)\rangle : \vartheta \in F\}$ be a normed space in which $N : F \times \mathbb{R}^+ \to [0,\ 1]$ and $\circ, \diamond$ respectively are $t-$norm and $t-$conorm. The four touple $V = (F, N,\ \circ, \diamond\ )$ is called a neutrosophic normed spaces (NNS) briefly it for every $p, q \in F$, $\rho\ \mu > 0$ and for every $\varsigma \neq 0$ we have

(i) $0 \leq \mathcal{H}(p,\ \rho) \leq 1$, $0 \leq \mathcal{I}(p,\ \rho) \leq 1$, $0 \leq \mathcal{J}(p,\ \rho) \leq 1$ for every $\rho \in \mathbb{R}^+$;

(ii) $\mathcal{H}(p,\ \rho) +\ \mathcal{I}(p,\ \rho) + \mathcal{J}(p,\ \rho) \leq 3$ for $\rho \in \mathbb{R}^+$;

(iii) $\mathcal{H}(p,\ \rho) = 1 \quad (\text{for } \rho > 0)$ iff $p = 0$;

(iv) $\mathcal{H}(\varsigma p, \rho) = \mathcal{H}\left(p,\ \frac{\rho}{|\varsigma|}\right)$;

(v) $\mathcal{H}(p,\ \mu) \circ \mathcal{H}(q,\ \rho) \leq \mathcal{H}(p+q,\ \mu+\rho)$;

(vi) $\mathcal{H}(p,\ .)$ is a non-decreasing function that runs continuously;

(vii) $\lim_{\rho\to\infty} \mathcal{H}(p,\ \rho) = 1$;

(viii) $\mathcal{I}(p, \rho) = 0\ \ (\text{for} \rho > 0)$ iff $p = 0$;

(ix) $\mathcal{I}(\varsigma p, \rho) = \mathcal{I}\left(p,\ \frac{\rho}{|\varsigma|}\right)$;

(x) $\mathcal{I}(p,\ \mu) \diamond \mathcal{I}(q,\ \rho) \geq \mathcal{I}(p+q, \rho+\mu)$;

(xi) $\mathcal{I}(p,\ .)$ is a non-decreasing function that runs continuously;

(xii) $\lim_{\lambda\to\infty} \mathcal{I}(p, \rho) = 0$;

(xiii) $\mathcal{J}(p, \rho) = 0 \quad (\text{for } \rho > 0)$ iff $p = 0$;

(xiv) $\mathcal{J}(\varsigma p, \rho) = \mathcal{J}\left(p,\ \frac{\rho}{|\varsigma|}\right)$;

(xv) $\mathcal{J}(p,\ \mu) \diamond \mathcal{J}(q,\ \rho) \geq \mathcal{J}(p+q, \rho+\mu)$;

(xvi) $\mathcal{J}(p,\ .)$ is a non-decreasing function that runs continuously;

(xvii) $\lim_{\lambda\to\infty} \mathcal{J}(p, \rho) = 0$;

(xviii) If $\rho \leq 0$, then $\mathcal{H}(p, \rho) = 0$, $\mathcal{I}(p, \rho) = 1$ and $\mathcal{J}(p, \rho) = 1$.

We call $N(\mathcal{H}, \mathcal{I}, \mathcal{J})$ the neutrosophic norm.

We next give the notions of statistical convergence and statistical Cauchy sequences in neutrosophic norm spaces as introduced in [13].

Definition 2.4 [13] Let V be a NNS. Choose $0 < \epsilon < 1$ and $\rho > 0$. A sequence (v_k) in V is said to be statistical convergent if $\exists\ v_0 \in F$ s.t. $\lim_n \frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_0, \rho) \leq 1 - \varepsilon$ or $\mathcal{I}(v_k - v_0, \rho) \geq \varepsilon$ and $\mathcal{J}(v_k - v_0, \rho) \geq \varepsilon\}| = 0$; or equivalently, the set's natural density $A(\varepsilon, \rho) = \{k \leq n :\ \mathcal{H}(v_k - v_0; \rho) \leq 1 - \varepsilon$ or $\mathcal{I}(v_k - v_0; \rho) \geq \varepsilon$ and $\mathcal{J}(v_k - v_0, \rho) \geq \varepsilon\}$ is zero, i.e., $\delta(A(\varepsilon, \rho)) = 0$. we can write it as $S(N) - \lim_{k\to\infty} v_k = v_0$.

Definition 2.5 [13] Let V be a NNS. Choose $0 < \epsilon < 1$ and $\rho > 0$. A sequence (v_k) in V is said to be statistical Cauchy if $\exists \ p \in \mathbb{N}$ s.t. $\lim_n \frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_p, \rho) \leq 1-\varepsilon$ or $\mathcal{I}(v_k - v_p, \rho) \geq \varepsilon$ and $\mathcal{J}(v_k - v_p, \rho) \geq \varepsilon\}| = 0$; or equivalently, the natural density of the set $A(\varepsilon, \rho) = \{k \leq n : \ \mathcal{H}(v_k - v_p, \rho) \leq 1-\varepsilon$ or $\mathcal{I}(v_k - v_p, \rho) \geq \varepsilon$ and $\mathcal{J}\ (v_k - v_p, \rho) \geq \varepsilon\}$ is zero, i.e., $\delta\left(A\left(\varepsilon, \rho\right)\right) = 0$.

We now turn towards the paper [9] and would like to quote the idea of two norm.

Definition 2.6 [9] Let V be a $d-$dimensional real vector space, where $2 \leq d < \infty$. A $2-$norm on V is a function $\|.,.\| : V \times V \to \mathbb{R}$ fulfilling the below listed requirements:

For all $p, q \in V$, and scalar α, we have

(i) $||p,q|| = 0$ iff p and q are linearly dependent;

(ii) $||p,q|| = ||p,q||$;

(iii)$||\alpha p,q|| = |\alpha|||p,q||$ and

(iv) $||p,q+r|| \leq ||p,q|| + ||p,r||$.

The pair $(V, ||.,.||)$ is known as $2-$normed space in this case.

Let $V = \mathbb{R}^2$ and for $p = (p_1,p_2)$ and $q = (q_1,q_2)$ we define $||p,q|| = |p_1q_2 - p_2q_1|$, then $||p,q||$ is a $2-$ norm on $V = \mathbb{R}^2$.

We now proceed with our main results.

3. Neutrosophic-2-norm spaces ($N-2-NS$)

This section starts with the following definition of neutrosophic$-2-$norm spaces.

Definition 3.1 Let F is a vector space, $N_2 = (\{(p,q), \mathcal{H}(p,q), \mathcal{I}(p,q), \mathcal{J}(p,q)\} : (p,q) \in F \times F)$ be a $2-$norm space s.t. $N_2 : F \times F \times \mathbb{R}^+ \to [0,1]$. If $\circ, \diamond$ respectively denotes $t-norm$ and $t-conorm$, then four-tuple $V = (F, N_2, \circ, \diamond)$ is known as neutrosophic $2-$norm spaces (briefly $N-2-NS$) if for every $p,q,w \in V$, $\rho, \mu \geq 0$ and $\varsigma \neq 0$:

(i) $0 \leq \mathcal{H}(p,q;\rho) \leq 1$, $0 \leq \mathcal{I}(p,q;\rho) \leq 1$ and $0 \leq \mathcal{J}(p,q;\rho) \leq 1$ for every $\rho \in R^+$;

(ii) $\mathcal{H}(p,q;\rho) + \mathcal{I}(p,q;\rho) + \mathcal{J}(p,q;\rho) \leq 3$;

(iii) $\mathcal{H}(p,q;\rho) = 1$ iff p,q are linearly dependent;

(iv) $\mathcal{H}(\varsigma p,\ q;\ \rho) = \mathcal{H}(p,q;\frac{\rho}{|\varsigma|})$ for each $\varsigma \neq 0$;

(v) $\mathcal{H}(p,q;\rho) \circ \mathcal{H}(p,w;\mu) \leq \mathcal{H}(p,q+w;\rho+\mu)$;

(vi) $\mathcal{H}(p,q;.) : (0,\infty) \to [0,1]$ is a non-increasing function that runs continuously;

(vii) $\lim\limits_{\rho\to\infty} \mathcal{H}(p,q;\rho) = 1$;

(viii) $\mathcal{H}(p,q;\rho) = \mathcal{H}(q,p;\rho)$

(ix) $\mathcal{I}(p,q;\rho) = 0$ iff p,q are linearly dependent;

(x) $\mathcal{I}(\varsigma p, q; \rho) = \mathcal{I}(p, q; \frac{\rho}{|\varsigma|})$ for each $\varsigma \neq 0$;

(xi) $\mathcal{I}(p, q; \rho) \diamond \mathcal{I}(p, w; \mu) \geq \mathcal{I}(p, q + w; \rho + \mu)$;

(xii) $\mathcal{I}(p, q; .) : (0, \infty) \to [0, 1]$ is a non-increasing function that runs continuously;

(xiii) $\lim_{\rho \to \infty} \mathcal{I}(p, q; \rho) = 0$;

(xiv) $\mathcal{I}(p, q; \rho) = \mathcal{I}(q, p; \rho)$

(xvi) $\mathcal{J}(p, q; \rho) = 0$ iff p, q are linearly dependent;

(xv)$\mathcal{J}(\varsigma p,\ q;\ \rho) = \mathcal{J}(p,\ q;\ \frac{\rho}{|\varsigma|})$ for each $\varsigma \neq 0$;

(xvi) $\mathcal{J}(p, q; \rho) \diamond \mathcal{J}(p, w; \mu) \geq \mathcal{J}(p, q + w; \rho + \mu)$;

(xvii) $\mathcal{J}(p, q; .) : (0, \infty) \to [0, 1]$ is a non-increasing function that runs continuously;

(xviii) $\lim_{\lambda \to \infty} \mathcal{J}(p, q; \rho) = 0$;

(xix) $\mathcal{J}(p, q; \rho) = \mathcal{J}(q, p; \rho)$

(xx) if $\rho \leq 0$, then $\mathcal{H}(p, q; \rho) = 0$, $\mathcal{I}(p, q; \rho) = 1$, $\mathcal{J}(p, q\rho) = 1$.

In this case, we call $N_2(\mathcal{H}, \mathcal{I}, \mathcal{J})$ a neutrosophic $2-$norm on F and is denoted by N_2.

Example 3.1 Let $(F, ||.,.||)$ be a $N-2-NS$. We define the continuous $t-norm$ and $t-conorm$ by

$p \circ q = pq$ and $p \diamond q = p + q - pq$.

For $p, q \in F$, $\rho > 0$ with $\rho > \|p, q\|$, we define

$\mathcal{H}(p, q; \rho) = \frac{\rho}{\rho + \|p, q\|}$, $\mathcal{I}(p, q; \rho) = \frac{\|p, q\|}{\rho + \|p, q\|}$, and $\mathcal{J}(p, q; \rho) = \frac{\|p, q\|}{\rho}$.

If we take $||p, q|| \geq \rho$, then $\mathcal{H}(p, q; \rho) = 0$, $\mathcal{I}(p, q; \rho) = 1$, $\mathcal{J}(p, q; \rho) = 1$ and $(F, N_2, \circ, \diamond)$ is a $N-2-NS$ where $N_2 : F \times F \times R^+ \to [0, 1]$.

We now define convergence structure and Cauchy sequences in $N-2-NS$.

Definition 3.2 Let V be a $N-2-NS$. Choose $0 < \epsilon < 1$ and $\rho > 0$. A sequence (v_k) in a V is said to be convergent if $\exists$ a positive integer m and $v_0 \in F$ s.t. $\mathcal{H}(v_k - v_0, w; \rho) > 1 - \epsilon$, $\mathcal{I}(v_k - v_0, w; \rho) < \epsilon$ and $\mathcal{J}(v_k - v_0, w; \rho) < \epsilon$ for all $k \geq m$ and $w \in V$ which is equivalently to say $\lim_{k\to\infty} \mathcal{H}(v_k - v_0, w; \rho) = 1$, $\lim_{k\to\infty} \mathcal{I}(v_k - v_0, w; \rho) = 0$ and $\lim_{k\to\infty} \mathcal{J}(v_k - v_0, w; \rho) = 0$. In this case, we write $N_2 - \lim_{k\to\infty} v_k = v_0$.

Theorem 3.1 Let V be a $N-2-NS$ (u_k) and (v_k) be two sequences in V and α being any scalar.

(i) If (u_k) is convergent w.r.t. N_2, then its limit is unique.

(ii) If $N_2 - \lim_{k\to\infty} u_k = u_0$, then $N_2 - \lim_{k\to\infty} \alpha u_k = \alpha u_0$.

(iii) If $N_2 - \lim_{k\to\infty} u_k = u_0$ and $N_2 - \lim_{k\to\infty} v_k = v_0$, then $N_2 - \lim_{k\to\infty}(u_k + v_k) = (u_0 + v_0)$.

Proof. Omitted. □

Definition 3.3 Let V be a $N-2-NS$. Choose $0<\epsilon<1$ and $\rho>0$. A sequence (v_k) in a V if $\exists$ a positive integer m is said to be Cauchy s.t. $\mathcal{H}(v_k-v_n,w;\rho)>1-\epsilon$, $\mathcal{I}(v_k-v_n,w;\rho)<\epsilon$ and $\mathcal{J}(v_k-v_n,w;\rho)<\epsilon\ \forall\ k,n\geq m$ and $\forall\ w\in V$.

Definition 3.4 A $N-2-NS$ V is said to be complete if and only if each Cauchy sequence in V is converget in V.

Theorem 3.2 Every convergent sequence in a $N-2-NS$, V is Cauchy however converse is not true.

Proof. Let $\epsilon>0$ and choose $r>0$ s.t. $(1-\epsilon)\circ(1-\epsilon)>1-r$ and $\epsilon\diamond\epsilon<r$. For $\rho>0$, if we take (v_k) be any convergent in V with $N_2-\lim_{k\to\infty}v_k=v_0$. There is an integer m s.t. $\mathcal{H}(v_k-v_0,w;\rho)>1-\epsilon$, $\mathcal{I}(v_k-v_0,w;\rho)<\epsilon$ and $\mathcal{J}(v_k-v_0,w;\rho)<\epsilon$ for all $k\geq m$ and $w\in V$. Now, for all $k,n\geq m$ we have $\mathcal{H}(v_k-v_n,w;\rho)\geq\mathcal{H}(v_k-v_0,w;\frac{\rho}{2})\circ\ \mathcal{H}(v_n-v_0,w;\frac{\rho}{2})$ $>(1-\epsilon)\circ(1-\epsilon)>r$. Similarly one can easily get $\mathcal{I}(v_k-v_n,w;\rho)<r$ and $\mathcal{J}(v_k-v_n,w;\rho)<r$ for every $k,n\geq m$. This prove that the (v_k) sequence is Cauchy. $\square$

Example 3.2 Let $F=\{z_{mn}=(\frac{1}{m},\frac{1}{n}):m,n\in\mathbb{N}\}\subseteq\mathbb{R}^2$ be a $2-$normed space with $\|(m,n)\|=|\frac{1}{m}-\frac{1}{n}|$. If we define the neutrosophic norm N_2 as in Example 3.1 then $V=(F,N_2,\circ,\diamond)$ is a $N-2-NS$. Further, the sequence z_{mn} is Cauchy but not convergent as $\lim_{k\to\infty}\mathcal{I}\left(v_k-v_0,w;\ \rho\right)\neq 0$. $\square$

4. Statistical Convergence in $N-2-NS$

This section explore the statistical convergence and its properties in a $N-2-NS$.

Definition 4.1 Let V be a $N-2-NS$. Choose $0<\epsilon<1$ and $\rho>0$. A sequence (v_k) in V is said to be statistical convergent to v_0 provided that $\lim_n\frac{1}{n}|\{k\leq n:\mathcal{H}(v_k-v_0,w;\rho)\leq 1-\varepsilon$ or $\mathcal{I}(v_k-v_0,w;\rho)\geq\varepsilon$ and $\mathcal{J}(v_k-v_0,w;\rho)\geq\varepsilon\}|=0$ for every $w\in V$ or equivalently, $\delta\left(A\left(\varepsilon,\rho\right)\right)=0$. where $A(\varepsilon,\rho)=\{k\leq n:\ \mathcal{H}(v_k-v_0,w;\rho)\leq 1-\varepsilon$ or $\mathcal{I}(v_k-v_0,w;\rho)\geq\varepsilon$ and $\mathcal{J}(v_k-v_0,w;\rho)\geq\varepsilon\}$ and we write $S(N_2)-\lim_{k\to\infty}v_k=v_0$.

Theorem 4.1 Let V be a $N-2-NS$ and (v_k) be any sequence in V. If $N_2-\lim_{k\to\infty}v_k=v_0$, then $S(N_2)-\lim_{k\to\infty}v_k=v_0$.

Proof According to the hypothesis, for every $\epsilon>0$ and $\rho>0$, there is an integer $k_0\in\mathbb{N}$ s.t. $\mathcal{H}(v_k-v_0,w;\rho)>1-\epsilon$ and $\mathcal{I}(v_k-v_0,w;\rho)<\epsilon$, $\mathcal{J}(v_k-v_0,w;\rho)<\epsilon$ for all $k\geq k_0$ and every $w\in V$. This guarantees that the set $\{k\in\mathbb{N}:\mathcal{H}(v_k-v_0,w;\rho)\leq 1-\epsilon$ or $\mathcal{I}(v_k-v_0,w;\rho)<\epsilon$,

$\mathcal{J}(v_k - v_0, w; \rho) < \epsilon\}$ has a finite number of terms whose density is zero. This immediately shows that $S(N_2) - \lim_{k\to\infty} v_k = v_0$. □

In general, The converse of the theorem is false.

Example 4.1 Let $(\mathbb{R}, |.|)$ be the real space with the usual norm. For $f, g \in [0, 1]$. Let the $t - norm$ and $t - conorm$ are defined by $f \circ g = fg$ and $f \diamond g = \min\{f + g, 1\}$. Choose $p, q \in F$ and $\rho > 0$ with $\rho > \|p, q\|$. If we define $\mathcal{H}(p, q; \rho) = \frac{\rho}{\rho+\|p,q\|}$, $\mathcal{I}(p, q; \rho) = \frac{\|p,q\|}{\rho+\|p,q\|}$ and $\mathcal{J}(p, q; \rho) = \frac{\|p,q\|}{\rho}$, then $N_2(\mathcal{H}, \mathcal{I}, \mathcal{J})$ is a neutrosophic-2-norm and $V = (F, N_2, \circ, \diamond)$ is a $N - 2 - NS$. Define a sequence (v_k) by

$$v_k = \begin{cases} (0,\ 1), & \text{if } k = m^2,\ m \in \mathbb{N}; \\ (0,\ 0), & \text{otherwise.} \end{cases} \tag{1}$$

Let, $A_n(\varepsilon, \rho) = \{k \le n : \ \mathcal{H}(v_k, w;\ \rho) \le 1 - \varepsilon$ or $\mathcal{I}(v_k, w; \rho) \ge \varepsilon,\ \mathcal{J}(v_k, w; \rho) \ge \varepsilon\}$, then $A_n(\varepsilon, \rho) = \{k \le n : \ \frac{\rho}{\rho+||p,q||} \le 1 - \varepsilon$ or $\frac{||p,q||}{\rho+||p,q||} \ge \varepsilon,\ \frac{||p,q||}{\rho} \ge \varepsilon\} = \{k \le n : ||p, q|| \ge \frac{\rho\epsilon}{1-\epsilon}$ or $||p, q|| \ge \rho\epsilon\} = \{k \le n : v_k = (0, 1)\} = \{k \le n : k = m^2\}$ and therefore we have $\lim_n \frac{1}{n}|A_n(\varepsilon, \rho)| = \{k \le n : k = m^2\} \le \frac{\sqrt{n}}{n} = \frac{1}{n} \to 0$. Thus $S(N_2) - \lim_{k\to\infty} v_k = 0$. However, the sequence (v_k) is not usual convergent. □

Lemma 4.1 Let V be a $N - 2 - NS$. Then for every $0 < \epsilon < 1$, $\rho > 0$ and for every $w \in V$, The statements below are equivalents:

(i) $S(N_2) - \lim_{k\to\infty} v_k = v_0$;

(ii) $\delta\{k \in \mathbb{N} : \mathcal{H}(v_k - v_0, w; \rho) \le 1 - \epsilon\} = \delta\{k \in \mathbb{N} : \mathcal{I}(v_k - v_0, w; \rho) \ge \epsilon\} = \delta\{k \in \mathbb{N} : \mathcal{J}(v_k - v_0, w; \rho) \ge \epsilon\} = 0$;

(iii) $\delta\{k \in \mathbb{N} : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \epsilon$ and $\mathcal{I}(v_k - v_0, w; \rho) < \epsilon$, $\mathcal{J}(v_k - v_0, w; \rho) < \epsilon\} = 1$;

(iv) $\delta\{k \in \mathbb{N} : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \epsilon\} = \delta\{k \in \mathbb{N} : \mathcal{I}(v_k - v_0, w; \rho) < \epsilon\} = \delta\{k \in \mathbb{N} : \mathcal{J}(v_k - v_0, w; \rho) < \epsilon\} = 1$ and

(v) $S(N_2) - \lim_{k\to\infty} \mathcal{H}(v_k - v_0, w; \rho) = 1$ or $S(N_2) - \lim_{k\to\infty} \mathcal{I}(v_k - v_0, w; \rho) = 0$ and $S(N_2) - \lim_{k\to\infty} \mathcal{J}(v_k - v_0, w; \rho) = 0$.

Proof. Omitted. □

Theorem 4.2 Let V be a $N - 2 - NS$. For any sequence (v_k), if $S(N_2) - \lim_{k\to\infty} v_k$ exists then it must be unique.

Proof Assume that $S(N_2) - \lim_{k\to\infty} v_k = v_1$ and $S(N_2) - \lim_{k\to\infty} v_k = v_2$. For a given $\epsilon > 0$, choose $l > 0$ s.t. $(1 - l) \circ (1 - l) > 1 - \epsilon$ and $q \diamond q < \epsilon$. For any $\rho > 0$ and any $w \in V$ The following sets are defined: $K_{\mathcal{H},1}(l, \rho) = \{k \in \mathbb{N} : \mathcal{H}(v_k - v_1, w; \rho) \le 1 - l\}$, $K_{\mathcal{H},2}(l, \rho) = \{k \in \mathbb{N} : \mathcal{H}(v_k - v_2, w; \rho) \le 1 - l\}$; $K_{\mathcal{I},1}(l, \rho) = \{k \in \mathbb{N} : \mathcal{I}(v_k - v_1, w; \rho) \ge l\}$, $K_{\mathcal{I},2}(l, \rho) = \{k \in \mathbb{N} : \mathcal{I}(v_k - v_2, w; \rho) \ge l\}$; $K_{\mathcal{J},1}(l, \rho) = \{k \in \mathbb{N} : \mathcal{Y}(v_k - v_1, w; \rho) \ge l\}$, $K_{\mathcal{J},2}(l, \rho) =$

$\{k \in \mathbb{N} : \mathcal{J}(v_k - v_2, w; \rho) \geq l\}$. Since $S(N_2) - \lim_{k\to\infty} v_k = v_1$, so by Lemma 4.1, we have $\delta\{K_{\mathcal{H},1}(l,\rho)\} = \delta\{K_{\mathcal{I},1}(l,\rho)\} = \delta\{K_{\mathcal{J},1}(l,\rho)\} = 0$. Furthermore, using $S(N_2) - \lim_{k\to\infty} v_k = v_2$, we get, $\delta\{K_{\mathcal{H},2}(l,\lambda)\} = \delta\{K_{\mathcal{I},2}(l,\rho)\} = \delta\{K_{\mathcal{J},2}(l,\rho)\} = 0$. Now let $K_{\mathcal{H}}, \mathcal{I}, \mathcal{J}(\epsilon,\rho) = \{K_{\mathcal{H},1}(\epsilon,\rho) \cup K_{\mathcal{H},2}(\epsilon,\rho)\} \cap \{K_{\mathcal{I},1}(\epsilon,\rho) \cup K_{\mathcal{I},2}(\epsilon,\rho)\} \cap \{K_{\mathcal{J},1}(\epsilon,\rho) \cup K_{\mathcal{J},2}(\epsilon,\rho)\}$. Then observe that $\delta(\{K_{\mathcal{H},\mathcal{I},\mathcal{J}}(\epsilon,\rho)\}) = 0$ which implies $\delta(\{\mathbb{N}/K_{\mathcal{H},\mathcal{I},\mathcal{J}}(\epsilon,\rho)\}) = 1$. If $k \in \mathbb{N}/K_{\mathcal{H},\mathcal{I},\mathcal{J}}(\epsilon,\rho)$, then we have the following possibilities.

Case 1 $k \in \mathbb{N}/\{K_{\mathcal{H},1}(\epsilon,\rho) \cup K_{\mathcal{H},2}(\epsilon,\rho)\}$,

Case 2 $k \in \mathbb{N}/\{K_{\mathcal{I},1}(\epsilon,\rho) \cup K_{\mathcal{I},2}(\epsilon,\rho)\}$,

Case 3 $k \in \mathbb{N}/\{K_{\mathcal{J},1}(\epsilon,\rho) \cup K_{\mathcal{J},2}(\epsilon,\rho)\}$.

We prove the result only for case 1 as other cases can be obtain similarly. Assume, $k \in \mathbb{N}/\{K_{\mathcal{H},1}(\epsilon,\rho) \cup K_{\mathcal{H},2}(\epsilon,\rho)\}$. Then for any $w \in V$ we have $\mathcal{H}(v_k - v_1, w; \rho) > 1 - l$ and $\mathcal{H}(v_k - v_2, w; \rho) > 1 - l$. Now $\mathcal{H}(v_1 - v_2, w; \rho) \geq \mathcal{H}(v_k - v_1, w, ; \frac{\rho}{2}) \circ \mathcal{H}(v_k - v_2, w; \frac{\rho}{2}) > (1-l) \circ (1-l) > 1 - \epsilon$ (by choice of q). i.e., $\mathcal{H}(v_1 - v_2, w; \rho) > 1 - \epsilon$. Since $\epsilon > 0$ is arbitrary so we have $\mathcal{H}(v_1 - v_2, w; \rho) = 1$, and therefore $v_1 - v_2 = 0$. This shows that $v_1 = v_2$. Similarly in case 2 and case 3, we obtain $\mathcal{I}(v_1 - v_2, w; \rho) < \epsilon$ and $\mathcal{J}(v_1 - v_2, w; \rho) < \epsilon$ which gives $\mathcal{I}(v_1 - v_2, w; \rho) = 0$ and $\mathcal{J}(v_1 - v_2, w; \rho) = 0$. The complete proof of the theorem. □

Theorem 4.3 Let V be a $N-2-NS$; (u_k) and (v_k) be two sequences in V and α being any scalar.
(i) If $S(N_2) - \lim_{k\to\infty} u_k = u_0$, then $S(N_2) - \lim_{k\to\infty} \alpha u_k = \alpha u_0$.
(ii) If $S(N_2) - \lim_{k\to\infty} u_k = u_0$ and $S(N_2) - \lim_{k\to\infty} v_k = v_0$, then $S(N_2) - \lim_{k\to\infty}(u_k + v_k) = (u_0 + v_0)$.
Proof. Omitted. □

Theorem 4.4 Let V be a $N-2-NS$ and (v_k) be any sequence in V, then $S(N_2) - \lim_{k\to\infty} v_k = v_0$ iff an ascending index sequence of natural numbers $K = \{k_n : n \in \mathbb{N}\}$ exists with $\delta\{K\} = 1$ and $N_2 - \lim\limits_{n\to\infty} v_{k_n} = v_0$.
Proof Necessity: Assume that $S(N_2) - \lim_{k\to\infty} v_k = v_0$. For any $\rho > 0$, $j \in \mathbb{N}$ and $w \in V$, let, $K_{N_2}(j,\rho) = \{n \in \mathbb{N} : \mathcal{H}(v_n - v_0, w; \rho) > 1 - \frac{1}{j}$ and $\mathcal{I}(v_n - v_0, w; \rho) < \frac{1}{j}$, $\mathcal{J}(v_n - v_0, w; \rho) < \frac{1}{j}\}$. Then it is clear that $K_{N_2}(j+1,\rho) \subset K_{N_2}(j,\rho)$. Since $S(N_2) - \lim_{k\to\infty} v_k = v_0$, so we have $\delta\{K_{N_2}(j,\rho)\} = 1$. Let m_1 be an arbitrary number in $K_{N_2}(1,\rho)$. Then, $\exists$ a number $m_2 \in K_{N_2}(2,\rho), (m_2 > m_1)$, such that for all $n \geq m_2$, $\frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \frac{1}{2}$ and $\mathcal{I}(v_k - v_0, w; \rho) < \frac{1}{2}$, $\mathcal{J}(v_k - v_0, w; \rho) < \frac{1}{2}\}| > \frac{1}{2}$. Again on the similar lines there is another number $m_3 \in K_{N_2}(3,\rho), (m_3 > m_2)$, such that for all $n \geq m_3$, $\frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \frac{1}{3}$ and $\mathcal{I}(v_k - v_0, w; \rho) < \frac{1}{3}$, $\mathcal{J}(v_k - v_0, w; \rho) < \frac{1}{3}\}| > \frac{2}{3}$ and so on. Thus we can set a

sequence $\{m_j\}_{j\in\mathbb{N}}$ of positive integers satisfying $m_j \in K_{N_2}(j,\rho)$ and for all $n \geq m_j (j \in \mathbb{N})$: $\frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \frac{1}{j}$ and $\mathcal{I}(v_k - v_0, w; \rho) < \frac{1}{j}$, $\mathcal{J}(v_k - v_0, w; \rho) < \frac{1}{j}\}| > \frac{j-1}{j}$.

Define $K = \{n \in \mathbb{N} : 1 < n < m_1\} \cup \{\bigcup_{j\in\mathbb{N}} \{n \in K_{N_2}(j,\rho) : m_j \leq n < m_{j+1}\}\}$, Then it is obvious that, for all n satisfying $(m_j \leq n < m_j + 1)$, we have $\frac{1}{n}|\{k \leq n : k \in K\}| \geq \frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_0, w; \rho) > 1 - \frac{1}{j}$ and $\mathcal{I}(v_k - v_0, w; \rho) < \frac{1}{j}$, $\mathcal{J}(v_k - v_0, w; \rho) < \frac{1}{j}\}| > \frac{j-1}{j}$. By taking limit on both side, we have $\delta(K) = 1$. It remains to prove that the subsequence of the sequence (v_k) over K is N_2-convergent to v_0. For this, let $\epsilon > 0$ be any number and select a number $j \in \mathbb{N}$ with $\frac{1}{j} < \epsilon$. Moreover, let $n \geq m_j$ as well as $n \in K$ Then, according to the definition of K, $\exists$ a number $l \geq j$ s.t, $m_l \leq n < m_{l+1}$ and $n \in K_{N_2}(j,\rho)$. Thus, for every $\epsilon > 0$, and for every $w \in V$ we have $\mathcal{H}(v_n - v_0, w; \rho) > 1 - \frac{1}{j} > 1 - \epsilon$ and $\mathcal{I}(v_n - v_0, w; \rho) < \frac{1}{j} < \epsilon$, $\mathcal{J}(v_n - v_0, w; \rho) < \frac{1}{j} < \epsilon$ for all $n \geq h_w$ and $n \in K$. This shows that $N_2 - \lim_{n\in K} v_n = v_0$.

Sufficiency: In second part, we assume that there is a set $K = \{k_n\}_{n\in\mathbb{N}} \subseteq \mathbb{N}$ with $\delta\{K\} = 1$ and $N_2 - \lim_{n\in K} v_n = v_0$. We shall show that $S(N_2) - \lim_{k\to\infty} v_k = v_0$. Let $\epsilon > 0$ and $\rho > 0$. Since, $N_2 - \lim_{n\in K} v_n = v_0$ so there exist positive integer n_0 such that $\mathcal{H}(v_{k_n} - v_0, w; \rho) > 1 - \epsilon$ and $\mathcal{I}(v_{k_n} - v_0, w; \rho) < \epsilon$, $\mathcal{J}(v_{k_n} - v_0, w; \rho) < \epsilon$ for every $k_n \geq k_{n_0}$ and every $w \in V$. This implies the containment: $T_{N_2}(\epsilon,\rho) = \{n \in \mathbb{N} : \mathcal{H}(v_n - v_0, w; \rho) \leq 1 - \epsilon$ and $\mathcal{I}(v_n - v_0, w; \rho) \geq \epsilon$, $\mathcal{J}(v_n - v_0, w; \rho) \geq \epsilon\} \subseteq \mathbb{N} - \{v_{n_0}, v_{n_0+1}, v_{n_0+2}, ...\}$. and therefore $\delta\{T_{N_2}(\epsilon,\rho)\} \leq \delta\{\mathbb{N} - \{v_{n_0}, v_{n_0+1}, v_{n_0+2}, ...\}$. As $\delta\{K\} = 1$, so $\delta\{T_{N_2}(\epsilon,\rho)\} = 0$. This shows that $S(N_2) - \lim_{k\to\infty} v_k = v_0$ and therefore the complete proof of the Theorem. □

Finally we define statistical Cauchy sequence in $N-2-NS$ and obtain the Cauchy convergence criteria in these spaces.

Definition 4.2 Let V be a $N-2-NS$, $\epsilon > 0$ and $\lambda > 0$. A sequence (v_k) in V is said to be statistical Cauchy if $\exists\ p \in \mathbb{N}$ s.t. $\lim_n \frac{1}{n}|\{k \leq n : \mathcal{H}(v_k - v_p, w; \rho) \leq 1 - \varepsilon$ or $\mathcal{I}(v_k - v_p, w; \rho) \geq \varepsilon$ and $\mathcal{J}(v_k - v_p, w; \rho) \geq \varepsilon\}| = 0$ for every $w \in V$ or equivalently, the natural density of the set $A(\varepsilon,\rho) = \{k \leq n :\ \mathcal{H}(v_k - v_p, w; \rho) \leq 1 - \varepsilon$ or $\mathcal{I}(v_k - v_p, w; \rho) \geq \varepsilon$ and $\mathcal{J}\ (v_k - v_p, w; \rho) \geq \varepsilon\}$ is zero, i.e., $\delta\left(A\left(\varepsilon,\ \rho\right)\right) = 0$.

Theorem 4.5 Let V be a $N-2-NS$, then every statistical convergent sequence in V is statistical Cauchy.

Proof Let (v_k) be a statistical convergent to v_0 and $\epsilon > 0$ be given. Chose $\mu > 0$ s.t. $(1-\epsilon)\circ(1-\epsilon) > 1-\mu$ and $\epsilon\diamond\epsilon < \mu$. For $\rho > 0$, if we define $A(\varepsilon,\ \rho) = \{k \leq n :\ \mathcal{H}(v_k - v_0, w;\ \frac{\rho}{2}) \leq 1 - \varepsilon$ or $\mathcal{I}(v_k - v_0, w; \frac{\rho}{2}) \geq \varepsilon$ and $\mathcal{J}(v_k - v_0, w; \frac{\rho}{2}) \geq \varepsilon\}$, then $\delta(A(\varepsilon,\rho)) = 0$ and therefore $\delta(A^C(\varepsilon,\rho)) = 1$. Let $p \in A^c(\epsilon,\rho)$ then for any $w \in V$ we have $\mathcal{H}(v_p - v_0, w; \frac{\rho}{2}) > 1 - \epsilon$ and $\mathcal{I}(v_p - v_0, w; \frac{\rho}{2}) < \epsilon, \mathcal{J}(v_p - v_0, w; \frac{\rho}{2}) < \epsilon$.

Define $\mathcal{B}(\mu,\rho) = \{k \leq n : \mathcal{H}(v_k - v_p, w;\rho) \leq 1-\mu$ or $\mathcal{I}(v_k - v_p, w;\rho) \geq \mu, \mathcal{J}(v_k - v_p, w;\rho) \geq \mu\}$. We claim that $\mathcal{B}(\mu,\rho) \subset A(\epsilon,\rho)$. Let $q \in \mathcal{B}(\mu,\rho)$. Then we have $\mathcal{H}(v_q - v_p, w;\rho) \leq 1-\mu$ or $\mathcal{I}(v_q - v_p, w;\rho) \geq \mu, \mathcal{J}(v_q - v_p, w;\rho) \geq \mu$.

Case (i): Suppose $\mathcal{H}(v_q - v_p, w;\rho) \leq 1-\mu$, Then we have $\mathcal{H}(v_q - v_0, w;\frac{\rho}{2}) \leq 1-\epsilon$ and therefore $q \in A(\epsilon,\rho)$ (as otherwise, i.e, if $\mathcal{H}(v_q - v_0, w;\frac{\rho}{2}) > 1-\epsilon$, then $1-\mu \geq \mathcal{H}(v_q - v_p, w;\rho) \geq \mathcal{H}(v_q - v_0, w;\frac{\rho}{2}) \circ \mathcal{H}(v_p - v_0, w;\frac{\rho}{2}) > (1-\epsilon) \circ (1-\epsilon) > 1-\mu$ which is not possible). Hence $\mathcal{B}(\mu,\rho) \subset A(\epsilon,\rho)$.

Case (ii): Suppose $\mathcal{I}(v_q - v_p, w;\rho) \geq \mu, \mathcal{J}(v_q - v_p, w;\rho) \geq \mu$. We first consider $\mathcal{I}(v_q - v_p, w;\rho) \geq \mu$, then we have $\mathcal{I}(v_q - v_0, w;\frac{\rho}{2}) \geq \epsilon$ as otherwise, i.e, if $\mathcal{I}(v_q - v_0, w;\frac{\rho}{2}) < \epsilon$, then $\mu \leq \mathcal{I}(v_q - v_p, w;\rho) \leq \mathcal{I}(v_q - v_0, w;\frac{\rho}{2}) \diamond \mathcal{I}(v_p - v_0, w;\frac{\rho}{2}) < \epsilon \diamond \epsilon < \mu$ which is not possible. On the same lines we have $\mathcal{J}(v_q - v_0, w;\frac{\rho}{2}) \geq \epsilon$. Hence $\mathcal{B}(\mu,\rho) \subset A(\epsilon,\rho)$ and therefore the Theorem is proved. □

Definition 4.3 A neutrosophic $2-$normed space V is said to be statistically complete if every statistical Cauchy sequence in V is statistical convergent in V.

Theorem 4.6 Every neutrosophic $2-$normed space V is statistically complete.

Proof Let (v_k) be statistical Cauchy sequence in V. To prove the Theorem, we have to show that (v_k) is statistical convergent in V. Suppose that (v_k) is not statistical convergent. Let $\epsilon > 0$ and $\rho > 0$. Then $\exists\, p \in \mathbb{N}$ such that $w \in V$ if we take $A(\epsilon,\rho) = \{k \leq n : \mathcal{H}(v_k - v_p, w;\rho) \leq 1-\epsilon$ or $\mathcal{I}(v_k - v_p, w;\rho) \geq \epsilon$, $\mathcal{J}(v_k - v_p, w;\rho) \geq \epsilon\}$ and $\mathcal{B}(\epsilon,\rho) = \{k \leq n : \mathcal{H}(v_k - v_0, w;\frac{\rho}{2}) > 1-\epsilon$ or $\mathcal{I}(v_k - v_0, w;\frac{\rho}{2}) < \epsilon$, $\mathcal{J}(v_k - v_0, w;\frac{\rho}{2}) < \epsilon\}$, then $\delta(A(\epsilon,\rho)) = \delta(B(\epsilon,\rho)) = 0$ and therefore we have $\delta(A^C(\epsilon,\rho)) = \delta(B^C(\epsilon,\rho)) = 1$.

Since $\mathcal{H}(v_k - v_p, w;\rho) \geq 2\mathcal{H}(v_k - v_0, w;\frac{\rho}{2}) > 1-\epsilon$ and $\mathcal{I}(v_k - v_p, w;\rho) \leq 2\mathcal{I}(v_k - v_0, w;\frac{\rho}{2}) < \epsilon$, $\mathcal{J}(v_k - v_p, w;\rho) \leq 2\mathcal{J}(v_k - v_0, w;\frac{\rho}{2}) < \epsilon$ if $\mathcal{H}(v_k - v_0, w;\frac{\rho}{2}) > \frac{1-\epsilon}{2}$ and $\mathcal{I}(v_k - v_0, w;\frac{\rho}{2}) < \frac{\epsilon}{2}$, $\mathcal{J}(v_k - v_0, w;\frac{\rho}{2}) < \frac{\epsilon}{2}$. We have $\delta(\{k \leq n : \mathcal{H}(v_k - v_p, w;\rho) > 1-\epsilon$ and $\mathcal{I}(v_k - v_p, w;\rho) < \epsilon$, $\mathcal{J}(v_k - v_p, w;\rho) < \epsilon\}) = 0$. i.e.,$\delta(A^C(\epsilon,\rho)) = 0$. In this way we obtain a contradiction as $\delta(A^C(\epsilon,\rho)) = 1$. Hence, (v_k) is statistically convergent w.r.t. $2-$norm N_2. □

Theorem 4.7 Let V be a $N-2-NS$ and (v_k) be a sequence in V, then the following statements are equivalents.

(i) (v_k) is a statistically cauchy sequence w.r.t. N_2.

(ii) There is a set $K = \{k_n\} \subseteq \mathbb{N}$ with $\delta\{K\} = 1$ and the associated subsequence $\{v_{k_n}\}_{n\in\mathbb{N}}$ is a cauchy sequence w.r.t. N_2.

5. Conclusion

Fuzzy sets and its generalizations have been frequently used in many branches of science, engineering and technology, especially, in control theory and mathematical modeling of various systems. In present work, we define a neutrosophic $2-$normed space as a generalization of fuzzy normed space and study a generalized limit in a more general setting. The results and definitions presented here will provide a new framework to resolve divergence related problems in these spaces.

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