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# Application Of Some Topological Indices In Nover Topologized Graphs

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**Abstract:** In this paper Nover top $Z_1$ and $Z_2$ $index$ and Nover top $H-index$, $Rd-index$, $GA-index$, $CON-index$ are investigated and some of the related theorems are discussed.These indices are also calculated for some specific types of Nover top graphs such as $2-regular$ , $K_2$ and $K_{2,2}$ Nover Top graphs.

**Keywords:** Nover top $Z_1$ and $Z_2$ $index$ , $H-index$, $Rd-index$, $GA-index$, $CON-index$

# 1 Introduction

There are many applications of graph theory to a wide variety of subjects which include operation Research, Physics, chemistry, Economics, Genetics, Engineering, computer Science etc.,In a classical graph for each vertex or edge there are two possibilities arises that is either in the graph or not in the graph and this will not classical graph model for uncertain problems. Fuzzy set [14] is a generalized version of the classical set in which objects have different membership degrees between zero and one. More work has already been done on fuzzy graphs. Zadeh introduced the degree of membership/Truth($T$) in 1965 and defined the fuzzy set. Atanassov [2] introduced the degree of non-memebership/Falsehood($F$) in 1983 and defined the intuitionstic fuzzy set. Smarandache [13,14,15,16,18,19] introduced the degree of Indeterminancy/Neutrality($I$) as an independent component in 1995 and defined the neutrosophic set on there compenents ($T$,$I$,$F$).Smarandache has introduced in 2020 the n-SuperHyperGraph, with super-vertices [that are groups of vertices] and hyper-edges defined on power-set of power-set... that is the most general form of graph as today, and n-HyperAlgebra.A SuperHyperGraph, is a HyperGraph (where a group of Edges form a HyperEdge) such that a group of vertices are united all together into a SuperVertex like a group of people (=vertices) that are united all together into an organization (=SuperVertex) ;and further on the n-SuperHyperGraph where many groups (=SuperVertices) are united all together to form a group-of-groups (called 2-SuperVertex, or Type-2 SuperVertex ), then a group of Type-2 SuperVertices forms a Type-3 SuperVertex, . . . , and so on up to Type-n SuperVertex, for any n 1, which better reflects our reality. Later Narmada Devi[5,6,7,8,9,10] worked on new type of neutrosophic over,off graph and minimal domination via neutrosophic over graph and neutrosophic over topologized graph. [20,21,23] A lot of topological indices are available in chemical-graph theory and H. Wiener proposed the first index to estimate the boiling point of alkanes called 'Wiener index'. Many topological indices exist only in the crisp but it's new to the Nover graph environment. The main aim of this paper is to define the topological indices in Nover graphs.The various topology indices such as Zagreb index, Randic index, Geometric-arithmetic, Hormonic are discussed them. Neutrosophic over graphs in addition to the degree of accuracy of each membership function, the degree of its membership is uncertain, as well as its inaccuracy. so in many cases, it may be more logical to use this model than graphs in real-world problems. Since that neutrosophic over graphs are more efficient than fuzzy graphs for modelling real problems. In this paper , we try to calculate some Neutrosophic over topological indices for this type of graphs.

# 2 Preliminiaries

**Definition 2.1.** **[4]** A set $\mathscr{D}$ of vertices of $\mathscr{G}$ is said to be a *topologized domination set* $\mathscr{D}$ if $\mathscr{G}$ is a *topologized graph* and every vertex in $\mathscr{V} - \mathscr{D}$ is adjacent to atleast one vertex of in $\mathscr{D}$.

**Definition 2.2.** **[6,7,9]** A Neutrosophic Over set $\mathscr{D}$ is defined as
$\mathscr{D} = (x, <(T(x), I(x), F(x)>), x \in X$ such that there exist some element in $\mathscr{D}$ that have atleast one neutrosophic component that is $> 1$ and no element has neutrosophic component that are $< 0$ and $((T(x), I(x), F(x)) \in [0, \Omega]$ where $\Omega$ is called Overlimit such that $0 < 1 < \Omega$.

**Definition 2.3.** **[5,6,7]** A Nover graph is a pair $\mathscr{G} = (A, B)$ of a crisp graph $\mathscr{G}^* = (\mathscr{V}, \mathscr{E})$ where $A$ is Nvertex over set on $\mathscr{V}$ and $B$ is a Nedge over set on $\mathscr{E}$ such that $\mathscr{T}_{\mathscr{B}}(xy) \leq (T_A(x) \wedge T_A(y))$, $I_B(xy) \leq (I_A(x) \wedge I_A(y))$, $F_B(xy) \geq (F_A(x) \vee F_A(y))$.

**Definition 2.4.** **[9,10]** A topologized graph is a topological space $\mathscr{H}$ such that

(i) every singleton is open or closed

(ii) $\forall h \in \mathscr{H}$, $|\eth(h)| \leq 2$, since $\eth(h)$ is denoted by the boundary of a point $h$.

**Definition 2.5.** **[9]** A *Nover graph* $\mathscr{G} = (A, B)$ is called *NOver Top graph* if $\mathscr{G}^*$ satisfy the following condition

(i) every singleton is open or closed in $\mathscr{V}$.

(ii) $\forall f \in \mathscr{F}$, $|\partial(f)| \leq 2$ where $\partial(f)$ is denoted by the boundary of a point $x$

**Definition 2.6.** **[9]** Let $\mathscr{G}$ be a Nover top graph. Let $x, y \in \mathscr{V}$. Then $x$ dominate $y$ in $\mathscr{G}$ if edge $xy$ is effective edge $T_B(xy) = (T_A(x) \wedge T_A(y))$, $I_B(xy) = (I_A(x) \wedge I_A(y))$, $F_B(xy) = (F_A(x) \vee F_A(y))$.
A subset $\mathscr{D}_{\mathscr{N}}$ of $\mathscr{V}$ is called a Nover top dominating set in $\mathscr{G}$ if every vertex $\mathscr{V} \notin \mathscr{D}_{\mathscr{N}}$ there exists $u \in \mathscr{D}_{\mathscr{N}}$ such that $u$ dominates $\mathscr{V}$.

# 3 $Z_1$ and $Z_2$ index in Nover top graphs

**Definition 3.1.** Let $\mathscr{G} = (A, B)$ be the Nover top graph with non-empty vertex set. The $Z_1$ index is denoted by $\mathscr{B}_{\text{Nov}}(\mathscr{G})$ and defined as

$$\mathscr{B}_{\text{Nov}}(\mathscr{G}) = \sum_{i=1}^{n} (T_A(u_i), I_A(u_i), F_A(u_i)) d_2(u_i), \forall u_i \in \mathscr{V}$$

**Definition 3.2.** The $Z_2$ index is denoted by by $\mathscr{B}^*{}_{\text{Nov}}(\mathscr{G})$ and defined as

$$\mathscr{B}^*{}_{\text{Nov}}(\mathscr{G}) = \frac{1}{2}\sum_{i=1}^{n}\left[(T_A(u_i), I_A(u_i), F_A(u_i))d(u_i)\right]\left[(T_A(v_j), I_A(v_j), F_A(u_i)d(v_j)\right],$$

$\forall i \neq j$, and $(u_i, v_j) \in \mathscr{E}$

**Example 3.1.** Consider the Nover top graph $\mathscr{G} = (A, B)$ as shown in Figure 1.

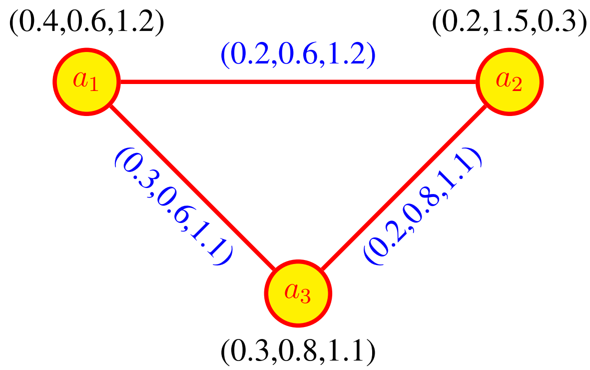



Figure 1: A Nover top graph $\mathscr{G}$

Let $a_1, a_2$ and $a_3$ denote the vertices and $(0.2, 0.6, 1.2), (0.2, 0.8, 1.1), (0.3, 0.6, 1.1)$ denote the edges which are labelled $f_u(0.2, 0.6, 1.2) = (a_1, a_2)$, $f_u(0.2, 0.8, 1.1) = (a_2, a_3)$, $f_u(0.3, 0.6, 1.1) = (a_1, a_3)$.

Let $\mathscr{X} = \Big\{a_1, a_2, a_3, (0.2, 0.6, 1.2), (0.2, 0.8, 1.1), (0.3, 0.6, 1.1)\Big\}$ be a topological space defined by the topology

$$\tau = \Big\{\emptyset, \mathscr{X}, \{a_1\}, \{a_2\}, \{a_3\}, \{a_1, a_2\}, \{a_1, a_3\}, \{a_2, a_3\}\Big\}$$

Here for every $x \in \mathscr{X}$, $\{x\}$ is open.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_3\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_1, a_2\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The $Z_1$ index is

$$d(a_1) = (.2 + .3, .6 + .6, 1.2 + 1.1) = (0.5, 1.2, 2.3)$$
$$d(a_2) = (.2 + .2, .6 + .8, 1.2 + 1.1) = (0.4, 1.4, 2.3)$$
$$d(a_3) = (.2 + .3, .8 + .6, 1.1 + 1.1) = (0.5, 1.4, 2.2)$$

Now, we have

$$d_2(a_1) = (0.04 + 0.09, 0.36 + 0.36, 1.44 + 1.21) = (0.13, 0.69, 2.65)$$
$$d_2(a_2) = (0.04 + 0.04, 0.36 + 0.64, 1.44 + 1.21) = (0.08, 1, 2.65)$$
$$d_2(a_3) = (0.04 + 0.09, 0.64 + 0.36, 1.21 + 1.21) = (0.13, 1, 1.42)$$

$$\mathscr{B}_{\text{Nov}}(\mathscr{G}) = \sum_{i=1}^{n}(T_A(u_i), I_A(u_i), F_A(u_i))d_2(u_i)$$

$$= (.4, .6, 1.2)(0.13, 0.69, 2.65) + (.2, 1.5, 0.3)(0.08, 1, 2.65) + (.3, .8, 1.1)(0.13, 1, 1.42)$$
$$= (.052 + 0.414 + 3.18) + (.016 + 1.5 + 0.795) + (0.039 + .8 + 1.562)$$
$$= 8.358$$

**Example 3.2.** Let $\mathscr{G}$ be a same Nover top graph as defined in example 3.1 Then $Z_2$ index is

$$\begin{aligned}
\mathscr{B}^{*}{}_{\text{Nov}}(\mathscr{G}) &= \frac{1}{2}[(0.4, 0.6, 1.2)(0.5, 1.2, 2.3) \times (0.2, 1.5, 0.3)(0.4, 1.4, 2.3)+ \\
&\quad (0.4, 0.6, 1.2)(0.5, 1.2, 2.3) \times (0.3, 0.8, 1.1)(0.5, 1.4, 2.2)+ \\
&\quad (0.2, 1.5, 0.3)(0.4, 1.4, 2.3) \times (0.3, 0.8, 1.1)(0.5, 1.4, 2.2)] \\
&= \frac{1}{2}[(0.2 + 0.72 + 2.76) \times (0.08 + 2.1 + 0.69)+ \\
&\quad (0.2 + 0.72 + 2.76) \times (0.15 + 1.12 + 2.42)+ \\
&\quad (0.08 + 2.1 + 0.69) \times (0.15 + 1.12 + 2.42)] \\
&= \frac{1}{2}[3.68 \times 2.87 + 3.68 \times 3.69 + 2.87 \times 3.69] \\
&= \frac{1}{2}[10.5616 + 13.5792 + 10.5903] \\
&= \frac{1}{2}[34.7311] = 17.3656
\end{aligned}$$

**Example 3.3.** Consider the Nover top graph $\mathscr{G} = (A, B)$ as shown in Fig. 2

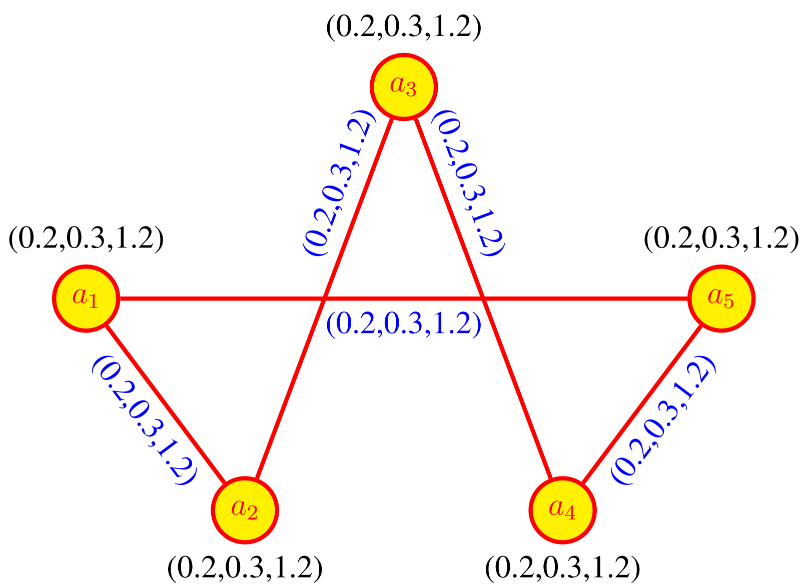



Figure 2: Storng Nover top graph $\mathscr{G}$

Let $a_1, a_2, a_3, a_4$ and $a_5$ denote the vertices and $(0.2, 0.3, 1.2)$, $(0.2, 0.3, 1.2)$, $(0.2, 0.3, 1.2)$, $(0.2, 0.3, 1.2)$ and $(0.2, 0.3, 1.2)$ denote the edges which are labelled $f_u(0.2, 0.3, 1.2) = (a_1, a_2)$, $f_u(0.2, 0.3, 1.2) = (a_2, a_3)$, $f_u(0.2, 0.3, 1.2) = (a_3, a_4)$, $f_u(0.2, 0.3, 1.2) = (a_4, a_5)$, $f_u(0.2, 0.3, 1.2) = (a_1, a_5)$.

Let $\mathscr{X} = \Big\{a_1, a_2, a_3, , a_4, a_5, (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2), (0.2, 0.3, 1.2)\Big\}$ be a topological space defined by the topology

$$\tau = \Big\{\emptyset, \mathscr{X}, \{a_1\}, \{a_2, a_3\}, \{a_4\}, \{a_5\}, \{a_1, a_2, a_3\}, \{a_1, a_4\}, \{a_1, a_5\}, \{a_1, a_2, a_3, a_4\}, \{a_1, a_2, a_3, a_5\}, \{a_2, a_3, a_4\}, \{a_2, a_3, a_5\}, \{a_2, a_3, a_4, a_5\}, \{a_4, a_5\}, \{a_1, a_4, a_5\}, \{a_1, a_2, a_4, a_5\}, \{a_1, a_3, a_4, a_5\}\Big\}$$

Here for every $x \in \mathscr{X}$, $\{x\}$ is open or closed.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_5\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_2, a_4\}$, $\partial(a_4) = \{a_3, a_5\}$, $\partial(a_5) = \{a_1, a_4\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The $Z_1$ index is

$$\begin{aligned}
d(a_1) &= (0.2 + 0.3, 0.3 + 0.3, 1.2 + 1.2) = (0.4, 0.6, 2.4)\\
d(a_2) &= (0.4, 0.6, 2.4)\\
d(a_3) &= (0.4, 0.6, 2.4)\\
d(a_4) &= (0.4, 0.6, 2.4)\\
d(a_5) &= (0.4, 0.6, 2.4)
\end{aligned}$$

Now, we have

$$\begin{aligned}
d_2(a_1) &= (0.04 + 0.04, 0.36 + 0.36, 1.44 + 1.44) = (0.08, 0.72, 2.88)\\
d_2(a_2) &= (0.08, 0.72, 2.88)\\
d_2(a_3) &= (0.08, 0.72, 2.88)\\
d_2(a_4) &= (0.08, 0.72, 2.88)\\
d_2(a_5) &= (0.08, 0.72, 2.88)
\end{aligned}$$

$$\begin{aligned}
\mathscr{B}_{\text{Nov}}(\mathscr{G}) &= \sum_{i=1}^{5}(T_A(u_i), I_A(u_i), F_A(u_i))d_2(u_i)\\
&= (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88)\\
&\quad + (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88) + (.2, .3, 1.2)(0.08, 0.72, 2.88)\\
&= (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456)\\
&\quad + (.016 + 0.216 + 3.456) + (.016 + 0.216 + 3.456)\\
&= 3.688 + 3.688 + 3.688 + 3.688 + 3.688\\
&= 18.44
\end{aligned}$$

$$\begin{aligned}
\mathscr{B}^*{}_{\text{Nov}}(\mathscr{G}) = \frac{1}{2}[&(.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)\\
&+ (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)\\
&+ (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)\\
&+ (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)
\end{aligned}$$

$$
\begin{aligned}
&+ (.2, .3, 1.2)(0.4, 0.6, 2.4) \times (.2, .3, 1.2)(0.4, 0.6, 2.4)] \\
&= \frac{1}{2}[(0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
&\quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
&\quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
&\quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
&\quad + (0.08 + 0.18 + 2.88) \times (0.08 + 0.18 + 2.88) \\
&= \frac{1}{2}[3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14 + 3.14 \times 3.14] \\
&= \frac{1}{2}[9.8596 + 9.8596 + 9.8596 + 9.8596 + 9.8596] \\
&= \frac{1}{2}[49.298] = 24.649
\end{aligned}
$$

**Definition 3.3.** Let $\mathscr{G} = (A, B)$ be an neutrosophic over top graph. $\mathscr{G}$ is a regular strong neutrosophic over top graph if it satisfies the following conditions.

$$T_B(a, b) = \min(T_A(a), T_A(b)), I_B(a, b) = \min(I_A(a), I_A(b)), F_B(a, b) = \max(F_A(a), F_A(b))$$

**Theorem 3.1.** Let $\mathscr{G}$ be the regular Nover top graph. Then, we have

$$\mathscr{B}_{\text{Nov}}(\mathscr{G}) = c^2 \times \sum_{i=1}^{n}[T_A(u_i) + I_A(u_i)] + c_1^2 \times \sum_{i=1}^{n} F_A(u_i), \forall u_i \in \mathscr{V}$$

where $\sum_{v \neq u} T_B(v, u) = c$, $\sum_{v \neq u} I_B(v, u) = c$, $\sum_{v \neq u} F_B(v, u) = c_1$.

**Proof:**

Given the degree of definition of each vertex

$$
\begin{aligned}
d(v) &= (d_T(v), d_I(v), d_F(v)) \\
&= \left( \sum_{\substack{v \in \mathscr{V} \\ v \neq u}} T_B(v, u), \sum_{\substack{v \in \mathscr{V} \\ v \neq u}} I_B(v, u), \sum_{\substack{v \in \mathscr{V} \\ v \neq u}} F_B(v, u) \right)
\end{aligned}
$$

On the other hand, for regular Nover top graphs, we know that

$$\sum_{v \neq u} T_B(v, u) = c, \sum_{v \neq u} I_B(v, u) = c, \sum_{v \neq u} F_B(v, u) = c_1$$

Therefore,

$$
\begin{aligned}
d(v) &= (d_T(v), d_I(v), d_F(v)) \\
&= (c, c, c_1)
\end{aligned}
$$

Now, by embedding the formula in the $Z_1$ index, we will get the desired result. The proof is complete.

**Theorem 3.2.** Let $\mathscr{G}$ be the regular Nover top graph. Then, we have

$$\mathscr{B}^*{}_{\text{Nov}}(\mathscr{G}) = \frac{1}{2}c^2\sum_{i=1}^{n}[T_A(u_i)+I_A(u_i)][T_A(v_j)+I_A(v_j)] + \frac{1}{2}c_1^2\sum_{i=1}^{n}[F_A(u_i)F_A(v_j)],$$

$\forall i \neq j$ and $(u_i, v_j) \in \mathscr{E}$
where $\sum_{v\neq u} T_B(u,v) = c$, $\sum_{v\neq u} I_B(u,v) = c$, $\sum_{v\neq u} F_B(u,v) = c_1$.

**Proof:** Assume $\mathscr{G}$ is regular Nover top graph, using the $Z_2$ index formula for $\mathscr{G}$, we have $\forall i \neq j$ and $(u_i, v_j) \in \mathscr{E}$.

$$\begin{aligned}
\mathscr{B}^*{}_{\text{Nov}}(\mathscr{G}) &= \frac{1}{2}\sum_{i=1}^{n}\left[(T_A(u_i), I_A(u_i), I_A(u_i))d(u_i)\right]\left[(T_A(v_j), I_A(v_j), F_A(v_J)d(v_j)\right] \\
&= \frac{1}{2}\sum\left[(T_A(u_i), I_A(u_i), I_A(u_i))d(d_T(u_i), d_I(u_i), d_F(u_i)\right] \\
&\qquad \left[(T_A(v_j), I_A(v_j), F_A(v_J)d(d_T(v_j), d_I(v_j), d_F(v_j)\right] \\
&= \frac{1}{2}\sum\left[(T_A(u_i), I_A(u_i), I_A(u_i))(c,c,c_1)\right]\left[(T_A(v_j), I_A(v_j), F_A(v_J)(c,c,c_1)\right] \\
&= \frac{1}{2}\sum\left[cT_A(u_i)+cI_A(u_i)+c_1I_A(u_i)\right]\times\left[cT_A(v_j)+cI_A(v_j)+c_1F_A(v_j)\right] \\
&= \frac{1}{2}\sum c\left[T_A(u_i)+I_A(u_i)\right]c_1[I_A(u_i)]c\left[T_A(v_j)+I_A(v_j)\right]c_1[F_A(v_j)] \\
&= \frac{1}{2}c^2\sum\left[T_A(u_i)+I_A(u_i)\right]\left[T_A(v_j)+I_A(v_j)\right] + \frac{1}{2}c_1^2\sum[I_A(u_i)F_A(v_j)]
\end{aligned}$$

The desired result was obtained.
These above two theorems are illustrated the following example.

**Example 3.4.** Consider the Nover top graph $G = (A, B)$ as shown in Fig

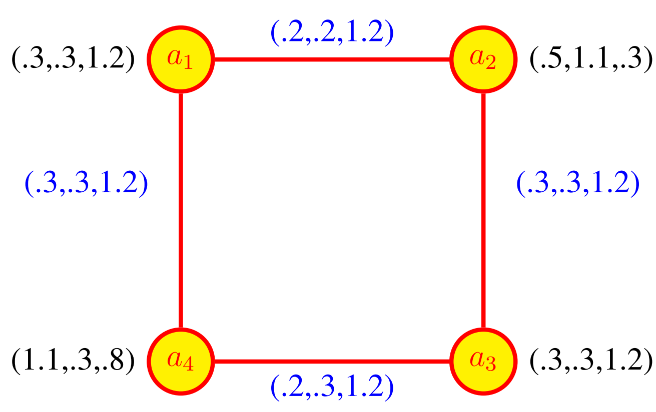



Figure 3: Regular Nover top graph

Let $a_1, a_2, a_3$ and $a_4$ denote the vertices and $(.2, .2, 1.2)$, $(.3, .3, 1.2)$, $(.2, .2, 1.2)$ and $(.3, .3, 1.2)$ denote the edges which are labelled $f_u(.2, .2, 1.2) = (a_1, a_2)$, $f_u(.3, .3, 1.2) = (a_2, a_3)$, $f_u(.2, .2, 1.2) = (a_3, a_4)$, $f_u(.3, .3, 1.2) = (a_4, a_1)$.

Let $X = \Big\{a_1, a_2, a_3, , a_4, (.2, .2, 1.2), (.3, .3, 1.2), (.2, .2, 1.2), (.3, .3, 1.2)\Big\}$ be a topological space defined by the topology

$$\tau = \Big\{\emptyset, \mathscr{X}, \{a_1\}, \{a_2\}, \{a_3\}, \{a_4\}, \{a_1, a_2\}, \{a_1, a_4\}, \{a_2, a_3\}, \{a_2, a_4\}, \{a_3, a_4\}, \{a_1, a_2, a_3\}, \{a_1, a_2, a_4\}, \{a_2, a_3, a_4\}, \{a_1, a_3, a_4\}\Big\}$$

Here for every $x \in \mathscr{X}$, $\{x\}$ is open.

By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a_1) = \{a_2, a_4\}$, $\partial(a_2) = \{a_1, a_3\}$, $\partial(a_3) = \{a_2, a_4\}$, $\partial(a_4) = \{a_1, a_3\}$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

The $Z_1$ index is

$$d(a_1) = (0.5, 0.5, 2.4)$$
$$d(a_2) = (0.5, 0.5, 2.4)$$
$$d(a_3) = (0.5, 0.5, 2.4)$$
$$d(a_4) = (0.5, 0.5, 2.4)$$

$$\begin{aligned}\mathscr{B}_{\text{Nov}}(\mathscr{G}) &= c^2 \sum_{i=1}^{n} [T_A(u_i) + I_A(u_i)] + c_1^2 \sum_{i=1}^{n} [I_A(u_i)] \\ &= (.5)^2[(.3 + .3) + (.5 + 1.1) + (.3 + .3) + (1.1 + .3)] + (2.4)^2[1.2 + .3 + 1.2 + .8] \\ &= (.5)^2[.6 + 1.6 + .6 + 1.4] + (2.4)^2[3.5] \\ &= (.5)^2 \times 4.2 + (2.4)^2 \times 3.5 \\ &= 1.05 + 20.16 \\ &= 21.21\end{aligned}$$

The $Z_2$ index is

$$\begin{aligned}\mathscr{B^*}_{\text{Nov}}(\mathscr{G}) &= \frac{1}{2}(.5)^2[.6 \times 1.6 + .6 \times 1.4 + 1.6 \times .6 + .6 \times 1.4] + \\ &\quad \frac{1}{2}(2.4)^2[1.2 \times .3 + 1.2 \times .8 + .3 \times 1.2 + 1.2 \times .8] \\ &= \frac{1}{2}(.5)^2(3.6) + \frac{1}{2}(2.4)^2(2.64) \\ &= \frac{1}{2} \times 0.9 + \frac{1}{2} \times 15.2064 \\ &= 0.45 + 7.6034 \\ &= 8.0532\end{aligned}$$

## 4 H-index and Rd-index in Nover top graphs

**Definition 4.1.** The H-index of Nover top graph $\mathscr{G}$ is defined as

$$\text{H}_{\text{Nov}}(\mathscr{G}) = \sum \frac{1}{[A(u_i) \cdot d(u_i)][A(v_j) \cdot d(v_j)]}, u_i, v_j \in \mathscr{E}, i \neq j$$

**Definition 4.2.** R-index in Nover top graph $\mathscr{G}$ is defined as

$$\mathrm{Rd}_{\mathrm{Nov}}(\mathscr{G}) = \frac{1}{\sum\{[A(u_i)d(u_i)][A(v_j)d(v_j)]\}^{\frac{1}{2}}}, u_i, v_j \in \mathscr{E}, \forall i \neq j$$

**Example 4.1.** For example (3.1), the H-index of Nover top graph $\mathscr{G}$ is

$$\begin{aligned}\mathrm{H}_{\mathrm{Nov}}(\mathscr{G}) &= \frac{1}{(.4,.6,1.2)(.5,1.2,2.3)+(.2,1.5,.3)(.4,1.4,2.3)}+\\ &\frac{1}{(.4,.6,1.2)(.5,1.2,2.3)+(.3,.8,1.1)(.5,1.4,2.2)}+\\ &\frac{1}{(.2,1.5,.3)(.4,1.4,2.3)+(.3,.8,1.1)(.5,1.4,2.2)}\\ &= \frac{1}{3.68+2.87}+\frac{1}{3.68+3.69}+\frac{1}{2.87+3.69}\\ &= \frac{1}{6.55}+\frac{1}{7.37}+\frac{1}{6.56}\\ &= 0.153+0.136+0.152\\ &= 0.441\end{aligned}$$

**Example 4.2.** For example (3.1), the Rd-index of Nover top graph $\mathscr{G}$ is

$$\begin{aligned}\mathrm{Rd}_{\mathrm{Nov}}(\mathscr{G}) &= \frac{1}{\sqrt{(.4..6,1.2)(.5,1.2,2.3)\times(.2,1.5,.3)(.4,1.4,2.3)}}+\\ &\frac{1}{\sqrt{(.4..6,1.2)(.5,1.2,2.3)\times(.3,.8,1.1)(.5,1.4,2.2)}}+\\ &\frac{1}{\sqrt{(.2.1.5,.3)(.4,1.4,2.3)\times(.3,.8,1.1)(.5,1.4,2.2)}}\\ &= \frac{1}{\sqrt{10.566}}+\frac{1}{\sqrt{13.5792}}+\frac{1}{\sqrt{10.5903}}\\ &= \frac{1}{3.24986}+\frac{1}{3.68499}+\frac{1}{3.2543}\\ &= 0.0300+0.27137+0.3073\\ &= 0.6086\end{aligned}$$

**Definition 4.3.** Let $\mathscr{G}_1$ and $\mathscr{G}_2$ be any neutrosophic over graphs isomorphism $f : \mathscr{G}_1 \to \mathscr{G}_2$ is bijective mapping $f : V_1 \to V_2$ which satisfies the following conditions

(a) $T_{A_1}(x_1) = T_{A_2}(f(x_1))$, $I_{A_1}(x_1) = I_{A_2}(f(x_1))$ and $F_{A_1}(x_1) = F_{A_2}(f(x_1))$

(b) $T_{B_1}(x_1, y_1) = T_{B_2}(f(x_1), f(y_1))$, $I_{B_1}(x_1, y_1) = I_{B_2}(f(x_1), f(y_1))$ and $F_{B_1}(x_1, y_1) = F_{B_2}(f(x_1), f(y_1))$ for all $x_1 \in V_1$, $x_1, y_1 \in E_1$

**Example 4.3.**

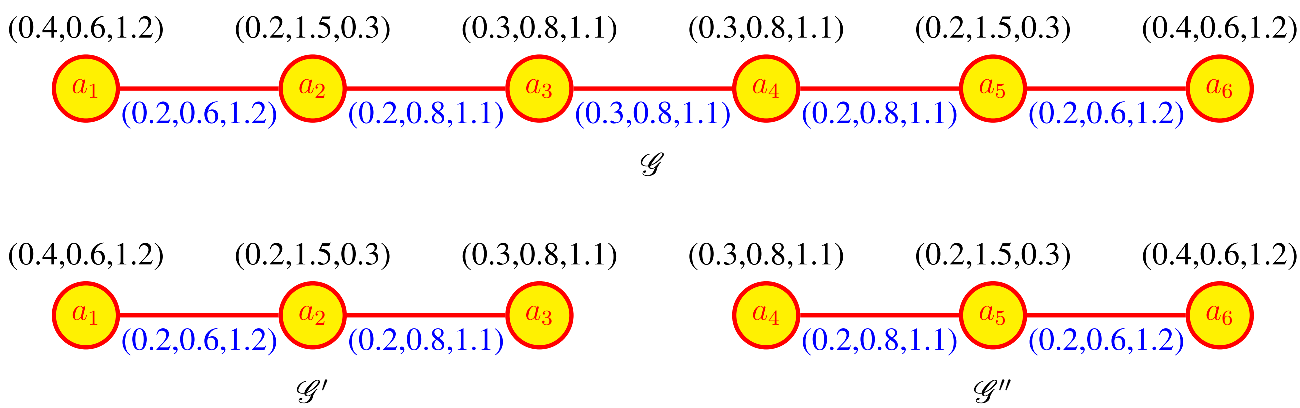



Figure 4: $\mathscr{G}'$ and $\mathscr{G}$" are isomorphic graphs

**Theorem 4.1.** Let $\mathscr{G}$ be the connected Nover top $T(u,v)$, $I(u,v)$, $F(u,v)$ graph and $(T,I,F)$ be the true membership, indeterminancy membership and falsity membership value of the chosen edge of $\mathscr{G}$ such that removal of $(\mu,\gamma,\sigma)$ from $\mathscr{G}$ splits into two Nover top graphs such that whose vertex set satisfies $|V_{\mathscr{G}'}| < |V_{\mathscr{G}''}|$ and therefore its disconnected. Then

(i) $\mathscr{B}_{Nov}(\mathscr{G}') < \mathscr{B}_{Nov}(\mathscr{G}'')$

(ii) $\mathscr{B}^*{}_{Nov}(\mathscr{G}') < \mathscr{B}^*{}_{Nov}(\mathscr{G}'')$

(iii) $H_{Nov}(\mathscr{G}') < H_{Nov}(\mathscr{G}'')$

(iv) $Rd_{Nov}(\mathscr{G}') < Rd_{Nov}(\mathscr{G}'')$

where $|V_{\mathscr{G}'}| < |V_{\mathscr{G}''}|$ denote the cardinality of $\mathscr{G}'$ & $\mathscr{G}''$ respectively.

**Proof:**

Let $\mathscr{G}$ be a connected Nover top graph where splitted into two Nover top graph $\mathscr{G}'$ and $\mathscr{G}''$ by removing the chosen membership value of the edge in $\mathscr{G}$. We know that $\mathscr{G}$ be the Nover top graph and $H$ be the Nover top sub graph of $\mathscr{G}$ such that $H = \mathscr{G} - u$ then $\mathscr{B}_{Nov}(\mathscr{G} - u) < \mathscr{B}_{Nov}(\mathscr{G})$ and $\mathscr{B}^*{}_{Nov}(\mathscr{G} - u) < \mathscr{B}^*{}_{Nov}(\mathscr{G})$.

Therefore, we get, $\mathscr{B}_{Nov}(\mathscr{G}') < \mathscr{B}_{Nov}(\mathscr{G}'') < \mathscr{B}_{Nov}(\mathscr{G})$ which implies that $\mathscr{B}_{Nov}(\mathscr{G}') < \mathscr{B}_{Nov}(\mathscr{G}'')$.

Hence the theorem proved.

The remaining cases are trivially true by the following above method.

# 5 GA- index in Nover top graphs

**Definition 5.1.** The GA-index of Nover top graph $\mathscr{G}$ is defined as

$$\mathrm{GA_{Nov}}(\mathscr{G}) = \frac{2\{[A(u_i)d(u_i)][A(v_j)d(v_j)]\}^{\frac{1}{2}}}{[A(u_i)d(v_j) + A(v_j)d(v_j)]^{\frac{1}{2}}}$$

**Theorem 5.1.** Let $\mathscr{G}$ be a Nover top graph with $m$ edges. Then $\mathrm{GA_{Nov}}(\mathscr{G}) \leq m$ with equality if and only if every component of $\mathscr{G}$ is regular.

**Example 5.1.** We have the previous example (3.1)

$$\begin{aligned}
\mathrm{GA}_{\mathrm{Nov}}(\mathscr{G}) &= \frac{2\begin{bmatrix} (.4,.6,1.2)(.5,1.2,2.3)+(.2,1.5,.3)(.4,1.4,2.3)+ \\ (.4,.6,1.2)(.5,1.2,2.3)+(.3,.8,1.1)(.5,1.4,2.3)+ \\ (.2,1.5,.3)(.4,1.4,2.3)+(.3,.8,1.1)(.5,1.4,2.2) \end{bmatrix}^{\frac{1}{2}}}{\begin{bmatrix} (.4,.6,1.2)+(.5,1.2,2.3)+(.2,1.5,.3)+(.4,1.4,2.3)+ \\ (.4,.6,1.2)+(.5,1.2,2.3)+(.3,.8,1.1)+(.5,1.4,2.2)+ \\ (.2,1.5,.3)+(.4,1.4,2.3)+(.3,.8,1.1)+(.5,1.4,2.2) \end{bmatrix}^{\frac{1}{2}}} \\
&= \frac{2[(3.68\times 2.87)+(3.68\times 3.69)+(2.87\times 3.69)]^{\frac{1}{2}}}{[2.2+4+2+4.1+2.2+4+2.2+4.1+2+4.1+2.2+4.1]^{\frac{1}{2}}} \\
&= \frac{2[10.562+13.579+10.5903]^{\frac{1}{2}}}{[37.2]^{\frac{1}{2}}} \\
&= \frac{2[34.7313]^{\frac{1}{2}}}{6.099} \\
&= \frac{2\times 5.8933}{6.099} \\
&= 1.9325
\end{aligned}$$

# 6 Connectivity Index in Nover top graphs

**Definition 6.1.** The strength of connectedness between $u_i$ and $v_j$ is defined as

$$\mathrm{CONN}_P(u_i,v_j)=\left(\min_{e\in P_{u_iv_j}} T_B(e),\ \min_{e\in P_{u_iv_j}} I_B(e),\ \max_{e\in P_{u_iv_j}} F_B(e)\right)$$

where $P_{u_iv_j}$ is the path between $u_i$ and $v_j$

$$|\mathrm{CONN}_P(u_i,v_j)|=2\left(\min_{e\in P_{u_iv_j}} T_B(e)\right)-\left(\min_{e\in P_{u_iv_j}} I_B(e)\right)-\left(\max_{e\in P_{u_iv_j}} F_B(e)\right)$$

Then $\mathrm{CONN}_P(u_i,v_j)=\max_p\{|\mathrm{CONN}_P(u_i,v_j)|\}$.

**Definition 6.2.** The Connectivity index (CI) of $\mathscr{G}$ is defined by

$$\mathrm{CI}_{\mathrm{Nov}}(\mathscr{G})=\sum_{u_iv_j\in\mathscr{V}} A(u_i)\cdot A(v_j)\times \mathrm{CONN}_{\mathscr{G}}(u_i,v_j)$$

Here $\mathrm{CONN}_{\mathscr{G}}(u_i,v_j)$ is the strength of connectedness between $u_i$ and $v_j$.

**Example 6.1.** For example (3.1), the strength of connectedness between $a_1$ and $a_2$ from the direct path $p_1=a_1a_2$ is $\mathrm{CONN}_{p_1}(a_1,a_2)=(0.2,0.6,1.2)$.

From path $p_2=a_1a_3a_2$

$$\mathrm{CONN}_{p_2}(a_1,a_2)=(\min\{0.3,0.2\},\min\{0.6,0.8\},\max\{1.1,1.1\})$$

$$= (0.2, 0.6, 1.1)$$

$a_1$ and $a_3$ from the direct path $p_1 = a_1a_3$ is

$$\text{CONN}_{p_1}(a_1, a_3) = (0.3, 0.6, 1.1)$$

From path $p_2 = a_1a_2a_3$

$$\begin{aligned}\text{CONN}_{p_2}(a_1, a_3) &= (\min\{0.2, 0.2\}, \min\{0.6, 0.8\}, \max\{1.2, 1.1\})\\ &= (0.2, 0.6, 1.2)\end{aligned}$$

$a_2$ and $a_3$ from the direct path $p_1 = a_2a_3$ is

$$\text{CONN}_{p_1}(a_2, a_3) = (0.2, 0.8, 1.1)$$

From path $p_2 = a_2a_1a_3$

$$\begin{aligned}\text{CONN}_{p_2}(a_2, a_3) &= (\min\{0.2, 0.3\}, \min\{0.6, 0.6\}, \max\{1.2, 1.1\})\\ &= (0.2, 0.6, 1.2)\end{aligned}$$

Then, we have for $a_1$ and $a_2$

$$\begin{aligned}|\text{CONN}_{p_1}(a_1, a_2)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4\\ |\text{CONN}_{p_2}(a_1, a_2)| &= 2 \times (0.2) - 0.6 - 1.1 = -1.3\end{aligned}$$

For $a_1$ and $a_3$

$$\begin{aligned}|\text{CONN}_{p_1}(a_1, a_3)| &= 2 \times (0.3) - 0.6 - 1.1 = -1.3\\ |\text{CONN}_{p_2}(a_1, a_3)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4\end{aligned}$$

For $a_2$ and $a_3$

$$\begin{aligned}|\text{CONN}_{p_1}(a_2, a_3)| &= 2 \times (0.2) - 0.8 - 1.1 = -1.5\\ |\text{CONN}_{p_2}(a_2, a_3)| &= 2 \times (0.2) - 0.6 - 1.2 = -1.4\end{aligned}$$

Since we have

$$\begin{aligned}\text{CONN}_{\mathscr{G}}(a_1, a_2) &= -1.3\\ \text{CONN}_{\mathscr{G}}(a_1, a_3) &= -1.3\\ \text{CONN}_{\mathscr{G}}(a_2, a_3) &= -1.4\end{aligned}$$

Then $\text{CI}_{\text{Nov}}(\mathscr{G})$ is calculated as follows.

$$\begin{aligned}\text{CI}_{\text{Nov}}(\mathscr{G}) &= \sum_{u_iv_j \in \mathscr{V}} (T_A(u_i), I_A(u_i), F_A(u_i))(T_A(v_j), I_A(v_j), F_A(v_J) \times \text{CONN}_{\mathscr{G}}(u_i, v_j)\\ &= (.4, .6, 1.2)(.2, 1.5, .3) \times (-1.3) + (.4, .6, 1.2)(.3, .8, 1.1) \times (-1.3) +\\ &\quad (.2, 1.5, .3)(.3, .8, 1.1) \times (-1.4)\end{aligned}$$

$$= (.08 + .9 + .36)(-1.3) + (.12 + .48 + 1.32)(-1.3) + (.06 + 1.2 + .33)(-1.4)$$
$$= (1.34)(-1.3) + (1.92)(-1.3) + (1.59)(-1.4)$$
$$= -1.742 - 2.496 - 2.226$$
$$= -6.464$$

Then CI of $\mathscr{G}$ is equal $-6.464$, which the negative sign indicates the high level of false and indeterminacy information in the problem.

**Theorem 6.1.** Let $\mathscr{G}$ and $\mathscr{G}_1$ be the two Nover top graphs are isomorphic, then the topological indices values of two Nover top graphs are equal.

**Proof:** Let $\mathscr{G} = (\mathscr{V}_{\mathscr{G}}, A_{\mathscr{G}}, B_{\mathscr{G}})$ and $\mathscr{G}_1 = (\mathscr{V}_{\mathscr{G}_1}, A_{\mathscr{G}_1}, B_{\mathscr{G}_1})$ be isomorphic Nover top graphs.

Hence there is an identity function

$$\mu_A : A_{\mathscr{G}}(u) \to A_{\mathscr{G}_1}(u^*) \text{ for all } u \in \mathscr{V}_{\mathscr{G}}, \exists u^* \in \mathscr{V}_{\mathscr{G}_1}$$

as well as

$$\mu_B : B_{\mathscr{G}}(u, v) \to B_{\mathscr{G}_1}(u^*, v^*),$$

then each vertex of $\mathscr{G}$ corresponds to an vertex in $\mathscr{G}_1$, with the same membership value and the same edges.

Hence, the Neutrosophic over top graph structure may differ but collection of vertices and edges are same gives the equal topological indices value.

**Theorem 6.2.** Let $G = (\mathscr{V}_G, A_G, B_G)$ is a Nover top graph and $H$ is the NOver top subgraph of $G$, such that $H$ is made by removing edge $uv \in B_G$ from $G$. Then, we have $\text{CI}_{\text{Nov}(H)} < \text{CI}_{\text{Nov}(\mathscr{G})}$ iff $uv$ is a bridge.

**Proof:** Now suppose that $uv$ is an edge that has maximum (or) minimum components, so they will have an effect on $\text{CONN}_{\mathscr{G}}(u, v)$.

Therefore, by removing edge $uv$, the value of $\text{CONN}_{\mathscr{G}}(u, v)$ will decrease, then we have $\text{CI}_{\text{Nov}(H)} < \text{CI}_{\text{Nov}(\mathscr{G})}$.

Since the bridge is called the edge that has its deletion reducing the $\text{CONN}_{\mathscr{G}}(u, v)$, however, $uv$ is a bridge.

Conversely, given that $uv$ is a bridge. By the definition of bridge we have, for the edge $uv$, $\text{CONN}_{\mathscr{G}}(u, v) > \text{CONN}_{G-uv}(u, v)$, so we conclude that, $\text{CI}_{\text{Nov}(H)} < \text{CI}_{\text{Nov}(\mathscr{G})}$.

**Example 6.2.**

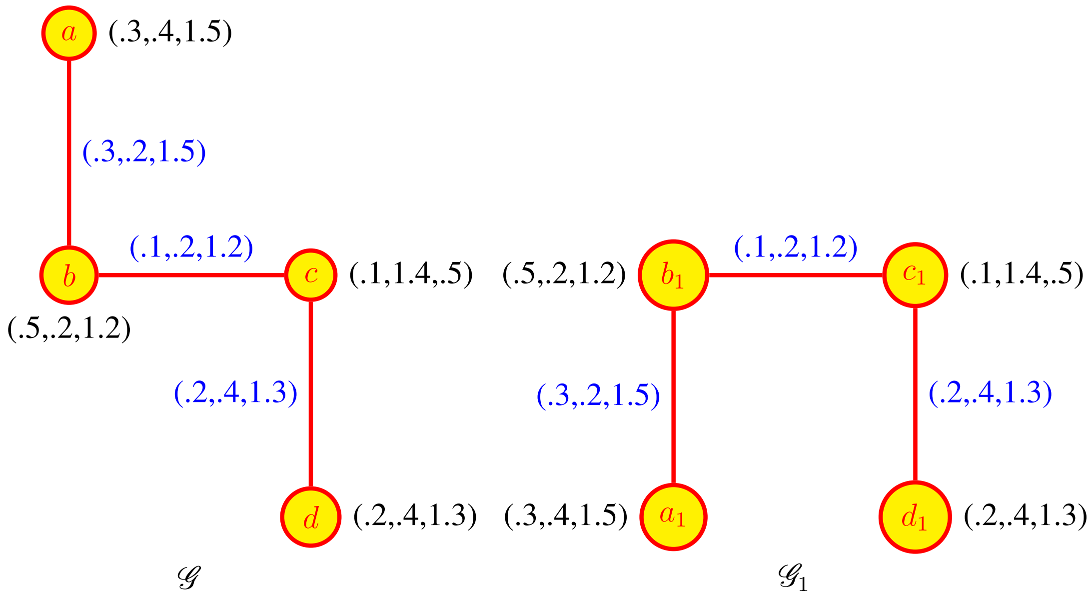
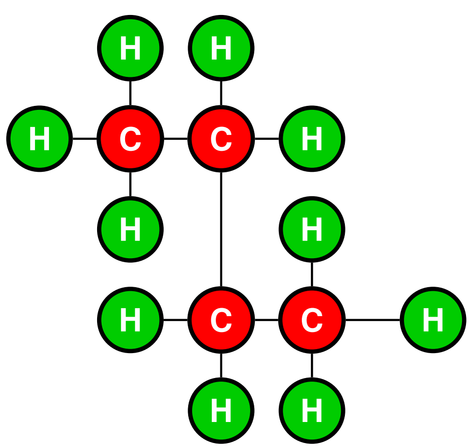



Figure 5: $\mathscr{G}$ and $\mathscr{G}_1$ are isomorphic Nover top graphs

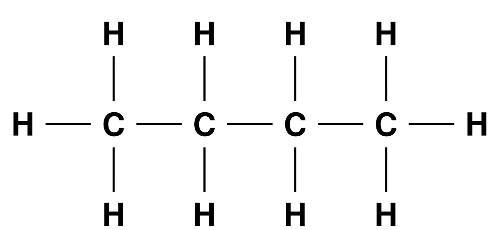
# 7 Illustration

Wiener was introduced two parameters for the specific purpose of correlating the boiling points of members of the alkane series of molecular structure which satisfied a linear formula $t_B = aW + bP + c$ where $t_B$ is the boiling point of a given alkane,$W$ is the wiener number, P is the polarity number and $a, b, c$ are constants. A topological representing of a molecule structure is called molecule graph which is a collection of points representing the atoms in the molecule and set of lines representing the covalent bonds. A hydrogen-detected graph is a molecular graph in which hydrogen atoms are not considered.

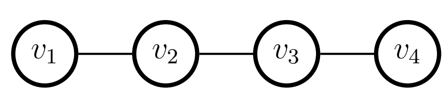
**Example 7.1.** Consider an alkane series butane where molecular and its hydrogen detected graph are given there.




Butane Structure




Butane



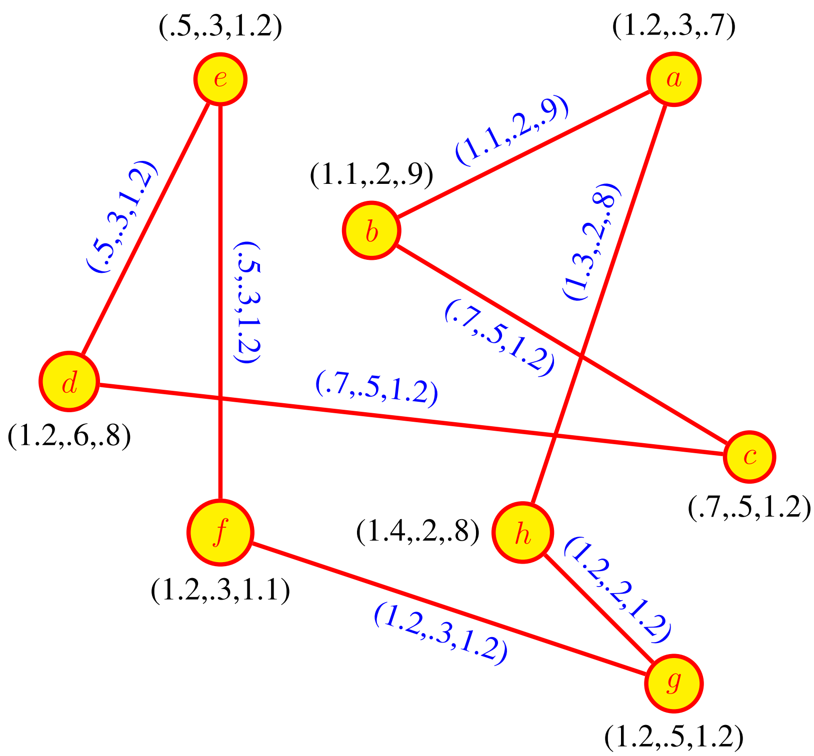

The path graph $P_4$

Suppose we can consider hydrogen detected graph $P_4$ as in Nover top graph structure, we can calculate the Nover top $Z_1$ index for $P_4$. Hence the boiling point of butane in Nover top graph is satisfies linear equation as $t_B = aW + bP + c$. Since butane is non-polar, $b = 0$. so,$t_B = aW + c$.If we can take for particular value of $a, b, c$ and get the boiling point of butane both of them is same.

# 8 Application

Most of the administration in the society, they depends the supporters than on themself. The application of dominating set and connecting index of Nover Top graph is expressed as a following example.An office decide to appoint a head under 8 employees.The employess are assumed to be a,b,c,d,e,f,g,h. In this Nover Top graph, the employees and strength between them are considered as vertex and edges.The true membership,inderterminancy and false membership is taken as a vertex.The true membership value is based on employees talent, work experience and salary basis.The inderterminancy membership function are considered as the persons with ability and skill but do not work in the suitable task.The false membership function is considred as be the lack of compatibility between educational major and occupation, lack of ability, non-skill and health issue.

Consider the following graphical structure




Let $a, b, c, d, e, f, g$ and $h$ denote the vertices and $(1.1, 0.2, 0.9)$, $(0.7, 0.5, 1.2)$, $(0.5, 0.3, 1.2)$, $(0.5, 0.3, 1.2)$,$(1.2, 0.3, 1.2)$,$(1.2, 0.2, 0.8)$, and $(1.3, 0.2, 0.8)$ denote the edges and there is a strong relationship between them.

Let $\mathscr{V} = \{a, b, c, d, e, f, g, h, (1.1, 0.2, 0.9), (0.7, 0.5, 1.2),$ $(0.5, 0.3, 1.2), (0.5, 0.3, 1.2), (1.2, 0.3, 1.2), (1.2, 0.2, 0.8), (1.3, 0.2, 0.8)\}$ be a topology

$\tau = \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{e\}, \{f\}, \{g\}, \{h\}, \{a, b\}, \{a, d\}\{b, c\}, \{e, f\}, \{f, g\}, \{g, h\}....$
$\{a, b, c, d\}, \{a, b, d, f\}, \{e, f, g, h\}, ....\{a, b, c, d, e, f, g\}, \}.$

Here for every $x \in X$, $\{x\}$ is open or closed. By the definition of Nover top graph, we have $|\partial(A)| \leq 2$ and $\partial(a) = (b, h)$, $\partial(b) = (a, c)$, $\partial(c) = (b, d)$, $\partial(d) = (e, c)$, $\partial(e) = (d, f)$ , $\partial(f) = (e, g)$, $\partial(g) = (f, h)$, $\partial(h) = (a, g)$ with $\partial(a_i) = 2$. Hence this graph is Nover top graph.

$$T_A(a) = \text{min}[T_B(a, b), T_B(a, h)] = \text{min}[1.1, 1.1] = 1.1$$
$$I_A(a) = \text{min}[I_B(a, b), I_B(a, h)] = \text{min}[0.2, 0.2] = 0.2$$
$$F_A(a) = \text{max}[F_B(a, b), F_B(a, h5)] = \text{max}[0.9, 0.8] = 0.9$$
$$T_A(b) = \text{min}[T_B(b, a), T_B(b, c)] = \text{min}[1.1, 0.7] = 0.7$$
$$I_A(b) = \text{min}[I_B(b, a), I_B(b, c)] = \text{min}[0.2, 0.5] = 0.2$$
$$F_A(b) = \text{max}[F_B(b, a), F_B(b, c)] = \text{max}[0.9, 1.2] = 1.2$$
$$T_A(c) = \text{min}[T_B(c, b), T_B(c, d)] = \text{min}[0.7, 0.7] = 0.7$$
$$I_A(c) = \text{min}[I_B(c, b), I_B(c, d)] = \text{min}[0.2, 0.5] = 0.2$$
$$F_A(c) = \text{max}[F_B(c, b), F_B(c, d)] = \text{max}[1.3, 1.3] = 1.3$$
$$T_A(d) = \text{min}[T_B(d, c), T_B(d, e)] = \text{min}[0.1, 0.1] = 0.1$$
$$I_A(d) = \text{min}[I_B(d, c), T_B(d, e)] = \text{min}[0.7, 0.7] = 0.7$$
$$F_A(d) = \text{max}[F_B(d, c), T_B(d, e)] = \text{max}[1.2, 1.2] = 1.2$$
$$T_A(e) = \text{min}[T_B(e, d), T_B(e, f)] = \text{min}[0.5, 0.5] = 0.5$$
$$I_A(e) = \text{min}[I_B(e, d), I_B(e, f)] = \text{min}[0.3, 0.3] = 0.3$$
$$F_A(e) = \text{max}[F_B(e, d), F_B(e, f)] = \text{max}[1.2, 1.2] = 1.2$$
$$T_A(f) = \text{min}[T_B(f, e), T_B(f, g)] = \text{min}[0.5, 1.2] = 0.5$$
$$I_A(f) = \text{min}[I_B(f, e), I_B(f, g)] = \text{min}[0.3, 0.3] = 0.3$$
$$F_A(f) = \text{max}[F_B(f, e), F_B(f, g)] = \text{max}[1.3, 1.3] = 1.3$$
$$T_A(g) = \text{min}[T_B(g, h), T_B(g, f)] = \text{min}[1.2, 1.2] = 1.2$$
$$I_A(g) = \text{min}[I_B(g, h), I_B(g, f)] = \text{min}[0.2, 0.3] = 0.2$$
$$F_A(g) = \text{max}[F_B(g, h), F_B(g, f)] = \text{max}[1.2, 1.2] = 1.2$$
$$T_A(h) = \text{min}[T_B(h, a), T_B(h, g)] = \text{min}[1.3, 1.2] = 0.1$$
$$I_A(h) = \text{min}[I_B((h, a), I_B(h, g)] = \text{min}[0.2, 0.2] = 0.2$$
$$F_A(h) = \text{max}[F_B((h, a), F_B(h, g)] = \text{max}[0.8, 1.2] = 1.2$$

Here $a$ dominates $b$ because

$$T_B(ab) \leq T_A(a) \wedge T_A(b), 1.1 \leq 1.1 \wedge 0.7$$
$$I_B(ab) \leq I_B(a) \wedge T_B(b), 0.2 \leq 0.2 \wedge 0.2$$
$$F_B(ab) \geq F_A(a) \vee T_B(b), 0.9 \geq 0.9 \vee 0.9$$

Here $b$ dominates $c$ because

$$T_B(bc) \leq T_A(b) \wedge T_A(c), 0.7 \leq 0.7 \wedge 0.7$$
$$I_B(bc) \leq I_B(b) \wedge T_B(c), 0.2 \leq 0.2 \wedge 0.2$$

$$F_B(bc) \geq F_A(b) \vee T_B(c), 1.2 \geq 0.9 \vee 1.2$$

Here $c$ dominates $d$ because

$$T_B(cd) \leq T_A(c) \wedge T_A(d), 0.7 \leq 0.7 \wedge 0.5$$
$$I_B(cd) \leq I_B(c) \wedge T_B(d), 0.5 \leq 0.2 \wedge 0.3$$
$$F_B(cd) \geq F_A(c) \vee T_B(d), 1.2 \geq 1.2 \vee 1.2$$

Here $d$ dominates $e$ because

$$T_B(de) \leq T_A(d) \wedge T_A(e), 0.5 \leq 0.5 \wedge 0.0.5$$
$$I_B(de) \leq I_B(d) \wedge T_B(e), 0.3 \leq 0.3 \wedge 0.3$$
$$F_B(de) \geq F_A(d) \vee T_B(e), 1.2 \geq 1.2 \vee 1.2$$

Here $e$ dominates $f$ because

$$T_B(ef) \leq T_A(e) \wedge T_A(f), 0.5 \leq 0.5 \wedge 0.5$$
$$I_B(ef) \leq I_B(e) \wedge I_B(f), 0.3 \leq 0.3 \wedge 0.3$$
$$F_B(ef) \geq F_A(e) \vee F_B(f), 1.2 \geq 1.2 \vee 1.2$$

Here $f$ dominates $g$ because

$$T_B(fg) \leq T_A(f) \wedge T_A(g), 1.2 \leq 0.5 \wedge 1.2$$
$$I_B(fg) \leq I_B(f) \wedge I_B(g), 0.3 \leq 0.3 \wedge 0.2$$
$$F_B(fg) \geq F_A(f) \vee F_B(g), 1.2 \geq 1.2 \vee 1.2$$

Here $g$ dominates $h$ because

$$T_B(gh) \leq T_A(g) \wedge T_A(h), 1.2 \geq 1.2 \vee 1.2$$
$$I_B(gh) \leq I_B(g) \wedge I_B(h), 0.2 \leq 0.2 \wedge 0.2$$
$$F_B(gh) \geq F_A(g) \vee F_B(h), 1.2 \geq 1.2 \vee 1.2$$

Here $h$ dominates $a$ because

$$T_B(ha) \leq T_A(a) \wedge T_A(h), 1.3 \geq 1.1 \vee 1.2$$
$$I_B(ha) \leq I_B(a) \wedge I_B(h), 0.2 \leq 0.2 \wedge 0.2$$
$$F_B(ha) \geq F_A(a) \vee F_B(h), 0.8 \geq 0.9 \vee 1.2$$

$V = \{a, b, c, d, e, f, g, h\}$, $D_N = \{c, e, h\}$ and $V - D_n = \{a, b, d, f, g\}$, $|D_N| = 3$.
Now we obtain the connectivity index for all paths

$$|\text{CONN}_{p_1}(a, b)| = 2 \times (1.1) - 0.5 - 0.9 = 0.8$$
$$|\text{CONN}_{p_1}(b, c)| = 2 \times (0.7) - 0.2 - 1.2 = 0$$
$$|\text{CONN}_{p_1}(c, d)| = 2 \times (0.7) - 0.5 - 1.2 = -0.3$$

$$|\text{CONN}_{p_1}(d,e)| = 2\times(0.5) - 0.3 - 1.2 = -0.5$$
$$|\text{CONN}_{p_1}(e,f)| = 2\times(0.5) - 0.3 - 1.2 = -0.5$$
$$|\text{CONN}_{p_1}(f,g)| = 2\times(1.2) - 0.3 - 1.1 = 1$$
$$|\text{CONN}_{p_1}(g,h)| = 2\times(1.2) - 0.2 - 1.1 = 1.1$$
$$|\text{CONN}_{p_1}(h,a)| = 2\times(1.3) - 0.2 - 0.8 = 1.6$$

Also, If needed, we can calculate the connectivity index for indirect relationship for Novertop graph.

Then $\text{CI}_{\text{Nov}}(\mathscr{G})$ we have,

$$\begin{aligned}
\text{CI}_{\text{Nov}}(\mathscr{G}) &= \sum_{u_iv_j\in\mathscr{V}} (T_A(u_i), I_A(u_i), F_A(u_i))(T_A(v_j), I_A(v_j), F_A(v_J) \times \text{CONN}_G(u_i,v_j) \\
&= (1.3,0.3,0.7)(1.1,0.2,0.9)\times(0.8) + (1.3,0.3,0.2)(1.4,0.2,0.8)\times(1.6)+ \\
&\quad (1.1,0.2,0.9)(0.7,0.5,1.2)\times(-0.3)(0.7,0.5,1.2)(1.2,0.6,0.8)\times(-0.3)+ \\
&\quad (1.2,0.6,0.8)(0.5,0.3,1.2)\times(-0.4)(0.5,0.3,1.2)(1.2,0.3,1.1)\times(-0.4)+ \\
&\quad (1.2,0.5,1.1)(1.4,0.2,0.8)\times(1) \\
&= (2.12)(0.8) + (2.23)(1.6) + (2.03)(-0.3) + (2.1)(-0.3) + (1.74)(-0.4)+ \\
&\quad (2.82)(-0.4) + (2.94)(1) \\
&= 4.478
\end{aligned}$$

Then connectivity index of $G$ is equal $4.478$, which the positive sign indicates the high level of true information in the problem.

Hence the employee (h) a relationship is high and good. so ”h” is the most dominating person.so we can select h is the head of the office.

# 9 Conclusion

In this paper has focussed an some topological indices for Neutrosophic over topologized graphs by using strong domination. It is an easy way to calculate the connectivity index.so we use the new method that is strong domination to find and calculate the connectivity index. The some topological indices for some standard neutrosophic over topologized graphs such as $2-regular$ , $K_2$ and $K_{2,2}$ are given.

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