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Hyperbolic Sine Similarity Measure of SVNSs for Open-Pit Mine Slope Stability Classification and Assessment

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Abstract: Slope instability is a common geological hazard in open-pit mines, which may cause huge economic losses and casualties. Thus, it is important to cluster and evaluate the stability of slopes effectively. This article proposes a hyperbolic sine similarity measure of single-valued neutrosophic sets (SVNSs) for a netting clustering method and a slope stability evaluation method to cluster and assess the stability of open-pit mine slopes. This study contains the following main content. First, we present a hyperbolic sine similarity measure between SVNSs. Second, slope stability impact factors are fuzzified into SVNSs by the utilization of true, indeterminate, and false membership functions, and then a netting clustering method using the proposed similarity measure is proposed to cluster the stability of open-pit mine slopes. Third, we propose a slope stability evaluation method based on the proposed similarity measure, where we give the SVNS knowledge of risk grades/patterns based on the clustering results of slope stability and then present the similarity measure values between the risk grades/patterns and the slope samples to assess that each slope sample with the larger measure value belongs to the corresponding slope risk grade. Finally, the proposed netting clustering and evaluation methods are applied to the clustering analysis and assessment of 20 open-pit mine slope samples to verify the rationality and effectivity of the proposed approaches in the scenario of SVNSs.

Keywords: single-valued neutrosophic set; netting clustering method; similarity measure; open-pit mine slope; slope stability assessment.

1. Introduction

Slope instability is a typical geological hazard in open-pit mines, so the disasters and losses caused by slope instability cannot be ignored. Thus, it is important to give some reasonable classification and evaluation methods for slope stability. Traditional qualitative classification methods for the slope stability grades include rock mass strength grading method, geological strength index method, slope failure probability grading method, and so on [1]. However, there are many factors that will affect the analysis of slope stability. Since the slope impact factors include a lot of uncertain and incomplete information, the traditional methods cannot effectively express the uncertain and incomplete information. Therefore, some indeterminate classification methods have been proposed, such as rainfall-induced landslides using ANN (artificial neural network) and fuzzy clustering methods [2], K-means and fuzzy c-means clustering algorithms [3], and a neuro-fuzzy inference system-based clustering methods [4]. But the existing indeterminate clustering methods difficultly express the true, indeterminate, and false information in the evaluation problems of slope stability.

In order to represent indeterminate and inconsistent information in the real world, Smarandache first proposed the concept of neutrosophic sets (NSs) [5] as a conceptual extension of fuzzy sets (FSs) [6] and (interval-valued) intuitionistic FSs (IFSs/IVFSs) [7, 8]. NS is characterized by a true membership function, an indeterminate membership function, and a false membership function independently. However, it is difficult to apply NSs in practical engineering fields because the values of their membership functions fall in the non-standard interval $]^{-}0, 1^{+}[$. As the subsets of NSs, Wang et al. [9, 10] introduced single-valued and interval-valued NSs (SVNSs and IVNSs) when the values of the three membership functions fall in the standard interval [0, 1] to describe indeterminate and inconsistent information in practical engineering issues. Recently, some researchers have applied SVNSs to the assessment of slope stability. Qin [11] proposed a SVNS adaptive neuro fuzzy inference system (SVNS-ANFIS) and applied it to the evaluation of open-pit mine slope stability. Then, Qin [12] further proposed a SVNS Gaussian process regression (SVNS-GPR) approach to predict the stability of open-pit mine slopes. However, SVNSs have not been applied to the clustering analysis of slope stability so far.

As another subclass of neutrosophic theory, a neutrosophic number ($N = a + bI$ for $I \in$ [inf I, sup I]) (NNs) [5, 13, 14] consists of a certain part a and an uncertain part bI, which is also called an uncertain number. Since similarity measures are one of the important research topics in neutrosophic theory, some similarity measures have been proposed and applied in slope stability evaluation problems in the environment of NNs [15]. Li et al. [15] proposed a slope stability evaluation approach based on the tangent and arctangent similarity measure of NNs. Li et al. [16] developed the vector similarity measures of NNs for the assessment of rock slope stability. However, these similarity measures lack the information of true, false, and indeterminate membership degrees. They cannot deal with indeterminate and inconsistent decision-making/evaluation problems in neutrosophic environments. Therefore, some researchers [17-19] presented various similarity measures of SVNSs/IVNSs to perform decision-making problems.

Regarding the current studies, the similarity measures of NNs cannot handle the actual clustering and evaluation problems of slope stability with SVNS information because NN cannot contain the true, false, indeterminate membership degrees. Then, the SVNS-ANFIS and SVNS-GPR methods [11, 12] require large amounts of learning data to train them, leading to complex learning operations and difficult update problems. Therefore, they are difficultly applied to actual clustering and evaluation problems of slope stability. Ye [15] also proposed a clustering method by the similarity measure of SVNSs, but it was not applied to actual clustering analysis and evaluation problems of slope stability because it is difficult to obtain true, false, indeterminate membership degrees from the data of slope samples. However, similarity metrics for clustering analysis and evaluation problems of slope stability have shown obvious superiority over neural networks in terms of data requirements, algorithms, and updated applications. Unfortunately, to date, the similarity measures of SVNS have not been applied to the clustering analysis and evaluation problems of slope stability. How to solve the clustering analysis and evaluation problems of slope stability by the similarity measure of SVNSs is a challenging problem in practical applications. Therefore, this paper will resolve this issue.

In this study, we present a hyperbolic sine similarity measure (HSSM) of SVNSs and its netting clustering analysis and evaluation methods of slope stability. Then, the proposed methods are applied to the clustering analysis and assessment of 20 open-pit mine slope samples. Through the comparative analysis with existing related methods, the proposed approaches reveal their rationality and effectivity in the clustering and evaluation application of the 20 open-pit mine slope samples in the scenario of SVNSs.

The remaining structure of this paper is arranged as follows. In section 2, some basic concepts of SVNSs are introduced. Section 3 proposes HSSM of SVNSs and netting clustering analysis and slope stability evaluation methods using the proposed HSSM. Section 4 applies the proposed clustering method to the slope stability clustering analysis of the 20 open-pit mine slope samples, and then the proposed evaluation method is applied to the stability evaluation of the 20 slope samples. Conclusions and further research are presented in Section 5.

2. Some Basic Concepts of SVNS

Smarandache first introduced NSs as the generalization of FSs, IVFSs, and IFSs. Then, Wang et al. [10] introduced SVNS as a subclass of NS to be applied in real scientific and engineering applications. The definition and operations of SVNSs are introduced below.

Definition 1 [10]. Let X be a universal set. A SVNS D in X can be denoted as $D = \{<x, D_T(x), D_I(x), D_F(x)> | x \in X\}$, where $D_T(x)$, $D_I(x)$, $D_F(x)$ are the true, indeterminate, and false membership functions for any $x \in X$, $D_T(x), D_I(x), D_F(x) \in [0, 1]$, and $0 \le D_T(x) + D_I(x) + D_F(x) \le 3$.

Then, the basic element of SVNS $d = <x, D_T(x), D_I(x), D_F(x)>$ is simply denoted as the single-valued neutrosophic number (SVNN) $d = <D_T, D_I, D_F>$ for the convenient representation.

Definition 2 [10]. Set two SVNSs as $d_1 = <D_{T1}, D_{I1}, D_{F1}>$ and $d_2 = <D_{T2}, D_{I2}, D_{F2}>$, then they follow the following operations.

(1) $d_1 \subseteq d_2$ if and only if $D_{T1} \le D_{T2}$, $D_{I1} \ge D_{I2}$, $D_{F1} \ge D_{F2}$;

(2) $d_1 = d_2$ if and only if $d_1 \subseteq d_2$ and $d_2 \subseteq d_1$;

(3) $d_1^c = <D_{F1}, 1 - D_{I1}, D_{T1}>$ (Complement of d_1);

(4) $d_1 \cup d_2 = <D_{T1} \vee D_{T2}, D_{I1} \wedge D_{I2}, D_{F1} \wedge D_{F2}>$;

(5) $d_1 \cap d_2 = <D_{T1} \wedge D_{T2}, D_{I1} \vee D_{I2}, D_{F1} \vee D_{F2}>$.

Definition 3 [20]. Let $D_1 = \{d_{11}, d_{12}, \ldots, d_{1n}\}$ and $D_2 = \{d_{21}, d_{22}, \ldots, d_{2n}\}$ be two SVNSs, where $d_{1i} = <D_{T1i}, D_{I1i}, D_{F1i}>$ and $d_{2i} = <D_{T2i}, D_{I2i}, D_{F2i}>$ ($i = 1, 2, \ldots, n$) are SVNNs. If the weight of d_{1i} and d_{2i} is specified by $g_i \in [0, 1]$ with $\sum_{i=1}^{n} g_i = 1$, the weighted generalized distance between D_1 and D_2 is defined as

$$G_\varphi(D_1, D_2) = \left\{ \frac{1}{3} \sum_{i=1}^{n} g_i \left[|D_{T1i} - D_{T2i}|^\varphi + |D_{I1i} - D_{I2i}|^\varphi + |D_{F1i} - D_{F2i}|^\varphi \right] \right\}^{1/\varphi} \text{ for } \varphi > 0. \quad (1)$$

Then, the above distance $G_\varphi(D_1, D_2)$ satisfies the following properties [20]:

(A1) $0 \le G_\varphi(D_1, D_2) \le 1$;

(A2) $G_\varphi(D_1, D_2) = 0$ if and only if $D_1 = D_2$;

(A3) $G_\varphi(D_1, D_2) = G_\varphi(D_2, D_1)$;

(A4) If $D_1 \subseteq D_2 \subseteq D_3$ for the SVNS D_3, then $G_\varphi(D_1, D_3) \ge G_\varphi(D_1, D_2)$ and $G_\varphi(D_1, D_3) \ge G_\varphi(D_2, D_3)$.

In view of the complementary relationship between the similarity measure and the distance, the weighted generalized distance-based similarity measure of SVNSs is presented as bellows [20]:

$$S_\varphi(D_1, D_2) = 1 - G_\varphi(D_1, D_2) = 1 - \left\{ \frac{1}{3} \sum_{i=1}^{n} g_i \left[|D_{T1i} - D_{T2i}|^\varphi + |D_{I1i} - D_{I2i}|^\varphi + |D_{F1i} - D_{F2i}|^\varphi \right] \right\}^{1/\varphi}. \quad (2)$$

Then, the weighted generalized distance-based similarity measure of SVNSs also implies the following properties [20]:

(B1) $0 \le S_\varphi(D_1, D_2) \le 1$;

(B2) $S_\varphi(D_1, D_2) = 1$ if and only if $D_1 = D_2$;

(B3) $S_\varphi(D_1, D_2) = S_\varphi(D_2, D_1)$;

(B4) If $D_1 \subseteq D_2 \subseteq D_3$ for the SVNS D_3, then $S_\varphi(D_1, D_2) \ge S_\varphi(D_1, D_3)$ and $S_\varphi(D_2, D_3) \ge S_\varphi(D_1, D_3)$.

3. Netting Clustering and Slope Stability Evaluation Methods Using HSSM of SVNSs

3.1. Netting Clustering Method Using HSSM of SVNSs

Considering the weighted generalized distance of SVNSs, this section further proposes HSSM between SVNSs and its netting clustering method for SVNSs.

First, we propose HSSM of SVNSs.

Definition 4. Let $D_1 = \{d_{11}, d_{12}, \ldots, d_{1n}\}$ and $D_2 = \{d_{21}, d_{22}, \ldots, d_{2n}\}$ be two SVNSs, where $d_{1i} = \langle D_{T1i}, D_{I1i}, D_{F1i}\rangle$ and $d_{2i} = \langle D_{T2i}, D_{I2i}, D_{F2i}\rangle$ ($i = 1, 2, \ldots, n$) are SVNNs. If the weight of d_{1i} and d_{2i} is specified by $g_i \in [0, 1]$ with $\sum_{i=1}^{n} g_i = 1$, the weighted HSSM between D_1 and D_2 is defined by

$$H_\varphi(D_1, D_2) = 1 - \sinh\{\ln(1+\sqrt{2})G_\varphi(D_1, D_2)\}$$
$$= 1 - \left\{\sinh\left(\frac{\ln(1+\sqrt{2})}{3}\sum_{i=1}^{n} g_i\left[\left|D_{T1i} - D_{T2i}\right|^\varphi + \left|D_{I1i} - D_{I2i}\right|^\varphi + \left|D_{F1i} - D_{F2i}\right|^\varphi\right]\right)\right\}^{1/\varphi} \quad \text{for } \varphi > 0. \quad (3)$$

Then, HSSM also contains the following properties:

(C1) $0 \le H_\varphi(D_1, D_2) \le 1$;

(C2) $H_\varphi(D_1, D_2) = 1$ if and only if $D_1 = D_2$;

(C3) $H_\varphi(D_1, D_2) = H_\varphi(D_2, D_1)$;

(C4) If $D_1 \subseteq D_2 \subseteq D_3$ for the SVNS D_3, then $H_\varphi(D_1, D_2) \ge H_\varphi(D_1, D_3)$ and $H_\varphi(D_2, D_3) \ge H_\varphi(D_1, D_3)$.

Proof: The properties (C1)-(C3) are obviously true. Therefore, we only prove the property (C4).

For $D_1 \subseteq D_2 \subseteq D_3$, in view of the above properties of the distance measure $G_\varphi(D_1, D_2)$ for SVNSs, there are $G_\varphi(D_1, D_3) \ge G_\varphi(D_1, D_2)$ and $G_\varphi(D_1, D_3) \ge G_\varphi(D_2, D_3)$. Since the sinh($x$) for $x \in [0, 1]$ is an increasing function, based on the compensatory relationship between the distance and the similarity measure, there are also $H_\varphi(D_1, D_2) \ge H_\varphi(D_1, D_3)$ and $H_\varphi(D_2, D_3) \ge H_\varphi(D_1, D_3)$.

Hence, the proof is completed.

In light of the proposed HSSM of SVNSs, we introduce a netting clustering method to cluster open-pit mine slopes in the environment of SVNSs.

In a clustering problem of open-pit mine slops, $D = \{D_1, D_2, \ldots, D_m\}$ is a set of m slopes and $Q = \{q_1, q_2, \ldots, q_n\}$ is a set of n impact factors (indices) of slope stability. The weight of each impact factor q_i is g_i subject to $g_i \in [0, 1]$ and $\sum_{i=1}^{n} g_i = 1$.

Using the suitable true, indeterminate, and false membership functions (MFs) (see Table 2), the measurement values of the slope stability impact indices for each slope sample are fuzzed as the true, indeterminate, and false fuzzy values, which is constructed as the SVNS $D_j = \{d_{j1}, d_{j2}, \ldots, d_{jn}\}$, where $d_{ji} = \langle D_{Tji}, D_{Iji}, D_{Fji}\rangle$ are SVNNs for $D_{Tji}, D_{Iji}, D_{Fji} \in [0, 1]$, $j = 1, 2, \ldots, m$, and $i = 1, 2, \ldots, n$.

In the clustering problem, the netting clustering method is used to cluster the open-pit mine slopes in the environment of SVNSs by the following steps:

Step 1: Establish the hyperbolic sine similarity matrix $H = (h_{ji})_{m\times m}$ ($i, j = 1, 2, \ldots, m$) through the similarity operations of Eq. (3) (usually taking $\varphi = 2$ as a typical parameter value) subject to $h_{ji} = H_\varphi(D_j, D_i)$, $h_{jj} = 1$, and $h_{ji} = h_{ij}$.

Step 2: Use the open-pit mine slope samples for replacing all the diagonal elements of the similarity matrix Y.

Step 3: Construct the β-cutting matrices $H^\beta = (h_{ji}^{\beta})_{m\times m}$ corresponding to different confidence levels of β by the following formula:

$$h_{ji}^{\beta} = \begin{cases} 0, h_{ji} < \beta \\ 1, h_{ji} \ge \beta \end{cases} \quad (i, j = 1, 2, \ldots, m). \quad (4)$$

All "0" is deleted in the β-cutting matrixes and "1" is replaced by "*", and then draw the vertical and horizontal lines from "*" to the diagonal elements. The slope samples connected by the same "*" are constructed as a type corresponding to the confidence level β. Update different confidence levels of β from big to small until the slope samples are clustered into the expected types.

3.2. Slope Stability Evaluation Method

In terms of the above clustering results of slope samples with SVNS information, we don't know which type belongs to which risk grade/pattern. Therefore, we must give the stability evaluation of the slope samples to recognize the corresponding risk patterns/grades of the slope stability. To do so, this subsection needs to give a slope stability evaluation method in the setting of SVNSs.

Based on the slope stability classification knowledge/experience, we can establish the expected slope stability patterns/risk grades expressed by their SVNSs $R_k = \{d_{k1}, d_{k2}, \ldots, d_{kn}\}$ that are composed of the SVNNs $d_{ki} = \langle D_{Tki}, D_{Iki}, D_{Fki}\rangle$ for $D_{Tki}, D_{Iki}, D_{Fki} \in [0, 1]$ ($k = 1, 2, \ldots, p$; $i = 1, 2, \ldots, n$). Suppose that there is a set of m slope samples $A = \{A_1, A_2, \ldots, A_m\}$ to require the risk evaluation of slope stability. Then, the slope samples can be represented by the SVNSs $D_j = \{d_{j1}, d_{j2}, \ldots, d_{jn}\}$ ($j = 1, 2, \ldots, m$) that are composed of the SVNNs $d_{ji} = \langle D_{Tji}, D_{Iji}, D_{Fji}\rangle$ for $D_{Tji}, D_{Iji}, D_{Fji} \in [0, 1]$ ($i = 1, 2, \ldots, n$).

Regarding the risk evaluation issue of slope stability, the similarity measure between each slope sample D_j ($j = 1, 2, \ldots, m$) and each slope stability pattern R_k ($k = 1, 2, \ldots, p$) is given by the following formula:

$$H_{\varphi}(D_j, R_k) = 1 - \left\{ \sinh\left(\frac{\ln(1+\sqrt{2})}{3} \sum_{i=1}^{n} g_i \left[\left|D_{Tji} - D_{Tki}\right|^{\varphi} + \left|D_{Iji} - D_{Iki}\right|^{\varphi} + \left|D_{Fji} - D_{Fki}\right|^{\varphi} \right] \right) \right\}^{1/\varphi} \text{ for } \varphi > 0. \quad (5)$$

Based on the HSSM values of Eq. (5), we can utilize $H_{\varphi}(D_j, R_{k^*}) = \underset{1 \le k \le m}{\mathrm{Max}}\left(H_{\varphi}(D_j, R_k)\right)$ to recognize that the stability grade of the slope sample D_j belongs to R_{k^*}.

4. Clustering Analysis and Stability Evaluation of Actual Open-Pit Mine Slopes

4.1. Clustering Analysis of Actual Cases

Table 1. Original data of 20 open-pit mine slope samples

D_j	q_1	q_2	q_3	q_4	q_5	q_6
D_1	62.0	47.0	32.0	0.115	43.6	29.1
D_2	40.0	55.0	31.0	0.0321	40.8	28.8
D_3	36.5	55.0	39.0	0.045	43.6	28.7
D_4	35.5	58.0	31.0	0.0273	39.2	28.9
D_5	66.0	57.0	40.0	0.0796	43.0	29.1
D_6	42.0	55.0	30.0	0.0157	37.6	29.1
D_7	43.5	54.0	33.0	0.0291	38.4	29.0
D_8	48.5	60.0	38.0	0.0522	40.5	29.2
D_9	46.5	57.0	40.0	0.0354	40.0	28.8
D_{10}	23.5	64.0	43.0	0.0285	39.8	29.0
D_{11}	59.5	71.0	37.0	0.0576	34.7	29.1
D_{12}	23.5	57.0	34.0	0.0125	31.4	28.9
D_{13}	25.0	65.0	48.0	0.0218	40.8	29.1
D_{14}	23.0	65.0	49.0	0.0141	43.7	28.9
D_{15}	18.0	70.0	41.0	0.0122	35.5	28.8
D_{16}	15.0	80.0	47.0	0.0074	37.8	29.2
D_{17}	16.5	70.0	60.0	0.0122	39.7	28.7
D_{18}	19.0	68.0	51.0	0.0103	37.1	28.9
D_{19}	17.0	70.0	60.0	0.0079	43.1	29.2
D_{20}	10.0	70.0	50.0	0.0044	34.7	29.1

In Zhejiang Province, China, many open-pit mines have slope instability problems, which will lead to a large number of economic losses and casualties. In order to reasonably classify and evaluate the slope stability, we collected 20 slope samples from field survey in Zhejiang Province. The slope height (q_1), slope angle (q_2), potential slip plane angle (q_3), cohesion (q_4), internal friction angle (q_5), and rock density (q_6) are considered as the 6 main impact factors of slope stability. The weight vector of the 6 impact factors is specified as $\boldsymbol{g}$ = (0.33, 0.22, 0.12, 0.1, 0.07, 0.16). In addition to the impact factors, we also collected the safety factor of each slope as the known knowledge/experience. The original data (six impact factors) of the 20 slope samples D_j (j = 1, 2, …, 20) is shown in Table 1.

Table 2. True, indeterminate, and false MFs for impact factors

Impact factor	MF		
	D_T	D_I	D_F
q_1	trapmf[0 0 15 80]	trimf[15 30 80]	trapmf[15 80 100 100]
q_2	trapmf[0 0 40 80]	trimf[40 60 80]	trapmf[40 80 100 100]
q_3	trapmf[0 0 20 60]	trimf[20 40 60]	trapmf[20 60 80 80]
q_4	trapmf[0.01 0.045 0.060 0.060]	trimf[0.01 0.0265 0.045]	trapmf[0 0 0.01 0.045]
q_5	trapmf[30 50 60 60]	trimf[30 40 50]	trapmf[0 0 30 50]
q_6	trimf[28.7 28.7 29.2]	trimf[28.8 28.9 29]	trapmf[28.7 29.2 29.5 29.5]

Table 3. SVNSs of 20 open-pit mine slope samples

D_j	q_1	q_2	q_3	q_4	q_5	q_6
D_1	<0.28,0.4,0.723>	(0.825,0.35,0.175)	(0.7,0.6,0.3)	(0,0,0)	(0.68,0.64,0.32)	(0.2,0,0.8)
D_2	(0.615,0.889,0.385)	(0.625,0.75,0.375)	(0.725,0.55,0.275)	(0.631,0.697,0.369)	(0.54,0.92,0.46)	(0.8,0,0.2)
D_3	(0.669,0.967,0.331)	(0.625,0.75,0.375)	(0.525,0.95,0.475)	(1,0,0)	(0.68,0.64,0.32)	(1,0,0)
D_4	(0.685,0.989,0.315)	(0.55,0.9,0.45)	(0.725,0.55,0.275)	(0.494,0.957,0.506)	(0.46,0.92,0.54)	(0.6,1,0.4)
D_5	(0.215,0.311,0.785)	(0.575,0.85,0.425)	(0.5,1,0.5)	(0,0,0)	(0.65,0.7,0.35)	(0.2,0,0.8)
D_6	(0.585,0.844,0.415)	(0.625,0.75,0.375)	(0.75,0.5,0.25)	(0.163,0.345,0.837)	(0.38,0.76,0.62)	(0.2,0,0.8)
D_7	(0.562,0.811,0.438)	(0.65,0.7,0.35)	(0.675,0.65,0.325)	(0.546,0.859,0.454)	(0.42,0.84,0.58)	(0.4,0,0.6)
D_8	(0.485,0.7,0.515)	(0.5,1,0.5)	(0.55,0.9,0.45)	(1,0,0)	(0.525,0.95,0.475)	(0,0,1)
D_9	(0.515,0.744,0.485)	(0.575,0.85,0.425)	(0.5,1,0.5)	(0.726,0.52,0.274)	(0.5,1,0.5)	(0.8,0,0.2)
D_{10}	(0.869,0.425,0.131)	(0.4,0.8,0.6)	(0.425,0.85,0.575)	(0.529,0.892,0.471)	(0.49,0.98,0.51)	(0.4,0,0.6)
D_{11}	(0.315,0.456,0.685)	(0.225,0.45,0.775)	(0.575,0.85,0.425)	(1,0,0)	(0.235,0.47,0.765)	(0.2,0,0.8)
D_{12}	(0.869,0.425,0.131)	(0.575,0.85,0.425)	(0.65,0.7,0.35)	(0.071,0.152,0.929)	(0.07,0.14,0.93)	(0.6,1,0.4)
D_{13}	(0.846,0.5,0.154)	(0.375,0.75,0.625)	(0.3,0.6,0.7)	(0.337,0.715,0.663)	(0.54,0.92,0.46)	(0.2,0,0.8)
D_{14}	(0.877,0.4,0.123)	(0.375,0.75,0.625)	(0.275,0.55,0.725)	(0.117,0.248,0.883)	(0.685,0.63,0.315)	(0.6,1,0.4)
D_{15}	(0.954,0.15,0.046)	(0.25,0.5,0.75)	(0.475,0.95,0.525)	(0.063,0.133,0.937)	(0.275,0.55,0.725)	(0.8,0,0.2)
D_{16}	(1,0,0)	(0,0,1)	(0.325,0.65,0.675)	(0,0,1)	(0.39,0.78,0.61)	(0,0,1)
D_{17}	(0.977,0.075,0.023)	(0.25,0.5,0.75)	(0,0,1)	(0.063,0.133,0.937)	(0.485,0.97,0.515)	(1,0,0)
D_{18}	(0.938,0.2,0.062)	(0.3,0.6,0.7)	(0.225,0.45,0.775)	(0.009,0.018,0.991)	(0.355,0.71,0.645)	(0.6,1,0.4)
D_{19}	(0.969,0.1,0.031)	(0.25,0.5,0.75)	(0,0,1)	(0,0,1)	(0.655,0.69,0.345)	(0,0,1)
D_{20}	(1,0,0)	(0.25,0.5,0.75)	(0.25,0.5,0.75)	(0,0,1)	(0.235,0.47,0.765)	(0.2,0,0.8)

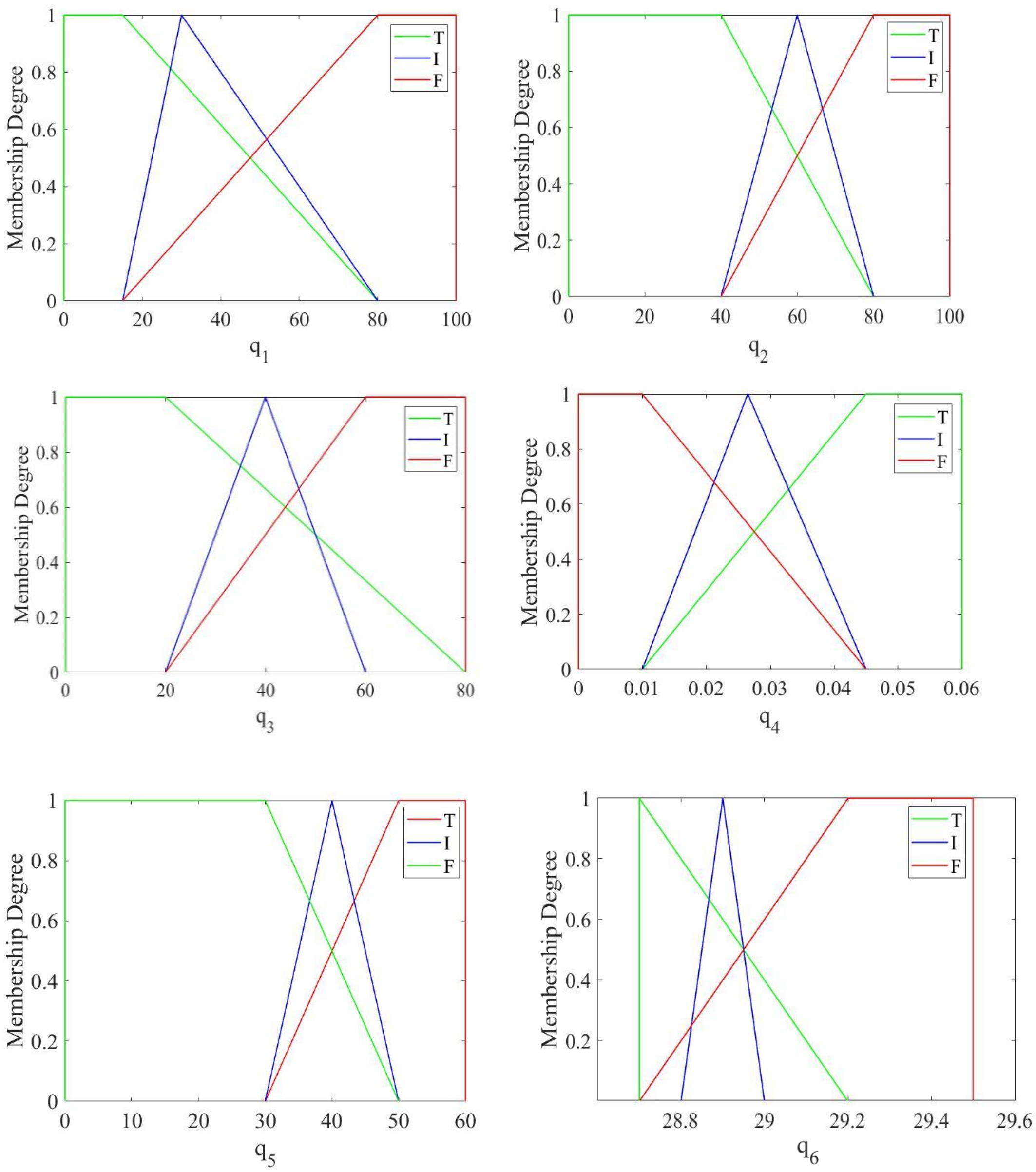

Figure 1. MFs of 6 impact factors

First, we chose appropriate true, indeterminate, and false MFs to fuzzify each impact factor in Table 1 into the form of SVNN. The different MFs for impact factors are shown in Table 2, then Figure 1 shows the curves of 18 MFs for six impact factors. Thus, the data (six impact factors) of the 20 slope samples D_k (k = 1, 2, …, 20) are fuzzified into SVNNs, which are given in Table 3.

Then, we use the proposed netting clustering method to classify the 20 slope samples with SVNS information. By the clustering analysis based on Eqs. (3) and (4) for $\varphi = 2$, the similarity matrix is obtained and shown in Figure 2, and then the slope samples are classified into 4 types when we specify the interval range $0.88899 \le \beta \le 1$, which are shown in Figure 3. Obviously, the set of slope samples {D_1, D_5, D_{11}} is classified into the same type; the set of slope samples {D_2, D_3, D_4, D_6, D_7, D_8, D_9} is classified into the same type; the set of slope samples {D_{10}, D_{12}, D_{13}, D_{14}} is classified into the same type; the set of slope samples {D_{15}, D_{16}, D_{17}, D_{18}, D_{19}, D_{20}} is classified into the same type.

H=

1	0.644762	0.585407	0.568223	0.936656	0.678179	0.703028	0.784727	0.752237	0.56052	0.960426	0.56052	0.575388	0.554675	0.474981	0.411722	0.444608	0.493716	0.455023	0.411722
0.644762	1	0.944582	0.928725	0.576828	0.968344	0.944582	0.865077	0.896953	0.697063	0.686481	0.697063	0.738776	0.682964	0.536873	0.444509	0.491166	0.566843	0.506505	0.444509
0.585407	0.944582	1	0.984174	0.516003	0.91285	0.888995	0.80895	0.841084	0.684017	0.627923	0.684017	0.72728	0.669471	0.519943	0.425927	0.473394	0.550505	0.489009	0.425927
0.568223	0.928725	0.984174	1	0.49836	0.896953	0.873057	0.792814	0.825039	0.678498	0.61099	0.678498	0.721939	0.663898	0.51385	0.41951	0.467142	0.544516	0.48281	0.41951
0.936656	0.576828	0.516003	0.49836	1	0.61099	0.636354	0.7195	0.686481	0.507952	0.896953	0.507952	0.519181	0.503351	0.435999	0.379395	0.409051	0.452354	0.418345	0.379395
0.678179	0.968344	0.91285	0.896953	0.61099	1	0.97626	0.896953	0.928725	0.69985	0.7195	0.69985	0.739968	0.6862	0.543326	0.452312	0.498325	0.572779	0.513436	0.452312
0.703028	0.944582	0.888995	0.873057	0.636354	0.97626	1	0.92079	0.952506	0.699628	0.744078	0.699628	0.738228	0.686407	0.546566	0.456774	0.502209	0.575541	0.517114	0.456774
0.784727	0.865077	0.80895	0.792814	0.7195	0.896953	0.92079	1	0.968344	0.684856	0.825039	0.684856	0.716978	0.673511	0.547221	0.462797	0.505716	0.574059	0.519712	0.462797
0.752237	0.896953	0.841084	0.825039	0.686481	0.928725	0.952506	0.968344	1	0.693282	0.792814	0.693282	0.728184	0.68112	0.548841	0.462037	0.50607	0.57663	0.520469	0.462037
0.56052	0.697063	0.684017	0.678498	0.507952	0.69985	0.699628	0.684856	0.693282	1	0.590467	1	0.958367	0.986126	0.846796	0.761953	0.804544	0.874811	0.818662	0.761953
0.960426	0.686481	0.627923	0.61099	0.896953	0.7195	0.744078	0.825039	0.792814	0.590467	1	0.590467	0.607964	0.583737	0.495697	0.428206	0.463126	0.515987	0.474255	0.428206
0.56052	0.697063	0.684017	0.678498	0.507952	0.69985	0.699628	0.684856	0.693282	1	0.590467	1	0.958367	0.986126	0.846796	0.761953	0.804544	0.874811	0.818662	0.761953
0.575388	0.738776	0.72728	0.721939	0.519181	0.739968	0.738228	0.716978	0.728184	0.958367	0.607964	0.958367	1	0.944476	0.804544	0.718951	0.761953	0.832745	0.776192	0.718951
0.554675	0.682964	0.669471	0.663898	0.503351	0.6862	0.686407	0.673511	0.68112	0.986126	0.583737	0.986126	0.944476	1	0.860817	0.776192	0.818662	0.888782	0.832745	0.776192
0.474981	0.536873	0.519943	0.51385	0.435999	0.543326	0.546566	0.547221	0.548841	0.846796	0.495697	0.846796	0.804544	0.860817	1	0.916661	0.958367	0.972249	0.972249	0.916661
0.411722	0.444509	0.425927	0.41951	0.379395	0.452312	0.456774	0.462797	0.462037	0.761953	0.428206	0.761953	0.718951	0.776192	0.916661	1	0.958367	0.888782	0.944476	1
0.444608	0.491166	0.473394	0.467142	0.409051	0.498325	0.502209	0.505716	0.50607	0.804544	0.463126	0.804544	0.761953	0.818662	0.958367	0.958367	1	0.930576	0.986126	0.958367
0.493716	0.566843	0.550505	0.544516	0.452354	0.572779	0.575541	0.574059	0.57663	0.874811	0.515987	0.874811	0.832745	0.888782	0.972249	0.888782	0.930576	1	0.944476	0.888782
0.455023	0.506505	0.489009	0.48281	0.418345	0.513436	0.517114	0.519712	0.520469	0.818662	0.474255	0.818662	0.776192	0.832745	0.972249	0.944476	0.986126	0.944476	1	0.944476
0.411722	0.444509	0.425927	0.41951	0.379395	0.452312	0.456774	0.462797	0.462037	0.761953	0.428206	0.761953	0.718951	0.776192	0.916661	1	0.958367	0.888782	0.944476	1

Figure 2. The 20×20 similarity matrix *H*

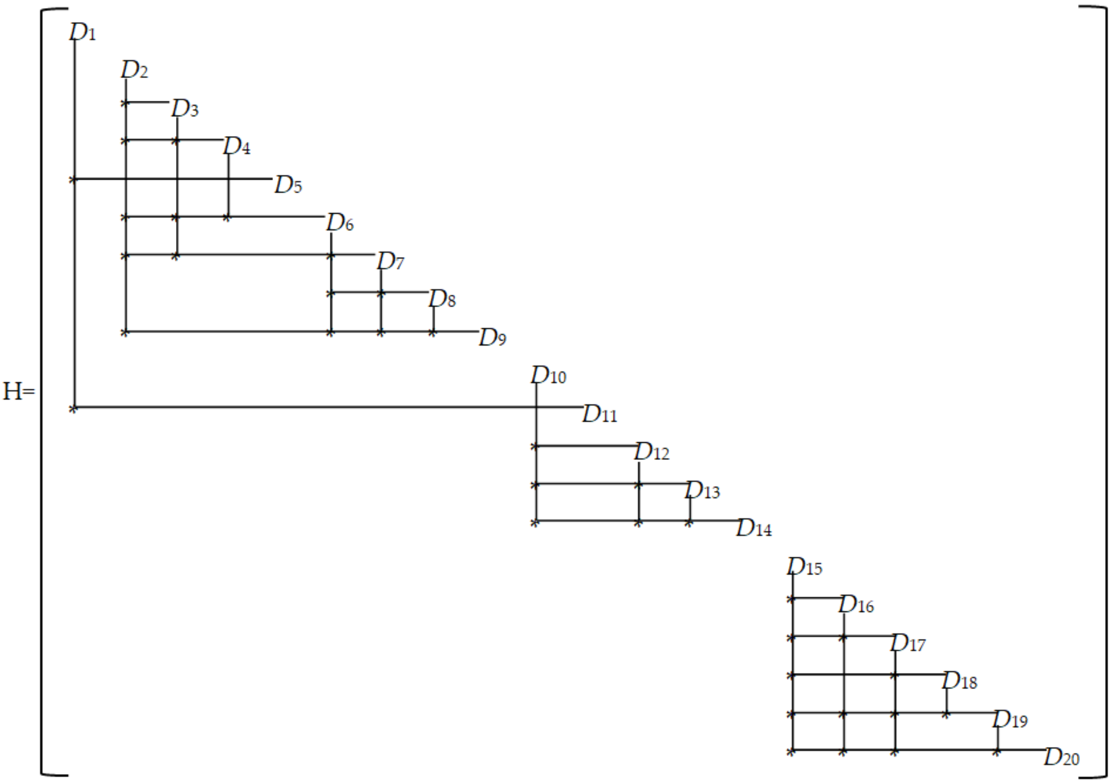

Figure 3. Netting clustering analysis results based on the proposed HSSM

Although the above 20 slope samples are clustered into the four types of slope stability, we don't know which type belongs to which risk grade/pattern. In this case, we must give the stability evaluation of the 20 slope samples to recognize the corresponding risk patterns/grades of the slope stability.

4.2. Clustering Analysis of Actual Cases

According to the above clustering results of the 20 slope samples, there are the four risk patterns/grades. Based on the risk knowledge/experience of the open-pit mine slope stability, we can establish the slope stability four risk patterns/grades: stability (R_1), basic stability (R_2), relative stability (R_3), and instability (R_4), which are expressed by SVNNs in Table 4.

In view of SVNNs in Table 4, we can give the following SVNSs of the four risk patterns/grades:

$R_1 = \{d_{11}, d_{12}, d_{13}, d_{14}, d_{15}, d_{16}\}$ = {<0.25, 0.36, 0.75>, <0.81, 0.38, 0.19>, <0.88, 0.25, 0.13>, <0.78, 0.42, 0.15>, <0.88, 0.25, 0.13>, <0.2, 0, 0.8>};

Table 4. Risk patterns of slope stability in the setting of SVNNs

q_i	R_1			R_2			R_3			R_4		
	D_{T1i}	D_{I1i}	D_{F1i}	D_{T2i}	D_{I2i}	D_{F2i}	D_{T3i}	D_{I3i}	D_{F3i}	D_{T4i}	D_{I4i}	D_{F4i}
q_1	0.25	0.36	0.75	0.56	0.81	0.44	0.74	0.83	0.26	0.9	0.34	0.1
q_2	0.81	0.38	0.19	0.59	0.83	0.41	0.38	0.75	0.63	0.13	0.25	0.88
q_3	0.88	0.25	0.13	0.63	0.75	0.38	0.43	0.85	0.58	0.18	0.35	0.83
q_4	0.78	0.42	0.15	0.56	0.84	0.39	0.33	0.7	0.64	0.09	0.2	0.9
q_5	0.88	0.25	0.13	0.63	0.75	0.38	0.38	0.75	0.63	0.13	0.25	0.88
q_6	0.2	0	0.8	0.5	0.5	0.38	0.7	0.5	0.3	0.9	0	0.1

$R_2 = \{d_{21}, d_{22}, d_{23}, d_{24}, d_{25}, d_{26}\}$ = {<0.56, 0.81, 0.44>, <0.59, 0.83, 0.41>, <0.63, 0.75, 0.38>, <0.56, 0.84, 0.39>, <0.63, 0.75, 0.38>, <0.5, 0.5, 0.38>};

$R_3 = \{d_{31}, d_{32}, d_{33}, d_{34}, d_{35}, d_{36}\}$ = {<0.74, 0.83, 0.26>, <0.38, 0.75, 0.63>, <0.43, 0.85, 0.58>, <0.33, 0.7, 0.64>, <0.38, 0.75, 0.63>, <0.7, 0.5, 0.3>};

$R_4 = \{ d_{41}, d_{42}, d_{43}, d_{44}, d_{45}, d_{46}\}$ = {<0.9, 0.334, 0.1>, <0.13, 0.25, 0.88>, <0.18, 0.35, 0. 83>, <0.09, 0.2, 0.9>, <0.13, 0.25, 0.88>, <0.9, 0, 0.1>}.

Table 5. Results of the proposed HSSM

D_j	$H_\varphi(D_j, R_1)$	$H_\varphi(D_j, R_2)$	$H_\varphi(D_j, R_3)$	$H_\varphi(D_j, R_4)$	Risk grade
D_1	**0.896665**	0.703826	0.608403	0.517216	R_1
D_2	0.677753	**0.891382**	0.823401	0.640984	R_2
D_3	0.607045	**0.812724**	0.788762	0.615625	R_2
D_4	0.583422	**0.869008**	0.838668	0.550995	R_2
D_5	**0.796918**	0.734908	0.657685	0.515831	R_1
D_6	0.732703	**0.851046**	0.792102	0.617296	R_2
D_7	0.732119	**0.916111**	0.813484	0.60338	R_2
D_8	0.70132	**0.793529**	0.72165	0.513288	R_2
D_9	0.670875	**0.87848**	0.816791	0.637876	R_2
D_{10}	0.620938	0.787465	**0.836347**	0.7285	R_3
D_{11}	**0.775396**	0.696152	0.676804	0.617519	R_1
D_{12}	0.547174	0.758627	**0.764204**	0.734565	R_3
D_{13}	0.644711	0.759293	**0.834798**	0.73285	R_3
D_{14}	0.537107	0.731683	**0.809379**	0.775532	R_3
D_{15}	0.508192	0.640297	0.742197	**0.856468**	R_4
D_{16}	0.480595	0.496986	0.578814	**0.734005**	R_4
D_{17}	0.446757	0.560614	0.655778	**0.842855**	R_4
D_{18}	0.483972	0.646724	0.742483	**0.794429**	R_4
D_{19}	0.530704	0.551393	0.606451	**0.740304**	R_4
D_{20}	0.549504	0.570076	0.651012	**0.786229**	R_4

Then, the slope stability of the 20 slope samples D_j (j = 1, 2, …, 20) is assessed by Eq. (5) for φ = 2, and the HSSM values between the slope samples D_j and the slope stability risk patterns R_k (k = 1, 2, 3, 4) are given in Table 5. The maximum measure value between D_j and R_k reflects the corresponding slope stability risk pattern/grade. From the evaluation results, it can be found that the four types of the 20 slope samples obtained by the proposed clustering method are consistent with the four risk patterns/levels:

(i) The set of slope samples {D_1, D_5, D_{11}} is the risk grade R_1;
(ii) The set of slope samples {D_2, D_3, D_4, D_6, D_7, D_8, D_9} is the risk grade R_2;
(iii) The set of slope samples {D_{10}, D_{12}, D_{13}, D_{14}} is the risk grade R_3;
(iv) The set of slope samples {D_{15}, D_{16}, D_{17}, D_{18}, D_{19}, D_{20}} is the risk grade R_4.

The above results prove the accuracy and validity of the proposed netting clustering method and the proposed evaluation method for the 20 slope samples.

4.3. Comparative Analysis

Regarding comparative analysis, we use the weighted generalized distance-based similarity measure of Eq. (2) [20] to assess the stability risk grades of the 20 slope samples. All the evaluation results are given in Table 6. It is obvious that the risk grade of each slope sample assessed by Eq. (2) for φ = 2 [20] is the same as that evaluated by the proposed HSSM of SVNSs. Therefore, the slope stability evaluation method using the proposed HSSM of SVNSs verifies its effectiveness and accuracy in the open-pit mine slope stability evaluation problems.

Table 6. Evaluation results based on Eq. (2)

D_j	$S_\varphi(D_j, R_1)$	$S_\varphi(D_j, R_2)$	$S_\varphi(D_j, R_3)$	$S_\varphi(D_j, R_4)$	Risk grade
D_1	**0.884997**	0.671241	0.56761	0.471864	R_1
D_2	0.641967	**0.87775**	0.801116	0.603543	R_2
D_3	0.568019	**0.792424**	0.765862	0.578501	R_2
D_4	0.54286	**0.852145**	0.818222	0.507299	R_2
D_5	**0.77512**	0.706827	0.62318	0.470381	R_1
D_6	0.703056	**0.833838**	0.76785	0.577951	R_2
D_7	0.70058	**0.905584**	0.790263	0.561491	R_2
D_8	0.667514	**0.771982**	0.694343	0.471249	R_2
D_9	0.635115	**0.863307**	0.793656	0.60041	R_2
D_{10}	0.581658	0.761448	**0.816313**	0.699008	R_3
D_{11}	**0.751333**	0.662475	0.643164	0.581591	R_1
D_{12}	0.505493	0.732561	**0.735527**	0.709053	R_3
D_{13}	0.608234	0.730184	**0.815707**	0.704047	R_3
D_{14}	0.493782	0.700375	**0.785899**	0.753015	R_3
D_{15}	0.463014	0.602347	0.712527	**0.838671**	R_4
D_{16}	0.440282	0.455418	0.539813	**0.707829**	R_4
D_{17}	0.402051	0.519151	0.619739	**0.823512**	R_4
D_{18}	0.439269	0.609585	0.712951	**0.772755**	R_4
D_{19}	0.491309	0.510861	0.568517	**0.715473**	R_4
D_{20}	0.511998	0.528949	0.614943	**0.76203**	R_4

5. Conclusions

The paper proposed HSSM of SVNSs and established its netting clustering analysis and risk evaluation methods for open-pit mine slopes in the scenario of SVNSs. Then, the proposed netting clustering analysis and risk evaluation methods were used for the clustering analysis and risk evaluation of open-pit mine slopes. In the applications of the clustering analysis and risk evaluation methods of slope samples, they contain the following techniques. First, appropriate true, indeterminate, and false membership functions for the impact factors of slope stability were fuzzified into the true, indeterminate, and false fuzzy values, which are constructed as the form of SVNNs. Then, the proposed netting clustering method based on the proposed HSSM was used to cluster the slope samples. Further, based on the clustering results and risk knowledge of slope stability, we gave the corresponding risk patterns/grades to evaluate the risk grades of slope stability by the HSSM values between the slope samples and the slope stability patterns in the scenario of SVNSs. Finally, the proposed netting clustering analysis and risk evaluation methods were applied to the clustering analysis and risk evaluation of 20 slope samples. The comparative results proved the accuracy, validity and rationality of the proposed netting clustering analysis and risk evaluation methods.

The main advantage of this study is that the proposed clustering method and the slope stability assessment approach can simply and effectively process the clustering analysis and evaluation problems of open-pit mine slopes; while the existing evaluation methods using ANN, ANFIS, and SVNANFIS [2, 4, 9] imply the defects of both the complex learning algorithms and the requirement of larger-scale sample data. It is obvious that the proposed methods effectively overcome the defects of the existing evaluation methods [2, 4, 9] and are more convenient and more reasonable than the existing clustering analysis and evaluation methods [1-4].

Regarding future research, more slope samples and more impact factors will be considered to further verify the accuracy and efficiency of the proposed clustering and evaluation methods. Then, new similarity measures and clustering and evaluation methods will be further proposed to make their clustering and evaluation methods more effective and reasonable in the setting of SVNSs.

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