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A Novel Approach Towards Parameter Reduction Based on Bipolar Hypersoft Set and Its Application to Decision-Making

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Abstract. For a mathematical model to describe vague (uncertain) problems effectively, it must have the ability to explain the links between the objects and parameters in the problem in the most precise way. There is no suitable model that can handle such scenarios in the literature. This deficiency serves as motivation for this study. In this article, the bipolar hypersoft set (abbreviated, BHSS) is considered since the parameters and their opposite play a symmetrical role. We present a novel theoretical technique for solving decision-making problems using BHSS and investigate parameter reductions for these sets. Algorithms for parameter reduction are provided and explained with examples. The findings demonstrate that our suggested parameter reduction strategies remove unnecessary parameters and still retain the same decision-making options.

Keywords: bipolar hypersoft set; hypersoft set; soft set; parameter reduction; decision-making; algorithm

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1. Introduction

Many real-world challenges in disciplines such as engineering, environmental sciences, information knowledge, medical sciences, and social sciences include varying degrees of uncertainty. It is well known that this type of uncertainty cannot be represented using conventional analytical methods. Despite this, they are effectively handled by theories ranging from the fuzzy set [43] to the intuitionistic fuzzy set [4] and the rough set [28, 29], as well as probability theory. However, all of these ideas have inherent problems, some of which were pointed out in Molodtsov (1999). Molodtsov [18, 19] offered a unique theory to address these problems, and the central idea of it is known as a soft set. From this starting point, a novel formal method for modeling uncertainties was created, and because of its adaptability, it has been discussed in the context of intelligent systems, operations research, information science, the theory of probability and measurement theory.

The study of decision-making procedures related to soft sets is an additional topic of interest in this subject. To address problems with decision-making, Maji et al. [17] suggested parameter reduction of soft sets. Subsequently, the Maji et al [17] approach was criticized by Chen et al. [10], who also gave a different idea for parameter reduction of soft sets. Ali [1] studied another point of view on parameter reduction in soft sets. Kong et al. [13] first proposed the idea of normal parameter reduction of soft sets in [10], which was intended to address the problem of suboptimal selection. However, the idea is too abstract and the procedure is difficult to understand and takes a long time. An improved approach is provided in [13], while Ma et al. [14] studied the normal parameter reduction. Xie [42] investigated parameter reduction by attribute reduction in information systems. Maharana and Mohanty [15] focused on the application of parametric reduction of soft set in decision-making problem. Zhan and Alcantud [44] discussed a variety of parameter reduction techniques based on soft (fuzzy) set types. Furthermore, they contrasted the algorithms to highlight their various benefits and drawbacks and provided examples to explain their differences. Using the notion of σ-algebraic soft sets, Khan et al. [11] have developed a novel approach for the normal parameter reduction. Applications in decision-making based on soft set and its extensions can be seen at [2, 3, 5, 6, 12, 16, 36].

In 2018, Smarandache [37, 38] extended the soft set to the hypersoft set. Then, in [39, 40], he extended the soft set and hypersoft set to IndetermSoft Set and IndetermHyperSoft Set, respectively. In addition, he introduced TreeSoft Set [41] as an extension to MultiSoft Set. The authors in [7, 9, 20, 21, 30–35] presented the principles of the hypersoft set and its application. Recently, Musa and Asaad [22] introduced the notion of BHSS as a combination of hypersoft set with bipolarity setting and investigated some of its fundamental operations. They also discussed some topological notions in the frame of bipolar hypersoft setting [8, 23–27].

1.1. *Motivation*

In many real-world decision-making challenges, we come into situations where each attribute needs to be further categorized into its appropriate attribute-valued set. In order to deal with such eventualities, a hypersoft set is projected, using the cartesian product of disjoint attribute-valued sets as the approximate function's domain. To handle uncertainties with this form of approximate function, the current models are insufficient. Therefore, in this paper, the BHSS is taken into consideration because the parameters and their opposite play a symmetric role.

1.2. *Main Contributions*

The following list highlights the study's main contributions:

(1) Some basic definitions are reviewed from the literature.
(2) Theory of BHSSs is used to present a novel theoretical technique for solving decision-making problems and investigate parameter reductions for these sets.
(3) Suggested algorithm is then tested by using it to solve a problem from daily life that involves decision-making.
(4) The scope and future directions of the paper are summarized in order to inspire the reader to pursue further extensions.

1.3. *Paper Layout*

Following is the structure for this paper: The second section, before getting into this article, some background information and ideas are given. In the third section, by utilizing BHSSs, we suggest a new technique for parameter reduction, which will then be followed by an example. The last section, section 4, provides the paper's conclusion and future directions.

2. **Preliminaries**

This part will show a few results that will be useful in the subsequent section. Let $\Re$ represent a finite universe of objects, $2^{\Re}$ the power set of $\Re$, and $\Sigma = \Sigma_1 \times \Sigma_2 \times ... \times \Sigma_n$ the set of parameters. Let $\Lambda = \Lambda_1 \times \Lambda_2 \times ... \times \Lambda_n$ and $\Delta = \Delta_1 \times \Delta_2 \times ... \times \Delta_n$ with $\Lambda_i, \Delta_i \subseteq \Sigma_i$ for each $i = 1, 2, ..., n$.

Definition 2.1. [22] A triple $(\mathcal{g}, \widehat{\mathcal{g}}, \Sigma)$ is called a BHSS over $\Re$, where $\mathcal{g}$ and $\widehat{\mathcal{g}}$ are mappings given by $\mathcal{g} : \Sigma \to 2^{\Re}$ and $\widehat{\mathcal{g}} : \neg\Sigma \to 2^{\Re}$ with $\mathcal{g}(s) \cap \widehat{\mathcal{g}}(\neg s) = \phi$ for all $s \in \Sigma$.

In other words, a BHSS $(\mathcal{g}, \widehat{\mathcal{g}}, \Sigma)$ over $\Re$ provides two parametrized families of subsets of $\Re$, with the consistency requirement $\mathcal{g}(s) \cap \widehat{\mathcal{g}}(\neg s) = \phi$ for all $s \in \Sigma$. From now on, a BHSS $(\mathcal{g}, \widehat{\mathcal{g}}, \Sigma)$ will be represented as follows:

$$(\mathcal{g}, \widehat{\mathcal{g}}, \Sigma) = \{(s, \mathcal{g}(s), \widehat{\mathcal{g}}(\neg s)) : s \in \Sigma \text{ and } \mathcal{g}(s) \cap \widehat{\mathcal{g}}(\neg s) = \phi\}.$$

Example 2.2. Suppose $\Re = \{r_1, r_2, r_3, r_4, r_5, r_6, r_7\}$ is the set of seven applicants that applying for a job in a company. Let, The Director $= \Sigma_1 = \{s_1 =$goal-oriented,$s_2 =$ risk-taking,$s_3 =$ good under stress $\}$, The Thinker $= \Sigma_2 = \{s_4 =$ logical,$s_5 =$ prepared $\}$, and The Supporter $= \Sigma_3 = \{s_7 =$ stabilizing, $s_8 =$ cautious $\}$ be the set of three personality types and $\Sigma = \Sigma_1 \times \Sigma_2 \times \Sigma_3 = \{\ell_1 = (s_1, s_4, s_7), \ell_2 = (s_1, s_4, s_8), \ell_3 = (s_1, s_5, s_7), \ell_4 = (s_1, s_5, s_8), \ell_5 = (s_2, s_4, s_7), \ell_6 = (s_2, s_4, s_8), \ell_7 = (s_2, s_5, s_7), \ell_8 = (s_2, s_5, s_8), \ell_9 = (s_3, s_4, s_7), \ell_{10} = (s_3, s_4, s_8), \ell_{11} = (s_3, s_5, s_7), \ell_{12} = (s_3, s_5, s_8)\}$. A BHSS

$(g,\widehat{g},\Sigma)$ can be defined to describe "Analysis of Applicants' Personality" as: $(g,\widehat{g},\Sigma) = \{(\ell_1,\Re,\phi),(\ell_2,\{r_2,r_3,r_4,r_6\},\{r_5,r_7\}),(\ell_3,\phi,\{r_4,r_5,r_6,r_7\}),(\ell_4,\{r_1,r_2,r_3,r_4\},\{r_5,r_6,r_7\}), (\ell_5,\{r_1,r_2,r_3\},\phi),(\ell_6,\{r_1,r_4,r_6\},\{r_2,r_7\}),(\ell_7,\Re,\phi),(\ell_8,\{r_1,r_7\},\{r_2,r_6\}),(\ell_9,\{r_1,r_6\},\{r_2,r_4\}), (\ell_{10},\phi,\Re),(\ell_{11},\Re,\phi),(\ell_{12},\phi,\Re)\}$.

Musa and Asaad [22] represented a BHSS by a binary table to store it in computer memory. The (i,j)-th entry in table is:

$$m_{ij} = \begin{cases} 1 & \text{if } r_i \in g(\ell_j) \\ 0 & \text{if } r_i \in \Re \setminus \{g(\ell_j) \cup \widehat{g}(\neg\ell_j)\} \\ -1 & \text{if } r_i \in \widehat{g}(\neg\ell_j) \end{cases}$$

Table 1 provides a tabular representation of the BHSS $(g,\widehat{g},\Sigma)$ with referring to Example 2.2.

TABLE 1. Tabular form of $(g,\widehat{g},\Sigma)$

$(g,\widehat{g},\Sigma)$	ℓ_1	ℓ_2	ℓ_3	ℓ_4	ℓ_5	ℓ_6	ℓ_7	ℓ_8	ℓ_9	ℓ_{10}	ℓ_{11}	ℓ_{12}
r_1	1	0	0	1	1	1	1	1	1	−1	1	−1
r_2	1	1	0	1	1	−1	1	−1	−1	−1	1	−1
r_3	1	1	0	1	1	0	1	0	0	−1	1	−1
r_4	1	1	−1	1	0	1	1	0	−1	−1	1	−1
r_5	1	−1	−1	−1	0	0	1	0	0	−1	1	−1
r_6	1	1	−1	−1	0	1	1	−1	1	−1	1	−1
r_7	1	−1	−1	−1	0	−1	1	1	0	−1	1	−1

Definition 2.3. [22] Let $(g_1,\widehat{g}_1,\Lambda)$ and $(g_2,\widehat{g}_2,\Delta)$ be two BHSSs. Then

(1) $(g_1,\widehat{g}_1,\Lambda)$ is a bipolar hypersoft subset of $(g_2,\widehat{g}_2,\Delta)$, denoted by $(g_1,\widehat{g}_1,\Lambda) \,\tilde{\tilde{\sqsubseteq}}\, (g_2,\widehat{g}_2,\Delta)$, if $\Lambda \subseteq \Delta$ and $g_1(s) \subseteq g_2(s)$, $\widehat{g}_2(\neg s) \subseteq \widehat{g}_1(\neg s)$ for all $s \in \Lambda$.

(2) $(g_1,\widehat{g}_1,\Lambda)$ and $(g_2,\widehat{g}_2,\Delta)$ are bipolar hypersoft equal, if $(g_1,\widehat{g}_1,\Lambda) \,\tilde{\tilde{\sqsubseteq}}\, (g_2,\widehat{g}_2,\Delta)$ and $(g_2,\widehat{g}_2,\Delta) \,\tilde{\tilde{\sqsubseteq}}\, (g_1,\widehat{g}_1,\Lambda)$.

(3) If $g_1(s) = \phi$ and $\widehat{g}_1(\neg s) = \Re$ for all $s \in \Lambda$, then $(g_1,\widehat{g}_1,\Lambda)$ is called a relative null BHSS and denoted by $(\widetilde{\phi},\widetilde{\Re},\Lambda)$.

(4) If $g_1(s) = \Re$ and $\widehat{g}_1(\neg s) = \phi$ for all $s \in \Lambda$, then $(g_1,\widehat{g}_1,\Lambda)$ is called a relative whole BHSS and denoted by $(\widetilde{\Re},\widetilde{\phi},\Lambda)$.

(5) The complement of $(g_1,\widehat{g}_1,\Lambda)$ is a BHSS $(g_1,\widehat{g}_1,\Lambda)^c = (g_1^c,\widehat{g}_1^c,\Lambda)$ where $g_1^c(s) = \widehat{g}_1(\neg s)$ and $\widehat{g}_1^c(\neg s) = g_1(s)$ for all $s \in \Lambda$.

(6) The union of $(g_1,\widehat{g}_1,\Lambda)$ and $(g_2,\widehat{g}_2,\Delta)$, denoted by $(g_1,\widehat{g}_1,\Lambda) \,\tilde{\tilde{\sqcup}}\, (g_2,\widehat{g}_2,\Delta)$, is a BHSS $(g,\widehat{g},\Gamma)$, where $\Gamma = \Lambda \cap \Delta$ and for all $s \in \Gamma$: $g(s) = g_1(s) \cup g_2(s)$ and $\widehat{g}(\neg s) = \widehat{g}_1(\neg s) \cap \widehat{g}_2(\neg s)$.

(7) The intersection of $(\mathcal{g}_1,\widehat{\mathcal{g}}_1,\Lambda)$ and $(\mathcal{g}_2,\widehat{\mathcal{g}}_2,\Delta)$, denoted by $(\mathcal{g}_1,\widehat{\mathcal{g}}_1,\Lambda)\ \widetilde{\widetilde{\sqcap}}\ (\mathcal{g}_2,\widehat{\mathcal{g}}_2,\Delta)$, is a BHSS $(\mathcal{g},\widehat{\mathcal{g}},\Gamma)$, where $\Gamma=\Lambda\cap\Delta$ and for all $s\in\Gamma$: $\mathcal{g}(s)=\mathcal{g}_1(s)\cap\mathcal{g}_2(s)$ and $\widehat{\mathcal{g}}(\neg s)=\widehat{\mathcal{g}}_1(\neg s)\cup\widehat{\mathcal{g}}_2(\neg s)$.

Definition 2.4. [28]

(1) Suppose B is a set of attributes with $B\subseteq A$. We identify a binary relation $\mathcal{IND}(B)$, known as indiscernibility, with expression $\mathcal{IND}(B)=\{(x,y)\in\Re\times\Re : b(x)=b(y),\forall b\in B\}$. Also, $\mathcal{IND}(B)=\cap_{b\in B}\mathcal{IND}(b)$.

(2) We call an element $b\in B$ dispensable if $\mathcal{IND}(B)=\mathcal{IND}(B-\{b\})$. Otherwise, b is called indispensable in B.

3. Parameter Reduction and Decision-Making Problem

In this section, we discuss the idea of reduction of parameters and decision-making problem in case of BHSS. Examples are provided to assist readers comprehend the key findings.

Definition 3.1. Let $\pi:\Sigma\to 2^{\Re\times\Re}$ be mapping. Then a hypersoft binary relation over $\Re$ is the hypersoft set (π,Σ) over $\Re\times\Re$.

In fact, (π,Σ) is a parametrized subsets of binary relations on $\Re$, i.e., there is a binary relation $\pi(s)$ on $\Re$ for each parameter $s\in\Sigma$.

Definition 3.2. If $\pi(s)\neq\phi$ is an equivalence relation over $\Re$ for all $s\in\Sigma$. Then a hypersoft binary relation (π,Σ) over a set $\Re$ is called a hypersoft equivalence relation over $\Re$.

Definition 3.3. The decision value of an object $r_i\in\Re$, denoted by $\mathcal{d}_i$, is defined as:

$$\mathcal{d}_i=\sum_j m_{ij}$$

where m_{ij} is the (i,j)-th element in the BHSS table. The decision table is constructed by joining the column of decision parameter $\mathcal{d}$ with values $\mathcal{d}_i$ to the table of the BHSS $(\mathcal{g},\widehat{\mathcal{g}},\Sigma)$.

Now, we provide a definition for the concept of indiscernibility relations related to a BHSS.

Definition 3.4. Let $(\mathcal{g},\widehat{\mathcal{g}},\Sigma)$ be a BHSS over $\Re$, then:

(1) If $\phi\neq\mathcal{g}(s)\subset\Re$ and $\phi\neq\widehat{\mathcal{g}}(\neg s)\subset\Re$ with $\mathcal{g}(s)\cup\widehat{\mathcal{g}}(\neg s)\neq\Re$, then $(\mathcal{g},\widehat{\mathcal{g}},\Sigma)$ divides $\Re$ into three classes.

(2) If $\mathcal{g}(s)=\phi$, $\widehat{\mathcal{g}}(\neg s)\subset\Re$ or $\widehat{\mathcal{g}}(\neg s)=\phi$, $\mathcal{g}(s)\subset\Re$, then $(\mathcal{g},\widehat{\mathcal{g}},\Sigma)$ divides $\Re$ into two classes.

(3) If $\mathcal{g}(s)=\Re$ or $\widehat{\mathcal{g}}(\neg s)=\Re$, then it provides the universal equivalence relation $\Re\times\Re$.

In any of the foregoing three cases, these classes represent an equivalence relation on $\Re$. As a result, we can note that we have an equivalence relation on $\Re$ for each parameter $s\in\Sigma$. If we denote this equivalence relation as $\lambda(s)$ for all $s \in \Sigma$, then (λ, Σ) is a hypersoft equivalence relation over $\Re$. We write

$$\mathcal{IND}(g, \widehat{g}, \Sigma) = \cap_{s\in\Sigma}\lambda(s).$$

It is obvious that $\mathcal{IND}(g, \widehat{g}, \Sigma)$ is an equivalence relation over $\Re$. The classes of $\mathcal{IND}(g, \widehat{g}, \Sigma)$ are fundamental types of knowledge that are shown by a BHSS over $\Re$. We could also consider that, $\mathcal{IND}(\Sigma) = \mathcal{IND}(g, \widehat{g}, \Sigma)$.

Definition 3.5. We call a decision table of $(g, \widehat{g}, \Sigma)$ consistent if and only if $\mathcal{IND}(\Sigma) \subseteq \mathcal{IND}(\mathcal{D})$, where $\mathcal{IND}(\mathcal{D})$ is the equivalence relation that divides $\Re$ into categories with similar decision values.

Definition 3.6. Suppose that $\mathcal{T} = (\Re, \Sigma, \Lambda, \mathcal{D})$ is a consistent decision table of BHSS $(g, \widehat{g}, \Sigma)$ and $\mathcal{T}_\epsilon = (\Re, \Sigma, \Lambda - \epsilon, \mathcal{D}_\epsilon)$ is a decision table genrated from $\mathcal{T}$ by removing some column $\epsilon \in \Lambda$. Then ϵ is dispensable in $\mathcal{T}$ if

(1) $\mathcal{T}_\epsilon$ is consistent, that is, $\Lambda - \epsilon \Rightarrow \mathcal{D}_\epsilon$.
(2) $\mathcal{IND}(\mathcal{D}) = \mathcal{IND}(\mathcal{D}_\epsilon)$.

Otherwise, ϵ is indispensable or core parameter. The set of all core parameters of Λ is denoted by $\mathcal{CORE}(\Lambda)$.

Now, we suggest the following algorithm based on a BHSS.

Algorithm 1.

(1) Identify the BHSS $(g, \widehat{g}, \Sigma)$.
(2) Identify $\Lambda \subseteq \Sigma$ as the set of choice parameters.
(3) Input $d \in \mathcal{D}$, $d_i = \sum_j m_{ij}$ and place it in the last column of the obtained choice parameters table.
(4) Place the objects that share the same value for d next to each other to rearrange the input.
(5) Specify core parameters as defined in Definition 3.6. Remove each dispensable parameter individually to get a table having a minimum number of condition parameters that has the same classification ability for d as the original table.
(6) Find k such that $d_k = \max d_i$. Then r_k is the best choice object. Any one of r_k's can be chosen if k has multiple values.

Below, we illustrate the proposed algorithm according to Example 2.2:

(1) Identify the BHSS $(g, \widehat{g}, \Sigma)$ given by Table 1.
(2) Let $\Lambda = \{\ell_1, \ell_3, \ell_5, \ell_7, \ell_9, \ell_{11}\}$ where $\Lambda = \Lambda_1 \times \Lambda_2 \times \Lambda_3$ and $\Lambda_1 = \{s_1, s_2, s_3\}$, $\Lambda_2 = \{s_4, s_5\}$, and $\Lambda_3 = \{s_7\}$.
(3) Table 2 gives the decision table of BHSS $(g, \widehat{g}, \Lambda)$. We observe that

TABLE 2. Tabular form of $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$

$(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	$\mathcal{d}$
r_1	1	0	1	1	1	−1	3
r_2	1	1	1	−1	−1	−1	0
r_3	1	1	1	0	0	−1	2
r_4	1	1	0	1	−1	−1	1
r_5	1	−1	0	0	0	−1	−1
r_6	1	1	0	1	1	−1	3
r_7	1	−1	0	−1	0	−1	−2

$$\begin{aligned}\mathcal{IND}(\Lambda) &= \{(r_1,r_1),(r_2,r_2),(r_3,r_3),(r_4,r_4),(r_5,r_5),(r_6,r_6),(r_7,r_7)\}\\ &\subset \{(r_1,r_1),(r_2,r_2),(r_3,r_3),(r_4,r_4),(r_5,r_5),(r_6,r_6),(r_7,r_7),(r_1,r_6),(r_6,r_1)\}\\ &= \mathcal{IND}(\mathcal{D}).\end{aligned}$$

Therefore, the decision table is consistent.

(4) Table 3 is obtained by rearranging Table 2 using the same values for $\mathcal{d}$.

TABLE 3. Rearrangement of Table 2

$(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$	ℓ_1	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	$\mathcal{d}$
r_1	1	0	1	1	1	−1	3
r_6	1	1	0	1	1	−1	3
r_3	1	1	1	0	0	−1	2
r_4	1	1	0	1	−1	−1	1
r_2	1	1	1	−1	−1	−1	0
r_5	1	−1	0	0	0	−1	−1
r_7	1	−1	0	−1	0	−1	−2

(5) In order to determine $\mathcal{CORE}(\Lambda)$. First, we remove ℓ_1 from Table 3, we obtain Table 4. We observe that removing ℓ_1 has no effect on the classification ability of the decision parameter $\mathcal{d}$, thus ℓ_1 is dispensable in Table 3. Then, if we remove ℓ_3 from Table 3, we are left with Table 5. Due to the removal of ℓ_3, $\mathcal{d}$'s classification is different from that in Table 3. Eliminating ℓ_3 therefore disturbs $\mathcal{d}$'s ability for classification; as a result, ℓ_3 is a core parameter. Continuing in the same manner we determine set of core parameters:

$$\mathcal{CORE}(\Lambda) = \{\ell_3, \ell_5, \ell_7, \ell_9\} \tag{1}$$

Hence, we can conclude that the removal of ℓ_1 and ℓ_{11} has no effect on $\mathcal{d}$'s classification ability, as seen in Table 3. The same classification is presented in Table 6 with minimum condition parameters.

TABLE 4. Tabular form of $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$ after eliminating ℓ_1

$(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$	ℓ_3	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	d_{ℓ_1}
r_1	0	1	1	1	-1	2
r_6	1	0	1	1	-1	2
r_3	1	1	0	0	-1	1
r_4	1	0	1	-1	-1	0
r_2	1	1	-1	-1	-1	-1
r_5	-1	0	0	0	-1	-2
r_7	-1	0	-1	0	-1	-3

TABLE 5. Tabular form of $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$ after eliminating ℓ_3

$(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$	ℓ_1	ℓ_5	ℓ_7	ℓ_9	ℓ_{11}	d_{ℓ_3}
r_1	1	1	1	1	-1	3
r_6	1	0	1	1	-1	2
r_3	1	1	0	0	-1	1
r_4	1	0	1	-1	-1	0
r_5	1	1	-1	-1	-1	0
r_2	1	0	0	0	-1	-1
r_7	1	0	-1	0	-1	-1

TABLE 6. Tabular form of $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$ after eliminating ℓ_1 and ℓ_{11}

$(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$	ℓ_3	ℓ_5	ℓ_7	ℓ_9	$d_{(\ell_1, \ell_{11})}$
r_1	0	1	1	1	3
r_6	1	0	1	1	3
r_3	1	1	0	0	2
r_4	1	0	1	-1	1
r_2	1	1	-1	-1	0
r_5	-1	0	0	0	-1
r_7	-1	0	-1	0	-2

Now, we define weighted table of the BHSS $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$. The reason is that some of the parameters are less important than others, so they must be prioritized lower. So, we propose that the column of that parameter have the following entries:

$$n_{ij} = \begin{cases} m_{ij} \times \varpi_j & \text{if } m_{ij} = 1 \\ 0 & \text{if } m_{ij} = 0 \\ m_{ij} \times (1 - \varpi_j) & \text{if } m_{ij} = -1 \end{cases}$$

instead of 0 and 1 and -1 only, where m_{ij} are the entries in the table of the BHSS $(\mathcal{g}, \widehat{\mathcal{g}}, \Lambda)$.

Definition 3.7. The weighted decision value of $r_i \in \Re$ is defined as:

$$\mathscr{d}_i = \sum_j n_{ij}$$

Next, we present the revised algorithm:

Algorithm 2.

(a) Identify the BHSS $(\mathscr{g}, \widehat{\mathscr{g}}, \Sigma)$.

(b) Identify $\Lambda \subseteq \Sigma$ as the set of choice parameters.

(c) Determine the weighted table of the BHSS $(\mathscr{g}, \widehat{\mathscr{g}}, \Lambda)$ based on the chosen weights.

(d) Input $\mathscr{d} \in \mathcal{D}$, $\mathscr{d}_i = \sum_j n_{ij}$ and place it in the last column of the weighted table $\mathcal{T}_{\varpi}$.

(e) Place the objects that share the same value for $\mathscr{d}$ next to each other to rearrange the input.

(f) Specify core parameters. Remove each dispensable parameter individually to get a table having a minimum number of condition parameters that has the same classification ability for $\mathscr{d}$ as the original table.

(g) Find k such that $\mathscr{d}_k = \max \mathscr{d}_i$. Then r_k is the optimal choice object. Any one of r_k's can be chosen if k has multiple values.

Now, the original problem is resolved utilizing the new algorithm. Assume that the selection committee assigns the following weights to the parameters of Λ, beginning with the third step:

$$\ell_3\colon\ \varpi_3 = 0.8$$
$$\ell_5\colon\ \varpi_5 = 0.5$$
$$\ell_7\colon\ \varpi_7 = 0.9$$
$$\ell_9\colon\ \varpi_9 = 0.9$$

Table 7 gives the weighted decision table of BHSS $(\mathscr{g}, \widehat{\mathscr{g}}, \Lambda)$. We note that

TABLE 7. Weighted Decision Table for $(\mathscr{g}, \widehat{\mathscr{g}}, \Lambda)$

$(\mathscr{g}, \widehat{\mathscr{g}}, \Lambda)_{\varpi}$	ℓ_3	ℓ_5	ℓ_7	ℓ_9	$\mathscr{d}$
r_1	0	0.5	0.9	0.9	2.3
r_2	0.8	0.5	−0.1	−0.1	1.1
r_3	0.8	0.5	0	0	1.3
r_4	0.8	0	0.9	−0.1	1.6
r_5	−0.2	0	0	0	−0.2
r_6	0.8	0	0.9	0.9	2.6
r_7	−0.2	0	−0.1	0	−0.3

$$\mathcal{IND}(\Lambda) = \{(r_1, r_1), (r_2, r_2), (r_3, r_3), (r_4, r_4), (r_5, r_5), (r_6, r_6), (r_7, r_7)\}$$
$$= \mathcal{IND}(\mathcal{D}).$$

Therefore, the decision table is consistent. Table 8 is obtained by rearranging Table 7 based on the descending values for d. We find that

TABLE 8. Table of weighted BHSS $(g, \widehat{g}, \Lambda)$ after rearrangement

$(g, \widehat{g}, \Lambda)_\varpi$	ℓ_3	ℓ_5	ℓ_7	ℓ_9	d
r_6	0.8	0	0.9	0.9	2.6
r_1	0	0.5	0.9	0.9	2.3
r_4	0.8	0	0.9	−0.1	1.6
r_3	0.8	0.5	0	0	1.3
r_2	0.8	0.5	−0.1	−0.1	1.1
r_5	−0.2	0	0	0	−0.2
r_7	−0.2	0	−0.1	0	−0.3

$$\mathcal{CORE}(\Lambda) = \Lambda. \tag{2}$$

The values of d indicate that $d_6 = \max d_i = 2.6$ and hence $k = 6$. Thus r_6 is the best candidate to choose since it is the best choice object. We observe that the change occurs in the place of r_1. In the first case, r_1 ranked 1st out of 7, however under the weighted criterion r_1 ranks 2nd overall. Similarly, we can identify the position of each object based on the weighted criteria.

4. Conclusions and Discussion

BHSS theory is a valuable mathematical model for expressing uncertainty concerns since it takes into consideration both NOT parameters and parameters sets. In this study, a novel method to decision-making using BHSS was presented, and a decision-making problem was solved to show the technique's validity. Parameter reduction techniques were created and described through examples. The study demonstrated that our recommended parameter reduction procedures minimize the unneeded parameters while keeping the same decision-making choices. The novelty of this work is that this is the first work that employed BHSS to reduce the parameters in decision-making problems. Although the proposed work are flexible and reliable as the findings demonstrated that, this model has limitations regarding some situations that deal with operators having some degree of indeterminacy of our world. Therefore, the future work may include the extension of this study (i.e. IndetermSoft Set, IndetermHyperSoft Set, etc.).

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