NSS

Neutrosophic Sets and Systems, Vol. 55, 2023

University of New Mexico

Extension of G-Algebras to SuperHyper G-Algebras

Marzieh Rahmati[1,*] and Mohammad Hamidi [2]

[1,2]Department of Mathematics, University of Payame Noor, Tehran, Iran. P.O. Box 19395-4697.

[1] m.rahmati@pnu.ac.ir

[2] m.hamidi@pnu.ac.ir

[*]Correspondence: m.rahmati@pnu.ac.ir

Abstract. The theory of superhyperalgebras is a new concept in the study of all branches of algebra structures. In this paper, we introduce a novel concept of (m, n)-superhyper G-algebra and present several results from the study of certain properties of (m, n)-superhyper G-algebras. The purpose of this paper is the study an extension of G-algebras to (m, n)-superhyper G-algebras, as a generalization of a logic algebra. The main motivation of this work was obtained based on an extension of G-algebra to superhyper G-algebra based on the n^{th}-power set of a set.

Keywords: (m, n)-superhyperoperation, (m, n)-superhyperalgebra, (m, n)-superhyper G-algebra.

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1. **Introduction**

The concept of superhyperalgebra has been introduced by Smarandache in [12]. Smarandache presented the n^{th}-power set of a set, superhyper operation, superhyper axiom, superhyper algebra, their corresponding neutrosophic superhyper operation, neutrosophic superhyper axiom, and neutrosophic superhyper algebra. In general, in any field of knowledge, he analyzes to encounter superhyper structures (or more accurately (m, n)-SuperHyperStructures). He studied related concepts, for example, the concepts of superhyperoperation, superhyperaxiom, superhyperstructure, superhyperalgebra, superhyperfunction, superhypergroup, superhypertopology, superhypergraph, and their corresponding neutrosophic superhyperoperation, neutrosophic superhyperaxiom, and neutrosophic superhyperalgebra in [10–14] between 2016-2022. Recently Hamidi et al. investigated some research in this scope such as the spectrum of superhypergraphs via flows [3], on neutro-d-subalgebras [4], neutro-BCK-algebra [5], on neutro G-subalgebra [7], single-valued neutro hyper BCK-subalgebras [6] and superhyper BCK-algebra [8]. The superhyperalgebra theory both extends some well-known algebra results and introduces new topics. The notion of superhyperalgebra is a natural generalization of the notion of algebra and the development of its fundamental properties. In 2012, the concept of G-algebra was introduced by Bandaru and Rafi [2]. They proved that QS-algebras are G-algebras, but the opposite is not necessarily true. The concept of G-algebra is a generalization of Q-algebra, which has many applications in algebra. We can read more about G-algebras in [1, 9]. In this paper, (m, n)-superhyper G-algebras is defined and considered. Examples of (m, n)-superhyper G-algebras are given and some of their properties are described. The concept of (m, n)-superhyper G-algebra is a generalization of G-algebra. The purpose of this paper is the study an extension of G-algebras to (m, n)-superhyper G-algebras, as a generalization of a logic algebra. The main motivation of this work was obtained based on an extension of G-algebra to superhyper G-algebra based on the powerset. In this regard, the notation of n^{th}-power set of a set, superhyper operation, superhyper axiom play the main role in the construction of (m, n)-superhyper G-algebras.

2. Preliminaries

In this section, we recall some concepts that need for our work.

Definition 2.1. [2] Let $X \neq \emptyset$ and $0 \in X$ be a constant. Then a universal algebra $(X, *, 0)$ of type $(2, 0)$ is called a *G-algebra*, if for all $x, y \in X$:
(G-1) $x * x = 0$,
(G-2) $x * (x * y) = y$.

Proposition 2.2. [2] *If $(X, *, 0)$ is a G-algebra. Then, for all $x, y \in X$, the following conditions hold:*
(i) $x * 0 = x$,
(ii) $0 * (0 * x) = x$,
(iii) $(x * (x * y))y = 0$,
(iv) $x * y = 0$ *impliesx* $= y$,
(v) $0 * x = 0 * y$ *implies* $x = y$.

Theorem 2.3. [2] *Let $(X, *, 0)$ be a G-algebra. Then the following are equivalent.*
(i) $(x * y) * z = (x * z) * y$ *for all* $x, y \in X$,
(ii) $(x * y) * (x * z) = z * y$ *for all* $x, y \in X$.

Theorem 2.4. [2]*Let $(X, *, 0)$ be a G-algebra.*
(i) If $(x * y) * (0 * y) = x$ *for all* $x, y \in X$, *then* $x * z = y * z$ *implies* $x = y$.
(ii) $a * x = a * y$ *implies* $x = y$ *for all* $a, x, y \in X$.

Definition 2.5. [14] Let X be a nonempty set. Then $(X, \circ^*_{(m,n)})$ is called an (m, n)-super hyperalgebra, where $\circ^*_{(m,n)} : X^m \to P^n_*(X)$ is called an (m, n)-super hyperoperation, $P^n_*(X)$

TABLE 1. G-algebra $(X, *, 0)$

$*$	0	1	2	3	4	5
0	0	2	1	3	4	5
1	1	0	3	2	5	4
2	2	3	0	1	5	4
3	3	2	1	0	4	5
4	4	5	3	2	0	1
5	5	4	2	3	1	0

is the n^{th}-powerset of the set X, $\emptyset \notin P_*^n(X)$, for any subset A of $P_*^n(X)$, we identify $\{A\}$ with A, $m, n \geq 1$ and $X^m = \underbrace{X \times X \times \ldots \times X}_{m\ times}$.

Let $\circ^*_{(m,n)}$ be an (m,n)-super hyperoperation on X and $A_1, \ldots, A_m$ subsets of X. We define

$$\circ^*_{(m,n)}(A_1, \ldots, A_m) = \bigcup_{x_i \in A_i} \circ^*_{(m,n)}(x_1, \ldots, x_m).$$

3. Superhyper G-Algebras

At the beginning of this section, we construct a G-algebra on every nonempty set. Then we give an example of G-algebra.

Theorem 3.1. *Let X be a nonempty set and $0 \in X$ be a constant. Then there exists $*$ on X such that $(X, *, 0)$ is a G-Algebra.*

$$x * y = \begin{cases} 0 & x = y \\ y & o.w. \end{cases}$$

Proof. (G-1) is true because $x * x = 0$. According to the definition $x * y = y$, therefore $x * (x * y) = x * y = y$, and (G-2) also hold. So $(X, *, 0)$ is a G-algebra. □

Example 3.2. Let $X = \{0, 1, 2, 3, 4, 5\}$ which $*$ is defined in Table 1. Then $(X, *, 0)$ is a G-algebra.

Example 3.3. Let $X = \{0, 1, 2, 3\}$ which $*$ is defined in Table 2. Then $(X, *, 0)$ is not a G-algebra, , since $0 * (0 * 2) = 0 * 0 \neq 2$.

In this section, we introduce the concept of (m,n)-superhyper G-algebra based on the n^{th}-power set of a set. Also, investigate the properties of this concept.

Definition 3.4. Let X be a nonempty set and $0 \in X$ be a constant. Then $(X, \circ^*_{(m,n)}, 0)$ is called an (m,n)-superhyper G-algebra, if for all $x, y \in X$:

TABLE 2

$*$	0	1	2	3
0	0	0	0	3
1	1	0	3	0
2	2	2	0	1
3	3	3	3	0

TABLE 3. superhyper G-algebra $(X, \circ^*_{(2,1)}, x)$

$\circ^*_{(2,1)}$	x	y	z
x	x	$\{x, y\}$	$\{x, z\}$
y	y	x	$\{y, z\}$
z	$\{x, z\}$	$\{x, y, z\}$	x

TABLE 4. superhyper G-algebra $(X, \circ^*_{(2,2)}, a)$

$\circ^*_{(2,2)}$	$\{a\}$	$\{b\}$
$\{a\}$	$\{\{a\}, \{a, b\}\}$	$\{\{a\}, \{b\}, \{a, b\}\}$
$\{b\}$	$\{a, b\}$	a

$(G_{sh}\text{-}1)$ $0 \in \circ^*(\underbrace{x, x, \ldots, x}_{m})$,

$(G_{sh}\text{-}2)$ $y \in \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y))$.

Example 3.5. (i) Let $X = \{x, y, z\}$ and x be a constant. $P_*(X) = \{x, y, z, \{x, y, z\}, \{x, y\}, \{x, z\}, \{y, z\}\}$. Then $(X, \circ^*_{(2,1)}, x)$ is called a $(2,1)$-superhyper G-algebra as shown in Table 3.

(ii) Let $X = \{a, b\}$, a be a constant and
$P^2_*(X) = \{\{a\}, \{b\}, \{a, b\}, \{\{a\}, \{a, b\}\}, \{\{b\}, \{a, b\}\}, \{\{a\}, \{b\}, \{a, b\}\}\}$. Then $(X, \circ^*_{(2,2)}, a)$ is called a $(2,2)$-superhyper G-algebra as shown in Table 4.

(iii) Let $X = \{0, 1, 2\}$ and $P_*(X) = \{0, 1, 2, \{0, 1, 2\}, \{0, 1\}, \{0, 2\}, \{1, 2\}\}$. Then $(X, \circ^*_{(3,1)}, 0)$ is called a $(3,1)$-superhyper G-algebra as shown in Table 5.

We see that two axioms $(G_{sh}\text{-}1)$ and $(G_{sh}\text{-}2)$ are independent. Let $X = \{0, 1, 2\}$ be a set with Table 6 and Table 7. In Table 6, the axiom $(G_{sh}\text{-}1)$ is valid but $(G_{sh}\text{-}2)$ does not, because $1 \notin \circ^*(2, \circ^*(2, 1))$, and in Table 7, the axiom $(G_{sh}\text{-}2)$ is valid, but the axiom $(G_{sh}\text{-}1)$ is not, because $0 \notin \circ^*(1, 1)$.

Table 5. superhyper G-algebra $(X, \circ^*_{(3,1)}, 0)$

$\circ^*_{(3,1)}$	0	1	2
$(0,0)$	0	1	2
$(0,1)$	1	$\{0,2\}$	$\{1,2\}$
$(0,2)$	2	$\{1,2\}$	$\{0,1\}$
$(1,0)$	1	$\{0,2\}$	$\{1,2\}$
$(2,0)$	2	$\{1,2\}$	$\{0,1\}$
$(1,1)$	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$
$(1,2)$	$\{1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
$(2,1)$	$\{0,1,2\}$	$\{0,1,2\}$	$\{0,1,2\}$
$(2,2)$	$\{0,1\}$	$\{0,1,2\}$	$\{0,2\}$

Table 6

$\circ^*_{(2,1)}$	0	1	2
0	0	$\{0,1,2\}$	$\{0,2\}$
1	$\{0,1\}$	$\{0,1,2\}$	2
2	$\{0,2\}$	$\{0,2\}$	0

Table 7

$\circ^*_{(2,1)}$	0	1	2
0	0	$\{0,1\}$	$\{0,2\}$
1	$\{0,1\}$	1	2
2	$\{0,2\}$	$\{0,1\}$	$\{0,1,2\}$

The following theore, we construct an (m,n)-superhyper G-algebra on each nonempty set.

Theorem 3.6. *Let X be a nonempty set and $0 \in X$ be a constant. Then there exists $\circ^*_{(m,n)}$ on X such that $(X, \circ^*_{(m,n)}, 0)$ is an (m,n)-superhyper G-algebra.*

$$\circ^*(x_1, x_2, \ldots, x_m) = \begin{cases} \{0\} & \forall i \neq j;\ x_i = x_j \\ \{0, y\} & o.w. \end{cases}$$

Proof. (G_{sh}-1) is true because $0 \in \circ^*(\underbrace{x, x, \ldots, x}_{m})$. According to the definition $y \in \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y)$, therefore $y \in \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y))$ and (G_{sh}-2) also hold. So, the proof is complete. □

Proposition 3.7. *Let $(X, \circ^*_{(m,n)}, 0)$ be an (m,n)-superhyper G-algebra. Then for any $x \in X$, the following conditions hold:*

(i) $\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, 0) \subseteq \circ^*\big(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m})\big)$,

(ii) $x \in \circ^*\big(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x)\big)$.

Proof. (i) By $(G_{sh}$-1$)$, $0 \in \circ^*(\underbrace{x, x, \ldots, x}_{m})$. Then we get $\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, 0) \subseteq \circ^*\big(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m})\big)$.

(ii) If we put $x = 0$ and $y = x$ in $(G_{sh}$-2$)$, then we get (ii). □

Proposition 3.8. *Let $(X, \circ^*_{(m,n)}, 0)$ be an (m,n)-superhyper G-algebra. Then for any $x, y \in X$,* $0 \in \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y)), \underbrace{y, y, \ldots, y}_{m-1}\big)$.

Proof. According to $(G_{sh}$-2$)$, $y \in \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y))$. Now we have according to $(G_{sh}$-1$)$, $0 \in \circ^*(\underbrace{y, y, \ldots, y}_{m}) \subseteq \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y)), \underbrace{y, y, \ldots, y}_{m-1}\big)$ and therefore the proof is complete. □

Theorem 3.9. *Let $(X, \circ^*_{(m,n)}, 0)$ be an (m,n)-superhyper G-algebra. If for any $x, y, z \in X$,* $\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \underbrace{z, z, \ldots, z}_{m-1}\big) = \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z), \underbrace{y, y, \ldots, y}_{m-1}\big)$. *Then* $\circ^*(\underbrace{z, z, \ldots, z}_{m-1}, y) \subseteq \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z), \underbrace{x, x, \ldots, x}_{m-2}\big)$.

Proof. By $(G_{sh}$-2$)$, $z \in \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z))$. Now we have $\circ^*(\underbrace{z, z, \ldots, z}_{m-1}, y) \subseteq \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z)), \underbrace{y, y, \ldots, y}_{m-1}\big)$. According to the assumption

$\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z)), \underbrace{y, y, \ldots, y}_{m-1}\big) =$

$\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z), \underbrace{x, x, \ldots, x}_{m-2}\big)$. Thus it is obtained. □

Theorem 3.10. *Let $(X, \circ^*_{(m,n)}, 0)$ be an (m,n)-superhyper G-algebra. If for any $x, y, z \in X$,* $\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z), \underbrace{x, x, \ldots, x}_{m-2}\big) = \circ^*(\underbrace{z, z, \ldots, z}_{m-1}, y)$.

Then $\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \underbrace{z, z, \ldots, z}_{m-1}\big) = \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, z), \underbrace{y, y, \ldots, y}_{m-1}\big)$.

Proof. By (G_{sh}-2), $z \in \circ^*(\underbrace{x,x,\ldots,x}_{m-1},\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z))$. Now by the assumption, we have
$\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\underbrace{z,z,\ldots,z}_{m-1}) \subseteq$
$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)),\underbrace{x,x,\ldots,x}_{m-2}\big) =$
$\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{y,y,\ldots,y}_{m-1})$.
Conversely, by (G_{sh}-2), $y \in \circ^*(\underbrace{x,x,\ldots,x}_{m-1},\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y))$. Therefore by the assumption,
$\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{y,y,\ldots,y}_{m-1}) \subseteq$
$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y)),\underbrace{x,x,\ldots,x}_{m-2}\big)$
$= \circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\underbrace{z,z,\ldots,z}_{m-1})$. □

Theorem 3.11. *Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra. If for any $x,y,z \in X$, $\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y) = \circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$. Then $\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z) \subseteq \circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y),\underbrace{x,x,\ldots,x}_{m-2}\big)$.*

Proof. According to (G_{sh}-2), $\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z) \subseteq$
$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}),\underbrace{x,x,\ldots,x}_{m-2}\big)$. By the assumption, we have $\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}) = \circ^*(\underbrace{z,z,\ldots,z}_{m-1},y)$.
Therefore it is obtained. □

Theorem 3.12. *Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra. If for any $x,y,z \in X$, $\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y),\underbrace{x,x,\ldots,x}_{m-2}\big) = \circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)$. Then $\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y) \subseteq \circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$.*

Proof. According to (G_{sh}-2), $\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z) \subseteq$
$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}),\underbrace{x,x,\ldots,x}_{m-2}\big)$ and by the assumption, $\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y),\underbrace{x,x,\ldots,x}_{m-2}) = \circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)$. Therefore
$\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y),\underbrace{x,x,\ldots,x}_{m-2}) \subseteq$

$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}),\underbrace{x,x,\ldots,x}_{m-2}\big)$. Thus $\circ^*(\underbrace{z,z,\ldots,z}_{m-1},y)\subseteq\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$. □

Definition 3.13. Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra and $A,B\in\circ^*_{(m,n)}(x_1,\ldots,x_m)$. Then A and B are called adjacent.

Proposition 3.14. *Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra. Then for any $a,x,y\in X$, $\circ^*(\underbrace{a,a,\ldots,a}_{m-1},x)=\circ^*(\underbrace{a,a,\ldots,a}_{m-1},y)$ implies x and y are adjacent.*

Proof. Let $a,x,y\in X$ and $\circ^*(\underbrace{a,a,\ldots,a}_{m-1},x)=\circ^*(\underbrace{a,a,\ldots,a}_{m-1},y)$. It follows that $\circ^*\big(\underbrace{a,a,\ldots,a}_{m-1},\circ^*(\underbrace{a,a,\ldots,a}_{m-1},x)\big)=\circ^*\big(\underbrace{a,a,\ldots,a}_{m-1},\circ^*(\underbrace{a,a,\ldots,a}_{m-1},y)\big)$. Thus according to ($G_{sh}$-2), x and y are adjacent. □

Theorem 3.15. *Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra. Then for any $x,y\in X$, $\circ^*(\underbrace{0,0,\ldots,0}_{m-1},x)=\circ^*(\underbrace{0,0,\ldots,0}_{m-1},y)$ implies x and y are adjacent.*

Proof. According to the assumption, $\circ^*(\underbrace{0,0,\ldots,0}_{m-1},x)=\circ^*(\underbrace{0,0,\ldots,0}_{m-1},y)$, So we have $\circ^*\big(\underbrace{0,0,\ldots,0}_{m-1},\circ^*(\underbrace{0,0,\ldots,0}_{m-1},x)\big)=\circ^*\big(\underbrace{0,0,\ldots,0}_{m-1},\circ^*(\underbrace{0,0,\ldots,0}_{m-1},y)\big)$. Therefore by Theorem 3.7 (ii), $x\in\circ^*\big(\underbrace{0,0,\ldots,0}_{m-1},\circ^*(\underbrace{0,0,\ldots,0}_{m-1},x)\big)$ and $y\in\circ^*\big(\underbrace{0,0,\ldots,0}_{m-1},\circ^*(\underbrace{0,0,\ldots,0}_{m-1},y)\big)$. By definition x and y are adjacent. □

Theorem 3.16. *Let $(X,\circ^*_{(m,n)},0)$ be an (m,n)-superhyper G-algebra. Then for any $x,y\in X$, $x\in\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},y),\circ^*(\underbrace{0,0,\ldots,0}_{m-1},y),\underbrace{y,y,\ldots,y}_{m-2}\big)$ and $\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)=\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)$, implies x and y are adjacent.*

Proof. If $\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z)=\circ^*(\underbrace{y,y,\ldots,y}_{m-1},z)$, then

$\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\circ^*(\underbrace{0,0,\ldots,0}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)=$

$\circ^*\big(\circ^*(\underbrace{y,y,\ldots,y}_{m-1},z),\circ^*(\underbrace{0,0,\ldots,0}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$. By the assumption

$x\in\circ^*\big(\circ^*(\underbrace{x,x,\ldots,x}_{m-1},z),\circ^*(\underbrace{0,0,\ldots,0}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$ and

$y\in\circ^*\big(\circ^*(\underbrace{y,y,\ldots,y}_{m-1},z),\circ^*(\underbrace{0,0,\ldots,0}_{m-1},z),\underbrace{x,x,\ldots,x}_{m-2}\big)$. It follows that x and y are adjacent. □

Definition 3.17. A non-empty subset Y of an (m, n)-superhyper G-algebra X is called an (m, n)-superhyper G-subalgebra if for all $a_1, a_2, \ldots, a_m \in Y$, implies $\circ^*_{(m,n)}(a_1, a_2, \ldots, a_m) \in P^n_*(Y)$.

Definition 3.18. Let $(X, \circ^*_{(m,n)}, 0_X)$ and $(X', \circ'^*_{(m,n)}, 0_{X'})$ be (m, n)-superhyper G-algebras. A mapping $\phi : X \longrightarrow X'$ is called a homomorphism if

(i) $\phi(\circ^*(x_1, x_2, \ldots, x_m)) = \circ'^*(\phi(x_1), \phi(x_2), \ldots, \phi(x_m))$, for $x_1, x_2, \ldots, x_m \in X$.

(ii) $0_{X'} \in \phi(0_X)$.

The homomorphism ϕ is said to be a monomorphism (resp., an epimorphism) if it is injective (resp., surjective). If the map ϕ is both injective and surjective then X and X' are said to be isomorphic, written $X \cong X'$. For any homomorphism $\phi : X \longrightarrow X'$, the set $\{x \in X | 0_{X'} \in \phi(x)\}$ is called the kernel of ϕ and is denoted by $Ker\phi$.

Lemma 3.19. *Let $\phi : (X, \circ^*_{(m,n)}, 0_X) \longrightarrow (X', \circ'^*_{(m,n)}, 0_{X'})$ be a homomorphism of (m, n)-superhyper G-algebras, then we have the following:*

(i) *$Ker\phi$ is an (m, n)-superhyper G-algebra of X,*

(ii) *$Im\phi = \{y \in X' | y = \phi(x), \text{for some } x \in X\}$ is an (m, n)-superhyper G-subalgebra of X.*

Proof. (i) Since $0_X \in Ker\phi$, then $Ker\phi \neq \emptyset$. Suppose $x_1, x_2, \ldots, x_m \in Ker\phi$. So $0_{X'} \in \phi(x_i)$ for $i = 1, \ldots, m$. From $\phi(\circ^*(x_1, x_2, \ldots, x_m)) = \circ'^*(\phi(x_1), \phi(x_2), \ldots, \phi(x_m))$. Because $0_{X'} \in \circ'^*(\phi(x_1), \phi(x_2), \ldots, \phi(x_m))$, Implies that $0_{X'} \in \phi(\circ^*(x_1, x_2, \ldots, x_m))$. It follows that, $\circ^*(x_1, x_2, \ldots, x_m) \in Ker\phi$.
(ii) Direct to prove. □

Definition 3.20. An (m, n)-superhyper G-algebra $(X, \circ^*_{(m,n)}, 0)$ is said to be 0-commutative if for any $x, y \in X$, $\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, y)) = \circ^*(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x))$.

Theorem 3.21. *Let $(X, \circ^*_{(m,n)}, 0)$ be an 0-commutative (m, n)-superhyper G-algebra. Then for any $x, y \in X$, $\circ^*(\underbrace{y, y, \ldots, y}_{m-1}, x) \subseteq \circ^*\big(\circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x), \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, y), \underbrace{x, x, \ldots, x}_{m-2}\big)$.*

Proof. Because
X be a 0-commutative, implies that $\circ^*\big(\circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x), \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, y), \underbrace{x, x, \ldots, x}_{m-2}\big) = \circ^*\big(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x))\big)$. By Theorem 3.7 (ii), $\circ^*(\underbrace{y, y, \ldots, y}_{m-1}, x) \subseteq \circ^*\big(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, x))\big)$ and the result is obtained. □

Theorem 3.22. *Let* $(X, \circ^*_{(m,n)}, 0)$ *be a* 0*-commutative* (m, n)*-superhyper* G*-algebra satisfying* $\circ^*\big(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y)\big) = \circ^*(\underbrace{y, y, \ldots, y}_{m-1}, x)$*. Then for any* $x, y \in X$*,*
$x \in \circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, y), \underbrace{x, x, \ldots, x}_{m-2}\big)$.

Proof. Because X be a 0-commutative, implies that
$\circ^*\big(\circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y), \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, y), \underbrace{x, x, \ldots, x}_{m-2}\big) =$
$\circ^*\big(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y))\big)$. By the assumption and (G_{sh}-2),
$x \in \circ^*(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{y, y, \ldots, y}_{m-1}, x)) = \circ^*\big(\underbrace{y, y, \ldots, y}_{m-1}, \circ^*(\underbrace{0, 0, \ldots, 0}_{m-1}, \circ^*(\underbrace{x, x, \ldots, x}_{m-1}, y))\big)$. Thus it is obtained. □

4. **Conclusions**

In this paper, we have introduced the novel concept of (m, n)-superhyper G-algebras based on a powerset and studied their properties. We have presented some basic results and examples of this superhyperalgebra. The basis of our work is the extension of G-algebras to superhyper G-algebras using a powerset. We wish that these results are helpful for further studies in the theory of superhyperalgebra. For future work, we hope to investigate the idea of neutrosophic superhyper G-algebras, fuzzy superhyper G-algebras, and soft superhyper G-algebras and obtain some results in this regard and their applications.

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Received: January 07, 2023 / Accepted: April 10, 2023