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Some Aggregation Operators of Credibility Interval Trapezoidal Fuzzy Neutrosophic Numbers and Their Decision-Making Application of Landslide Control Design Schemes

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Abstract: As a generalization of trapezoidal fuzzy neutrosophic numbers (TFNNs), credibility trapezoidal fuzzy neutrosophic numbers (C-TFNNs) can independently describe true, false, and indeterminate membership degrees and their credibility levels in uncertain and inconsistent scenarios. Since the true, false, and indeterminate membership degrees are closely related to their credibility levels, C-TFNN can ensure the credibility of TFNN, which shows its clear merit. However, C-TFNNs cannot expresses the interval membership degrees of the truth, falsity and indeterminacy and the uncertain credibility levels, which are produced due to human cognitive vagueness, incompleteness, and uncertainty. Furthermore, existing decision models of C-TFNNs cannot perform such a DM issue with both ITFNNs and uncertain credibility levels, which reveals a gap. To compensates for this gap. this paper extends C-TFNNs to credibility interval TFNNs (C-ITFNNs), which strengthens the expression capability of uncertain information. Then, the operational laws and score function of C-ITFNNs are defined to solve the aggregation and sorting issues of C-ITFNNs in decision-making (DM) problems. Subsequently, the C-ITFNN weighted geometric averaging (C-ITFNNWGA) and C-ITFNN weighted arithmetic averaging (C-ITFNNWAA) operators are proposed in view of operational laws of C-ITFNNs. Furthermore, a multi-attribute DM model is established in terms of the two aggregation operators and the score function in the C-ITFNN circumstance. Finally, a DM case of landslide control design schemes is used to reveal the applicability of the proposed DM model in the C-ITFNN scenario. By comparative analysis, the main superiority of our new DM model is that it not only compensates for the gap of existing DM models, but also is more reliable and versatile than existing DM models.

Keywords: Credibility interval trapezoidal fuzzy neutrosophic number, credibility interval trapezoidal fuzzy neutrosophic number weighted arithmetic averaging operator, credibility interval trapezoidal fuzzy neutrosophic number weighted geometric averaging operator, decision making, landslide control design scheme

1. Introduction

In real life, there are many uncertainties and ambiguities, which it is difficult to measure by crisp concepts. Then, fuzzy sets [1] can represent them by membership degrees belonging to [0, 1]. Due to the uncertainty of the membership degrees, they are difficultly described by exact fuzzy values, so the concept of interval-valued fuzzy sets (IVFSs) was proposed to solve this issue [2]. However, since there is true and false information in real life, Atanassov [3] proposed the concept of intuitionistic fuzzy sets (IFSs). Subsequently, IFSs were generalized to interval-valued IFSs (IVIFSs) [4] to facilitate the representation of incomplete and uncertain information. Although IFSs and IVIFSs can better express true and false membership degrees belonging to [0, 1], they cannot represent true, false, and indeterminate membership degrees independently and indeterminate and inconsistent information. To solve these issues, the neutrosophic sets (NSs) presented by Smarandache [5] are the extension of various fuzzy sets. Since NS can easily describe indeterminate and inconsistent information in terms of true, false, and indeterminate membership degrees, it reveals obvious merits. Due to the diversity of neutrosophic expressions (including fuzzy sets, IVFSs, IFSs, IVIFSs), neutrosophic theory has also become a research hotspot of scholars in recent years and has been widely used in many fields, such as risk assessment [6, 7], image processing (segmentation, denoising, and thresholding) [8-10], decision-making (DM) [11-14], and so on. As subsets of NSs, interval NSs (INSs) and single-valued NSs (SVNSs) were proposed by Wang et al. [15, 16]. Then SVNSs and INSs have been widely used in DM problems [17-21], risk assessment [22, 23], and medical diagnosis [24-26] in neutrosophic environments.

In order to extend discrete fuzzy information to continuous fuzzy information, Wang and Zhong [27] proposed the concept of intuitionistic trapezoidal fuzzy numbers (ITFNs) based on true and false trapezoidal fuzzy numbers (TFNs) and defined their operational laws and the weighted geometric and arithmetic averaging operators for DM. Wan and Dong [28] proposed a multi-attribute group DM approach of ITFNs. Li [29] proposed interval-valued intuitionistic trapezoidal fuzzy numbers (IVITFNs) as a further extension of ITFNs. Subsequently, Ye [30] proposed the concept of trapezoidal fuzzy neutrosophic numbers (TFNNs) in view of true, false and indeterminate TFNs and defined some weighted aggregation operators of TNNs for DM. Then, the concept of interval trapezoidal fuzzy neutrosophic numbers (ITFNNs) [31] were proposed to solve the problems of multi-attribute DM [32-34] in the setting of ITFNNs. As a special case of TFNNs, Deli and Şubaş [35] introduced the weighted geometric operators of triangular fuzzy neutrosophic numbers and applied them to DM problems.

However, decision makers are not completely familiar with various attributes in a DM problem when they evaluate them. The accuracy of the evaluation given by decision makers to unfamiliar attributes is not as high as that of familiar attributes, so it will affect the accuracy of the DM results. For this case, we need to consider the decision maker's credibility level to ensure the credibility of the assessment value to each attribute in a DM problem. Therefore, Ye et al. [36] proposed the concept of fuzzy credibility numbers (FCNs) to enrich the evaluation information of multi-attribute DM problems and to ensure their DM credibility. Then, Ye et al. [37] further proposed the concept of credibility TFNNs (C-TFNNs) and established a multi-attribute DM model using the C-TFNN weighted geometric averaging (C-TFNNWGA) and C-TFNN weighted arithmetic averaging (C-TFNNWAA) operators to solve DM problems with C-TFNNs.

In the C-TFNN situation, it is difficult for decision makers to give exact C-TFNNs, but they easily provide ITFNNs and uncertain credibility levels in indeterminate DM problems to meet the uncertain judgments and expressions of decision makers. However, the existing various DM techniques cannot handle such a DM issue with both ITFNNs and uncertain credibility levels. Therefore, we need to make up for this gap. To do so, this paper aims to: (a) propose the concept of credibility ITFNNs (C-ITFNNs) and the C-ITFNN score function and sorting rules, (b) present two basic aggregation operators of C-ITFNNs, (c) establish a multi-attribute DM model in the scenario of C-ITFNNs, and

(d) apply the established DM model to an actual DM case of landslide control design schemes (LCDSs) in the C-ITFNN scenario.

In this original study, the contributions and advantages of this paper are revealed as follows:

(1) The proposed C-ITFNNs can overcome the defect of single-valued/exact C-TFNNs in the expression of indeterminate information.

(2) The proposed C-ITFNN makes the expression of uncertain information more reasonable and reliable.

(3) The proposed C-ITFNNWAA and C-ITFNNWGA operators and score function provide effective DM tools for handling multi-attribute DM problems with C-ITFNNs.

(4) The DM model established in the C-ITFNN setting has stronger DM credibility and more general DM capabilities.

The rest of the paper is given as follows. Section 2 introduces the related concepts of INS and C-TFNNs, the weighted geometric and arithmetic averaging operators of C-TFNNs, and the scoring function of C-TFNN as preliminaries in this study. Section 3 presents some new concepts of C-ITFNNs, including operational laws, score function, and sorting rules of C-ITFNNs. Section 4 introduces the weighted geometric averaging (C-ITFNNWGA) and weighted arithmetic averaging (C-ITFNNWAA) operators for C-ITFNNs and their characteristics. In Section 5, a multi-attribute DM model is established in light of the C-ITFNNWAA and C-ITFNNWGA operators and score function. Section 6 demonstrates the applicability of the proposed DM model through an actual DM case of LCDSs in the C-ITFNN scenarios. Section 7 summarizes the conclusions of this article and future research.

2. Preliminaries

Definition 1 [15]. Let X be a non-empty set. An INS $\tilde{P}$ in X is given by

$$\tilde{P} = \left\{ x, \left\langle T_{\tilde{P}}(x), I_{\tilde{P}}(x), F_{\tilde{P}}(x), \right\rangle x \in X \right\},$$

where $T_{\tilde{P}}(x) \subseteq [0,1]$, $I_{\tilde{P}}(x) \subseteq [0,1]$, and $F_{\tilde{P}}(x) \subseteq [0,1]$ are the true, indeterminate, and false membership functions and then their membership degrees are subject to $0 \le \sup(T_{\tilde{P}}(x)) + \sup(I_{\tilde{P}}(x)) + \sup(F_{\tilde{P}}(x)) \le 3$.

Definition 2 [37]. Let X be a non-empty set. A C-TFNN s is denoted by $s = \left(\left\langle (g_1, g_2, g_3, g_4); T_N(x), I_N(x), F_N(x) \right\rangle, \left\langle (h_1, h_2, h_3, h_4); T_L(x), I_L(x), F_L(x) \right\rangle \right)$. Then, it's true, indeterminate, and false membership functions are denoted as follows:

$$T_N(x) = \begin{cases} \dfrac{x - g_1}{g_2 - g_1} T_N, g_1 \le x < g_2, \\ T_N, g_2 \le x \le g_3, \\ \dfrac{g_4 - x}{g_4 - g_3} T_N, g_3 < x \le g_4, \\ 0, \text{otherwise} \end{cases}$$

$$I_N(x)=\begin{cases}\dfrac{g_2-x+I_N(x-g_1)}{g_2-g_1}, & g_1\le x<g_2,\\ I_N, g_2\le x\le g_3, \\ \dfrac{x-g_3+I_N(g_4-x)}{g_4-g_3}, g_3<x\le g_4, \\ 1, \text{otherwise}\end{cases}$$

$$F_N(x)=\begin{cases}\dfrac{g_2-x+F_N(x-g_1)}{g_2-g_1}, & g_1\le x<g_2,\\ F_N, g_2\le x\le g_3, \\ \dfrac{x-g_3+F_N(g_4-x)}{g_4-g_3}, g_3<x\le g_4, \\ 1, \text{otherwise}\end{cases}$$

and it's true, indeterminate, and false credibility measure functions are denoted as follows:

$$T_L(x)=\begin{cases}\dfrac{x-h_1}{h_2-h_1}T_L, h_1\le x<h_2,\\ T_L, h_2\le x\le h_3, \\ \dfrac{h_4-x}{h_4-h_3}T_L, h_3<x\le h_4, \\ 0, \text{otherwise}\end{cases}$$

$$I_L(x)=\begin{cases}\dfrac{h_2-x+I_L(x-h_1)}{h_2-h_1}, h_1\le x<h_2,\\ I_L, h_2\le x\le h_3, \\ \dfrac{x-h_3+I_L(h_4-x)}{h_4-h_3}, h_3<x\le h_4, \\ 1, \text{otherwise}\end{cases}$$

$$F_L(x)=\begin{cases}\dfrac{h_2-x+F_L(x-h_1)}{h_2-h_1}, h_1\le x<h_2,\\ F_L, h_2\le x\le h_3, \\ \dfrac{x-h_3+F_L(h_4-x)}{h_4-h_3}, h_3<x\le h_4, \\ 1, \text{otherwise}\end{cases}$$

where $T_N, I_N, F_N \in [0,1]$, $T_L, I_L, F_L \in [0,1]$, $0\le T_N+I_N+F_N\le 3$, $0\le T_L+I_L+F_L\le 3$, and g_k, h_k $\in \Re$ (k = 1, 2, 3, 4). Then, a C-TFNN s is simply denoted as $s=\left(\left\langle (g_1,g_2,g_3,g_4);T_N,I_N,F_N\right\rangle,\left\langle (h_1,h_2,h_3,h_4);T_L,I_L,F_L\right\rangle\right)$.

Definition 3 [37]. Let $s_1 = \begin{pmatrix} \langle (g_{11}, g_{12}, g_{13}, g_{14}); T_{N1}, I_{N1}, F_{N1} \rangle, \\ \langle (h_{11}, h_{12}, h_{13}, h_{14}); T_{L1}, I_{L1}, F_{L1} \rangle \end{pmatrix}$ and $s_2 = \begin{pmatrix} \langle (g_{21}, g_{22}, g_{23}, g_{24}); T_{N2}, I_{N2}, F_{N2} \rangle, \\ \langle (h_{21}, h_{22}, h_{23}, h_{24}); T_{L2}, I_{L2}, F_{L2} \rangle \end{pmatrix}$ be two C-TFNNs and $\lambda > 0$. The operational laws are defined as follows:

(1) $$s_1 \oplus s_2 = \begin{pmatrix} \left\langle \begin{matrix} (g_{11}+g_{21}, g_{12}+g_{22}, g_{13}+g_{23}, g_{14}+g_{24}); \\ T_{N1}+T_{N2}-T_{N1}T_{N2}, I_{N1}I_{N2}, F_{N1}F_{N2} \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} (h_{11}+h_{21}, h_{12}+h_{22}, h_{13}+h_{23}, h_{14}+h_{24}); \\ T_{L1}+T_{L2}-T_{L1}T_{L2}, I_{L1}I_{L2}, F_{L1}F_{L2} \end{matrix} \right\rangle \end{pmatrix},$$

(2) $$s_1 \otimes s_2 = \begin{pmatrix} \left\langle \begin{matrix} (g_{11}g_{21}, g_{12}g_{22}, g_{13}g_{23}, g_{14}a_{24}); \\ T_{N1}T_{N2}, I_{N1}+I_{N2}-I_{N1}I_{N2}, \\ F_{N1}+F_{N2}-F_{N1}F_{N2} \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} (h_{11}h_{21}, h_{12}h_{22}, h_{13}h_{23}, h_{14}h_{24}); \\ T_{L1}T_{L2}, I_{L1}+I_{L2}-I_{L1}I_{L2}, \\ F_{L1}+F_{L2}-F_{L1}F_{L2} \end{matrix} \right\rangle \end{pmatrix},$$

(3) $$\lambda s_1 = \begin{pmatrix} \left\langle \begin{matrix} (\lambda g_{11}, \lambda g_{12}, \lambda g_{13}, \lambda g_{14}); \\ 1-(1-T_{N1})^{\lambda}, I_{N1}^{\lambda}, F_{N1}^{\lambda} \end{matrix} \right\rangle, \\ \left\langle \begin{matrix} (\lambda h_{11}, \lambda h_{12}, \lambda h_{13}, \lambda h_{14}); \\ 1-(1-T_{L1})^{\lambda}, I_{L1}^{\lambda}, F_{L1}^{\lambda} \end{matrix} \right\rangle \end{pmatrix},$$

(4) $$(s_1)^{\lambda} = \begin{pmatrix} \left\langle (g_{11}^{\lambda}, g_{12}^{\lambda}, g_{13}^{\lambda}, g_{14}^{\lambda}); T_{N1}^{\lambda}, 1-(1-I_{N1})^{\lambda}, 1-(1-F_{N1})^{\lambda} \right\rangle, \\ \left\langle (h_{11}^{\lambda}, h_{12}^{\lambda}, h_{13}^{\lambda}, h_{14}^{\lambda}); T_{L1}^{\lambda}, 1-(1-I_{L1})^{\lambda}, 1-(1-F_{L1})^{\lambda} \right\rangle \end{pmatrix}.$$

Regarding a series of C-TFNNs $s_i = \begin{pmatrix} \langle (g_{i1}, g_{i2}, g_{i3}, g_{i4}); T_{Ni}, I_{Ni}, F_{Ni} \rangle, \\ \langle (h_{i1}, h_{i2}, h_{i3}, h_{i4}); T_{Li}, I_{Li}, F_{Li} \rangle \end{pmatrix}$ (*i = 1, 2, …, J*) subject to the weight λ_i of *si* with $0 \le \lambda_i \le 1$ and $\sum_{i=1}^{J} \lambda_i = 1$, the C-TFNNWAA and C-TFNNWGA operators [37] are introduced as follows:

$$\text{C-TFNNWAA}\left(s_1,s_2,...,s_J\right)=\sum_{i=1}^{J}\lambda_i s_i=\begin{pmatrix}\left\langle \left(\sum_{i=1}^{J}\lambda_i g_{i1},\sum_{i=1}^{J}\lambda_i g_{i2},\sum_{i=1}^{J}\lambda_i g_{i3},\sum_{i=1}^{J}\lambda_i g_{i4}\right); \\ 1-\prod_{i=1}^{J}\left(1-T_{Ni}\right)^{\lambda_i},\prod_{i=1}^{J}I_{Ni}^{\lambda_i},\prod_{i=1}^{J}F_{Ni}^{\lambda_i} \right\rangle, \\ \left\langle \left(\sum_{i=1}^{J}\lambda_i h_{i1},\sum_{i=1}^{J}\lambda_i h_{i2},\sum_{i=1}^{J}\lambda_i h_{i3},\sum_{i=1}^{J}\lambda_i h_{i4}\right); \\ 1-\prod_{i=1}^{J}\left(1-T_{Li}\right)^{\lambda_i},\prod_{i=1}^{J}I_{Li}^{\lambda_i},\prod_{i=1}^{J}F_{Li}^{\lambda_i} \right\rangle \end{pmatrix}, \quad (1)$$

$$\text{C-TFNNWGA}\left(s_1,s_2,...,s_J\right)=\prod_{i=1}^{J}s_i^{\lambda_i}=\begin{pmatrix}\left\langle \left(\prod_{i=1}^{J}g_{i1}^{\lambda_i},\prod_{i=1}^{J}g_{i2}^{\lambda_i},\prod_{i=1}^{J}g_{i3}^{\lambda_i},\prod_{i=1}^{J}g_{i4}^{\lambda_i}\right); \\ \prod_{i=1}^{J}T_{Ni}^{\lambda_i},1-\prod_{i=1}^{J}\left(1-I_{Ni}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-F_{Ni}\right)^{\lambda_i} \right\rangle, \\ \left\langle \left(\prod_{i=1}^{J}h_{i1}^{\lambda_i},\prod_{i=1}^{J}h_{i2}^{\lambda_i},\prod_{i=1}^{J}h_{i3}^{\lambda_i},\prod_{i=1}^{J}h_{i4}^{\lambda_i}\right); \\ \prod_{i=1}^{J}T_{Li}^{\lambda_i},1-\prod_{i=1}^{J}\left(1-I_{Li}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-F_{Li}\right)^{\lambda_i} \right\rangle \end{pmatrix}. \quad (2)$$

Definition 4 [37]. To compare C-TFNNs, the score function of C-TFNN is defined as

$$S(s)=\frac{1}{144}\begin{pmatrix}\left(g_1+g_2+g_3+g_4\right)\times\left(2+T_N-I_N-F_N\right)\times \\ \left(h_1+h_2+h_3+h_4\right)\times\left(2+T_L-I_L-F_L\right)\end{pmatrix},\ 0\le S\left(s\right)\le 1. \quad (3)$$

The following sorting rules are given by the score function:

(1) If $S(s_1)<S(s_2)$, then $s_1<s_2$;

(2) If $S(s_1)=S(s_2)$, then $s_1\cong s_2$.

Particularly, when we ignore credibility degrees in C-TFNNs, C-TFNNs becomes TFNNs. Thus, Eqs. (1)-(3) become the TFNN weighted arithmetic averaging (TFNNWAA) and TFNN weighted geometric averaging (TFNNWGA) operators and the score function [30]:

$$\begin{aligned}&\text{TFNNWAA}\left(s_1,s_2,...,s_J\right)=\sum_{i=1}^{J}\lambda_i s_i \\ &=\left(\left\langle\sum_{i=1}^{J}\lambda_i g_{i1},\sum_{i=1}^{J}\lambda_i g_{i2},\sum_{i=1}^{J}\lambda_i g_{i3},\sum_{i=1}^{J}\lambda_i g_{i4}\right\rangle;1-\prod_{i=1}^{J}\left(1-T_{Ni}\right)^{\lambda_i},\prod_{i=1}^{J}I_{Ni}^{\lambda_i},\prod_{i=1}^{J}F_{Ni}^{\lambda_i}\right)',\end{aligned} \quad (4)$$

$$\begin{aligned}&\text{TFNNWGA}\left(s_1,s_2,...,s_J\right)=\prod_{i=1}^{J}s_i^{\lambda_i} \\ &=\left(\left\langle\prod_{i=1}^{J}g_{i1}^{\lambda_i},\prod_{i=1}^{J}g_{i2}^{\lambda_i},\prod_{i=1}^{J}g_{i3}^{\lambda_i},\prod_{i=1}^{J}g_{i4}^{\lambda_i}\right\rangle;\prod_{i=1}^{J}T_{Ni}^{\lambda_i},1-\prod_{i=1}^{J}\left(1-I_{Ni}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-F_{Ni}\right)^{\lambda_i}\right)',\end{aligned} \quad (5)$$

$$S''(s)=\frac{1}{12}\left(\left(g_1+g_2+g_3+g_4\right)\times\left(2+T_N-I_N-F_N\right)\right), S''(s)\in[0,1]. \tag{6}$$

If there are no credibility degrees, $g_1 = g_2 = g_3 = g_4 = 1$ and $h_1 = h_2 = h_3 = h_4 = 1$ (without considering TFNs) in C-TFNNs, C-TFNNs become single-valued neutrosophic numbers (SVNNs). Thus, Eqs. (1)-(3) become the SVNN weighted arithmetic averaging (SVNNWAA) and SVNN weighted geometric averaging (SVNNWGA) operators and the score function [19]:

$$\mathrm{SVNNWAA}\left(s_1,s_2,...,s_J\right)=\sum_{i=1}^{J}\lambda_i s_i=\left(1-\prod_{i=1}^{J}\left(1-T_{Ni}\right)^{\lambda_i},\prod_{i=1}^{J}I_{Ni}^{\lambda_i},\prod_{i=1}^{J}F_{Ni}^{\lambda_i}\right), \tag{7}$$

$$\mathrm{SVNNWGA}\left(s_1,s_2,...,s_J\right)=\prod_{i=1}^{J}s_i^{\lambda_i}=\left(\prod_{i=1}^{J}T_{Ni}^{\lambda_i},1-\prod_{i=1}^{J}\left(1-I_{Ni}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-F_{Ni}\right)^{\lambda_i}\right), \tag{8}$$

$$S'(s)=\frac{1}{3}\left(2+T_N-I_N-F_N\right), S'(s)\in[0,1]. \tag{9}$$

3. C-ITFNNs

As an extension of C-TFNNs, this section proposes C-ITFNNs, some operational laws of C-ITFNNs, and a score function for comparing C-TFNNs.

Definition 5. Set X as a non-empty set. A C-ITFNN $\tilde{a}$ can be defined as $\tilde{a}=\left(\left\langle (g_1,g_2,g_3,g_4);\tilde{T}_N(x),\tilde{I}_N(x),\tilde{F}_N(x)\right\rangle,\left\langle (h_1,h_2,h_3,h_4);\tilde{T}_L(x),\tilde{I}_L(x),\tilde{F}_L(x)\right\rangle\right)$ for $x \in X$. Then, it's true, indeterminate, and false membership functions and their corresponding credibility measure functions are indicated, respectively, as follows:

$$\tilde{T}_N(x)=\begin{cases}\left[\dfrac{x-g_1}{g_2-g_1}\tilde{T}_N^-,\dfrac{x-g_1}{g_2-g_1}\tilde{T}_N^+\right], g_1\le x<g_2,\\ \left[\tilde{T}_N^-,\tilde{T}_N^+\right], g_2\le x\le g_3,\\ \left[\dfrac{g_4-x}{g_4-g_3}\tilde{T}_N^-,\dfrac{g_4-x}{g_4-g_3}\tilde{T}_N^+\right], g_3<x\le g_4,\\ [0,0], \text{otherwise}\end{cases},$$

$$\tilde{I}_N(x)=\begin{cases}\left[\dfrac{g_2-x+\tilde{I}_N^-(x-g_1)}{g_2-g_1},\dfrac{g_2-x+\tilde{I}_N^+(x-g_1)}{g_2-g_1}\right], g_1\le x<g_2,\\ \left[\tilde{I}_N^-,\tilde{I}_N^+\right], g_2\le x\le g_3,\\ \left[\dfrac{x-g_3+\tilde{I}_N^-(g_4-x)}{g_4-g_3},\dfrac{x-g_3+\tilde{I}_N^+(g_4-x)}{g_4-g_3}\right], g_3<x\le g_4,\\ [1,1], \text{otherwise}\end{cases},$$

$$\tilde{F}_N(x)=\begin{cases}\left[\dfrac{g_2-x+\tilde{F}_N^-(x-g_1)}{g_2-g_1},\dfrac{g_2-x+\tilde{F}_N^+(x-g_1)}{g_2-g_1}\right], g_1\le x<g_2,\\ \left[\tilde{F}_N^-,\tilde{F}_N^+\right], g_2\le x\le g_3,\\ \left[\dfrac{x-g_3+\tilde{F}_N^-(g_4-x)}{g_4-g_3},\dfrac{x-g_3+\tilde{F}_N^+(g_4-x)}{g_4-g_3}\right], g_3<x\le g_4,\\ [1,1], \text{otherwise}\end{cases},$$

and it's true, indeterminate and false credibility measure functions are indicated as follows:

$$\tilde{T}_L(x)=\begin{cases}\left[\dfrac{x-h_1}{h_2-h_1}\tilde{T}_L^-,\dfrac{x-h_1}{h_2-h_1}\tilde{T}_L^+\right], h_1\le x<h_2,\\ \left[\tilde{T}_L^-,\tilde{T}_L^+\right], h_2\le x\le h_3,\\ \left[\dfrac{h_4-x}{h_4-h_3}\tilde{T}_L^-,\dfrac{h_4-x}{h_4-h_3}\tilde{T}_L^+\right], h_3<x\le h_4,\\ [0,0], \text{otherwise}\end{cases},$$

$$\tilde{I}_L(x)=\begin{cases}\left[\dfrac{h_2-x+\tilde{I}_L^-(x-h_1)}{h_2-h_1},\dfrac{h_2-x+\tilde{I}_L^+(x-h_1)}{h_2-h_1}\right], h_1\le x<h_2,\\ \left[\tilde{I}_L^-,\tilde{I}_L^+\right], h_2\le x\le h_3,\\ \left[\dfrac{x-h_3+\tilde{I}_L^-(h_4-x)}{h_4-h_3},\dfrac{x-h_3+\tilde{I}_L^+(h_4-x)}{h_4-h_3}\right], h_3<x\le h_4,\\ [1,1], \text{otherwise}\end{cases},$$

$$\tilde{F}_L(x)=\begin{cases}\left[\dfrac{h_1-x+\tilde{F}_L^-(x-h_1)}{h_2-h_1},\dfrac{h_1-x+\tilde{F}_L^+(x-h_1)}{h_2-h_1}\right], \quad h_1\le x<h_2,\\ \left[\tilde{F}_L^-,\tilde{F}_L^+\right], h_2\le x\le h_3,\\ \left[\dfrac{x-h_3+\tilde{F}_L^-(h_4-x)}{h_4-h_3},\dfrac{x-h_3+\tilde{F}_L^+(h_4-x)}{h_4-h_3}\right], h_3<x\le h_4,\\ [1,1], \text{otherwise}\end{cases},$$

where $\left[\tilde{T}_N^-,\tilde{T}_N^+\right]\subseteq[0,1],\left[\tilde{I}_N^-,\tilde{I}_N^+\right]\subseteq[0,1],\left[\tilde{F}_N^-,\tilde{F}_N^+\right]\subseteq[0,1]$ and $\left[\tilde{T}_L^-,\tilde{T}_L^+\right]\subseteq[0,1],\left[\tilde{I}_L^-,\tilde{I}_L^+\right]\subseteq[0,1],\left[\tilde{F}_L^-,\tilde{F}_L^+\right]\subseteq[0,1]$ are the true, indeterminate, false interval membership degrees and credibility levels in the C-ITFNN $\tilde{a}$ subject to $0\le\tilde{T}_N^++\tilde{I}_N^++\tilde{F}_N^+\le 3$ and $0\le\tilde{T}_L^++\tilde{I}_L^++\tilde{F}_L^+\le 3$, then g_k, $h_k \in \Re$ (k = 1, 2, 3, 4) are the parameters of ITFNNs in the C-ITFNN $\tilde{a}$.

For the convenient representation of the C-ITFNN $\tilde{a}$, it is simply expressed as $\tilde{a}=\left(\left\langle(g_1,g_2,g_3,g_4);\tilde{T}_N,\tilde{I}_N,\tilde{F}_N\right\rangle,\left\langle(h_1,h_2,h_3,h_4);\tilde{T}_L,\tilde{I}_L,\tilde{F}_L\right\rangle\right)$.

Then, the two special cases of C-ITFNN are indicated below:

(1) If the upper and lower endpoints of the interval values $\left[\tilde{T}_N^-,\tilde{T}_N^+\right]$, $\left[\tilde{I}_N^-,\tilde{I}_N^+\right]$, $\left[\tilde{F}_N^-,\tilde{F}_N^+\right]$, $\left[\tilde{T}_L^-,\tilde{T}_L^+\right]$, $\left[\tilde{I}_L^-,\tilde{I}_L^+\right]$, and $\left[\tilde{F}_L^-,\tilde{F}_L^+\right]$ in the C-ITFNN $\tilde{a}$ are equal, C-ITFNN becomes C-TFNN.

(2) If $g_1 = g_2 = g_3 = g_4 = 1$ and $h_1 = h_2 = h_3 = h_4 = 1$ in the C-ITFNN $\tilde{a}$, C-ITFNN becomes the credibility interval neutrosophic number.

Definition 6. Let $\tilde{a}_1 = \begin{pmatrix} \left\langle (g_{11}, g_{12}, g_{13}, g_{14}); \left[\tilde{T}_{N1}^-, \tilde{T}_{N1}^+\right], \left[\tilde{I}_{N1}^-, \tilde{I}_{N1}^+\right], \left[\tilde{F}_{N1}^-, \tilde{F}_{N1}^+\right] \right\rangle \\ \left\langle (h_{11}, h_{12}, h_{13}, h_{14}); \left[\tilde{T}_{L1}^-, \tilde{T}_{L1}^+\right], \left[\tilde{I}_{L1}^-, \tilde{I}_{L1}^+\right], \left[\tilde{F}_{L1}^-, \tilde{F}_{L1}^+\right] \right\rangle \end{pmatrix}$ and $\tilde{a}_2 = \begin{pmatrix} \left\langle (g_{21}, g_{22}, g_{23}, g_{24}); \left[\tilde{T}_{N2}^-, \tilde{T}_{N2}^+\right], \left[\tilde{I}_{N2}^-, \tilde{I}_{N2}^+\right], \left[\tilde{F}_{N2}^-, \tilde{F}_{N2}^+\right] \right\rangle \\ \left\langle (h_{21}, h_{22}, h_{23}, h_{24}); \left[\tilde{T}_{L2}^-, \tilde{T}_{L2}^+\right], \left[\tilde{I}_{L2}^-, \tilde{I}_{L2}^+\right], \left[\tilde{F}_{L2}^-, \tilde{F}_{L2}^+\right] \right\rangle \end{pmatrix}$ be two C-ITFNNs and $\lambda > 0$. Then, their operational laws are satisfied below:

$$(1)\ \tilde{a}_1 \oplus \tilde{a}_2 = \begin{pmatrix} \left\langle \begin{array}{l} (g_{11}+g_{21}, g_{12}+g_{22}, g_{13}+g_{23}, g_{14}+g_{24}); \\ \left[\tilde{T}_{N1}^- + \tilde{T}_{N2}^- - \tilde{T}_{N1}^-\tilde{T}_{N2}^-, \tilde{T}_{N1}^+ + \tilde{T}_{N2}^+ - \tilde{T}_{N1}^+\tilde{T}_{N2}^+\right], \\ \left[\tilde{I}_{N1}^-\tilde{I}_{N2}^-, \tilde{I}_{N1}^+\tilde{I}_{N2}^+\right], \left[\tilde{F}_{N1}^-\tilde{F}_{N2}^-, \tilde{F}_{N1}^+\tilde{F}_{N2}^+\right] \end{array} \right\rangle, \\ \left\langle \begin{array}{l} (h_{11}+h_{21}, h_{12}+h_{22}, h_{13}+h_{23}, h_{14}+h_{24}); \\ \left[\tilde{T}_{L1}^- + \tilde{T}_{L2}^- - \tilde{T}_{L1}^-\tilde{T}_{L2}^-, \tilde{T}_{L1}^+ + \tilde{T}_{L2}^+ - \tilde{T}_{L1}^+\tilde{T}_{L2}^+\right], \\ \left[\tilde{I}_{L1}^-\tilde{I}_{L2}^-, \tilde{I}_{L1}^+\tilde{I}_{L2}^+\right], \left[\tilde{F}_{L1}^-\tilde{F}_{L2}^-, \tilde{F}_{L1}^+\tilde{F}_{L2}^+\right] \end{array} \right\rangle \end{pmatrix};$$

$$(2)\ \tilde{a}_1 \otimes \tilde{a}_2 = \begin{pmatrix} \left\langle \begin{array}{l} (g_{11}g_{21}, g_{12}g_{22}, g_{13}g_{23}, g_{14}g_{24}); \left[\tilde{T}_{N1}^-\tilde{T}_{N2}^-, \tilde{T}_{N1}^+\tilde{T}_{N2}^+\right], \\ \left[\tilde{I}_{N1}^- + \tilde{I}_{N2}^- - \tilde{I}_{N1}^-\tilde{I}_{N2}^-, \tilde{I}_{N1}^+ + \tilde{I}_{N2}^+ - \tilde{I}_{N1}^+\tilde{I}_{N2}^+\right], \\ \left[\tilde{F}_{N1}^- + \tilde{F}_{N2}^- - \tilde{F}_{N1}^-\tilde{F}_{N2}^-, \tilde{F}_{N1}^+ + \tilde{F}_{N2}^+ - \tilde{F}_{N1}^+\tilde{F}_{N2}^+\right] \end{array} \right\rangle, \\ \left\langle \begin{array}{l} (h_{11}h_{21}, h_{12}h_{22}, h_{13}h_{23}, h_{14}h_{24}); \left[\tilde{T}_{L1}^-\tilde{T}_{L2}^-, \tilde{T}_{L1}^+\tilde{T}_{L2}^+\right], \\ \left[\tilde{I}_{L1}^- + \tilde{I}_{L2}^- - \tilde{I}_{L1}^-\tilde{I}_{L2}^-, \tilde{I}_{L1}^+ + \tilde{I}_{L2}^+ - \tilde{I}_{L1}^+\tilde{I}_{L2}^+\right], \\ \left[\tilde{F}_{L1}^- + \tilde{F}_{L2}^- - \tilde{F}_{L1}^-\tilde{F}_{L2}^-, \tilde{F}_{L1}^+ + \tilde{F}_{L2}^+ - \tilde{F}_{L1}^+\tilde{F}_{L2}^+\right] \end{array} \right\rangle \end{pmatrix};$$

$$(3)\ \lambda\tilde{a}_1 = \begin{pmatrix} \left\langle \begin{array}{l} (\lambda g_{11}, \lambda g_{12}, \lambda g_{13}, \lambda g_{14}); \left[1-\left(1-\tilde{T}_{N1}^-\right)^\lambda, 1-\left(1-\tilde{T}_{N1}^+\right)^\lambda\right], \\ \left[\left(\tilde{I}_{N1}^-\right)^\lambda, \left(\tilde{I}_{N1}^+\right)^\lambda\right], \left[\left(\tilde{F}_{N1}^-\right)^\lambda, \left(\tilde{F}_{N1}^+\right)^\lambda\right] \end{array} \right\rangle, \\ \left\langle \begin{array}{l} (\lambda h_{11}, \lambda h_{12}, \lambda h_{13}, \lambda h_{14}); \left[1-\left(1-\tilde{T}_{L1}^-\right)^\lambda, 1-\left(1-\tilde{T}_{L1}^+\right)^\lambda\right], \\ \left[\left(\tilde{I}_{L1}^-\right)^\lambda, \left(\tilde{I}_{L1}^+\right)^\lambda\right], \left[\left(\tilde{F}_{L1}^-\right)^\lambda, \left(\tilde{F}_{L1}^+\right)^\lambda\right] \end{array} \right\rangle \end{pmatrix};$$

(4) $$\left(\tilde{a}_1\right)^{\lambda}=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(g_{11}^{\lambda},g_{12}^{\lambda},g_{13}^{\lambda},g_{14}^{\lambda}\right);\left[\left(\tilde{T}_{N1}^{-}\right)^{\lambda},\left(\tilde{T}_{N1}^{+}\right)^{\lambda}\right],\\ \left[1-\left(1-\tilde{I}_{N1}^{-}\right)^{\lambda},1-\left(1-\tilde{I}_{N1}^{+}\right)^{\lambda}\right],\\ \left[1-\left(1-\tilde{F}_{N1}^{-}\right)^{\lambda},1-\left(1-\tilde{F}_{N1}^{+}\right)^{\lambda}\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(h_{11}^{\lambda},h_{12}^{\lambda},h_{13}^{\lambda},h_{14}^{\lambda}\right);\left[\left(\tilde{T}_{L1}^{-}\right)^{\lambda},\left(\tilde{T}_{L1}^{+}\right)^{\lambda}\right],\\ \left[1-\left(1-\tilde{I}_{L1}^{-}\right)^{\lambda},1-\left(1-\tilde{I}_{L1}^{+}\right)^{\lambda}\right],\\ \left[1-\left(1-\tilde{F}_{L1}^{-}\right)^{\lambda},1-\left(1-\tilde{F}_{L1}^{+}\right)^{\lambda}\right]\end{array}\right\rangle\end{array}\right).$$

To compare C-ITFNNs, the score function and sorting rules of C-ITFNNs are defined in terms of the score function of C-TFNN [37].

Definition 7. Set $\tilde{a}=\left(\begin{array}{l}\left\langle\left(g_1,g_2,g_3,g_4\right);\left[\tilde{T}_N^{-},\tilde{T}_N^{+}\right],\left[\tilde{I}_N^{-},\tilde{I}_N^{+}\right],\left[\tilde{F}_N^{-},\tilde{F}_N^{+}\right]\right\rangle\\ \left\langle\left(h_1,h_2,h_3,h_4\right);\left[\tilde{T}_L^{-},\tilde{T}_L^{+}\right],\left[\tilde{I}_L^{-},\tilde{I}_L^{+}\right],\left[\tilde{F}_L^{-},\tilde{F}_L^{+}\right]\right\rangle\end{array}\right)$ as C-ITFNN. The score function is defined below:

$$\begin{aligned}S\left(\tilde{a}\right)&=\frac{1}{24}\left(\left(g_1+g_2+g_3+g_4\right)\times\left(4+\tilde{T}_N^{-}+\tilde{T}_N^{+}-\tilde{I}_N^{-}-\tilde{I}_N^{+}-\tilde{F}_N^{-}-\tilde{F}_N^{+}\right)\right)\times\\ &\frac{1}{24}\left(\left(h_1+h_2+h_3+h_4\right)\times\left(4+\tilde{T}_L^{-}+\tilde{T}_L^{+}-\tilde{I}_L^{-}-\tilde{I}_L^{+}-\tilde{F}_L^{-}-\tilde{F}_L^{+}\right)\right)\\ &=\frac{1}{576}\left(\left(g_1+g_2+g_3+g_4\right)\times\left(4+\tilde{T}_N^{-}+\tilde{T}_N^{+}-\tilde{I}_N^{-}-\tilde{I}_N^{+}-\tilde{F}_N^{-}-\tilde{F}_N^{+}\right)\right)\times\\ &\left(\left(h_1+h_2+h_3+h_4\right)\times\left(4+\tilde{T}_L^{-}+\tilde{T}_L^{+}-\tilde{I}_L^{-}-\tilde{I}_L^{+}-\tilde{F}_L^{-}-\tilde{F}_L^{+}\right)\right).\end{aligned}\tag{10}$$

For two C-ITFNNs $\tilde{a}_1$ and $\tilde{a}_2$, if $S\left(\tilde{a}_1\right)>S\left(\tilde{a}_2\right)$, then $\tilde{a}_1>\tilde{a}_2$; if $S\left(\tilde{a}_1\right)=S\left(\tilde{a}_2\right)$, then $\tilde{a}_1\cong\tilde{a}_2$.

Example 1. Set two C-ITFNNs as $\tilde{a}_1=\left(\begin{array}{l}\left\langle\left(0.4,0.5,0.6,0.7\right);\left[0.7,0.9\right],\left[0.1,0.3\right],\left[0.2,0.3\right]\right\rangle,\\ \left\langle\left(0.3,0.4,0.5,0.6\right);\left[0.5,0.6\right],\left[0.2,0.3\right],\left[0.1,0.2\right]\right\rangle\end{array}\right)$ and $\tilde{a}_2=\left(\begin{array}{l}\left\langle\left(0.5,0.6,0.7,0.8\right);\left[0.6,0.8\right],\left[0.3,0.5\right],\left[0.1,0.2\right]\right\rangle;\\ \left\langle\left(0.6,0.7,0.8,0.9\right);\left[0.5,0.7\right],\left[0.2,0.3\right],\left[0.1,0.3\right]\right\rangle\end{array}\right)$. Thus, the two C-ITFNNs are sorted by the score function of Eq. (10):

Since $S(\tilde{a}_1)=\dfrac{1}{576}\left(\begin{array}{l}\left(0.4+0.5+0.6+0.7\right)\times\left(4+0.7+0.9-0.1-0.3-0.2-0.3\right)\\ \times\left(0.3+0.4+0.5+0.6\right)\times\left(4+0.5+0.6-0.2-0.3-0.1-0.2\right)\end{array}\right)=0.1389$

and $S(\tilde{a}_2)=\dfrac{1}{576}\left(\begin{array}{l}\left(0.5+0.6+0.7+0.8\right)\times\left(4+0.6+0.8-0.3-0.5-0.1-0.2\right)\\ \times\left(0.6+0.7+0.8+0.9\right)\times\left(4+0.5+0.7-0.2-0.3-0.1-0.3\right)\end{array}\right)=0.2504$, the sorting of both is $\tilde{a}_1<\tilde{a}_2$.

4. Two Basic Aggregation Operators of C-ITFNNs

As an important tool of aggregation operators for DM modeling, this section proposes two basic aggregation operators for C-ITFNNs by extending the weighted arithmetic and geometric averaging operators of C-TFNNs proposed by Ye et al. [37].

Definition 8. Set $\tilde{a}_i = \begin{pmatrix} \left\langle (g_{i1}, g_{i2}, g_{i3}, g_{i4}); \left[\tilde{T}_{Ni}^-, \tilde{T}_{Ni}^+\right], \left[\tilde{I}_{Ni}^-, \tilde{I}_{Ni}^+\right], \left[\tilde{F}_{Ni}^-, \tilde{F}_{Ni}^+\right] \right\rangle \\ \left\langle (h_{i1}, h_{i2}, h_{i3}, h_{i4}); \left[\tilde{T}_{Li}^-, \tilde{T}_{Li}^+\right], \left[\tilde{I}_{Li}^-, \tilde{I}_{Li}^+\right], \left[\tilde{F}_{Li}^-, \tilde{F}_{Li}^+\right] \right\rangle \end{pmatrix}$ (*i* = 1, 2, …, *J*) as a group of C-ITFNNs with the weight λ_i of $\tilde{a}_i$ for $0 \le \lambda_i \le 1$ and $\sum_{i=1}^{J} \lambda_i = 1$. The C-ITFNNWAA operator is defined as follows:

$$C-ITFNNWAA(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_J) = \sum_{i=1}^{J} \lambda_i \tilde{a}_i \,. \tag{11}$$

In view of the operational laws of C-ITFNNs and Eq. (11), we can obtain the C-ITFNNWAA operator.

Theorem 1. Set $\tilde{a}_i = \begin{pmatrix} \left\langle (g_{i1}, g_{i2}, g_{i3}, g_{i4}); \left[\tilde{T}_{Ni}^-, \tilde{T}_{Ni}^+\right], \left[\tilde{I}_{Ni}^-, \tilde{I}_{Ni}^+\right], \left[\tilde{F}_{Ni}^-, \tilde{F}_{Ni}^+\right] \right\rangle \\ \left\langle (h_{i1}, h_{i2}, h_{i3}, h_{i4}); \left[\tilde{T}_{Li}^-, \tilde{T}_{Li}^+\right], \left[\tilde{I}_{Li}^-, \tilde{I}_{Li}^+\right], \left[\tilde{F}_{Li}^-, \tilde{F}_{Li}^+\right] \right\rangle \end{pmatrix}$ (*i* = 1, 2, …, *J*) as a group of C-ITFNNs with the weight λ_i of $\tilde{a}_i$ for $0 \le \lambda_i \le 1$ and $\sum_{\mathrm{i}=1}^{J} \lambda_i = 1$. On the basis of Eq. (11) and the operational laws of C-ITFNNs, the C-ITFNNWAA operator can be expressed as follows:

$$C-ITFNNWAA(\tilde{a}_1, \tilde{a}_2, ...\tilde{a}_J) = \sum_{i=1}^{J} \lambda_i \tilde{a}_i = \begin{pmatrix} \left\langle \begin{array}{l} \left(\sum_{i=1}^{J} \lambda_i g_{i1}, \sum_{i=1}^{J} \lambda_i g_{i2}, \sum_{i=1}^{J} \lambda_i g_{i3}, \sum_{i=1}^{J} \lambda_i g_{i4} \right); \\ \left[1 - \prod_{i=1}^{J} \left(1 - \tilde{T}_{Ni}^-\right)^{\lambda_i}, 1 - \prod_{i=1}^{J} \left(1 - \tilde{T}_{Ni}^+\right)^{\lambda_i} \right], \\ \left[\prod_{i=1}^{J} \left(\tilde{I}_{Ni}^-\right)^{\lambda_i}, \prod_{i=1}^{J} \left(\tilde{I}_{Ni}^+\right)^{\lambda_i} \right], \left[\prod_{i=1}^{J} \left(\tilde{F}_{Ni}^-\right)^{\lambda_i}, \prod_{i=1}^{J} \left(\tilde{F}_{Ni}^+\right)^{\lambda_i} \right] \end{array} \right\rangle, \\ \left\langle \begin{array}{l} \left(\sum_{i=1}^{J} \lambda_i h_{i1}, \sum_{i=1}^{J} \lambda_i h_{i2}, \sum_{i=1}^{J} \lambda_i h_{i3}, \sum_{i=1}^{J} \lambda_i h_{i4} \right); \\ \left[1 - \prod_{i=1}^{J} \left(1 - \tilde{T}_{Li}^-\right)^{\lambda_i}, 1 - \prod_{i=1}^{J} \left(1 - \tilde{T}_{Li}^+\right)^{\lambda_i} \right], \\ \left[\prod_{i=1}^{J} \left(\tilde{I}_{Li}^-\right)^{\lambda_i}, \prod_{i=1}^{J} \left(\tilde{I}_{Li}^+\right)^{\lambda_i} \right], \left[\prod_{i=1}^{J} \left(\tilde{F}_{Li}^-\right)^{\lambda_i}, \prod_{i=1}^{J} \left(\tilde{F}_{Li}^+\right)^{\lambda_i} \right] \end{array} \right\rangle \end{pmatrix}. \tag{12}$$

Proof: Here, Theorem 1 is proved in light of mathematical induction.

(1) When *J* = 2, the aggregated result of the two C-ITFNNs is obtained as follows:

$$
\begin{aligned}
& C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2)=\sum_{i=1}^{2}\lambda_i\tilde{a}_i=\lambda_1\tilde{a}_1\oplus\lambda_2\tilde{a}_2 \\
&=\left(\begin{array}{l}
\left\langle\begin{array}{l}
\left(\lambda_1 g_{11}+\lambda_2 g_{21},\lambda_1 g_{12}+\lambda_2 g_{22},\lambda_1 g_{13}+\lambda_2 g_{23},\lambda_1 g_{14}+\lambda_2 g_{24}\right);\\
\left[\begin{array}{l}
1-\left(1-\tilde{T}_{N1}^{-}\right)^{\lambda_1}+1-\left(1-\tilde{T}_{N2}^{-}\right)^{\lambda_2}-\left(1-\left(1-\tilde{T}_{N1}^{-}\right)^{\lambda_1}\right)\left(1-\left(1-\tilde{T}_{N2}^{-}\right)^{\lambda_2}\right),\\
1-\left(1-\tilde{T}_{N1}^{+}\right)^{\lambda_1}+1-\left(1-\tilde{T}_{N2}^{+}\right)^{\lambda_2}-\left(1-\left(1-\tilde{T}_{N1}^{+}\right)^{\lambda_1}\right)\left(1-\left(1-\tilde{T}_{N2}^{+}\right)^{\lambda_2}\right)
\end{array}\right],\\
\left[\left(\tilde{I}_{N1}^{-}\right)^{\lambda_1}\left(\tilde{I}_{N2}^{-}\right)^{\lambda_2},\left(\tilde{I}_{N1}^{+}\right)^{\lambda_1}\left(\tilde{I}_{N2}^{+}\right)^{\lambda_2}\right],\left[\left(\tilde{F}_{N1}^{-}\right)^{\lambda_1}\left(\tilde{F}_{N2}^{-}\right)^{\lambda_2},\left(\tilde{F}_{N1}^{+}\right)^{\lambda_2}\left(\tilde{F}_{N2}^{+}\right)^{\lambda_2}\right]
\end{array}\right\rangle,\\
\left\langle\begin{array}{l}
\left(\lambda_1 h_{11}+\lambda_2 h_{21},\lambda_1 h_{12}+\lambda_2 h_{22},\lambda_1 h_{13}+\lambda_2 h_{23},\lambda_1 h_{14}+\lambda_2 h_{24}\right);\\
\left[\begin{array}{l}
1-\left(1-\tilde{T}_{L1}^{-}\right)^{\lambda_1}+1-\left(1-\tilde{T}_{L2}^{-}\right)^{\lambda_2}-\left(1-\left(1-\tilde{T}_{L1}^{-}\right)^{\lambda_1}\right)\left(1-\left(1-\tilde{T}_{L2}^{-}\right)^{\lambda_2}\right),\\
1-\left(1-\tilde{T}_{L1}^{+}\right)^{\lambda_1}+1-\left(1-\tilde{T}_{L2}^{+}\right)^{\lambda_2}-\left(1-\left(1-\tilde{T}_{L1}^{+}\right)^{\lambda_1}\right)\left(1-\left(1-\tilde{T}_{L2}^{+}\right)^{\lambda_2}\right)
\end{array}\right],\\
\left[\left(\tilde{I}_{L1}^{-}\right)^{\lambda_1}\left(\tilde{I}_{L2}^{-}\right)^{\lambda_2},\left(\tilde{I}_{L1}^{+}\right)^{\lambda_1}\left(\tilde{I}_{L2}^{+}\right)^{\lambda_2}\right],\left[\left(\tilde{F}_{L1}^{-}\right)^{\lambda_1}\left(\tilde{F}_{L2}^{-}\right)^{\lambda_2},\left(\tilde{F}_{L1}^{-}\right)^{\lambda_1}\left(\tilde{F}_{L2}^{-}\right)^{\lambda_2}\right]
\end{array}\right\rangle
\end{array}\right)\\
&=\left(\begin{array}{l}
\left\langle\begin{array}{l}
\left(\sum_{i=1}^{2}\lambda_i g_{i1},\sum_{i=1}^{2}\lambda_i g_{i2},\sum_{i=1}^{2}\lambda_i g_{i3},\sum_{i=1}^{2}\lambda_i g_{i4}\right);\\
\left[1-\prod_{i=1}^{2}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{2}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\\
\left[\prod_{i=1}^{2}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{2}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{2}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{2}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]
\end{array}\right\rangle,\\
\left\langle\begin{array}{l}
\left(\sum_{i=1}^{2}\lambda_i h_{i1},\sum_{i=1}^{2}\lambda_i h_{i2},\sum_{i=1}^{2}\lambda_i h_{i3},\sum_{i=1}^{2}\lambda_i h_{i4}\right);\\
\left[1-\prod_{i=1}^{2}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{2}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right],\\
\left[\prod_{i=1}^{2}\left(\tilde{I}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{2}\left(\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{2}\left(\tilde{F}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{2}\left(\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]
\end{array}\right\rangle
\end{array}\right).
\end{aligned}
\qquad (13)
$$

(2) If J = n, the aggregated result of n C-ITFNNs is given as follows:

$$C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...\tilde{a}_n)=\sum_{i=1}^{n}\lambda_i\tilde{a}_i=\left(\begin{array}{c}\left\langle\begin{array}{l}\left(\sum_{i=1}^{n}\lambda_i g_{i1},\sum_{i=1}^{n}\lambda_i g_{i2},\sum_{i=1}^{n}\lambda_i g_{i3},\sum_{i=1}^{n}\lambda_i g_{i4}\right);\\ \left[1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{n}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\sum_{i=1}^{n}\lambda_i h_{i1},\sum_{i=1}^{n}\lambda_i h_{i2},\sum_{i=1}^{n}\lambda_i h_{i3},\sum_{i=1}^{n}\lambda_i h_{i4}\right);\\ \left[1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{n}\left(\tilde{I}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n}\left(\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n}\left(\tilde{F}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n}\left(\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle\end{array}\right). \quad (14)$$

(3) If $J = n + 1$, according to Eqs. (12) and (13), we can get the following result:

$$C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n,\tilde{a}_{n+1})=\sum_{i=1}^{n+1}\lambda_i\tilde{a}_i=\sum_{i=1}^{n}\lambda_i\tilde{a}_i\oplus\lambda_{n+1}\tilde{a}_{n+1}$$

$$=\left(\begin{array}{c}\left\langle\begin{array}{l}\left(\sum_{i=1}^{n+1}\lambda_i g_{i1},\sum_{i=1}^{n+1}\lambda_i g_{i2},\sum_{i=1}^{n+1}\lambda_i g_{i3},\sum_{i=1}^{n+1}\lambda_i g_{i4}\right);\\ \left[\begin{array}{l}1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i}+1-\left(1-\tilde{T}_{Nn+1}^{-}\right)^{\lambda_{n+1}}-\left(1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i}\right)\left(1-\left(1-\tilde{T}_{Nn+1}^{-}\right)^{\lambda_{n+1}}\right),\\ 1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}+1-\left(1-\tilde{T}_{Nn+1}^{+}\right)^{\lambda_{n+1}}-\left(1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right)\left(1-\left(1-\tilde{T}_{Nn+1}^{+}\right)^{\lambda_{n+1}}\right)\end{array}\right],\\ \left[\prod_{i=1}^{n+1}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n+1}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\sum_{i=1}^{n+1}\lambda_i h_{i1},\sum_{i=1}^{n+1}\lambda_i h_{i2},\sum_{i=1}^{n+1}\lambda_i h_{i3},\sum_{i=1}^{n+1}\lambda_i h_{i4}\right);\\ \left[\begin{array}{l}1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i}+1-\left(1-\tilde{T}_{Ln+1}^{-}\right)^{\lambda_{n+1}}-\left(1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i}\right)\left(1-\left(1-\tilde{T}_{Ln+1}^{-}\right)^{\lambda_{n+1}}\right),\\ 1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}+1-\left(1-\tilde{T}_{Ln+1}^{+}\right)^{\lambda_{n+1}}-\left(1-\prod_{i=1}^{n}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right)\left(1-\left(1-\tilde{T}_{Ln+1}^{+}\right)^{\lambda_{n+1}}\right)\end{array}\right],\\ \left[\prod_{i=1}^{n+1}\left(\tilde{I}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n+1}\left(\tilde{F}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle\end{array}\right) \quad (15)$$

$$=\left(\left\langle\begin{array}{l}\left(\sum_{i=1}^{n+1}\lambda_i g_{i1},\sum_{i=1}^{n+1}\lambda_i g_{i2},\sum_{i=1}^{n+1}\lambda_i g_{i3},\sum_{i=1}^{n+1}\lambda_i g_{i4}\right);\\ \left[1-\prod_{i=1}^{n+1}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{n+1}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{n+1}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n+1}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle,\left\langle\begin{array}{l}\left(\sum_{i=1}^{n+1}\lambda_i h_{i1},\sum_{i=1}^{n+1}\lambda_i h_{i2},\sum_{i=1}^{n+1}\lambda_i h_{i3},\sum_{i=1}^{n+1}\lambda_i h_{i4}\right);\\ \left[1-\prod_{i=1}^{n+1}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{n+1}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{n+1}\left(\tilde{I}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{n+1}\left(\tilde{F}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{n+1}\left(\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle\right).$$

In view of the above results, Eq. (12) is true for any J.

Then, the C-ITFNNWAA operator of Eq. (12) has the following characteristics:

(1) Idempotency: Let $\tilde{a}_i$ (i = 1, 2, …, J) be a group of C-ITFNNs. If $\tilde{a}_i=\tilde{a}$ for i = 1, 2, …, J, there is $C-ITFNNWAA\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J\right)=\tilde{a}$.

(2) Boundedness: Let $\tilde{a}_i$ (i = 1, 2, …, J) be a group of C-ITFNNs and let the minimum and maximum C-ITFNNs:

$$\tilde{a}_{\min}=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(\min_i(g_{i1}),\min_i(g_{i2}),\min_i(g_{i3}),\min_i(g_{i4})\right);\\ \left[\min_i\left(\tilde{T}_{Ni}^-\right),\min_i\left(\tilde{T}_{Ni}^+\right)\right],\left[\max_i\left(\tilde{I}_{Ni}^-\right),\max_i\left(\tilde{I}_{Ni}^+\right)\right],\left[\max_i\left(\tilde{F}_{Ni}^-\right),\max_i\left(\tilde{F}_{Ni}^+\right)\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\min_i(h_{i1}),\min_i(h_{i2}),\min_i(h_{i3}),\min_i(h_{i4})\right);\\ \left[\min_i\left(\tilde{T}_{Li}^-\right),\min_i\left(\tilde{T}_{Li}^+\right)\right],\left[\max_i\left(\tilde{I}_{Li}^-\right),\max_i\left(\tilde{I}_{Li}^+\right)\right],\left[\max_i\left(\tilde{F}_{Li}^-\right),\max_i\left(\tilde{F}_{Li}^+\right)\right]\end{array}\right\rangle\end{array}\right),$$

$$\tilde{a}_{\max}=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(\max_i(g_{i1}),\max_i(g_{i2}),\max_i(g_{i3}),\max_i(g_{i4})\right);\\ \left[\max_i\left(\tilde{T}_{Ni}^-\right),\max_i\left(\tilde{T}_{Ni}^+\right)\right],\left[\min_i\left(\tilde{I}_{Ni}^-\right),\min_i\left(\tilde{I}_{Ni}^+\right)\right],\left[\min_i\left(\tilde{F}_{Ni}^-\right),\min_i\left(\tilde{F}_{Ni}^+\right)\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\max_i(h_{i1}),\max_i(h_{i2}),\max_i(h_{i3}),\max_i(h_{i4})\right);\\ \left[\max_i\left(\tilde{T}_{Li}^-\right),\max_i\left(\tilde{T}_{Li}^+\right)\right],\left[\min_i\left(\tilde{I}_{Li}^-\right),\min_i\left(\tilde{I}_{Li}^+\right)\right],\left[\min_i\left(\tilde{F}_{Li}^-\right),\min_i\left(\tilde{F}_{Li}^+\right)\right]\end{array}\right\rangle\end{array}\right).$$

Then $\tilde{a}_{\min}\le C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J)\le\tilde{a}_{\max}$.

(3) Monotonicity: Let $\tilde{a}_i$ (i = 1, 2, …, J) be a group of C-ITFNNs. If $\tilde{a}_i\le\tilde{a}_i'$ for i = 1, 2, …, J, there is $C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J)\le C-ITFNNWAA\left(\tilde{a}_1',\tilde{a}_2',...,\tilde{a}_J'\right)$.

Proof: (1) Let $\tilde{a}_i=\tilde{a}$ for i = 1, 2, …, J. Then, the aggregated result of Eq. (12) is given below:

$$C-ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J)=\sum_{i=1}^{J}\lambda_i\tilde{a}_i$$

$$=\begin{pmatrix}\left\langle\begin{matrix}\left(\sum_{i=1}^{J}\lambda_i g_{i1},\sum_{i=1}^{J}\lambda_i g_{i2},\sum_{i=1}^{J}\lambda_i g_{i3},\sum_{i=1}^{J}\lambda_i g_{i4}\right);\\ \left[1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{J}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{J}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{matrix}\right\rangle,\\ \left\langle\begin{matrix}\left(\sum_{i=1}^{J}\lambda_i h_{i1},\sum_{i=1}^{J}\lambda_i h_{i2},\sum_{i=1}^{J}\lambda_i h_{i3},\sum_{i=1}^{J}\lambda_i h_{i4}\right);\\ \left[1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right],\\ \left[\prod_{i=1}^{J}\left(\tilde{I}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\left[\prod_{i=1}^{J}\left(\tilde{F}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]\end{matrix}\right\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\left\langle\begin{matrix}\left(g_1\sum_{i=1}^{J}\lambda_i,g_2\sum_{i=1}^{J}\lambda_i,g_3\sum_{i=1}^{J}\lambda_i,g_4\sum_{i=1}^{J}\lambda_i\right);\\ \left[1-\left(1-\tilde{T}_{N}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},1-\left(1-\tilde{T}_{N}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right],\\ \left[\left(\tilde{I}_{N}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},\left(\tilde{I}_{N}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right],\left[\left(\tilde{F}_{N}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},\left(\tilde{F}_{N}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right]\end{matrix}\right\rangle,\\ \left\langle\begin{matrix}\left(h_1\sum_{i=1}^{J}\lambda_i,h_2\sum_{i=1}^{J}\lambda_i,h_3\sum_{i=1}^{J}\lambda_i,h_4\sum_{i=1}^{J}\lambda_i\right);\\ \left[1-\left(1-\tilde{T}_{L}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},1-\left(1-\tilde{T}_{L}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right],\\ \left[\left(\tilde{I}_{L}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},\left(\tilde{I}_{L}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right],\left[\left(\tilde{F}_{L}^{-}\right)^{\sum_{i=1}^{J}\lambda_i},\left(\tilde{F}_{L}^{+}\right)^{\sum_{i=1}^{J}\lambda_i}\right]\end{matrix}\right\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\left\langle\left(g_1,g_2,g_3,g_4\right);\left[1-\left(1-\tilde{T}_N^{-}\right),1-\left(1-\tilde{T}_N^{+}\right)\right],\left[\tilde{I}_N^{-},\tilde{I}_N^{+}\right],\left[\tilde{F}_N^{-},\tilde{F}_N^{+}\right]\right\rangle,\\ \left\langle\left(h_1,h_2,h_3,h_4\right);\left[1-\left(1-\tilde{T}_L^{-}\right),1-\left(1-\tilde{T}_L^{+}\right)\right],\left[\tilde{I}_L^{-},\tilde{I}_L^{+}\right],\left[\tilde{F}_L^{-},\tilde{F}_L^{+}\right]\right\rangle\end{pmatrix}=\tilde{a}.$$

(2) Since $\tilde{a}_{\min}$ and $\tilde{a}_{\max}$ are the minimum and maximum C-ITFNNs, respectively, there is $\tilde{a}_{\min}\le\tilde{a}_i\le\tilde{a}_{\max}$, then $\sum_{i=1}^{J}\lambda_i\tilde{a}_{\min}\le\sum_{i=1}^{J}\lambda_i\tilde{a}_i\le\sum_{i=1}^{J}\lambda_i\tilde{a}_{\max}$ also exists. On the basis of the characteristic (1), there is $\tilde{a}_{\min}\le\sum_{i=1}^{J}\lambda_i\tilde{a}_i\le\tilde{a}_{\max}$, i.e., $\tilde{a}_{\min}\le C-ITFNNWAA\left(\tilde{a}_1,\tilde{a}_2,..,\tilde{a}_J\right)\le\tilde{a}_{\max}$.

(3) Owning to $\tilde{a}_i\le\tilde{a}_i^{'}$ for i = 1, 2, …, J, there is $\sum_{i=1}^{J}\lambda_i\tilde{a}_i\le\sum_{i=1}^{J}\lambda_i\tilde{a}_i^{'}$, namely, $C-ITFNNWAA\left(\tilde{a}_1,\tilde{a}_2,..,\tilde{a}_J\right)\le C-ITFNNWAA\left(\tilde{a}_1',\tilde{a}_2',..,\tilde{a}_J'\right)$.

Example 2. Set two C-ITFNNs as $\tilde{a}_1=\begin{pmatrix}\langle(0.2,0.3,0.4,0.5);[0.1,0.2],[0.2,0.3],[0.3,0.4]\rangle,\\ \langle(0.3,0.4,0.5,0.6);[0.1,0.3],[0.2,0.4],[0.3,0.5]\rangle\end{pmatrix}$ and $\tilde{a}_2=\begin{pmatrix}\langle(0.4,0.5,0.6,0.7);[0.2,0.4],[0.1,0.3],[0.5,0.7]\rangle,\\ \langle(0.5,0.6,0.7,0.8);[0.2,0.3],[0.3,0.4],[0.4,0.5]\rangle\end{pmatrix}$ with their weight values 0.3 and 0.7. Then, the calculational result using Eq. (12) is given below:

$$C-ITFNVWAA(\tilde{a}_1,\tilde{a}_2)=\sum_{i=1}^{2}\lambda_i\tilde{a}_i=\lambda_1\tilde{a}_1\oplus\lambda_2\tilde{a}_2$$

$$=\begin{pmatrix}\left\langle\begin{matrix}\begin{pmatrix}0.2\times0.3+0.4\times0.7,0.3\times0.3+0.5\times0.7,\\0.4\times0.3+0.6\times0.7,0.5\times0.3+0.7\times0.7\end{pmatrix};\\ \left[1-(1-0.1)^{0.3}\times(1-0.2)^{0.7},1-(1-0.2)^{0.3}\times(1-0.4)^{0.7}\right],\\ \left[0.2^{0.3}\times0.1^{0.7},0.3^{0.3}\times0.3^{0.7}\right],\left[0.3^{0.3}\times0.5^{0.7},0.4^{0.3}\times0.7^{0.7}\right]\end{matrix}\right\rangle,\\ \left\langle\begin{matrix}\begin{pmatrix}0.3\times0.3+0.5\times0.7,0.4\times0.3+0.6\times0.7,\\0.5\times0.3+0.7\times0.7,0.6\times0.3+0.8\times0.7\end{pmatrix};\\ \left[1-(1-0.1)^{0.3}\times(1-0.2)^{0.7},1-(1-0.3)^{0.3}\times(1-0.3)^{0.7}\right],\\ \left[0.2^{0.3}\times0.3^{0.7},0.4^{0.3}\times0.4^{0.7}\right],\left[0.3^{0.3}\times0.4^{0.7},0.5^{0.3}\times0.5^{0.7}\right]\end{matrix}\right\rangle\end{pmatrix}$$

$$=\begin{pmatrix}\langle(0.34,0.44,0.54,0.64);[0.171,0.346],[0.123,0.3],[0.429,0.592]\rangle,\\ \langle(0.44,0.54,0.64,0.74);[0.171,0.3],[0.266,0.4],[0.367,0.5]\rangle\end{pmatrix}.$$

Definition 4.2. Set $\tilde{a}_i=\begin{pmatrix}\langle(g_{i1},g_{i2},g_{i3},g_{i4});[\tilde{T}_{Ni}^-,\tilde{T}_{Ni}^+],[\tilde{I}_{Ni}^-,\tilde{I}_{Ni}^+],[\tilde{F}_{Ni}^-,\tilde{F}_{Ni}^+]\rangle\\ \langle(h_{i1},h_{i2},h_{i3},h_{i4});[\tilde{T}_{Li}^-,\tilde{T}_{Li}^+],[\tilde{I}_{Li}^-,\tilde{I}_{Li}^+],[\tilde{F}_{Li}^-,\tilde{F}_{Li}^+]\rangle\end{pmatrix}$ (i = 1, 2, …, J) as a group of C-ITFNNs with the weight λ_i of $\tilde{a}_i$ for $0\le\lambda_i\le1$ and $\sum_{i=1}^{J}\lambda_i=1$. The C-ITFNNWGA operator is defined as follows:

$$C-ITFNNWGA(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J)=\prod_{i=1}^{J}\tilde{a}^{\lambda_i}. \tag{16}$$

In terms of the operational laws of C-ITFNNs and Eq. (16), we can obtain the C-ITFNNWGA operator.

Theorem 2. Set $\tilde{a}_i=\begin{pmatrix}\langle(g_{i1},g_{i2},g_{i3},g_{i4});[\tilde{T}_{Ni}^-,\tilde{T}_{Ni}^+],[\tilde{I}_{Ni}^-,\tilde{I}_{Ni}^+],[\tilde{F}_{Ni}^-,\tilde{F}_{Ni}^+]\rangle\\ \langle(h_{i1},h_{i2},h_{i3},h_{i4});[\tilde{T}_{Li}^-,\tilde{T}_{Li}^+],[\tilde{I}_{Li}^-,\tilde{I}_{Li}^+],[\tilde{F}_{Li}^-,\tilde{F}_{Li}^+]\rangle\end{pmatrix}$ (i = 1, 2, …, J) as a group of C-ITFNNs with the weight λ_i of $\tilde{a}_i$ for $0\le\lambda_i\le1$ and $\sum_{i=1}^{J}\lambda_i=1$. On the basis of Eq. (16) and the operational laws of C-ITFNNs, the C-ITFNNWGA operator can be expressed as follows:

$$C-ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J\right)=\prod_{i=1}^{J}\tilde{a}_i^{\lambda_i}$$
$$=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(\prod_{i=1}^{J}g_{i1}^{\lambda_i},\prod_{i=1}^{J}g_{i2}^{\lambda_i},\prod_{i=1}^{J}g_{i3}^{\lambda_i},\prod_{i=1}^{J}g_{i4}^{\lambda_i}\right);\\ \left[\prod_{i=1}^{J}\left(\tilde{T}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\left[1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\\ \left[1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\prod_{i=1}^{J}h_{i1}^{\lambda_i},\prod_{i=1}^{J}h_{i2}^{\lambda_i},\prod_{i=1}^{J}h_{i3}^{\lambda_i},\prod_{i=1}^{J}h_{i4}^{\lambda_i}\right);\\ \left[\prod_{i=1}^{J}\left(\tilde{T}_{Li}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{T}_{Li}^{+}\right)^{\lambda_i}\right],\left[1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Li}^{+}\right)^{\lambda_i}\right],\\ \left[1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Li}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Li}^{+}\right)^{\lambda_i}\right]\end{array}\right\rangle\end{array}\right). \tag{17}$$

Since the proof method of Theorem 2 is similar to that of Theorem 1, it can also be proved in light of mathematical induction, which will not be repeated here.

Then, the C-ITFNNWGA operator has the following characteristics:

Idempotency: Let $\tilde{a}_i$ (i = 1, 2, …, J) be a group of C-ITFNNs. If $\tilde{a}_i=\tilde{a}$ for i = 1, 2, …, J, there is $C-ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_j\right)=\tilde{a}$.

Boundedness: Let $\tilde{a}_i$ (i = 1, 2, …, J) be a group of C-ITFNNs and let the minimum and maximum C-ITFNNs:

$$\tilde{a}_{\min}=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(\min_i\left(g_{i1}\right),\min_i\left(g_{i2}\right),\min_i\left(g_{i3}\right),\min_i\left(g_{i4}\right)\right);\\ \left[\min_i\left(\tilde{T}_{Ni}^{-}\right),\min_i\left(\tilde{T}_{Ni}^{+}\right)\right],\left[\max_i\left(\tilde{I}_{Ni}^{-}\right),\max_i\left(\tilde{I}_{Ni}^{+}\right)\right],\left[\max_i\left(\tilde{F}_{Ni}^{-}\right),\max_i\left(\tilde{F}_{Ni}^{+}\right)\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\min_i\left(h_{i1}\right),\min_i\left(h_{i2}\right),\min_i\left(h_{i3}\right),\min_i\left(h_{i4}\right)\right);\\ \left[\min_i\left(\tilde{T}_{Li}^{-}\right),\min_i\left(\tilde{T}_{Li}^{+}\right)\right],\left[\max_i\left(\tilde{I}_{Li}^{-}\right),\max_i\left(\tilde{I}_{Li}^{+}\right)\right],\left[\max_i\left(\tilde{F}_{Li}^{-}\right),\max_i\left(\tilde{F}_{Li}^{+}\right)\right]\end{array}\right\rangle\end{array}\right),$$

$$\tilde{a}_{\max}=\left(\begin{array}{l}\left\langle\begin{array}{l}\left(\max_i\left(g_{i1}\right),\max_i\left(g_{i2}\right),\max_i\left(g_{i3}\right),\max_i\left(g_{i4}\right)\right);\\ \left[\max_i\left(\tilde{T}_{Ni}^{-}\right),\max_i\left(\tilde{T}_{Ni}^{+}\right)\right],\left[\min_i\left(\tilde{I}_{Ni}^{-}\right),\min_i\left(\tilde{I}_{Ni}^{+}\right)\right],\left[\min_i\left(\tilde{F}_{Ni}^{-}\right),\min_i\left(\tilde{F}_{Ni}^{+}\right)\right]\end{array}\right\rangle,\\ \left\langle\begin{array}{l}\left(\max_i\left(h_{i1}\right),\max_i\left(h_{i2}\right),\max_i\left(h_{i3}\right),\max_i\left(h_{i4}\right)\right);\\ \left[\max_i\left(\tilde{T}_{Li}^{-}\right),\max_i\left(\tilde{T}_{Li}^{+}\right)\right],\left[\min_i\left(\tilde{I}_{Li}^{-}\right),\min_i\left(\tilde{I}_{Li}^{+}\right)\right],\left[\min_i\left(\tilde{F}_{Li}^{-}\right),\min_i\left(\tilde{F}_{Li}^{+}\right)\right]\end{array}\right\rangle\end{array}\right).$$

Then $\tilde{a}_{\min}\le C-ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J\right)\le\tilde{a}_{\max}$.

For $\tilde{a}_i \le \tilde{a}_i^{'}$ (i = 1, 2, …, J), there is $\sum_{i=1}^{J}\lambda_i\tilde{a}_i \le \sum_{i=1}^{J}\lambda_i\tilde{a}_i^{'}$, namely, $C-ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2,..,\tilde{a}_J\right) \le C-ITFNNWGA\left(\tilde{a}_1^{'},\tilde{a}_2^{'},..,\tilde{a}_J^{'}\right)$.

However, the proof method of the C-ITFNNWGA operator is similar to that of the C-ITFNNWAA operator, which is omitted here.

Example 3. Set two C-ITFNNs as $\tilde{a}_1 = \begin{pmatrix} \langle (0.4,0.5,0.6,0.7);[0.1,0.4],[0.2,0.5],[0.3,0.6]\rangle, \\ \langle (0.5,0.6,0.7,0.8);[0.2,0.3],[0.3,0.4],[0.4,0.5]\rangle \end{pmatrix}$ and $\tilde{a}_2 = \begin{pmatrix} \langle (0.3,0.4,0.5,0.6);[0.1,0.2],[0.1,0.3],[0.1,0.4]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.2,0.5],[0.3,0.6],[0.4,0.7]\rangle \end{pmatrix}$ with their weight values 0.4 and 0.6. Then, the calculational result using Eq. (17) is given below:

$$C-ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2\right) = \prod_{i=1}^{2}\tilde{a}_i^{\lambda_i} = \tilde{a}_1^{\lambda_1}\otimes\tilde{a}_2^{\lambda_2}$$

$$= \begin{pmatrix} \left\langle \begin{array}{l} \left(0.4^{0.4}\times 0.3^{0.6}, 0.5^{0.4}\times 0.4^{0.6}, 0.6^{0.4}\times 0.5^{0.6}, 0.7^{0.4}\times 0.6^{0.6}\right); \\ \left[0.1^{0.4}\times 0.1^{0.6}, 0.4^{0.4}\times 0.2^{0.6}\right], \\ \left[1-(1-0.2)^{0.4}\times(1-0.1)^{0.6}, 1-(1-0.5)^{0.4}\times(1-0.3)^{0.6}\right], \\ \left[1-(1-0.3)^{0.4}\times(1-0.1)^{0.6}, 1-(1-0.6)^{0.4}\times(1-0.4)^{0.6}\right] \end{array} \right\rangle, \\ \left\langle \begin{array}{l} \left(0.5^{0.4}\times 0.6^{0.6}, 0.6^{0.4}\times 0.7^{0.6}, 0.7^{0.4}\times 0.8^{0.6}, 0.8^{0.4}\times 0.9^{0.6}\right); \\ \left[0.2^{0.4}\times 0.2^{0.6}, 0.3^{0.4}\times 0.5^{0.6}\right], \\ \left[1-(1-0.3)^{0.4}\times(1-0.3)^{0.6}, 1-(1-0.4)^{0.4}\times(1-0.6)^{0.6}\right], \\ \left[1-(1-0.4)^{0.4}\times(1-0.4)^{0.6}, 1-(1-0.5)^{0.4}\times(1-0.7)^{0.6}\right] \end{array} \right\rangle \end{pmatrix}$$

$$= \begin{pmatrix} \langle (0.337,0.437,0.538,0.638);[0.1,0.264],[0.141,0.388],[0.186,0.4898]\rangle, \\ \langle (0.558,0.658,0.758,0.859);[0.2,0.408],[0.3,0.53],[0.4,0.632]\rangle \end{pmatrix}.$$

Particularly, when there are no credibility degrees in C-ITFNNs, C-ITFNNs become ITFNNs. Thus, Eqs. (12), (17), and (10) become the ITFNN weighted arithmetic averaging (ITFNNWAA) and ITFNN weighted geometric averaging (ITFNNWGA) operators and the score function:

$$ITFNNWAA(\tilde{a}_1,\tilde{a}_2,...\tilde{a}_J) = \sum_{i=1}^{J}\lambda_i\tilde{a}_i$$

$$= \begin{pmatrix} \left\langle \sum_{i=1}^{J}\lambda_i g_{i1}, \sum_{i=1}^{J}\lambda_i g_{i2}, \sum_{i=1}^{J}\lambda_i g_{i3}, \sum_{i=1}^{J}\lambda_i g_{i4} \right\rangle; \left[1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Ni}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right], \\ \left[\prod_{i=1}^{J}\left(\tilde{I}_{Ni}^{-}\right)^{\lambda_i}, \prod_{i=1}^{J}\left(\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right], \left[\prod_{i=1}^{J}\left(\tilde{F}_{Ni}^{-}\right)^{\lambda_i}, \prod_{i=1}^{J}\left(\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right] \end{pmatrix}, \quad (18)$$

$$ITFNNWGA\left(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_J\right)=\prod_{i=1}^{J}\tilde{a}_i^{\lambda_i}$$
$$=\left(\begin{array}{l}\left\langle \prod_{i=1}^{J} g_{i1}^{\lambda_i},\prod_{i=1}^{J} g_{i2}^{\lambda_i},\prod_{i=1}^{J} g_{i3}^{\lambda_i},\prod_{i=1}^{J} g_{i4}^{\lambda_i}\right\rangle;\left[\prod_{i=1}^{J}\left(\tilde{T}_{Ni}^{-}\right)^{\lambda_i},\prod_{i=1}^{J}\left(\tilde{T}_{Ni}^{+}\right)^{\lambda_i}\right],\\ \left[1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Ni}^{+}\right)^{\lambda_i}\right],\left[1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Ni}^{-}\right)^{\lambda_i},1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Ni}^{+}\right)^{\lambda_i}\right]\end{array}\right), \quad (19)$$

$$S'\left(\tilde{a}\right)=\frac{1}{24}\left(\left(g_1+g_2+g_3+g_4\right)\times\left(4+\tilde{T}_N^{-}+\tilde{T}_N^{+}-\tilde{I}_N^{-}-\tilde{I}_N^{+}-\tilde{F}_N^{-}-\tilde{F}_N^{+}\right)\right), S'\left(\tilde{a}\right)\in\left[0,1\right]. \quad (20)$$

5. DM Model with C-ITFNNs

In this section, we use the C-ITFNNWAA and C-ITFNNWGA operators and the score function to establish the multi-attribute DM model in the C-ITFNN circumstance.

Let a set of alternatives be $E = \{e_1, e_2, \ldots, e_p\}$ and a set of attributes be $G = \{g_1, g_2, \ldots, g_J\}$. The weight vector of the attributes is $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_J)$, which indicates the importance of various attributes. Decision makers can give their satisfactory linguistic evaluation values and credibility degrees through the set of linguistic terms L = {Very bad, Bad, Fairly bad, Medium, Fairly good, Good, Very good}. In view of Table 1, we can obtain the linguistic values of each alternative er ($r = 1, 2, \ldots, p$) over the attributes g_i ($i = 1, 2, \ldots, J$) and express them as the following C-ITFNN:

$$\tilde{a}_{ri}=\left(\begin{array}{l}\left\langle\left(g_{ri1},g_{ri2},g_{ri3},g_{ri4}\right);\left[\tilde{T}_{Nri}^{-},\tilde{T}_{Nri}^{+}\right],\left[\tilde{I}_{Nri}^{-},\tilde{I}_{Nri}^{+}\right],\left[\tilde{F}_{Nri}^{-},\tilde{F}_{Nri}^{+}\right]\right\rangle,\\ \left\langle\left(h_{ri1},h_{ri2},h_{ri3},h_{ri4}\right);\left[\tilde{T}_{Lri}^{-},\tilde{T}_{Lri}^{+}\right],\left[\tilde{I}_{Lri}^{-},\tilde{I}_{Lri}^{+}\right],\left[\tilde{F}_{Lri}^{-},\tilde{F}_{Lri}^{+}\right]\right\rangle\end{array}\right).$$

Thus, we can establish the C-ITFNN decision matrix $N=\left(\tilde{a}_{ri}\right)_{p\times J}$.

Table 1. Linguistic terms and linguistic values of ITFNNs

Linguistic term	Linguistic value of ITFNNs
Very bad (VB)	<(0.1,0.1,0.1,0.1);[0.1,0.2],[0.9,1.0],[0.9,1.0]>
Bad (B)	<(0.2,0.3,0.4,0.5);[0.2,0.3],[0.8,0.9],[0.8,0.9]>
Fairy bad (FB)	<(0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]>
Medium (M)	<(0.4,0.5,0.6,0.7);[0.5,0.6],[0.4,0.6],[0.4,0.6]>
Fairy good (FG)	<(0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]>
Good (G)	<(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]>
Very good (VG)	<(1.0,1.0,1.0,1.0);[0.9,1],[0,0.1],[0,0.1]>

Then, we use C-ITFNNWAA and C-ITFNNWGA operators and the score function to establish the multi-attribute DM model with C-ITFNN information and to select the best alternative. The DM process is indicated below.

Step 1: Give the aggregated C-ITFNNs $\tilde{a}_r$ for $e_r\left(r=1,2,...,p\right)$ by the C-ITNNWAA operator or the C-ITNNWGA operator:

$$
\begin{aligned}
\tilde{a}_r &= C-ITFNNWAA(\tilde{a}_{r1}, \tilde{a}_{r2}, ...\tilde{a}_{rJ}) = \sum_{i=1}^{J} \lambda_i \tilde{a}_{rJ} \\
&= \left(\begin{array}{l}
\left\langle \begin{array}{l}
\left(\sum_{i=1}^{J} \lambda_i g_{ri1}, \sum_{i=1}^{J} \lambda_i g_{ri2}, \sum_{i=1}^{J} \lambda_i g_{ri3}, \sum_{i=1}^{J} \lambda_i g_{ri4} \right); \\
\left[1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Nri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Nri}^{+}\right)^{\lambda_i} \right], \\
\left[\prod_{i=1}^{J} \tilde{I}_{Nri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{I}_{Nri}^{+\ \lambda_i} \right], \left[\prod_{i=1}^{J} \tilde{F}_{Nri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{F}_{Nri}^{+\ \lambda_i} \right]
\end{array} \right\rangle, \\
\left\langle \begin{array}{l}
\left(\sum_{i=1}^{J} \lambda_i h_{ri1}, \sum_{i=1}^{J} \lambda_i h_{ri2}, \sum_{i=1}^{J} \lambda_i h_{ri3}, \sum_{i=1}^{J} \lambda_i h_{ri4} \right); \\
\left[1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Lri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{T}_{Lri}^{+}\right)^{\lambda_i} \right], \\
\left[\prod_{i=1}^{J} \tilde{I}_{Lri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{I}_{Lri}^{+\ \lambda_i} \right], \left[\prod_{i=1}^{J} \tilde{F}_{Lri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{F}_{Lri}^{+\ \lambda_i} \right]
\end{array} \right\rangle
\end{array} \right)
\end{aligned} \tag{21}
$$

or

$$
\begin{aligned}
\tilde{a}_r &= C-ITFNNWGA\left(\tilde{a}_{r1}, \tilde{a}_{r2}, ...\tilde{a}_{rJ}\right) = \prod_{i=1}^{J} \tilde{a}_{ri}^{\lambda_i} \\
&= \left(\begin{array}{l}
\left\langle \begin{array}{l}
\left(\prod_{i=1}^{J} g_{ri1}^{\lambda_i}, \prod_{i=1}^{J} g_{ri2}^{\lambda_i}, \prod_{i=1}^{J} g_{ri3}^{\lambda_i}, \prod_{i=1}^{J} g_{ri4}^{\lambda_i} \right); \\
\left[\prod_{i=1}^{J} \tilde{T}_{Nri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{T}_{Nri}^{+\ \lambda_i} \right], \left[1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Nri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Nri}^{+}\right)^{\lambda_i} \right], \\
\left[1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Nri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Nri}^{+}\right)^{\lambda_i} \right]
\end{array} \right\rangle, \\
\left\langle \begin{array}{l}
\left(\prod_{i=1}^{J} h_{ri1}^{\lambda_i}, \prod_{i=1}^{J} h_{ri2}^{\lambda_i}, \prod_{i=1}^{J} h_{ri3}^{\lambda_i}, \prod_{i=1}^{J} h_{ri4}^{\lambda_i} \right); \\
\left[\prod_{i=1}^{J} \tilde{T}_{Lri}^{-\ \lambda_i}, \prod_{i=1}^{J} \tilde{T}_{Lri}^{+\ \lambda_i} \right], \left[1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Lri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{I}_{Lri}^{+}\right)^{\lambda_i} \right], \\
\left[1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Lri}^{-}\right)^{\lambda_i}, 1-\prod_{i=1}^{J}\left(1-\tilde{F}_{Lri}^{+}\right)^{\lambda_i} \right]
\end{array} \right\rangle
\end{array} \right).
\end{aligned} \tag{22}
$$

Step 2: Calculate the score values of the aggregated values $\tilde{a}_r$ by Eq. (10).

Step 3: Sort the alternatives and determine the optimal one with the largest score value.

Step 4: End.

6. Actual DM Example

6.1 DM Case of LCDSs

With the rapid development of China economy in recent years, the scale of engineering activities has become larger and larger, and the problem of landslides has become more and more serious. This section gives an actual DM case of LCDSs to illustrate the feasibility and applicability of the proposed DM model in the C-ITFNN scenario. The terrain of the landslide area is high in the west and low in the east, high in the north and low in the south, generally inclined from west to east, and the terrain fluctuates greatly. Referring to the experience of landslide control, four potential LVDSs are provided for some landslide treatment in Shaoxing City, China, which are indicated by a set of the four alternatives $E = \{e_1, e_2, e_3, e_4\}$. In the scheme e_1, graded slope toes and retaining wall and lattices are used in the central and northern areas, while slope unloading, anti-sliding piles and anchor-cable anti-sliding piles in the southern area. In the scheme e_2, graded slope toes, retaining wall, and anchor rod lattice are used in the central and northern areas, and double-row anti-sliding piles and anchor-cable anti-sliding piles are used in the southern area. In the scheme e_3, graded slope toes and supporting anti-sliding piles are used in the central and northern areas, and anchor-cable anti-sliding piles are used in the southern area. In the scheme e_4, graded slope toes and retaining walls and anchor rod lattices are used in the central and northern areas, and anchor cable anti-sliding piles are used in the southern area. Then, a satisfactory evaluation of the four alternatives is subject to the three important conditions (attributes): technical difficulty (g_1), environmental impact (g_2), and governance cost (g_3). The importance of the three conditions is assigned by the weight vector $\lambda = (0.2, 0.3, 0.5)$.

In the DM issue of LCDSs, experts are invited to give the satisfactory degrees and credibility levels of the four alternatives with respect to the three attributes by the linguistic terms obtained from the set of linguistic terms L = {Very bad, Bad, Fairly bad, Medium, Fairly good, Good, Very good}, then the given linguistic terms are shown in Table 2.

Table 2. Linguistic terms of the satisfactory degrees and credibility levels

	g_1	g_2	g_3
e_1	(B, M)	(FB, G)	(FG, G)
e_2	(G, VG)	(FG, G)	(FB, G)
e_3	(B, FG)	(G, G)	(M, VG)
e_4	(FG, M)	(VB, FG)	(FB, FG)

Thus, the linguistic terms of the satisfactory degrees and credibility levels in Table 2 can be converted into C-ITFNNs in view of the corresponding linguistic values in Table 1, which are constructed as the decision matrix:

$$N = \begin{bmatrix} \begin{pmatrix} \langle (0.2,0.3,0.4,0.5);[0.2,0.3],[0.8,0.9],[0.8,0.9]\rangle, \\ \langle (0.4,0.5,0.6,0.7);[0.5,0.6],[0.4,0.6],[0.4,0.6]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle, \\ \langle (1.0,1.0,1.0,1.0);[0.9,1.0],[0,0.1],[0,0.1]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.2,0.3,0.4,0.5);[0.2,0.3],[0.8,0.9],[0.8,0.9]\rangle, \\ \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle, \\ \langle (0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.4,0.5,0.6,0.7);[0.5,0.6],[0.4,0.6],[0.4,0.6]\rangle, \\ \langle (1.0,1.0,1.0,1.0);[0.9,1.0],[0,0.1],[0,0.1]\rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle, \\ \langle (0.4,0.5,0.6,0.7);[0.5,0.6],[0.4,0.6],[0.4,0.6]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.1,0.1,0.1,0.1);[0.1,0.2],[0.9,1],[0.9,1]\rangle, \\ \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle, \\ \langle (0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle \end{pmatrix} \end{bmatrix}.$$

To deal with the DM problem, we give the following decision process.

Step 1: Utilize the C-ITFNNWAA operator of Eq. (21) and get the aggregated values $\tilde{a}_r$ for e_r (r =1, 2, …, p):

$$\tilde{a}_1 = \begin{pmatrix} \langle (0.38,0.48,0.58,0.68);[0.5293,0.6427],[0.3843,0.5016],[0.3843,0.5016]\rangle, \\ \langle (0.56,0.66,0.76,0.86);[0.7598,0.868],[0.132,0.2491],[0.132,0.2491]\rangle \end{pmatrix},$$

$$\tilde{a}_2 = \begin{pmatrix} \langle (0.42,0.52,0.62,0.72);[0.5774,0.6984],[0.3257,0.4517],[0.3257,0.4517]\rangle, \\ \langle (0.68,0.76,0.84,0.92);[0.8259,1],[0,0.1741],[0,0.1741]\rangle \end{pmatrix},$$

$$\tilde{a}_3 = \begin{pmatrix} \langle (0.42, 0.48, 0.58, 0.68); [0.5827, 0.7048], [0.3031, 0.468], [0.3031, 0.468] \rangle, \\ \langle (0.78, 0.83, 0.88, 0.93); [0.8466, 1], [0, 0.1534], [0, 0.1534] \rangle \end{pmatrix},$$

$$\tilde{a}_4 = \begin{pmatrix} \langle (0.28, 0.48, 0.58, 0.68); [0.3628, 0.4749], [0.5875, 0.703], [0.5875, 0.703] \rangle, \\ \langle (0.48, 0.58, 0.68, 0.78); [0.6677, 0.7703], [0.2297, 0.3446], [0.2297, 0.3446] \rangle \end{pmatrix}.$$

Or use the C-ITFNNWGA operator of Eq. (22) and get the aggregated values $\tilde{a}_r$ for e_r (r =1, 2, …, p):

$$\tilde{a}_1 = \begin{pmatrix} \langle (0.3571, 0.4625, 0.5658, 0.668); [0.4226, 0.5341], [0.5483, 0.6743], [0.5483, 0.6743] \rangle, \\ \langle (0.5533, 0.6544, 0.7553, 0.8559); [0.7282, 0.8299], [0.1701, 0.3036], [0.1701, 0.3036] \rangle \end{pmatrix},$$

$$\tilde{a}_2 = \begin{pmatrix} \langle (0.4017, 0.5052, 0.6076, 0.7093); [0.4707, 0.5792], [0.4984, 0.6157], [0.4984, 0.6157] \rangle, \\ \langle (0.6645, 0.7518, 0.8365, 0.9192); [0.8191, 0.9192], [0.0808, 0.1809], [0.0808, 0.1809] \rangle \end{pmatrix},$$

$$\tilde{a}_3 = \begin{pmatrix} \langle (0.3933, 0.4994, 0.6031, 0.7057); [0.4793, 0.5899], [0.4561, 0.6268], [0.4561, 0.6268] \rangle, \\ \langle (0.7469, 0.8113, 0.8709, 0.9266); [0.8262, 0.9266], [0.0734, 0.1738], [0.0734, 0.1738] \rangle \end{pmatrix},$$

$$\tilde{a}_4 = \begin{pmatrix} \langle (0.239, 0.2862, 0.33, 0.3713); [0.2556, 0.3732], [0.7375, 1], [0.7375, 1] \rangle, \\ \langle (0.4782, 0.5785, 0.6787, 0.7789); [0.6544, 0.7553], [0.2447, 0.3741], [0.2447, 0.3741] \rangle \end{pmatrix}.$$

Step 2: Use Eq. (10) and calculate the score values of the aggregated values $\tilde{a}_r$ (r =1, 2, …, p):

$S(\tilde{a}_1) = 0.1729$, $S(\tilde{a}_2) = 0.2582$, $S(\tilde{a}_3) = 0.2661$, and $S(\tilde{a}_4) = 0.0855$.

Or $S(\tilde{a}_1) = 0.1164$, $S(\tilde{a}_2) = 0.1802$, $S(\tilde{a}_3) = 0.1954$, and $S(\tilde{a}_4) = 0.0258$.

Step 3: Give the sorting of the four LCDSs: $e_3 > e_2 > e_1 > e_4$. Hence, the optimal choice is the scheme e_3.

It can be found that the sorting and optimal choice results obtained by the C-ITFNNWAAA operator and the C-ITFNNWGA operator are consistent.

6.2 Comparison of the Proposed DM Model with Previous DM Models in the Scenarios of C-TFNNs, ITFNNs, TFNNs, and SVNNs

To indicate the efficiency of the proposed DM model in the C-ITFNN scenarios, we compare the DM model proposed in this paper with the previous DM models in the C-TFNN, ITFNN, TFNN and SVNN scenarios. Since the previous DM models cannot perform the DM issue of C-ITFNNs, we only consider the situations of C-TFNNs, ITFNNs, TFNNs, and SVNNs as four special cases of C-ITFNNs for convenient comparison. Therefore, we assume that all interval values in the C-ITFNN decision matrix N are converted into their average values to produce the C-TFNN matrix $N_{C\text{-}TFNN}$:

$$N_{C-TFNN} = \begin{bmatrix} \begin{pmatrix} \langle (0.2, 0.3, 0.4, 0.5); 0.25, 0.85, 0.85 \rangle, \\ \langle (0.4, 0.5, 0.6, 0.7); 0.55, 0.5, 0.5 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3, 0.4, 0.5, 0.6); 0.35, 0.75, 0.75 \rangle, \\ \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle, \\ \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle, \\ \langle (1.0, 1.0, 1.0, 1.0); 0.95, 0.05, 0.05 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle, \\ \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3, 0.4, 0.5, 0.6); 0.35, 0.75, 0.75 \rangle, \\ \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.2, 0.3, 0.4, 0.5); 0.25, 0.85, 0.85 \rangle, \\ \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle, \\ \langle (0.6, 0.7, 0.8, 0.9); 0.85, 0.15, 0.15 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.4, 0.5, 0.6, 0.7); 0.55, 0.5, 0.5 \rangle, \\ \langle (1.0, 1.0, 1.0, 1.0); 0.95, 0.05, 0.05 \rangle \end{pmatrix} \\ \begin{pmatrix} \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle, \\ \langle (0.4, 0.5, 0.6, 0.7); 0.55, 0.5, 0.5 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.1, 0.1, 0.1, 0.1); 0.15, 0.95, 0.95 \rangle, \\ \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle \end{pmatrix} & \begin{pmatrix} \langle (0.3, 0.4, 0.5, 0.6); 0.35, 0.75, 0.75 \rangle, \\ \langle (0.5, 0.6, 0.7, 0.8); 0.75, 0.25, 0.25 \rangle \end{pmatrix} \end{bmatrix}.$$

If we do not consider the credibility levels in *N*, *N* becomes the ITFNN matrix N_{ITFNN}:

$$N_{ITFNN}=\begin{bmatrix}\langle(0.2,0.3,0.4,0.5);[0.2,0.3],[0.8,0.9],[0.8,0.9]\rangle & \langle(0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle & \langle(0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle\\ \langle(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle & \langle(0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle & \langle(0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle\\ \langle(0.2,0.3,0.4,0.5);[0.2,0.3],[0.8,0.9],[0.8,0.9]\rangle & \langle(0.6,0.7,0.8,0.9);[0.8,0.9],[0.1,0.2],[0.1,0.2]\rangle & \langle(0.4,0.5,0.6,0.7);[0.5,0.6],[0.4,0.6],[0.4,0.6]\rangle\\ \langle(0.5,0.6,0.7,0.8);[0.7,0.8],[0.2,0.3],[0.2,0.3]\rangle & \langle(0.1,0.1,0.1,0.1);[0.1,0.2],[0.9,1],[0.9,1]\rangle & \langle(0.3,0.4,0.5,0.6);[0.3,0.4],[0.7,0.8],[0.7,0.8]\rangle\end{bmatrix}.$$

If we do not consider the credibility levels in $N_{C\text{-}TFNN}$, $N_{C\text{-}TFNN}$ becomes the TFNN matrix N_{TFNN}:

$$N_{TFNN}=\begin{bmatrix}((0.2,0.3,0.4,0.5);0.25,0.85,0.85) & ((0.3,0.4,0.5,0.6);0.35,0.75,0.75) & ((0.5,0.6,0.7,0.8);0.75,0.25,0.25)\\ ((0.6,0.7,0.8,0.9);0.85,0.15,0.15) & ((0.5,0.6,0.7,0.8);0.75,0.25,0.25) & ((0.3,0.4,0.5,0.6);0.35,0.75,0.75)\\ ((0.2,0.3,0.4,0.5);0.25,0.85,0.85) & ((0.6,0.7,0.8,0.9);0.85,0.15,0.15) & ((0.4,0.5,0.6,0.7);0.55,0.5,0.5)\\ ((0.5,0.6,0.7,0.8);0.75,0.25,0.25) & ((0.1,0.1,0.1,0.1);0.15,0.95,0.95) & ((0.3,0.4,0.5,0.6);0.35,0.75,0.75)\end{bmatrix}.$$

If we do not consider TFNs in N_{TFNN}, N_{TFNN} becomes the SVNN matrix N_{SVNN}:

$$N_{SVNN}=\begin{bmatrix}(0.25,0.85,0.85) & (0.35,0.75,0.75) & (0.75,0.25,0.25)\\ (0.85,0.15,0.15) & (0.75,0.25,0.25) & (0.35,0.75,0.75)\\ (0.25,0.85,0.85) & (0.85,0.15,0.15) & (0.55,0.5,0.5)\\ (0.75,0.25,0.25) & (0.15,0.95,0.95) & (0.35,0.75,0.75)\end{bmatrix}.$$

In the scenarios of C-TFNNs, ITFNNs, TFNNs, and SVNNs, we apply Eqs. (1), (2), (18), (19), (4), (5), (7), and (8) for the decision matrices of C-TFNNs, ITFNNs, TFNNs, and SVNNs to obtain their aggregated values. Then, the score values of their aggregated values are obtained by the score functions of Eqs. (3), (6), (9), and (20) in the corresponding scenarios. For clear comparison, all decision results are given in Table 3.

Table 3. Decision results based on different DM models in the scenarios of SVNNs, TFNNs, ITFNNs, C-TFNNs, and C-ITFNNs

Method	Score value	Sorting	Optimal one
DM model using the SVNNWAA operator [19]	0.5657,0.618,0.6223,0.3752	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the SVNNWGA operator [19]	0.4203,0.4711,0.4856,0.2335	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the TFNNWAA operator [30]	0.2998, 0.3523,0.336,0.1895	$e_2 > e_3 > e_1 > e_4$	e_2
DM model using the TFNNWGA operator [30]	0.2158,0.2619,0.2673,0.0716	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the ITFNNWAA operator [34]	0.3004,0.3535,0.3371,0.1899	$e_2 > e_3 > e_1 > e_4$	e_2
DM model using the ITFNNWAA operator [34]	0.2149,0.2615,0.2663,0.059	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the C-TFNNWAA operator [37]	0.1725,0.2479,0.2597,0.0853	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the C-TFNNWGA operator [37]	0.1170,0.1805,0.1965,0.0313	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the C-ITFNNWAA operator	0.1729,0.2582,0.2661,0.0855	$e_3 > e_2 > e_1 > e_4$	e_3
DM model using the C-ITFNNWGA operator	0.1164,0.1802,0.1954,0.0258	$e_3 > e_2 > e_1 > e_4$	e_3

From the sorting results in Table 3, it can be seen that there are differences in the DM results obtained based on different aggregation operators of SVNNs, TFNNs, C-TFNNs, and C-ITFNNs. In the cases of TFNNs and ITFNNs, the optimal schemes obtained by different aggregation operators are inconsistent, where the optimal scheme obtained by using the weighted arithmetic averaging operators is e_2 and the optimal scheme obtained by using the weighted geometric averaging operators is e_3. Moreover, the optimal DM result obtained in the scenarios of SVNNs, C-TFNNs and C-ITFNNs is e_3. However, the level of credibility plays a key role in the sorting of the alternatives because it can ensure the credibility of the assessment information of TFNNs and ITFNNs. The previous DM models with SVNNs, TFNNs, ITFNNs [19, 30, 34] may result in unreasonable/distorted DM results because they are difficult to ensure the credibility of SVNNs, TFNNs, and ITFNNs. In addition, in the scenarios of C-TFNNs and C-ITFNNs, the proposed DM model of C-ITFNNs obtains the same DM results as the DM model of C-TFNNs [37], which also proves the rationality and efficiency of the proposed DM model in the scenario of C-ITFNNs. The reason is that the C-TFNN matrix obtained by taking the average value of the interval values in C-ITFNNs is only a special case of the C-ITFNN matrix. Therefore, it can be seen that the proposed DM model of C-ITFNNs generalizes the previous DM model of C-TFNNs [37], while the previous DM model of C-TFNNs [37] is only a special case of the proposed DM model of C-ITFNNs. In general, the proposed DM model of C-ITFNNs makes DM applications more general and practical, demonstrating the clear advantages in the setting of C-ITFNNs.

7. Conclusion

As an extension of C-TFNNs, this paper first proposed C-ITFNNs in view of ITFNNs and credibility levels, which are expressed by an ordered pair of ITFNNs. Then, we defined some operational laws of C-ITFNNs and the score function of C-ITFNN and presented the C-ITFNNWAA and C-ITFNNWGA operators and their properties. Furthermore, the C-ITFNNWAA and C-ITFNNWGA operators and the score function were used for a multi-attribute DM model of C-ITFNNs. Lastly, the proposed DM model was applied to the DM case of LCDSs in the scenario of C-ITFNNs and verified its feasibility. By comparative analysis of the different DM models in the scenarios of C-ITFNNs, C-TFNNs, TFNNs, and SVNNs, the proposed DM model revealed the superiority of DM generalization in the scenario of C-ITFNNs since the previous DM models are only the special cases of the proposed DM model of C-ITFNNs.

Generally, the information representation, operation and DM techniques of C-ITFNNs reveal their original contributions in this study. Then, the main superiority of our new DM model is that it not only compensates for the gap of existing DM models, but also is more reliable and versatile than existing DM models. As future research, the techniques proposed in this paper can be extended to slope stability/risk assessment, mine risk/safety assessment, and image processing in a C-ITFNN circumstance.

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References

1. Zadeh, L.A. Fuzzy sets. *Information and Control* **1965**, 8(3), 338–353.
2. Turksen, I.B. Interval valued fuzzy sets based on normal forms. *Fuzzy sets and systems* **1986**, 20(2), 191–210.
3. Intanssov, K. Intuitionistic fuzzy set. *Fuzzy Sets and Systems* 1986, 20 (1), 87–96.
4. Atanassov, K.T.; Gargov, G. Interval valued intuitionistic fuzzy sets. *Fuzzy Sets and Systems* **1989**, 31(3), 343–349.
5. Smarandache, F. *Neutrosophy: Neutrosophic probability, set, and logic*. American Research Press, Rehoboth, DE, USA, **1998**.

6. Junaid, M.; Xue, Y.; Syed, M. W.; Li, J. Z.; Ziaullah, M. A neutrosophic ahp and topsis framework for supply chain risk assessment in automotive industry of Pakistan. *Sustainability* **2019,** 12(1), 154.
7. Ayber, S.; Erginel, N. In Developing the neutrosophic fuzzy FMEA method as evaluating risk assessment tool, *International Conference on Intelligent and Fuzzy Systems*, Springer: **2019**, pp 1130–1137.
8. Guo, Y.; Sengur, A. A novel color image segmentation approach based on neutrosophic set and modified fuzzy c-means. *Circuits, Systems, and Signal Processing* **2013,** 32(4), 1699–1723.
9. Lina Espinioza Nery , María de los Angeles Galarza Pazmiño , Diana Jordan Fiallos , Said Broumi, Analysis of the Success Factors of the Quality of E-learning in the Medical School in a Neutrosophic Environment, International Journal of Neutrosophic Science, Vol. 18 , No. 3 , (2022) : 189-198
10. Singh, P. A neutrosophic-entropy based adaptive thresholding segmentation algorithm: A special application in MR images of Parkinson's disease. *Artificial Intelligence in Medicine* **2020,** 104, 101838.
11. Eroğlu, H.; Şahin, R. A neutrosophic VIKOR method-based decision-making with an improved distance measure and score function: case study of selection for renewable energy alternatives. *Cognitive Computation* **2020,** 12(6), 1338–1355.
12. Das, S.; Roy, B.K.; Kar, M.B.; Kar, S.; Pamučar, D. Neutrosophic fuzzy set and its application in decision making. *Journal of Ambient Intelligence and Humanized Computing* **2020,** 11(11), 5017–5029.
13. Khan, M.; Son, L.H.; Ali, M.; Chau, H.T.M.; Na, N.T.N.; Smarandache, F. Systematic review of decision making algorithms in extended neutrosophic sets. *Symmetry* **2018,** 10(8), 314.
14. Majumdar, P. Neutrosophic sets and its applications to decision making. *In Computational intelligence for big data analysis*, Springer **2015**, pp 97–115.
15. Wang, H.; Smarandache, F.; Sunderraman, R.; Zhang, Y.Q. *Interval neutrosophic sets and logic: Theory and applications in computing*. Hexis, Phoenix, AZ, **2005**.
16. Wang, H.; Smarandache, F.; Zhang, Y.; Sunderraman, R. Single valued neutrosophic sets. *Multispace Multistructure* **2010**, 4, 410–413.
17. Jana, C.; Pal, M. Multi-criteria decision making process based on some single-valued neutrosophic Dombi power aggregation operators. *Soft Computing* **2021,** 25(7), 5055–5072.
18. Juanjuan Ding , Wenhui Bai , Chao Zhang, A New Multi-Attribute Decision Making Method with Single-Valued Neutrosophic Graphs, International Journal of Neutrosophic Science, vol. 11 , No.2 , (2020) : 76-86
19. Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *International journal of systems science* **2016,** 47(10), 2342–2358.
20. Garg, H. Non-linear programming method for multi-criteria decision making problems under interval neutrosophic set environment. *Applied Intelligence* **2018,** 48(8), 2199–2213.
21. Yang, H.; Wang, X.; Qin, K. New similarity and entropy measures of interval neutrosophic sets with applications in multi-attribute decision-making. *Symmetry* **2019**, 11(3), 370.
22. Başhan, V.; Demirel, H.; Gul, M. An FMEA-based TOPSIS approach under single valued neutrosophic sets for maritime risk evaluation: the case of ship navigation safety. *Soft Computing* **2020,** 24(24), 18749–18764.
23. Gulum, P.; Ayyildiz, E.; Gumus, A.T. A two level interval valued neutrosophic AHP integrated TOPSIS methodology for post-earthquake fire risk assessment: An application for Istanbul. *International Journal of Disaster Risk Reduction* **2021,** 61, 102330.
24. Shahzadi, G.; Akram, M.; Saeid, A.B. An application of single-valued neutrosophic sets in medical diagnosis. *Neutrosophic sets and systems* **2017,** 18, 80–88.
25. Norzieha Mustapha , Suriana Alias , Roliza Md Yasin , Nurnisa Nasuha Mohd Yusof , Nurul Najiha Fakhrarazi , Nik Nur Aisyah Nik Hassan, New Entropy Measure Concept for Single Value Neutrosophic Sets with Application in Medical Diagnosis, International Journal of Neutrosophic Science, Vol. 19 , No. 1 , (2022) : 375-383
26. Broumi, S.; Deli, I.; Smarandache, F. N-valued interval neutrosophic sets and their application in medical diagnosis. *Critical Review, Center for Mathematics of Uncertainty, Creighton University, Omaha, NE, USA* **2015,** 10, 45–69.
27. Wang, J.Q; Zhang, Z. Aggregation operators on intuitionistic trapezoidal fuzzy number and its application to multi-criteria decision making problems. *Journal of Systems Engineering and Electronics* **2009,** 20(2), 321–326.

28. Wan, S.; Dong, J. Multi-attribute group decision making with trapezoidal intuitionistic fuzzy numbers and application to stock selection. *Informatica* **2014**, 25(4), 663–697.
29. Li, J.; Zeng, W.; Guo, P. *Interval-valued intuitionistic trapezoidal fuzzy number and its application*, In 2014 IEEE international conference on systems, man, and cybernetics (SMC), IEEE **2014**; pp 734–737.
30. Ye, J. Some weighted aggregation operators of trapezoidal neutrosophic numbers and their multiple attribute decision making method. *Informatica* **2017,** 28(2), 387–402.
31. Biswas, P.; Pramanik, S.; Giri, B.C. Distance measure based MADM strategy with interval trapezoidal neutrosophic numbers. *Neutrosophic Sets and Systems* **2018,** 19, 40–46.
32. Biswas, P.; Pramanik, S.; Giri, B.C. TOPSIS strategy for multi-attribute decision making with trapezoidal neutrosophic numbers. *Neutrosophic Sets and Systems* **2018,** 19, 29–39.
33. Jana, C.; Karaaslan, F. *Dice and Jaccard similarity measures based on expected intervals of trapezoidal neutrosophic fuzzy numbers and their applications in multicriteria decision making*. Optimization Theory Based on Neutrosophic and Plithogenic Sets, Elsevier, **2020**; pp 261–287.
34. Jana, C.; Pal, M.; Karaaslan, F.; Wang, J. Trapezoidal neutrosophic aggregation operators and their application to the multi-attribute decision-making process. *Scientia Iranica. Transaction E, Industrial Engineering* **2020,** 27(3), 1655–1673.
35. Deli, I.; Şubaş, Y. Some weighted geometric operators with SVTrN-numbers and their application to multi-criteria decision making problems. *Journal of Intelligent & Fuzzy Systems* **2017,** 32(1), 291–301.
36. Ye, J.; Song, J.; Du, S.; Yong, R. Weighted aggregation operators of fuzzy credibility numbers and their decision-making approach for slope design schemes. *Computational and Applied Mathematics* **2021,** 40(4), 1–14.
37. Ye, J.; Du, S.; Yong, R. Some aggregation operators of credibility trapezoidal fuzzy neutrosophic values and their decision-making application in the selection of slope design schemes. *Journal of Intelligent & Fuzzy Systems* **2022**, 43(3), 2809–2817.

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