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TOPSIS Method-Based Decision-Making Model of Simplified Neutrosophic Indeterminate Sets for Teaching Quality Evaluation

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Abstract: Recently, colleges/universities have paid a lot of attention to the teaching quality evaluation (TQE) of teachers in China. TQE is an essential way to improve teachers' teaching ability and quality in the teaching process. Then, the TQE of teaching supervisors is a multi-attribute decision-making (MADM) problem with vague, inconsistent, and indeterminate information. The simplified neutrosophic indeterminate set/element (SNIS/SNIE) is an appropriate form to express the indeterminate decision-making information in the TQE process. Therefore, this article presents an improved ranking method based on maximizing deviations principle and technique for order of preference by similarity (TOPSIS) for SNIS and applies it to evaluate teachers' teaching quality. First, the Hamming distance between two SNIEs is defined. Then, attribute weights are obtained by maximizing deviation method and the TOPSIS method-based decision-making model is developed for the MADM applications with unknown attribute weights. Finally, we perform the developed MADM model for a TQE case and compare it with existing related models to indicate the feasibility and rationality of the proposed model with unknown attribute weights in the SNIE circumstance.

Keywords: simplified neutrosophic indeterminate set; maximizing deviation; TOPSIS method; teaching quality evaluation

1. Introduction

Cultivating qualified talents is the central task of colleges/universities. During the talent training process, the teaching quality evaluation (TQE) is a key task. Then, TQE is one of the main tasks of teaching administration in various colleges/universities. A teaching evaluation system includes two aspects: evaluation framework and evaluation method. However, TQE is a multi-attribute decision-making (MADM) problem, which implies vagueness and uncertainty in the evaluation process. In recent years, various fuzzy evaluation methods have been applied to TQE [1–5].

Recently, neutrosophic set (NS) [6] has become the most popular topic for describing indeterminate and inconsistent information. The true, indeterminate, and false membership functions in NS are independent components. Compared with a fuzzy set (FS) [7] and an intuitionistic FS (IFS) [8, 9], NS can be used to express the corresponding inconsistent, indeterminate, and incomplete information in real decision-making (DM) problems. In practical applications, NS has been simplified into many forms, for example, single-valued NS (SvNS) [10], interval NS (INS) [11], and simplified NS (SNS) [12]. They are widely used in engineering and science fields. However, the true, indeterminate, and false membership degrees are specified by single values or interval values in SNS. In complicated MADM problems, the true, indeterminate, and false membership degrees may be partly certain and partly uncertain. In this case, a neutrosophic number (NN) [13] can describe them by $p = \rho + \mu\xi$ for $\rho, \mu \in \Re$ and $\xi \in [\xi^-, \xi^+]$, where ρ is the certain term and $\mu\xi$ is the uncertain term. Du et al. [14] put forward a simplified neutrosophic indeterminate set/element (SNIS/SNIE) combining SNS with NN. Each SNIE consists of the true NN, the false NN, and the indeterminate NN. SNIS can be transformed to SvNS or INS according to $\xi^-= \xi^+$ or $\xi^- \neq \xi^+$ with the value/range of indeterminacy $\xi \in [\xi^-, \xi^+]$.

The TOPSIS method proposed by Hwang and Yoon [15] is a kind of distance-based rank method. Better choices are closer to positive ideals and farther away from negative ideals. Then, it has been applied to various fuzzy DM environments. For example, the TOPSIS method was used to solve the supplier selection problem in intuitionistic fuzzy environment [16]. Later, many researchers [17–20] developed the DM methods using TOPSIS methods in hesitant FS, IFS, and interval-valued IFS environments. In indeterminate and inconsistent circumstances, Sahin and Yiider [21] introduced a modified TOPSIS method with SvNSs for the group DM. Chi and Liu [22] extended the TOPSIS method to the INS environment.

Although some researchers have developed several operational rules for SNIEs and SNIS. Existing SNIS decision-making (DM) methods [14, 23, 24] used a specified weight vector of attributes to solve the MADM problems of SNISs. So far, no researchers consider the influence of indeterminate degrees on attribute weights in MADM problems of SNISs. In a real DM situation, the weights of attributes may be indeterminate or unknown. In this article, we extend a MADM model which combines the determining method of unknown attribute weights with the TOPSIS method of SNISs and use it for TQE. The rest of the article is as follows.

In Section 2, the Hamming distance of SNIEs is introduced. Section 3 presents a method for determining unknown attribute weights and an extended TOPSIS method for SNISEs. In Section 4, we apply the proposed model to a TQE case and analyze the influence of indeterminate ranges in SNISEs on decision results. Then, the extend TOPSIS method is compared with the related models in Section 5. The article is summarized in Section 6.

2. Distance of SNIEs

This section presents the Hamming distance between SNIEs and its properties.

First, we introduce the notions of SNIS and SNIE [14].

Definition 1 [14]. Let S = {$s_1, s_2, \ldots, s_n$} be a universe set. A SNIS B in S is described as $B = \{<s_k, P(s_k, \xi), N(s_k, \xi), Q(s_k, \xi)> | s_k \in S\}$, where $P(s_k, \xi) = \rho_k+\mu_k\xi \subseteq [0, 1]$, $N(s_k, \xi) = \delta_k+\Phi_k\xi \subseteq [0, 1]$, and $Q(s_k, \xi) = \lambda_k+\nu_k\xi \subseteq [0, 1]$ for $s_k \in S$ (k = 1, 2, …, n) and $\xi \in [\xi^-, \xi^+]$. Then, $P(s_k, \xi)$, $N(s_k, \xi)$, and $Q(s_k, \xi)$ are the true NN, the indeterminate NN, and the false NN. Each component $<s_k, P(s_k, \xi), N(s_k, \xi), Q(s_k, \xi)>$ in B is called SNIE, which can be represented as the simple form $b_k = <P_k(\xi), N_k(\xi), Q_k(\xi)> = <\rho_k+\mu_k\xi, \delta_k+\Phi_k\xi, \lambda_k+\nu_k\xi>$.

Then, we present the Hamming distance of SNIEs below.

Definition 2. Suppose that $b_1 = <P_1(\xi), N_1(\xi), Q_1(\xi)> = <\rho_1+\mu_1\xi, \delta_1+\Phi_1\xi, \lambda_1+\nu_1\xi>$ and $b_2 = <P_2(\xi), N_2(\xi), Q_2(\xi)> = <\rho_2+\mu_2\xi, \delta_2+\Phi_2\xi, \lambda_2+\nu_2\xi>$ are two SNIEs for $\xi \in [\xi^-, \xi^+]$. Thus, the Hamming distance between b_1 and b_2 are defined as follows:

$$\begin{aligned} l_H(b_1,b_2) &= \frac{1}{6}\begin{pmatrix} \left|P_1(\xi^-) - P_2(\xi^-)\right| + \left|P_1(\xi^+) - P_2(\xi^+)\right| + \left|N_1(\xi^-) - N_2(\xi^-)\right| + \\ \left|N_1(\xi^+) - N_2(\xi^+)\right| + \left|Q_1(\xi^-) - Q_2(\xi^-)\right| + \left|Q_1(\xi^+) - Q_2(\xi^+)\right| \end{pmatrix} \\ &= \frac{1}{6}\begin{pmatrix} \left|\rho_1 - \rho_2 + (\mu_1 - \mu_2)\xi^-\right| + \left|\rho_1 - \rho_2 + (\mu_1 - \mu_2)\xi^+\right| + \left|\delta_1 - \delta_2 + (\phi_1 - \phi_2)\xi^-\right| + \\ \left|\delta_1 - \delta_2 + (\phi_1 - \phi_2)\xi^+\right| + \left|\lambda_1 - \lambda_2 + (\nu_1 - \nu_2)\xi^-\right| + \left|\lambda_1 - \lambda_2 + (\nu_1 - \nu_2)\xi^+\right| \end{pmatrix} \end{aligned} \tag{1}$$

Proposition 1. Set $b_1 = <P_1(\xi), N_1(\xi), Q_1(\xi)> = <\rho_1 + \mu_1\xi, \delta_1 + \Phi_1\xi, \lambda_1 + \nu_1\xi>$, $b_2 = <P_2(\xi), N_2(\xi), Q_2(\xi)> = <\rho_2 + \mu_2\xi, \delta_2 + \Phi_2\xi, \lambda_2 + \nu_2\xi>$, and $b_3 = <P_3(\xi), N_3(\xi), Q_3(\xi)> = <\rho_3 + \mu_3\xi, \delta_3 + \Phi_3\xi, \lambda_3 + \nu_3\xi>$ as three SNIEs for $\xi \in [\xi^-, \xi^+]$. The Hamming distance between them meets the following properties:

(1) $0 \le l_H(b_1, b_2) \le 1$;

(2) $l_H(b_1, b_2) = l_H(b_2, b_1)$;

(3) $l_H(b_1, b_3) \le l_H(b_1, b_2) + l_H(b_2, b_3)$.

Proof:

(1) Since $P(\xi), N(\xi), Q(\xi) \subseteq [0, 1]$, $|P_1(\xi^+) - P_2(\xi^+)|$, $|P_1(\xi^-) - P_2(\xi^-)|$, $|N_1(\xi^+) - N_2(\xi^+)|$, $|N_1(\xi^-) - N_2(\xi^-)|$, $|Q_1(\xi^+) - Q_2(\xi^+)|$, $|Q_1(\xi^-) - Q_2(\xi^-)| \subseteq [0, 1]$, then there is $0 \le l_H(b_1, b_2) \le 1$.

(2) The proof is obvious.

(3) Since there is the following inequality:

$$\begin{aligned} l_H(b_1,b_3) &= \frac{1}{6}\begin{pmatrix} \left|P_1(\xi^-) - P_3(\xi^-)\right| + \left|P_1(\xi^+) - P_3(\xi^+)\right| + \left|N_1(\xi^-) - N_3(\xi^-)\right| + \\ \left|N_1(\xi^+) - N_3(\xi^+)\right| + \left|Q_1(\xi^-) - Q_3(\xi^-)\right| + \left|Q_1(\xi^+) - Q_3(\xi^+)\right| \end{pmatrix} \\ &= \frac{1}{6}\begin{pmatrix} \left|P_1(\xi^-) - P_2(\xi^-) + P_2(\xi^-) - P_3(\xi^-)\right| + \left|P_1(\xi^+) - P_2(\xi^+) + P_2(\xi^+) - P_3(\xi^+)\right| + \\ \left|N_1(\xi^-) - N_2(\xi^-) + N_2(\xi^-) - N_3(\xi^-)\right| + \left|N_1(\xi^+) - N_2(\xi^+) + N_2(\xi^+) - N_3(\xi^+)\right| + \\ \left|Q_1(\xi^-) - Q_2(\xi^-) + Q_2(\xi^-) - Q_3(\xi^-)\right| + \left|Q_1(\xi^+) - Q_2(\xi^+) + Q_2(\xi^+) - Q_3(\xi^+)\right| \end{pmatrix} \\ &\le \frac{1}{6}\begin{pmatrix} \left|P_1(\xi^-) - P_2(\xi^-)\right| + \left|P_2(\xi^-) - P_3(\xi^-)\right| + \left|P_1(\xi^+) - P_2(\xi^+)\right| + \left|P_2(\xi^+) - P_3(\xi^+)\right| + \\ \left|N_1(\xi^-) - N_2(\xi^-)\right| + \left|N_2(\xi^-) - N_3(\xi^-)\right| + \left|N_1(\xi^+) - N_2(\xi^+)\right| + \left|N_2(\xi^+) - N_3(\xi^+)\right| + \\ \left|Q_1(\xi^-) - Q_2(\xi^-)\right| + \left|Q_2(\xi^-) - Q_3(\xi^-)\right| + \left|Q_1(\xi^+) - Q_2(\xi^+)\right| + \left|Q_2(\xi^+) - Q_3(\xi^+)\right| \end{pmatrix} \\ &= \frac{1}{6}\begin{pmatrix} \left|P_1(\xi^-) - P_2(\xi^-)\right| + \left|P_1(\xi^+) - P_2(\xi^+)\right| + \left|N_1(\xi^-) - N_2(\xi^-)\right| + \left|N_1(\xi^+) - N_2(\xi^+)\right| \\ + \left|Q_1(\xi^-) - Q_2(\xi^-)\right| + \left|Q_1(\xi^+) - Q_2(\xi^+)\right| \end{pmatrix} \\ &\quad + \frac{1}{6}\begin{pmatrix} \left|P_2(\xi^-) - P_3(\xi^-)\right| + \left|P_2(\xi^+) - P_3(\xi^+)\right| + \left|N_2(\xi^-) - N_3(\xi^-)\right| + \left|N_2(\xi^+) - N_3(\xi^+)\right| + \\ \left|Q_2(\xi^-) - Q_3(\xi^-)\right| + \left|Q_2(\xi^+) - Q_3(\xi^+)\right| \end{pmatrix} \\ &= l_H(b_1,b_2) + l_H(b_2,b_3) \end{aligned}$$

Thus $l_H(b_1, b_3) \le l_H(b_1, b_2) + l_H(b_2, b_3)$ holds.

3. An Extended TOPSIS Method for SNIEs

For a MADM problem, suppose that $E = \{E_1, E_2, \ldots, E_m\}$ and $C = \{C_1, C_2, \ldots, C_n\}$ are a set of alternatives and a set of attributes, respectively. But the attribute weight vector $\beta = \{\beta_1, \beta_2, \ldots, \beta_n\}$ is unknown for $\sum_{j=1}^{n}\beta_j = 1$ and $\beta_j \in [0,1]$. The decision matrix is $B = [b_{ij}]_{m\times n}$, where $b_{ij} = <P_{ij}(\xi), N_{ij}(\xi), Q_{ij}(\xi)> = <\rho_{ij}+\mu_{ij}\xi, \delta_{ij}+\Phi_{ij}\xi, \lambda_{ij}+\nu_{ij}\xi>$ is SNIE as the assessment value of the alternative E_i on the attribute C_j. The MADM method is described by the following steps.

Step 1. Normalize SNIEs for different attribute types.

In the DM problems, attributes were classified as benefit and cost. For the benefit type, it is better with a higher attribute value; For the cost type, it is worse with a higher attribute value. We generally adopt the beneficial type in most DM situations. If the attribute C_j is the cost type, the SNIE b_{ij} needs to be converted to its complement $\overline{b}_{ij}$ by $\overline{b}_{ij}$ = <[$\lambda_{ij}+\nu_{ij}\xi^-$, $\lambda_{ij}+\nu_{ij}\xi^+$], [1−($\delta_{ij}+\Phi_{ij}\xi^+$), 1−($\delta_{ij}+\Phi_{ij}\xi^-$)], [$\rho_{ij}+\mu_{ij}\xi^-$, $\rho_{ij}+\mu_{ij}\xi^+$]>.

Step 2. Determine attribute weights by the maximizing deviation model.

When the attribute weight information is incomplete or completely unknown in MADM problems. The maximizing deviation model [25] is a commonly used method to determine attribute weights. The model is based on the following principle. In MADM, the evaluation values of all alternatives for each attribute are generally different. For an attribute, the difference between the assessment values of all alternatives demonstrates the importance of the attribute. The greater the deviation between attribute values, the greater influence this attribute will have on the ranking of alternatives. Thus, this attribute is set as a higher weight. On the contrary, the smaller the deviation between attribute values, the lower the weight.

We determine the weight vector in view of the following steps:

(i) Define the deviation of E_i (i = 1, 2, …, m) to all alternatives for the attribute C_j (j = 1, 2, …, n) by

$$L_{ij}\left(\beta_j\right)=\sum_{k=1}^{m} l_H(b_{ij},b_{kj})\beta_j \tag{2}$$

(ii) Define the deviation of all the alternatives for the attribute C_j for (j = 1, 2, …, n) by

$$L_j\left(\beta_j\right)=\sum_{i=1}^{m} L_{ij}\left(\beta_j\right)=\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij},b_{kj})\beta_j \tag{3}$$

(iii) Define the deviation for all attributes by

$$L\left(\beta_j\right)=\sum_{j=1}^{n} L_j\left(\beta_j\right)=\sum_{j=1}^{n}\sum_{i=1}^{m} L_{ij}\left(\beta_j\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij},b_{kj})\beta_j \tag{4}$$

(iv) Construct a model that maximizes all deviations to determine the weights by

$$\max L\left(\beta_j\right)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij},b_{kj})\beta_j \tag{5}$$

$$s.t\ \begin{cases}\sum_{j=1}^{n}\beta_j^2=1\\ 0\le\beta_j\le 1,\ j=1,2,\ldots n\end{cases}$$

(v) Construct a Lagrange function by

$$S(\beta_j,\varphi)=\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij},b_{kj})\beta_j+\frac{\varphi}{2}(\sum_{j=1}^{n}\beta_j^2-1) \tag{6}$$

(vi) Take the partial derivative with respect to β_j and φ by

$$\frac{\partial S(\beta_j,\varphi)}{\partial\beta_j}=\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij},b_{kj})+\varphi\beta_j=0$$

$$\frac{\partial S(\beta_j,\varphi)}{\partial\varphi}=\frac{1}{2}(\sum_{j=1}^{n}\beta_j^2-1)=0$$

$$\beta_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij}, b_{kj})}{\sqrt{\sum_{j=1}^{n}\left(\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij}, b_{kj})\right)^2}} \tag{7}$$

(vii) Normalize the attribute weights by

$$\beta_j = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij}, b_{kj})}{\sum_{j=1}^{n}\sum_{i=1}^{m}\sum_{k=1}^{m} l_H(b_{ij}, b_{kj})} \tag{8}$$

Step 3. Rank all alternatives with the TOPSIS method.

(i) Determine two sets of positive and negative ideal solutions $b^+ = \{b_1^+, b_2^+, ..., b_n^+\}$ and $b^- = \{b_1^-, b_2^-, ..., b_n^-\}$ by the following equations:

$$b_j^+ = <\left[\max_i P_{ij}(\xi^-), \max_i P_{ij}(\xi^+)\right], \left[\min_i N_{ij}(\xi^-), \min_i N_{ij}(\xi^+)\right], \left[\min_i Q_{ij}(\xi^-), \min_i Q_{ij}(\xi^+)\right]>, \tag{9}$$

$$b_j^- = <\left[\min_i P_{ij}(\xi^-), \min_i P_{ij}(\xi^+)\right], \left[\max_i N_{ij}(\xi^-), \max_i N_{ij}(\xi^+)\right], \left[\max_i Q_{ij}(\xi^-), \max_i Q_{ij}(\xi^+)\right]> \tag{10}$$

(ii) Calculate the weighted distances of all alternatives to the positive ideal set $h^+ = \{h_1^+, h_2^+, ..., h_m^+\}$ and the weighted distances of all alternatives to the negative ideal set $h^- = \{h_1^-, h_2^-, ..., h_m^-\}$ by

$$\begin{cases} h_i^+ = \sum_{j=1}^{n} \beta_j l_H(b_{ij}, b_j^+) \\ h_i^- = \sum_{j=1}^{n} \beta_j l_H(b_{ij}, b_j^-) \end{cases} \tag{11}$$

(iii) Calculate the correlation coefficient h_i for each alternative by

$$h_i = \frac{h_i^-}{h_i^+ + h_i^-} \tag{12}$$

(iv) Rank alternatives

Alternatives are ranked according to their correlation coefficient values.

4. An Indeterminate DM Case about TQE

4.1 Problem Description of a TQE Case

This study is to apply the proposed TOPSIS method to TQE in the teaching assessment of teachers. Teaching skill competitions are often held in universities to promote teaching quality and improve teaching level. Teachers participating in the competition hold open classes based on their courses. In China, each university usually establishes a teaching evaluation system and specifies a group of teaching experts as an assessment committee. In the assessment process, teaching evaluation system commonly includes the attributes/criteria of teaching content, method, and attitude. Each attribute is briefly described as follows.

The teaching content means the rationality of teaching design. An excellent teaching design can strengthen the key points and important knowledge of the teaching content, which should be related to the knowledge levels and the learning ability of students.

The teaching method reflects that heuristic teaching is adopted. The teachers focus on teaching feedback and ability cultivation. Various teaching means are used in the classroom to maintain a good classroom atmosphere.

The teaching attitude is reflected in good appearance, good manners and fluent teaching language. Teachers must strictly adhere to the teaching norms. Teachers must be strict with students and manage classroom order well.

In the TQE case, the School of Mechanical and Electrical Engineering of Shaoxing University in China will offer teaching excellence awards to outstanding teachers among the four finalists E_1, E_2, E_3 and E_4 (the four alternatives) in the final round. Teaching supervisors observe their teaching one time in any class. Experts give their assessments according to the three attributes: C_1 (the teaching content design), C_2 (the teaching method), and C_3 (the teaching attitude). The teaching experts were invited to assess each teacher participating in the competition process. However, since the weights of the three attributes are not specified, they are unknown. Then, they give the assessment values of the SNIEs b_{ij} = $<P_{ij}(\xi), N_{ij}(\xi), Q_{ij}(\xi)>$ = $<\rho_{ij}+\mu_{ij}\xi, \delta_{ij}+\Phi_{ij}\xi, \lambda_{ij}+\nu_{ij}\xi>$ (i = 1, 2, 3, 4; j = 1, 2, 3) for $\xi \in [0, 1.5]$. The decision matrix is indicated as below.

$$B=\begin{bmatrix} <0.7+0.2\xi, 0.2+0.1\xi, 0.2+0.2\xi> & <0.7+0.2\xi, 0.1+0.3\xi, 0.1+0.1\xi> & <0.6+0.2\xi, 0.2+0.2\xi, 0.2+0.2\xi> \\ <0.7+0.2\xi, 0.2+0.1\xi, 0.3+0.1\xi> & <0.8+0.1\xi, 0.1+0.2\xi, 0.1+0.3\xi> & <0.7+0.1\xi, 0.2+0.2\xi, 0.1+0.1\xi> \\ <0.8+0.1\xi, 0.2+0.1\xi, 0.1+0.2\xi> & <0.7+0.1\xi, 0.2+0.1\xi, 0.1+0.2\xi> & <0.7+0.2\xi, 0.3+0.1\xi, 0.2+0.1\xi> \\ <0.7+0.1\xi, 0.1+0.2\xi, 0.2+0.1\xi> & <0.8+0.1\xi, 0.1+0.2\xi, 0.2+0.1\xi> & <0.7+0.1\xi, 0.2+0.1\xi, 0.2+0.2\xi> \end{bmatrix}$$

4.2 Ranking the Alternatives

The proposed TOPSIS method is applied to the TQE case and gives the following steps.

Step 1. Since the attributes are all benefit types in this case, their assessed values do not need to be converted.

Firstly, we assume that ξ is an indeterminate range of $\xi \in [0, 0.5]$, then the decision matrix is produced as follows:

$$B^*=\begin{bmatrix} <[0.7,0.8],[0.2,0.25],[0.2,0.3]> & <[0.7,0.8],[0.1,0.25],[0.1,0.15]> & <[0.6,0.7],[0.2,0.3],[0.2,0.3]> \\ <[0.7,0.8],[0.2,0.25],[0.3,0.35]> & <[0.7,0.85],[0.1,0.2],[0.1,0.25]> & <[0.7,0.75],[0.2,0.3],[0.1,0.15]> \\ <[0.8,0.85],[0.2,0.25],[0.1,0.2]> & <[0.7,0.75],[0.2,0.25],[0.1,0.2]> & <[0.7,0.8],[0.3,0.35],[0.2,0.25]> \\ <[0.7,0.75],[0.1,0.2],[0.2,0.25]> & <[0.8,0.85],[0.1,0.2],[0.2,0.25]> & <[0.7,0.75],[0.2,0.25],[0.2,0.3]> \end{bmatrix}$$

Step 2. Calculate and normalize the attribute weights β_j.

(i) According to Eq. (2), we calculate the Hamming distances between b_{ij} and b_{kj}, which are listed in Table 1.

(ii) According to Eq. (9), we can get the normalized attribute weights

β_1= 0.3554, β_2= 0.3140, and β_3= 0.3306.

Step 3. Rank the alternatives with the TOPSIS method.

(i) According to Eqs. (10)-(11) and the decision matrix B^*, we can determine two sets of the positive and negative ideal solutions b^+ and b^-:

b^+ = {<[0.8,0.85],[0.1,0.2],[0.1,0.2]>,<[0.8,0.85],[0.1,0.2],[0.1,0.15]>,<[0.7,0.8].[0.20,0.25],[0.10,0.15]>},

b^- = {<[0.7,0.75],[0.2,0.25],[0.3,0.35]>,<[0.7,0.75],[0.2,0.25],[0.2,0.25]>,<[0.6,0.7].[0.3,0.35],[0.2,0.3]>}

(ii) According to Eq. (12), we calculate the weighted distances of all alternatives to the positive and negative ideal solutions:

h_1^+ = 0.0676, h_2^+ = 0.0492, h_3^+ = 0.0519, and h_4^+ = 0.0477;

h_1^- = 0.0384, h_2^- = 0.0568, h_3^- = 0.0542, and h_4^- = 0.0583.

(iii) By Eq. (13), we get the correlation coefficient h_i as follows:

h_1 = 0.3623, h_2 = 0.5357, h_3 = 0.5110, and h_4 = 0.5500.

(iv) In terms of the ranking rules, the alternative is better if the coefficient value is greater. Therefore, the ranking order from the best to worst is $E_4 > E_2 > E_3 > E_1$. The best teacher is E_4 when the indeterminacy ξ is in the range of [0, 0.5].

Table 1. The Hamming distances between b_{ij} and b_{kj}

		$l_H(b_{ij},b_{kj})$		
		j = 1	j = 2	j = 3
i = 1	k = 1	0.0000	0.0000	0.0000
	k = 2	0.0250	0.0500	0.0667
	k = 3	0.0583	0.0333	0.0667
	k = 4	0.0417	0.0667	0.0333
i = 2	k =1	0.0250	0.0500	0.0667
	k = 2	0.0000	0.0000	0.0000
	k = 3	0.0833	0.0667	0.0667
	k = 4	0.0667	0.0167	0.0500
i = 3	k = 1	0.0583	0.0333	0.0667
	k = 2	0.0833	0.0667	0.0667
	k = 3	0.0000	0.0000	0.0000
	k = 4	0.0833	0.0833	0.0500
i = 4	k = 1	0.0417	0.0667	0.0333
	k = 2	0.0667	0.0167	0.0500
	k = 3	0.0833	0.0833	0.0500
	k = 4	0.0000	0.0000	0.0000

4.3 Sensitivity Analysis

In the MADM method proposed above, by sensitivity analysis, we reveal that different indeterminate ranges of ξ can change weight values and decision results.

The relationship between the indeterminate range of ξ and the weight value is shown in Fig. 1. The corresponding relationship between the indeterminate range of ξ and the decision result is exhibited in Fig. 2. Fig. 1 reflects that the weight values will change slightly as the indeterminate range of ξ changes. In Fig.2, when the range of ξ is less than [0, 0.6], the ranking order of the alternatives is $E_4 > E_2 > E_3 > E_1$. Then, the ranking order of the alternatives becomes $E_2 > E_4 > E_3 > E_1$ when the range of ξ is between [0, 0.7] and [0, 1.2]. In other ranges of ξ, the ranking order is $E_3 > E_2 > E_4 > E_1$.

5. Comparison with Existing Related MADM Models

We compare the developed TOPSIS method with existing MADM models of SNIEs [14, 23]. In the existing DM models [14, 23], the weight vector $\beta = (\beta_1, \beta_2, \beta_3)$ is specified as β = (0.30,0.36,0.34), which is not related to the indeterminate range of ξ. It is seen from Table 2 that the ranking results of the developed TOPSIS method are mostly different from those of existing DM methods [14, 23]. Comparing the results of Fig. 1 and Fig. 2, we find that the ranking order is $E_2 > E_4 > E_3 > E_1$ when the range of ξ is between [0, 0.7] and [0, 1.2], while the corresponding weight vector in Fig. 1 is gradually close to the specified weight vector β = (0.30,0.36,0.34). The decision result of this extended TOPSIS method is almost the same as that of other aggregation approaches regarding the same weight vector. It demonstrates that the developed method is effective.

In DM problems of SINEs, the decision results change with the change in the indeterminate range of ξ, but the existing DM models all use a specified weight vector, which is not related to the indeterminate range. Then, the developed DM model fully reflects the influence of the range of ξ on the weight vector and the ranking order. This new approach demonstrates the importance of decision makers' indeterminate levels. Obviously, there are three kinds of indeterminate levels in the TQE case, such as the low indeterminate level for $\xi \in \{[0, 0], [0, 0.6]\}$, the moderate indeterminate level for $\xi \in \{[0, 0.7], [0, 1.2]\}$, and the high indeterminate level for $\xi \in \{[0, 1.3], [0, 1.5]\}$. Since different indeterminate levels reflect different ranking orders, decision makers can choose the decision result based on some indeterminate level.

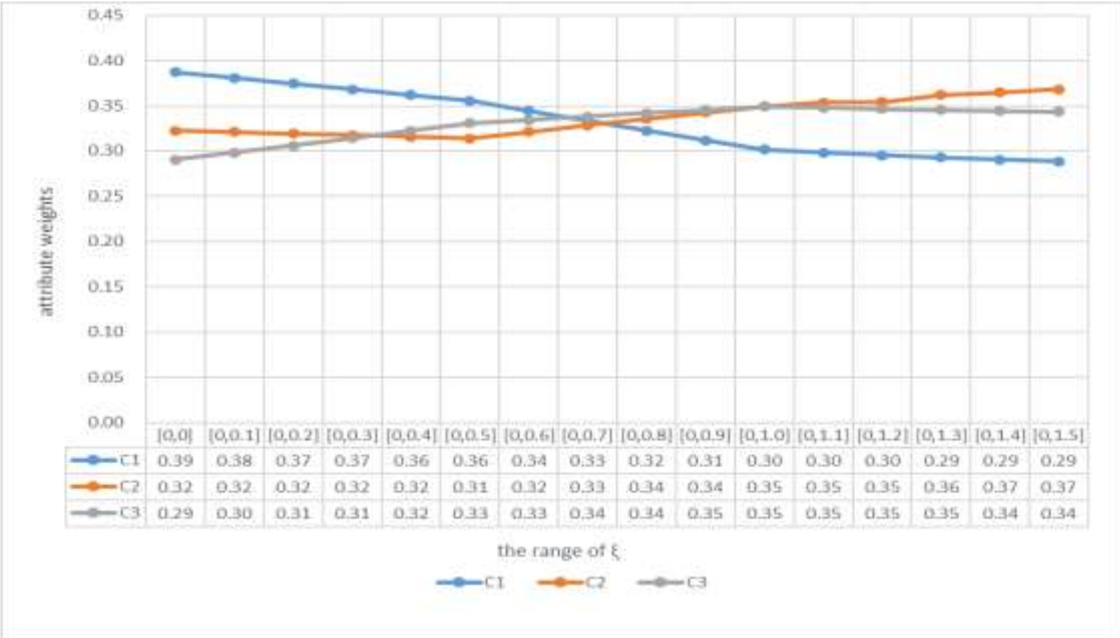

Fig. 1. Relationship between the range of ξ and the weight values

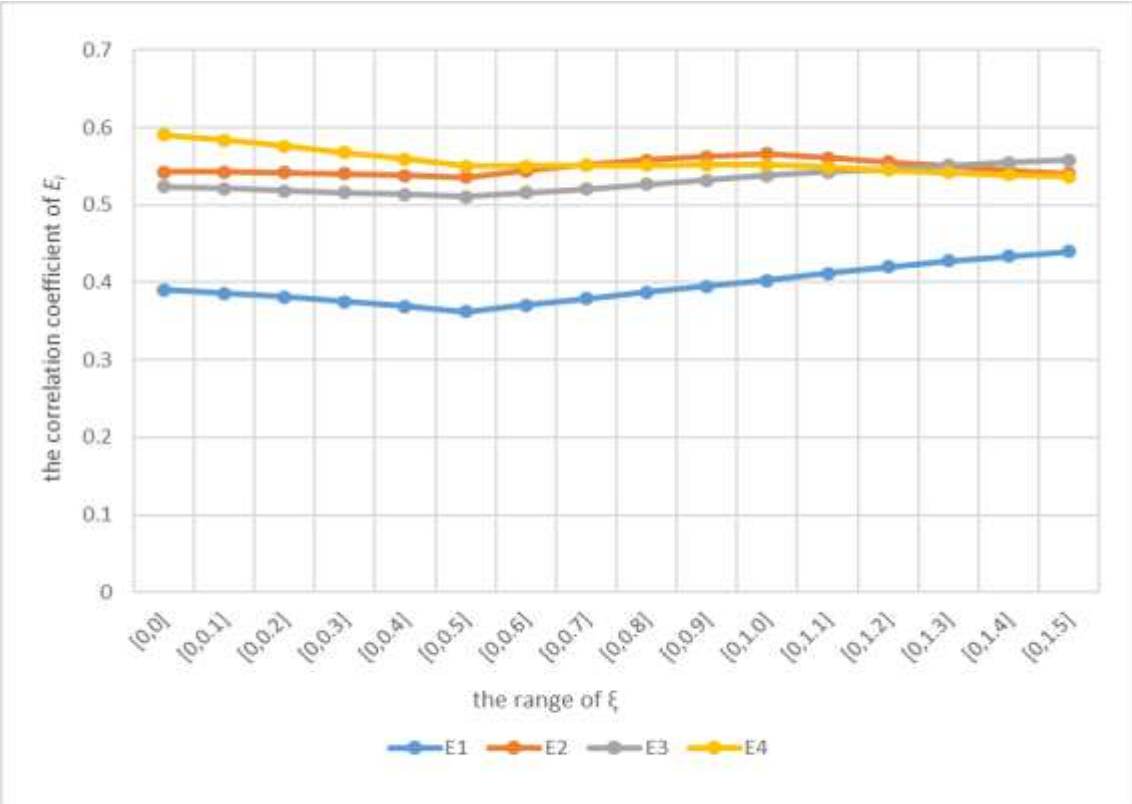

Fig. 2. Relationship between the range of ξ and the decision results

Table 2. Ranking results of different methods

Approaches	Ranking order with different indeterminacy ξ			
	ξ= [0,0]	ξ= [0,0.5]	ξ= [0,1]	ξ= [0,1.5]
SNIEWAA [14]	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$	$E_2 > E_3 > E_4 > E_1$
SNIEWGA [14]	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$.	$E_4 > E_2 > E_3 > E_1$	$E_3 > E_4 > E_2 > E_1$
SNIEEWA [23]	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$	$E_2 > E_3 > E_4 > E_1$
SNIEEWG [23]	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$	$E_3 > E_4 > E_2 > E_1$
TOPSIS	$E_4 > E_2 > E_3 > E_1$.	$E_4 > E_2 > E_3 > E_1$.	$E_2 > E_4 > E_3 > E_1$	$E_3 > E_2 > E_4 > E_1$

6. Conclusions

This paper first defined the Hamming distance between two SNIEs. According to the distance of SNIEs, we proposed the maximizing deviation method for determining the weight vector and the TOPSIS method for ranking alternatives. Then, the extended TOPSIS method-based MADM model was developed in the SNIE circumstance. Next, the developed MADM model was applied to a TQE case. By the case analysis, we not only obtained the decision results corresponding to the specified indeterminate ranges of ξ, but also analyzed the influence of the indeterminate range of ξ on the weight vector and the ranking order. Through comparing the developed model and the existing models, the results demonstrated that the developed model with unknown weights is valid by considering the weight vector obtained in the indeterminate range of ξ. The extended TOPSIS method-based MADM model can be widely used for these areas such as quality evaluation, service evaluation, project optimization in the environment of SNIEs.

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